

Proposal: Higher-Order Entanglement Type

1 Introduction

EN type is not expressive enough for Quantum Fourier Transformation. Consider an arbitrary 2-range locus $\{ x[0..n], y[0..m] \}$. Applying QFT to $|x\rangle$ is equivalent to

$$\begin{aligned} \sum_{i \in S} \alpha_i \omega(p_i, N) |x_i\rangle |y_i\rangle &\mapsto \frac{1}{\sqrt{N}} \sum_{i \in S} \sum_{k=0}^{N-1} \alpha_i \omega(p_i + x_i k, N) |k\rangle |y_i\rangle \\ &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} |k\rangle \left(\alpha_i \sum_{i \in S} \omega(p_i + x_i k, N) |y_i\rangle \right) \end{aligned}$$

which generalizes the semantics of EN type using amplitudes, a lower-level EN representation and a sequence of basis ket induced by a Qft operation

$$\llbracket \text{EN} \rrbracket_{n+1} = \text{seq} \langle \text{nat} \rangle \times \llbracket \text{EN} \rrbracket_n \times \text{seq} \langle \text{nat} \rangle$$

where

$$\llbracket \text{EN} \rrbracket_0 = \text{seq} \langle \text{nat} \rangle \times \text{nat}$$

standing for the exponents of a given root of unity's power.

Given EN_{n+1} , measuring the range corresponding to the tailing kets lowers the type to EN_n . However, when measuring the range corresponding to the inner representation, the order doesn't reduce until the inner representation's order is reduced. In another words, if an EN_{n+1} representation demotes to EN_n , the entire representation demotes from EN_{n+2} to EN_{n+1} .

Higher-order entanglement type, or the universe of entanglement type, may be too powerful. Here are two useful instantiation.

- EN_1 is identical to the current EN.
- EN_2 can be named as Qft type that corresponds to outcome of a Qft application.