SI 2013 Mark Scheme Q1

(i)	$y=\sqrt{x} \;\Rightarrow\; y^2+3y-\tfrac{1}{2}=0$	**B1**	for a correct quadratic eqn. in y or $\sqrt{x}$
	$(2y+3)^2=11$	**M1**	for use of a method for solving a quadratic eqn. (compl^{g}. the square, formula, etc.) If candidate fails to obtain a numerical answer for y (correct or not) then **M0**
	$y=\dfrac{-3\pm\sqrt{11}}{2}$	**A1**	
	$y\ge 0 \;\Rightarrow\; \sqrt{x}=\dfrac{\sqrt{11}-3}{2}$	**M1**	for clearly choosing the correct root: **FT** *provided* they have 1 $+_{ve}$ and 1 $-_{ve}$ root to choose from
	$x=\left(\dfrac{\sqrt{11}-3}{2}\right)^2$ or $\dfrac{20-6\sqrt{11}}{4}$ or $5-\tfrac{3}{2}\sqrt{11}$	**A1**	$\boxed{5}$
(ii) (a)	$y=\sqrt{x+2}$	**M1**	for clear indication of this substitution (or equivalent)
	$y^2+10y-24=0$	**A1**	for a correct quadratic
	y or $\sqrt{x+2}=-12,\ 2$	**M1**	for solution method of a suitable quadratic
	$y\ge 0 \;\Rightarrow\; \sqrt{x+2}=2$	**M1**	for choosing valid root: **FT** *provided* they have 1 $+_{ve}$ and 1 $-_{ve}$ root to choose from
	$x=2$	**A1**	$\boxed{5}$
(ii) (b)	$y=\sqrt{2x^2-8x-3}$	**M1**	for clear indication of this substitution (or equivalent)
	$y^2+2y-15=0$	**A1**	for a correct quadratic
	y or $\sqrt{2x^2-8x-3}=-5,\ 3$	**M1**	for solution of a suitable quadratic
	$y\ge 0 \;\Rightarrow\; \sqrt{2x^2-8x-3}=3$	**M1**	for choosing valid root: **FT** *provided* they have 1 $+_{ve}$ and 1 $-_{ve}$ root to choose from
	$2x^2-8x-3=9 \;\Rightarrow\; x^2-4x-6=0$	**M1A1**	for obtaining and solving a quadratic eqn. in x; A1 for the correct quadratic
	$x=2\pm\sqrt{10}$	**A1**	$\boxed{7}$

$x = 2 \pm \sqrt{10} \;\Rightarrow\; x^2 = 14 \pm 4\sqrt{10}$ **M1** for checking attempt (for at least one of the answers found)

so $x^2 - 4x - 9 = -3$ & $2x^2 - 8x - 3 = 9$

$\Rightarrow$ (both cases) $-3 + \sqrt{9} = 0$ **A1A1** one for each clearly shown (with working)

ALTERNATIVELY For validity, $2x^2 - 8x - 3 \ge 0$ also **M1** i.e. $(x-2)^2 \ge \frac{11}{2}$ **A1**

Since $(x-2)^2 = 10 > \frac{11}{2}$ both solns. valid **E1**

$\boxed{3}$

ALTERNATIVES

(i) $3\sqrt{x} = \frac{1}{2} - x$ and squaring **M1** $x^2 - 10x + \frac{1}{4} = 0$ **A1** correct quadratic **M1** for solution of a suitable quadratic **A1** $x = 5 \pm \frac{3}{2}\sqrt{11}$

However, *both* these roots are positive, so the final mark will be **E1** for checking both, with working, and correctly discarding the unsuitable answer

(ii) (a) $10\sqrt{x+2} = 22 - x$ and squaring **M1** $x^2 - 144x + 284 = 0$ **A1** correct quadratic **M1** for solution of a suitable quadratic **A1** $x = 142, 2$

E1 for checking both, with working, and correctly discarding the unsuitable answer (e.g. $x = 142$ gives LHS > 0 but RHS < 0 would suffice)

(ii) (b) $\sqrt{2x^2 - 8x - 3} = 9 + 4x - x^2$ and squaring **M1** $x^4 - 8x^3 - 4x^2 + 80x + 84 = 0$ **A1** correct quartic

