1. $p(x,y|z) = p(y|z)\,p(x|y,z)$ 普遍成立

$= p(x|z)\,p(y|z)$ x和y独立时成立

2. P48. 推导

已有 Data = $\{D_1, D_2, \ldots D_n\}$ $p(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

$\theta = (\mu, \sigma^2)$

$$L(\theta) = p(Data|\theta) = \prod_{i=1}^{n} p(D_i|\theta) = \prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right)$$

$$\log L(\theta) = \sum_{i=1}^{n} -\log\sqrt{2\pi}\sigma - \frac{(x_i-\mu)^2}{2\sigma^2}$$

$$= \sum_{i=1}^{n} -\frac{1}{2}\log(2\pi\sigma^2) - \frac{(x_i-\mu)^2}{2\sigma^2}$$

$$= -\frac{n}{2}\log(2\pi\sigma^2) - \frac{1}{2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{\sigma^2}$$

$$\frac{\partial[\log L(\theta)]}{\partial\mu} = -\frac{1}{2}\sum_{i=1}^{n}\frac{2(x_i-\mu)\times(-1)}{\sigma^2} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)$$

令上式 = 0. $\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}x_i$

$$\frac{\partial[\log L(\theta)]}{\partial\sigma^2} = -\frac{n}{2}\times\frac{2\pi}{2\pi\sigma^2} - \frac{1}{2}\sum_{i=1}^{n}(x_i-\mu)^2\cdot\left(-\frac{1}{\sigma^4}\right)$$

$$= -\frac{n}{2\sigma^2} + \frac{1}{2}\sum_{i=1}^{n}(x_i-\mu)^2\cdot\frac{1}{\sigma^4}$$

令上式 = 0 $\quad \frac{n}{\sigma^2} = \sum_{i=1}^{n}(x_i-\mu)^2\cdot\frac{1}{\sigma^4}$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

x的均值

代入 $\hat{\mu}$. 有 $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \frac{1}{n}\sum_{i=1}^{n}x_i\right)^2$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

就是方差.

2. $\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t}\,dt$, $\alpha > 0$. Gamma 分布函数

性质 $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$ $\Gamma(n+1) = n!$ $\Gamma(1) = 1$. $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$$\int_0^{\infty} \frac{t^{\alpha-1} e^{-t}}{\Gamma(\alpha)}\,dt = 1$$

Gamma 密度函数: $\mathrm{Gamma}(t|\alpha) = \frac{t^{\alpha-1} e^{-t}}{\Gamma(\alpha)}$

令 $t = \beta x$.

$$\mathrm{Gamma}(x|\alpha,\beta) = \frac{\beta^{\alpha} x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}$$

$$\int_0^{\infty} \frac{(\beta x)^{\alpha-1} e^{-t}}{\Gamma(\alpha)}\,d(\beta x) = 1$$

$$\int_0^{\infty} \frac{\beta^{\alpha} x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}\,dx = 1$$

故 $\mathrm{Gamma}(x|\alpha,\beta) = \frac{\beta^{\alpha} x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}$

α 决定分布曲线的形状
β 决定分布曲线陡峭

3. Beta 分布:

概率密度函数 $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$

$$\mathrm{Beta}(x|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$$