Proposal: Higher-Order Entanglement Type

1 Introduction

`EN` type is not expressive enough for Quantum Fourier Transformation. Consider an arbitrary 2-range locus `{ x[0..n], y[0..m] }`. Applying QFT to $|x\rangle$ is equivalent to

$$\begin{aligned}\sum_{i \in S} \alpha_i \omega(p_i, N) |x_i\rangle |y_i\rangle &\mapsto \frac{1}{\sqrt{N}} \sum_{i \in S} \sum_{k=0}^{N-1} \alpha_i \omega(p_i + x_i k, N) |y_i\rangle |k\rangle \\ &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left(\alpha_i \sum_{i \in S} \omega(p_i + x_i k, N) |y_i\rangle \right) |k\rangle\end{aligned}$$

which generalizes the semantics of `EN` type using amplitudes, a lower-level `EN` representation and a sequence of basis ket induced by a Qft operation

$$[\![\mathtt{EN}]\!]_{n+1} = \mathtt{seq}\,\langle\mathtt{nat}\rangle \times [\![\mathtt{EN}]\!]_n \times \mathtt{seq}\,\langle\mathtt{nat}\rangle$$

where

$$[\![\mathtt{EN}]\!]_0 = \mathtt{seq}\,\langle\mathtt{nat}\rangle \times \mathtt{nat}$$

standing for the exponents of a given root of unity's power.

Given $\mathtt{EN}_{n+1}$, measuring the range corresponding to the tailing kets lowers the type to $\mathtt{EN}_n$. However, when measuring the range corresponding to the inner representation, the order doesn't reduce until the inner representation's order is reduced. In anoter words, if an $\mathtt{EN}_{n+1}$ representation demotes to $\mathtt{EN}_n$, the entire representation demotes from $\mathtt{EN}_{n+2}$ to $\mathtt{EN}_{n+1}$.

Higher-order entanglement type, or the universe of entanglement type, may be too powerful. Here are two useful instantiation.

- $\mathtt{EN}_1$ is identical to the current `EN`.
- $\mathtt{EN}_2$ can be named as `Qft` type that corresponds to outcome of a `Qft` application.