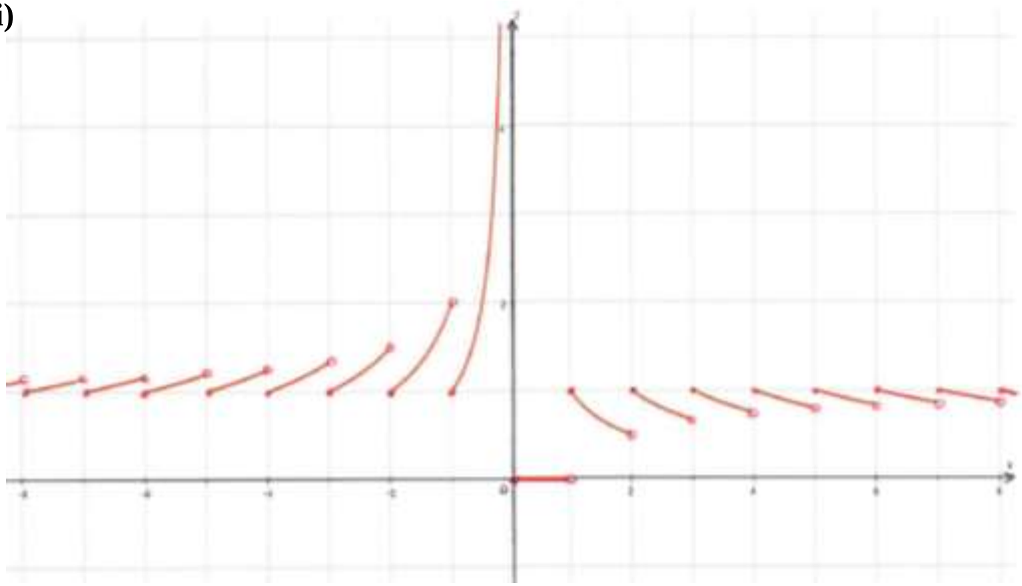
$(x-2)^4 - 28(x-2)^2 + 180 = 0$ **M1A1** $\Rightarrow$ $(x-2)^2 = 10, 18$ **M1A1**

Now $\sqrt{2}\sqrt{(x-2)^2 - \frac{11}{2}} = 13 - (x-2)^2 \;\Rightarrow\; \frac{11}{2} \le (x-2)^2 \le 13$ **M1A1A1** so the only valid solutions arise from $(x-2)^2 = 10$ and $x = 2 \pm \sqrt{10}$ **A1**

However, I cannot see candidates making this approach work. **M1A1** for getting the correct quartic may be all they can reasonably get.
Attempts to find linear factors (by the *Factor Theorem*, for instance) will go nowhere.
Some may attempt to find a pair of quadratic factors: $(x^2 + Ax + B)(x^2 + Cx + D) \equiv x^4 + (A+C)x^3 + (AC+B+D)x^2 + (AD+BC)x + BD = 0$ and compare terms ($A + C = -8$, $AC + B + D = -4$, $AD + BC = 80$ and $BD = 84$) but I do not want them to have any marks unless they can get to (by guessing/verifying … divine inspiration?) $(x^2 - 4x - 6)(x^2 - 4x - 14)$, at which point I would award them the next **M1A1** & **M1A1**.
They now have four answers to check for and I would propose a **B1** for each correctly checked (with working) and accepted/rejected appropriately.

SI 2013 Mark Scheme Q2

(i)



G1	Lots of "unit" segments
G1	$\left.\begin{matrix}\text{LH}\\\text{RH}\end{matrix}\right\}$ end clearly $\left\{\begin{matrix}\text{included}\\\text{excluded}\end{matrix}\right.$
G1	Each segment looks like a portion of a reciprocal curve
G1	Essentially correct $0 \le x \le 3$
G1	Essentially correct $-3 \le x < 0$
	Ignore endpoint issues for these last two Gs

5

(ii) Note that, for $n \le x < n+1$, $\lfloor x \rfloor = n$ so f(x) = $\frac{n}{x}$. Also, $\frac{n}{n+1} < \mathrm{f}(x) \le 1$ for $x > 0$, and f(x) ≥ 1 for $x < 0$, so …

f(x) = $\frac{7}{12}$ only in [1, 2). **E1** Sketch may show it so

$\mathrm{f}(x) = \frac{1}{x} = \frac{7}{12} \Rightarrow x = \frac{12}{7}$ **B1** B0 if extra answers appear

2

Similarly, $\frac{n}{n+1} > \frac{17}{24} \Rightarrow 24n > 17n + 17 \Rightarrow n > 2\frac{3}{7}$, i.e. $n \ge 3$; so f(x) = $\frac{17}{24}$ only in [1, 2) and [2, 3).

In [1, 2), $\mathrm{f}(x) = \frac{1}{x} = \frac{17}{24} \Rightarrow x = \frac{24}{17}$ **B1**

In [2, 3), $\mathrm{f}(x) = \frac{2}{x} = \frac{17}{24} \Rightarrow x = \frac{48}{17}$ **B1** Give max. B1 if extra answers appear

2

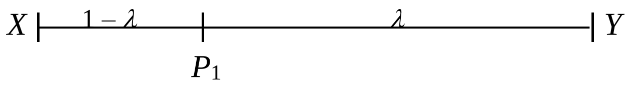
Now, for $x < 0$, $1 \le \mathrm{f}(x) < \dfrac{n}{n+1}$, and $\dfrac{-n}{-n-1} < \dfrac{4}{3} \Rightarrow -4n - 4 > -3n \Rightarrow n < -4$; so $\mathrm{f}(x) = \frac{4}{3}$ only in $[-4, -3)$, $[-3, -2)$, $[-2, -1)$ and $[-1, 0)$.

