SCOTTYRANK.JL: AN IMPLEMENTATION OF PAGERANK & HITS

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1. Mathematical Background

1.1. **Definitions.** Positive matrices are defined as matrices with positive entries.

Markov matrices are defined as square matrices with nonnegatives entries and column sum 1 across all of its columns. Note that for a $n \times n$ matrix M, the latter condition is equivalent to $M^T\vec{1} = \vec{1}$, where $\vec{1} \in \mathbb{R}^n$ has all ones as components.

Positive Markov matrices are defined as, well, positive Markov matrices.

1.2. **Facts.** (Perron-Frobenius theorem) Let A be a positive square matrix. Let λ_1 be A's maximum eigenvalue in terms of absolute values. Then λ_1 is positive and has algebraic (and subsequently geometric) multiplicity 1.

Let M be a Markov matrix. Let λ_1 be M's maximum eigenvalue in terms of absolute values. Then $\lambda_1 = 1$.

Let M' be a positive Markov matrix. Let λ_1 be M''s maximum eigenvalue in terms of absolute values. Then $\lambda_1 = 1$ and has algebraic (and subsequently geometric) multiplicity 1.

1.3. **Usage.** Let M be a $n \times n$ Markov matrix. Then M specifies a dicrete memoryless transition process between n states, namely the process where

$$(\forall (t, i, j) \in \mathbb{N} \times [n] \times [n]) \left[\Pr(\text{state } i \text{ at time } t + 1 \mid \text{state } j \text{ at time } t) = M_{ij}\right].$$

Let $\vec{v} \in \mathbb{R}^n$ such that $\vec{v}$ has nonnegative components and $\vec{v}^T\vec{1} = 1$ (a stochastic vector). Then $\vec{v}$ specifies an (initial) discrete probability distribution over the n states, namely the distribution where

$$(\forall i \in [n])[\Pr(\text{state } i \text{ at time } 0) = \vec{v}_i].$$

Then

$$(\forall (k, i) \in \mathbb{N} \times [n]) \left[\Pr(\text{state } i \text{ at time } k) = \left(M^k\vec{v}\right)_i\right].$$