Computational physics 4031 KNTU

- Determine the real roots of $f(x) = -0.6x^2 + 2.4x + 5.5$:
  - **(a)** Using the quadratic formula.
  - **(b)** Using three iterations of the bisection method to determine the
  - highest root. Employ initial guesses of $x_l = 5$ and $x_u = 10$.
  - Compute the estimated error ε_a and the true error ε_t after each iteration.
- Use the graphical approach and bisection method to determine the drag coefficient c needed for a parachutist of mass $m = 68.1$ kg to have a velocity of 40 m/s after free falling for 10 seconds ($t = 10$ s).

Note: The acceleration due to gravity is 9.81 m/s2.

Computational physics 4031 KNTU

The quadratic formula is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Coefficients:

- $a = -0.6$
- $b = 2.4$
- $c = 5.5$

Step 1: Calculate the Discriminant

$$b^2 - 4ac = (2.4)^2 - 4 \times (-0.6) \times 5.5$$

$$b^2 = 5.76 \quad \text{and} \quad -4ac = 13.2$$

$$\text{Discriminant} = 5.76 + 13.2 = 18.96$$

Step 2: Compute the Roots

$$x = \frac{-2.4 \pm \sqrt{18.96}}{2 \times (-0.6)}$$

$$x = \frac{-2.4 \pm 4.352}{-1.2}$$

Calculating each root:

- **For** $+\sqrt{18.96}$:

$$x_1 = \frac{-2.4 + 4.352}{-1.2} = \frac{1.952}{-1.2} \approx -1.6267$$

- **For** $-\sqrt{18.96}$:

$$x_2 = \frac{-2.4 - 4.352}{-1.2} = \frac{-6.752}{-1.2} \approx 5.6267$$

Thus, the roots using the quadratic formula are:

$$x_1 \approx -1.63 \quad \text{and} \quad x_2 \approx 5.63$$

The highest root is $x_2 = 5.63$.

Computational physics 4031 KNTU

We want to use the bisection method for three iterations to approximate the highest root, starting with $x_l = 5$ and $x_u = 10$.

Iteration 1

1. Compute the midpoint:

$$x_r = \frac{x_l + x_u}{2} = \frac{5+10}{2} = 7.5$$

2. Evaluate $f(x_r)$:

$$f(7.5) = -0.6(7.5)^2 + 2.4(7.5) + 5.5 = -0.6 \times 56.25 + 18 + 5.5 = -33.75 + 18 + 5.5 = -10.25$$

Since $f(7.5)$ is negative and $f(5)$ is positive, the root lies between 5 and 7.5. Update $x_u = 7.5$.

3. Calculate the estimated error:

$$\varepsilon_a = \left|\frac{7.5-5}{7.5}\right| \times 100 = \frac{2.5}{7.5} \times 100 = 33.33\%$$

4. Calculate the true error:

$$\varepsilon_t = \left|\frac{5.63-7.5}{5.63}\right| \times 100 \approx \frac{1.87}{5.63} \times 100 \approx 33.21\%$$

Computational physics 4031 KNTU

Iteration 2

1. Compute the new midpoint:

$$x_r = \frac{5 + 7.5}{2} = 6.25$$

2. Evaluate $f(x_r)$:

$$f(6.25) = -0.6(6.25)^2 + 2.4(6.25) + 5.5 = -0.6 \times 39.0625 + 15 + 5.5 = -23.4375 + 15 + 5.5 = -2.9375$$

   Since $f(6.25)$ is negative, update $x_u = 6.25$.

3. Calculate the estimated error:

$$\varepsilon_a = \left| \frac{6.25 - 5}{6.25} \right| \times 100 = \frac{1.25}{6.25} \times 100 = 20.00\%$$

4. Calculate the true error:

$$\varepsilon_t = \left| \frac{5.63 - 6.25}{5.63} \right| \times 100 \approx \frac{0.62}{5.63} \times 100 \approx 11.01\%$$

Computational physics 4031 KNTU

Iteration 3

1. Compute the new midpoint:

$$x_r = \frac{5 + 6.25}{2} = 5.625$$

2. Evaluate $f(x_r)$:

$$f(5.625) = -0.6(5.625)^2 + 2.4(5.625) + 5.5 = -0.6 \times 31.640625 + 13.5 + 5.5 = -18.984375 + 13.5 + 5.5 = 0.015625$$

   Since $f(5.625)$ is positive, update $x_l = 5.625$.