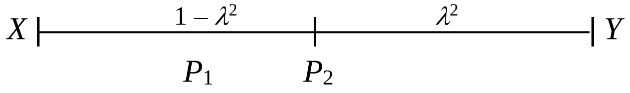
	$\mathrm{f}(-3) = 1$ so no solution in $[-4, -3)$	**B1**	Possibly implicitly, if just not there
	In $[-3, -2)$, $\mathrm{f}(x) = \dfrac{-3}{x} = \dfrac{4}{3} \Rightarrow x = -\frac{9}{4}$	**B1**	
	In $[-2, -1)$, $\mathrm{f}(x) = \dfrac{-2}{x} = \dfrac{4}{3} \Rightarrow x = -\frac{3}{2}$	**B1**	
	In $[-1, 0)$, $\mathrm{f}(x) = \dfrac{-1}{x} = \dfrac{4}{3} \Rightarrow x = -\frac{3}{4}$	**B1**	Give max. B1B1 if extra answers appear
			4
(iii)	$\dfrac{n}{n+1} > \dfrac{9}{10}$ for $n > 9$ so …		
	$\mathrm{f}(x_{\max}) = \frac{9}{10}$ in $[8, 9)$	**E1**	
	and $\mathrm{f}(x) = \dfrac{8}{x} = \dfrac{9}{10} \Rightarrow x = \frac{80}{9}$	**B1**	NB $\mathrm{f}(10) = 1$, so $x \ne 10$
			2
	$\mathrm{f}(x) = c$ has exactly n roots for …		
	for $x > 0$: $\dfrac{n}{n+1} < c \le \dfrac{n+1}{n+2}$	**B1B1**	LHS; RHS
	for $x < 0$: $\dfrac{n+1}{n} \le c < \dfrac{n}{n-1}$, $n \ge 2$	**B1B1**	LHS; RHS
	$c \ge 2$, $n = 1$	**B1**	
			5

SI 2013 Mark Scheme Q3

(i)	$X * Y = Y * X \iff \lambda\mathbf{x} + (1-\lambda)\mathbf{y} = \lambda\mathbf{y} + (1-\lambda)\mathbf{x}$	**M1**	Including correct $Y * X = \lambda\mathbf{y} + (1-\lambda)\mathbf{x}$	**3**
	$\iff (2\lambda - 1)(\mathbf{x} - \mathbf{y}) = \mathbf{0}$	**M1**		
	(Since $\mathbf{x} \neq \mathbf{y}$) $\lambda = \frac{1}{2}$	**A1**		
(ii)	$(X * Y) * Z = \lambda(\lambda\mathbf{x} + (1-\lambda)\mathbf{y}) + (1-\lambda)\mathbf{z}$	**M1**		**6**
	$= \lambda^2\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)\mathbf{z}$	**A1**		
	and			
	$X * (Y * Z) = \lambda\mathbf{x} + (1-\lambda)[\lambda\mathbf{y} + (1-\lambda)\mathbf{z}]$	**M1**		
	$= \lambda\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)^2\mathbf{z}$	**A1**		
	$(X * Y) * Z - X * (Y * Z) = \lambda(1-\lambda)(\mathbf{x} - \mathbf{z})$	**M1**		
	The two are distinct provided $\lambda \neq 0, 1$ or $X \neq Z$	**A1**		
(ii)	$(X * Y) * Z = \lambda^2\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)\mathbf{z}$			**5**
	$(X * Z) * (Y * Z) = [\lambda\mathbf{x} + (1-\lambda)\mathbf{z}] * [\lambda\mathbf{y} + (1-\lambda)\mathbf{z}]$	**M1**		
	$= \lambda^2\mathbf{x} + \lambda(1-\lambda)\mathbf{z} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)^2\mathbf{z}$			
	$= \lambda^2\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)\mathbf{z}$		**A1** (and the two are always equal)	
	$X * (Y * Z) = \lambda\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)^2\mathbf{z}$			
	$(X * Y) * (X * Z) = [\lambda\mathbf{x} + (1-\lambda)\mathbf{y}] * [\lambda\mathbf{x} + (1-\lambda)\mathbf{z}]$	**M1**		
	$= \lambda^2\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + \lambda(1-\lambda)\mathbf{x} + (1-\lambda)^2\mathbf{z}$			
	$= \lambda^2\mathbf{x} + \lambda(1-\lambda)\mathbf{y} + (1-\lambda)\mathbf{z}$		**A1**	
	Hence $X * (Y * Z) = (X * Y) * (X * Z)$	**E1**	Conclusion must be stated (before or after proof)	

