1. $G = \hat{\mathbb{Z}}$, F a topological generator

<u>Claim.</u> $\forall$ torsion G-module M,

$$H^1(G, M) \simeq M/(F-1)M.$$

$$H^{\geq 2}(G, M) = 0.$$

$\forall m, \exists n \quad n \cdot m = 0.$

$S := \{p \mid p \nmid n\}$

$\mathbb{N}(S) = \{n \mid p \nmid n; \text{if } p \notin S\}$

$S =$ all primes

$\mathbb{N}(S) =$ all natural numbers

$$M = \bigcup_{n \in \mathbb{N}} {}_n M$$

$H^1(G, M)$

$$M^G = \ker(A \xrightarrow{F-1} A).$$

$$\hat{\mathbb{Z}} = \prod_p \mathbb{Z}_p$$

$$\hat{\mathbb{Z}} = \varprojlim_n \mathbb{Z}/n\mathbb{Z}$$

$$H^1(\hat{\mathbb{Z}}, M) = \varinjlim_n H^1(\mathbb{Z}/n\mathbb{Z}, M^{n\mathbb{Z}})$$

<u>Claim.</u> $H^1(\mathbb{Z}/n\mathbb{Z}, M^{n\mathbb{Z}}) \simeq {}_{N_n}M^{n\mathbb{Z}}/(F-1)M^{n\mathbb{Z}}$

$$H^1(\mathbb{Z}/n\mathbb{Z}, M^{n\mathbb{Z}}) = \frac{\{\text{crossed-homos } \mathbb{Z}/n\mathbb{Z} \longrightarrow M^{n\mathbb{Z}}\}}{\{\text{principal ones}\}}$$

$\varphi: \mathbb{Z}/n\mathbb{Z} \longrightarrow M^{n\mathbb{Z}}$ determined by

$\varphi(\sigma)$, where $\mathbb{Z}/n\mathbb{Z} = \langle \sigma \rangle$.

φ crossed means $\varphi(a+b) = a \cdot \varphi(b) + \varphi(a)$, so
$\varphi(0+1) = 0 \cdot \varphi(1) + \varphi(0)$

$\varphi(0) = \varphi(0) + \varphi(0) \Rightarrow \varphi(0) = 0$. $\quad 0 \cdot \varphi(\sigma) = \varphi(\sigma)$.

Say $\varphi(\sigma) = m$. $\quad \varphi(a\sigma) = \varphi(\sigma + (a-1)\sigma))$

$$= \sigma \cdot \varphi((a-1)\sigma) + m$$

$$= \sigma(\sigma \cdot \varphi((a-2)\sigma) + m) + m$$

$$= \sigma^2 \varphi((a-2)\sigma) + \sigma m + m$$

$\sigma^a m = a\sigma m$ $\qquad = (\sigma^{a-1} + \cdots + \sigma + 1) m$

(writing the action multiplicatively)

$$\varphi(a\sigma + b\sigma) = a\sigma \cdot \varphi(b\sigma) + \varphi(a\sigma)$$

$$= \sigma^a(\sigma^{b-1} + \cdots + 1) m + (\sigma^{a-1} + \cdots + 1) m$$

$$0 = \varphi(0) = \varphi(n\sigma) = (\sigma^{n-1} + \cdots + \sigma + 1) m$$

Principal homo's are $\phi_m : a \longrightarrow am - id_G m$, i.e.

$$\sigma \longrightarrow \sigma m - id_G m.$$

crossed $\overset{?}{\simeq}$ $\binom{n}{2} M^{n\mathbb{Z}}$

principal $\simeq$ $(\sigma - id_G) M^{n\mathbb{Z}}$

So $\quad H^1(\mathbb{Z}/n\mathbb{Z}, M^{n\mathbb{Z}}) \simeq \binom{n}{2} M^{n\mathbb{Z}} / (\sigma - 1) M^{n\mathbb{Z}}$

We have

$$0 \to (\sigma_n - 1)_{\binom{n}{2}} M^{n\mathbb{Z}} \longrightarrow {}_{\binom{n}{2}} M^{n\mathbb{Z}} \longrightarrow {}_{\binom{n}{2}} M^{n\mathbb{Z}} / (\sigma_n - 1) M^{n\mathbb{Z}} \longrightarrow 0$$

$\varinjlim$ is exact, so to understand $\varinjlim_n {}_{\binom{n}{2}} M^{n\mathbb{Z}} / (\sigma - 1) M^{n\mathbb{Z}}$,

look at $\varinjlim_n (\sigma_n - 1)_{\binom{n}{2}} M^{n\mathbb{Z}}$, $\varinjlim_n {}_{\binom{n}{2}} M^{n\mathbb{Z}}$.

Say $n > m$

$${}_{\binom{m}{2}} M^{m\mathbb{Z}} \xrightarrow{?} {}_{\binom{n}{2}} M^{n\mathbb{Z}}$$

$$m \longrightarrow m$$