Proposal: Phases and Amplitude

1 Synopsis

We will allow phases as roots of unity and amplitudes as an integer associated with each phase.

The goal is to provide a native support for QFT operators as well as an effecient and accurate desription of post-measurement states and outcomes.

1.1 Example

2 Proposal

2.1 Representation

2.1.1 Root of Unity

Notation

$$\omega_N^k = \exp(2\pi i \frac{k}{N})$$

- $\omega_N^0 = 1$
- $\omega_2^1 = -1$
- $\omega_4^1 = i$
- $\omega_4^3 = -i$

2.1.2 Degree of Phases

For efficiency, a hierarchy of phases should be adopted to provide a pay-as-you-go model for phase description. The degree corresponds to the dimension of the underlying matrix.

- zero degree for cases where phases and amplitudes are uninterested
  For example, a simple `Nor` vector where phases can be omitted.
- first degree for cases where one root of unity suffices
  For example, $|-\rangle = \omega_2^0 |0\rangle + \omega_2^1 |1\rangle$.
- second degree for cases using a summation of roots of unity.
  For example,

$$\sum_k^{N-1} \omega_N^{xk} |k\rangle$$

2.1.3 Placement

Phases and amplitudes should be assigned on partition basis. The number of phases object should agree with that of rows per partition.

2.2 Phase Language

- `0`th-degree: Specification without explicit phases are of `0`-degree phases.
- 1st-degree: `ω(k, N)` for ω_N^k
- 2nd-degree: `Ω i . (f(i), N)` for

$$\sum_i \omega_N^{f(i)} |\phi\rangle$$

2.3 Data Representation

2.3.1 Zeroth-degree Phase

Zeroth-degree phases, as the name suggests, will not be emitted by the compiler and will be bookkept by the type system. The benefit of this is that there's no verification overhead imposed for a program that doesn't care about phases, e.g,. the GHZ program.

Implementationwise, the modification to the compiler to support the phase will be minimal and incremental.

2.3.2 First-degree Phase

WIP

2.3.3 Second-degree Phase

WIP