For $0 < \lambda < 1$, P_1 cuts XY in the ratio $(1 - \lambda) : \lambda$ **B1**





M1 Iterating

We see that P_n cuts XY in the ratio

$$1 - \lambda^n : \lambda^n$$

A1A1 Any correct form

$$\begin{aligned} P_{n+1} = P_n * Y &= \lambda\, \mathbf{p}_{\boldsymbol{n}} + (1 - \lambda)\mathbf{y} \\ &= \lambda \left\{\left(\lambda^n\right)\mathbf{x} + \left(1 - \lambda^n\right)\mathbf{y}\right\} + (1 - \lambda)\mathbf{y} \\ &= \lambda^{n+1}\mathbf{x} + \left(\lambda - \lambda^{n+1} + 1 - \lambda\right)\mathbf{y} \\ &= \lambda^{n+1}\mathbf{x} + \left(1 - \lambda^{n+1}\right)\mathbf{y} \end{aligned}$$

and proof follows by induction **M1A1** for attempt at an inductive proof; fully correct

6

SI 2013 Mark Scheme Q4

(i) $\int \tan^n x.\sec^2 x \; dx = \left[\frac{1}{n+1}\tan^{n+1} x\right]$ **M1** May be done via a substn. such as $t = \tan x$ (or by "parts")

$= \frac{1}{n+1}$ **A1** **ANSWER GIVEN**

2

$\int \sec^n x.\tan x \; dx = \int \sec^{n-1} x.\sec x \tan x \; dx$ **M1** May be done via a substn. such as $s = \sec x$ (or by parts)

$= \left[\frac{1}{n}\sec^n x\right]$ **A1**

$= \frac{\left(\sqrt{2}\right)^n - 1}{n}$ **A1** **ANSWER GIVEN**

3

(ii) (a) $\int_0^{\pi/4} x\sec^4 x\tan x \; dx = \left[x.\frac{\sec^4 x}{4}\right]_0^{\pi/4} - \int_0^{\pi/4} \frac{\sec^4 x}{4} dx$ **M1A1A1** for appropriate use of *parts*; correct (1st, 2nd)

$= \frac{\pi}{4} - \frac{1}{4}J$ where $J = \int_0^{\pi/4} \sec^4 x \; dx$

$J = \int_0^{\pi/4} \sec^2 x \; dx + \int_0^{\pi/4} \sec^2 x\tan^2 x \; dx$ **M1** for use of $\sec^2 x = 1 + \tan^2 x$ to split up the integral

$= \left[\tan x + \tfrac{1}{3}\tan^3 x\right]$ **A1 A1**

$= \frac{4}{3}$ NB Limits are ignored until the end, when numerical answers need to appear

Thus $I = \frac{\pi}{4} - \frac{1}{3}$ **A1**

7

(ii) (b) $\int x^2 (\sec^2 x . \tan x)\, dx$	**M1**	for appropriate splitting and attempted use of *integration by parts*
$= \left[x^2 . \tfrac{1}{2}\tan^2 x\right] - \int 2x . \tfrac{1}{2}\tan^2 x \; dx$	**A1A1**	
$= \dfrac{\pi^2}{32} - \int x(\sec^2 x - 1)$	**M1**	for use of $\tan^2 x = \sec^2 x - 1$
$= \dfrac{\pi^2}{32} - K + \int x\, dx$ where $K = \int_0^{\pi/4} x \sec^2 x \; dx$		
$K = [x . \tan x] - \int \tan x \; dx$	**M1**	
$= x \tan x - \ln(\sec x)$	**B1**	for $\int \tan x\, dx = \ln(\sec x)$
$= \dfrac{\pi}{4} - \dfrac{1}{2}\ln 2$	**A1**	
Thus $I = \dfrac{\pi^2}{32} - \left(\dfrac{\pi}{4} - \dfrac{1}{2}\ln 2\right) + \dfrac{\pi^2}{32}$		
$= \dfrac{\pi^2}{16} - \dfrac{\pi}{4} + \dfrac{1}{2}\ln 2$	**A1**	

Note that there are many ways to split these integrals in (ii) into parts.