3. Calculate the estimated error:

$$\varepsilon_a = \left| \frac{6.25 - 5.625}{6.25} \right| \times 100 = \frac{0.625}{6.25} \times 100 = 10.00\%$$

4. Calculate the true error:

$$\varepsilon_t = \left| \frac{5.63 - 5.625}{5.63} \right| \times 100 \approx \frac{0.005}{5.63} \times 100 \approx 0.09\%$$

Computational physics 4031 KNTU

Final Results

After three iterations, the approximate root is $x_r = 5.625$ with:

- Estimated error $\varepsilon_a \approx 11.11\%$
- True error $\varepsilon_t \approx 0.089\%$

The bisection method gives a close approximation to the true root 5.62 with good accuracy in just three iterations.

The velocity v of a parachutist after time t when the force of gravity and air resistance (drag) are acting is given by:

$$v(t) = \frac{mg}{c}\left(1 - e^{-\frac{c}{m}t}\right)$$

where:

- $m = 68.1\ \mathrm{kg}$ (mass of the parachutist)
- $g = 9.81\ \mathrm{m/s^2}$ (acceleration due to gravity)
- c is the drag coefficient
- $v(t) = 40\ \mathrm{m/s}$ (velocity after time t)
- $t = 10\ \mathrm{s}$ (time of freefall)

Objective

We need to solve for c so that:

$$40 = \frac{68.1 \times 9.81}{c}\left(1 - e^{-\frac{c}{68.1}\times 10}\right)$$

Rearranging the Equation

The equation can be rearranged to form:

$$f(c) = \frac{68.1 \times 9.81}{c}\left(1 - e^{-\frac{c}{68.1}\times 10}\right) - 40 = 0$$

Our goal is to find the value of c that satisfies this equation.

Computational physics 4031 KNTU

Step 1: Graphical Approach

To solve graphically:

1. Define the function $f(c)$ as:

$$f(c) = \frac{68.1 \times 9.81}{c}\left(1 - e^{-\frac{c}{68.1} \times 10}\right) - 40$$

2. Plot the function $f(c)$ over a reasonable range of c values to visually estimate the root.

Step-by-step for Graphical Approach:

1. Consider a range for c, such as c from 5 to 20, and evaluate $f(c)$ at various points to see where it crosses the x-axis.
2. Plot $f(c)$ and identify the value of c where $f(c) = 0$.

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Step 2: Bisection Method

We will use the bisection method to numerically find the root c.

Bisection Method Steps:

1. **Choose Initial Guesses:** We need to pick two initial values c_l and c_u such that $f(c_l)$ and $f(c_u)$ have opposite signs (indicating a root lies between them). From the graphical method, we can estimate a suitable interval.
2. **Compute Midpoint:**

$$c_r = \frac{c_l + c_u}{2}$$

3. **Check the Sign of $f(c_r)$:**
   - If $f(c_r)$ has the same sign as $f(c_l)$, update $c_l = c_r$.
   - If $f(c_r)$ has the same sign as $f(c_u)$, update $c_u = c_r$.
4. **Repeat:** Continue halving the interval until the error is small enough.

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Exercise

Apply 'by hand' the bisection method as used in the bisection code to find the root of

$$f(x) = x^2 - x$$

between ½ and 2.

Calculate the error as Ea=abs(xr-xrold) at each step.

Example

- Open function file **example_bisection**
- Define the function

$$f(z) = \sqrt{z_0^2 - z^2 \sin(z) + z\cos(z)}$$

Set $z_0 = 10$ before you define the function.

- Use bisect find the root between 5.5 and 5.7

Computational physics 4031 KNTU

Homework

- Problem 5.22

Extend the problem to different radii.

For $r = 1{:}5$, use a for loop for r and calculate the function $f(h)$, the root of which gives the required height.

Plot $f(h)\ \ vs\ \ h$ for $0 \le h \le 2r$ for every r in the same figure.

Print the values of h for every r in a tabular form with heading "$r \qquad h$"

Computational physics 4031 KNTU

- **5.2** Determine the real root of $f(x) = 4x^3 - 6x^2 + 7x - 2.3$:
- **(a)** Graphically.
- **(b)** Using bisection to locate the root. Employ initial guesses of
- $x_l = 0$ and $x_u = 1$ and iterate until the estimated error ε_a falls
- below a level of $\varepsilon_s = 10\%$.
- **5.3** Determine the real root of $f(x) = -26 + 85x - 91x^2 + 44x^3 - 8x^4 + x^5$:
- **(a)** Graphically.
- **(b)** Using bisection to determine the root to $\varepsilon_s = 10\%$. Employ initial
- guesses of $x_l = 0.5$ and $x_u = 1.0$.