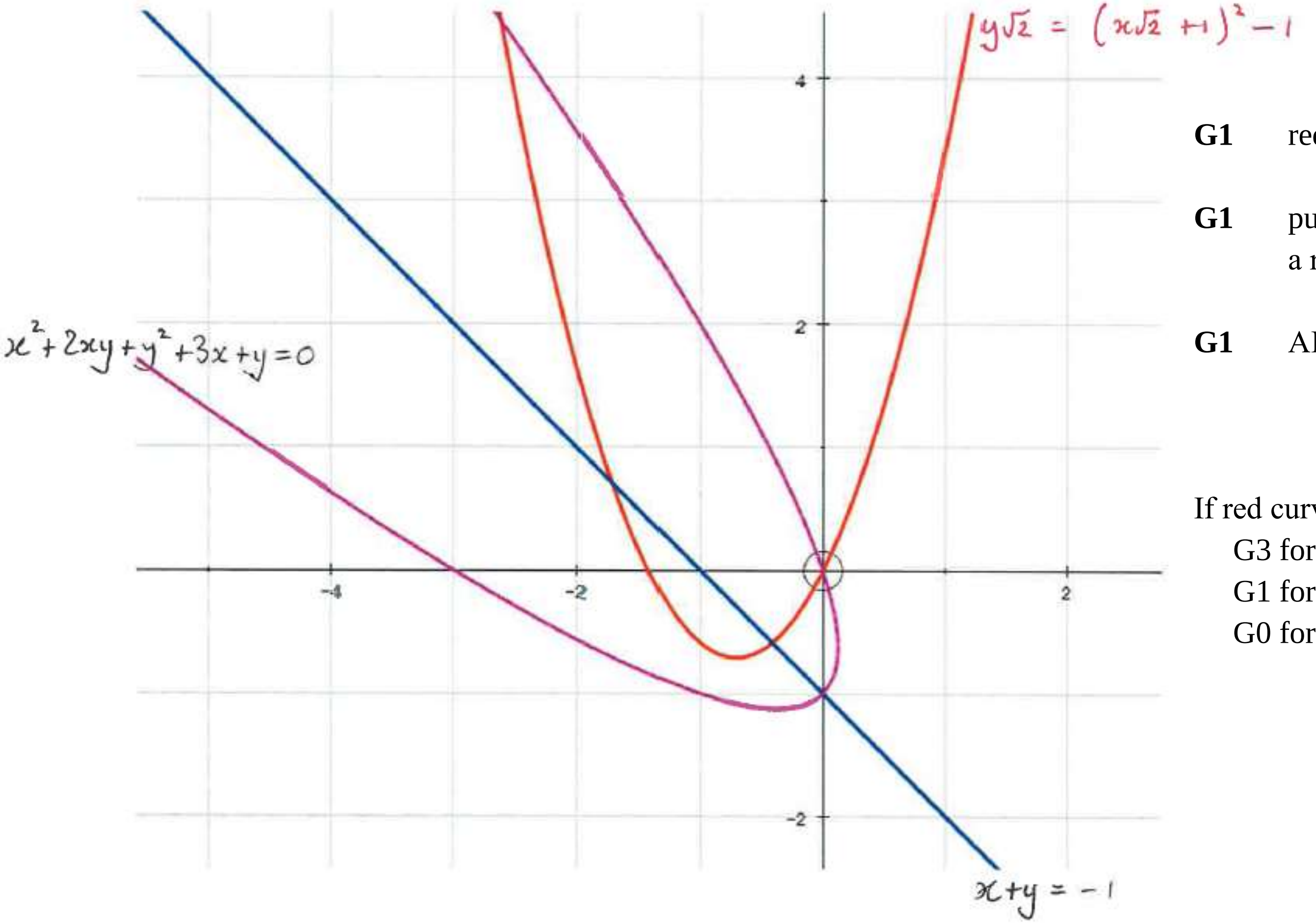
8

SI 2013 Mark Scheme Q5

(i)	For $k = 0$: $x^2 + 3x + y^2 + y = 0$			
	$\left(x + \frac{3}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \left(\frac{1}{2}\sqrt{10}\right)^2$	**M1**	for completing the square for both x and y	
	giving a CIRCLE	**G1**	for a circle drawn	**4**
	thro' (0, 0), (0, –1) & (–3, 0)	**B1B1**	passing thro' the origin; other 2 intercepts correct (noted on sketch or separately)	
(ii)	For $k = \frac{10}{3}$: $(3x + y)(x + 3y + 3) = 0$	**B1**	Must be the full thing (no marks for just factorising the given quadratic)	
	giving a LINE-PAIR	**G1**	for two (intersecting) lines drawn	
	Lines $y = -3x$ & $x + 3y = -3$		Statement of eqns. not actually required	
	1st line thro' O with $-_{ve}$ gradient	**B1**		
	2nd line not thro' O with $-_{ve}$ gradient	**M1**		**5**
	thro' (0, –1) & (–3, 0)	**A1**	for both stated or noted on sketch	
(iii)	For $k = 2$: $(x + y)^2 + 3x + y = 0$			
	When $\theta = 45^\circ$,			
	$x + y = X\sqrt{2}$ and $y - x = Y\sqrt{2}$	**B1**	noted or used anywhere	
	$\Rightarrow x = \dfrac{X - Y}{\sqrt{2}}$ and $y = \dfrac{X + Y}{\sqrt{2}}$	**M1A1**		
	$(x + y)^2 + 3x + y = 0$ becomes …			
	$2X^2 + \dfrac{3X - 3Y}{\sqrt{2}} + \dfrac{X + Y}{\sqrt{2}} = 0$	**M1**	for eliminating both x and y for X and Y	
	$\Rightarrow 2X^2 + 2\sqrt{2}X = Y\sqrt{2}$			**6**
	$\Rightarrow \left(\sqrt{2}X + 1\right)^2 - 1 = Y\sqrt{2}$	**M1A1**	for completing the square; correct **ANSWER GIVEN** obtained	

This line has axis of symmetry $X = \dfrac{-1}{\sqrt{2}}$ **M1** Stated or noted (explicitly) on sketch

$\Rightarrow \dfrac{x+y}{\sqrt{2}} = \dfrac{-1}{\sqrt{2}}$ i.e. $x + y = -1$ **A1**



G1 red curve correct

G1 purple curve and blue line obviously a rotation thro' 45° of their red curve

G1 ALL correct (incl. thro' O)

If red curve does not appear …

G3 for purple curve & blue line correct

G1 for obviously rotated 45° c/w

G0 for anything else

$\boxed{5}$

SI 2013 Mark Scheme Q6

(∗) Coefft. of x^r in $(1 + x)^{n+1}$ is $\binom{n+1}{r}$ **B1**

Coefft. of x^r in $(1 + x)(1 + x)^n$ is from

$$(1+x)\left(\ \ldots..+\binom{n}{r-1}x^{r-1}+\binom{n}{r}x^r+\ldots..\ \right)$$ **M1**

$$\Rightarrow \binom{n+1}{r}=\binom{n}{r-1}+\binom{n}{r}$$ **A1** **GIVEN ANSWER** legitimately obtained

3

For n even, writing $n = 2m$ … **M1** for attempting the even case

$$B_{2m} + B_{2m+1} = \binom{2m}{0}+\binom{2m-1}{1}+\binom{2m-2}{2}+\ldots..+\binom{m+1}{m-1}+\binom{m}{m}$$ **B1**

$$+\binom{2m+1}{0}+\binom{2m}{1}+\binom{2m-1}{2}+\binom{2m-2}{3}+\ldots..+\binom{m+1}{m}$$ **B1**

$$=\binom{2m+1}{0}+\left[\binom{2m}{0}+\binom{2m}{1}\right]+\left[\binom{2m-1}{1}+\binom{2m-1}{2}\right]+\ldots..+\left[\binom{m+1}{m-1}+\binom{m+1}{m}\right]+\binom{m}{m}$$ **M1** for suitable pairings (clear)

$$=\binom{2m+2}{0}+\left[\binom{2m+1}{1}\right]+\left[\binom{2m}{2}\right]+\ldots..+\left[\binom{m+2}{m}\right]+\binom{m+1}{m+1}$$ **M1** for use of first result, **(∗)**

using the result **(∗)** from above and since $\binom{2m+1}{0}=\binom{2m+2}{0}=\binom{m}{m}=\binom{m+1}{m+1}=1$ **M1** for noting the equality of the "1"s

$$=\sum_{j=0}^{m+1}\binom{2(m+1)-j}{j}=B_{2m+2}$$ **A1** Legitimately shown

7

For n odd, writing $n = 2m + 1$ … **M1** for attempting the odd case

$$B_{2m+1} + B_{2m+2} = \binom{2m+1}{0} + \binom{2m}{1} + \binom{2m-1}{2} + \ldots\ldots + \binom{m+2}{m-1} + \binom{m+1}{m}$$

Note that this appeared above

$$+ \binom{2m+2}{0} + \binom{2m+1}{1} + \binom{2m}{2} + \binom{2m-1}{3} + \ldots\ldots + \binom{m+2}{m} + \binom{m+1}{m+1}$$

B1

$$= \binom{2m+2}{0} + \left[\binom{2m+1}{0} + \binom{2m+1}{1}\right] + \left[\binom{2m}{1} + \binom{2m}{2}\right] + \ldots\ldots + \left[\binom{m+2}{m-1} + \binom{m+2}{m}\right] + \left[\binom{m+1}{m} + \binom{m+1}{m+1}\right]$$

M1 for suitable pairings (clear)

$$= \binom{2m+3}{0} + \left[\binom{2m+2}{1}\right] + \left[\binom{2m+1}{2}\right] + \ldots\ldots + \left[\binom{m+3}{m}\right] + \binom{m+2}{m+1}$$

M1 for use of first result, **(∗)**

using the result **(∗)** from above and since $\binom{2m+2}{0} = \binom{2m+3}{0} = 1$ **M1** for noting the equality of the "1"s

$$= \sum_{j=0}^{m+1} \binom{2(m+1)+1-j}{j} = B_{2m+3}$$

A1 Legitimately shown

6

B_0 , B_1 = $\binom{0}{0}, \binom{1}{0} = 1, 1$ **B1** Evaluating B_0 , B_1

Evaluating F_0 , F_1 & $F2$ **B1** Both sides must be evaluated, not just stated

Statement that $B_n = F_{n+1}$ **B1** At any point

For a clear justification of result (e.g. inductively) **E1**

(i.e. comment that since $B_0 = F_1$, $B_1 = F_2$ and B_n & F_n satisfy the same recurrence relation, we must have $B_n = F_{n+1}$ for all n)

4

SI 2013 Mark Scheme Q7

(i)	$y = ux \Rightarrow \dfrac{dy}{dx} = u + x\dfrac{du}{dx}$	**B1**	
	$\Rightarrow u + x\dfrac{du}{dx} = \dfrac{1}{u} + u$	**M1**	substituted in
	$\Rightarrow \int u\,du = \int \dfrac{1}{x}\,dx$	**M1**	variables separated and integration attempted
	$\Rightarrow \tfrac{1}{2}u^2 = \dfrac{y^2}{2x^2} = \ln x \quad (+C)$	**M1**	substituted back for x and y
	$\Rightarrow y^2 = x^2(2\ln x + 2C)$		
	$x = 1,\ y = 2$ substd. to determine C	**M1**	$C = 2$ (the lack of a constant of integration earlier $\Rightarrow$ M0 here)
	$\Rightarrow y = x\sqrt{2\ln x + 4}$		**ANSWER GIVEN** (all working and signs must be correct throughout)
		E1	Justification of $+_{ve}$ square-root required: $y > 0$ when $x = 1$ gives this (Given $x > e^{-2}$, so square-rooting valid – this does not need to be stated by candidates)
			6
(ii)	$y = ux$ used to get $u + x\dfrac{du}{dx} = \dfrac{1}{u} + 2u$	**B1**	
	$\Rightarrow \int \dfrac{2u}{1+u^2}\,du = \int \dfrac{2}{x}\,dx$	**M1**	variables separated and integration attempted
	$\Rightarrow \ln(1+u^2) = 2\ln x \quad (+\ln A)$	**A1**	
	$x = 1,\ y = 2$ substd. to determine A	**M1**	$A = 5$ (the lack of a constant of integration earlier $\Rightarrow$ M0 here)
	$1 + \dfrac{y^2}{x^2} = 5x^2$	**M1**	for eliminating u and correct use of log laws (*with* correct number of terms)
	$y = x\sqrt{5x^2 - 1}$	**A1**	No need to justify choice of $+_{ve}$ square-root, but don't allow $\pm$
	for $x > \frac{1}{\sqrt{5}}$	**B1**	**FT** sensible domains (provided $x > 0$)

(ii) **ALTERNATIVE**

$y = ux^2$ used to get $2ux + x^2\dfrac{du}{dx} = \dfrac{1}{ux} + 2ux$	**B1M1**	$\dfrac{dy}{dx}$ correct (LHS); full substitution into given d.e.
$\Rightarrow \int u\, du = \int \dfrac{1}{x^3}\, dx$	**M1**	variables separated and integration attempted
$\Rightarrow \frac{1}{2}u^2 = \dfrac{-1}{2x^2} \quad \left(+\frac{1}{2}C\right)$		
$x = 1,\ y = 2\ (u = 2)$ substd. to find C	**M1**	$C = l5$ (the lack of a constant of integration earlier $\Rightarrow$ M0 here)
$\dfrac{y^2}{x^4} = 5 - \dfrac{1}{x^2}$	**M1**	for eliminating u
$y = x\sqrt{5x^2 - 1}$	**A1**	No need to justify choice of $+_{ve}$ square-root, but don't allow $\pm$
for $x > \frac{1}{\sqrt{5}}$	**B1**	**FT** sensible domains (provided $x > 0$)

7

(iii)

$y = ux^2$ used to get $2ux + x^2\dfrac{du}{dx} = \dfrac{1}{u} + 2ux$	**B1M1**	$\dfrac{dy}{dx}$ correct (LHS); full substitution into given d.e.
$\Rightarrow \int u\, du = \int \dfrac{1}{x^2}\, dx$	**M1**	variables separated and integration attempted
$\Rightarrow \frac{1}{2}u^2 = \dfrac{-1}{x} \quad (+D)$		
$x = 1,\ y = 2\ (u = 2)$ substd. to find D	**M1**	$D = 3$ (the lack of a constant of integration earlier $\Rightarrow$ M0 here)
$\dfrac{y^2}{x^4} = 6 - \dfrac{2}{x}$	**M1**	for eliminating u
$y = x\sqrt{6x^2 - 2x}$ or $y = x\sqrt{2x}\sqrt{3x-1}$	**A1**	No need to justify choice of $+_{ve}$ square-root, but don't allow $\pm$
for $x > \frac{1}{3}$	**B1**	**FT** sensible domains (provided $x > 0$)

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SI 2013 Mark Scheme Q8

(i)

function	domain	range		
$\mathrm{cb}(x) = 2\ln x$	$x > 0$	$\mathbb{R}$	**B1**	
$\mathrm{ab}(x) = (\ln x)^2$	$x > 0$	$y \geq 0$	**B1**	
$\mathrm{da}(x) = \lvert x \rvert$	$\mathbb{R}$	$y \geq 0$	**B1**	Do not allow x or $\sqrt{x^2}$ here; but $\begin{cases} x & x > 0 \\ -x & x < 0 \end{cases}$ is ok
$\mathrm{ad}(x) = x$	$x \geq 0$	$y \geq 0$	**B1**	
				These B1s are for the functions.
			B1B1	1st for all domains correct; 2nd for all ranges correct

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(ii)

$\mathrm{fg}(x) = \lvert x \rvert$ or $\sqrt{x^2}$	$\mathbb{R}$	$y \geq 0$	**B1 B1**	Allow $\begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$ but not or $\pm x$
$\mathrm{gf}(x) = \lvert x \rvert$ or $\sqrt{x^2}$	$\lvert x \rvert \geq 1$	$y \geq 1$	**B1 B1**	

In each case, 1st B1 for the fn. & 2nd B1 for both domain and range

OR 1st B1 for both domains correct, 2nd B1 for both ranges correct

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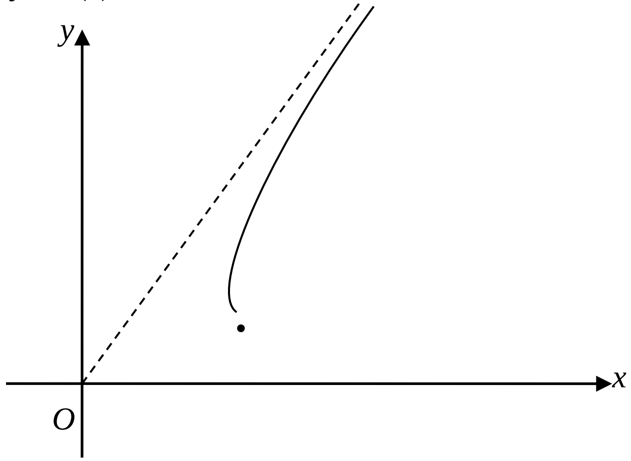
(iii) $y = \mathrm{h}(x) = x + \sqrt{x^2 - 1}$, $x \geq 1$



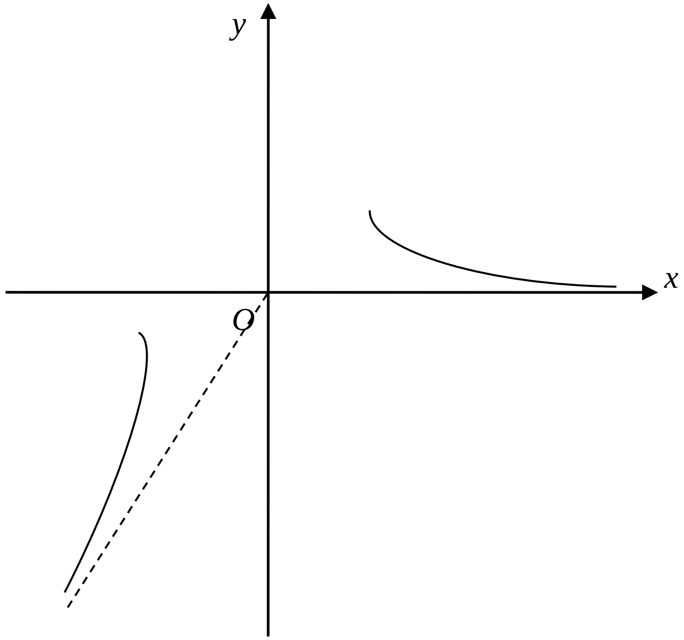
G1 Starts at (1, 1) – noted or clear on sketch

G1 Going upwards $(\to \infty)$

G1 Approaching asymptote $y = 2x$ (eqn. must be given somewhere)

NB – I will correct the diagram so it is the graph of a *function*

$y = \mathrm{k}(x) = x - \sqrt{x^2 - 1}\,, \ |x| \geq 1$



G1 Two branches

G1 (1, 1) and (–1, –1) noted or clear on sketch

G1 Branch in Q1 approaches (from above) asymptote $y = 0$ (or "x-axis") must be noted somewhere

G1 Branch in Q3 approaches asymptote (from above) $y = 2x$ must be noted somewhere

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Domain of kh is $x \geq 1$ **B1**

Range of h is $y \geq 1 \Rightarrow$ Range of kh is $0 < y \leq 1$ **M1A1**

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