Chapter 8: Index Structures

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Outline[1]

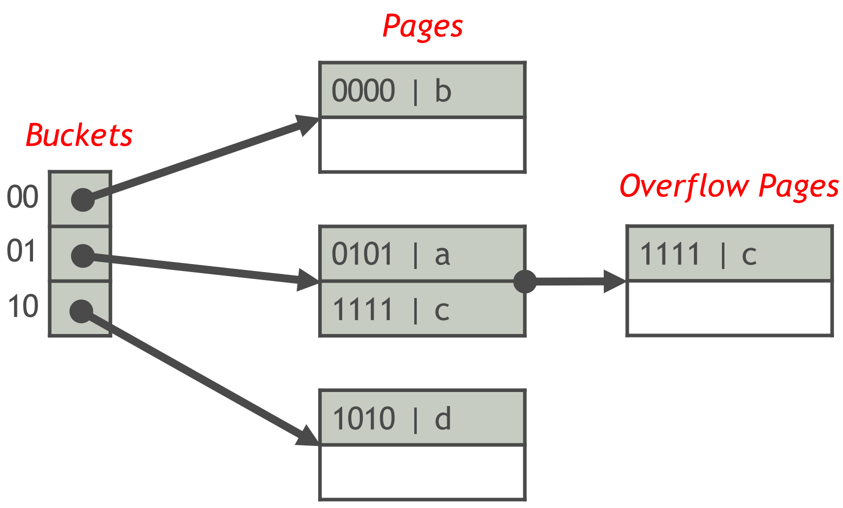
1. Hash-based Index Structures
   - Extensible Hash Tables
   - Linear Hash Tables
2. Tree-based Index Structures
   - B+ Trees

[1]Updated on April 2, 2020

Hash-based Index Structures

Secondary-Storage Hash Tables (外存哈希表)

- A secondary-storage hash table consists of a number of buckets
- An index entry with key K is put in the bucket numbered $\mathit{hash}(K)$, where *hash* is a hash function
- Each bucket stores a pointer to a linked list of pages holding the index entries in the bucket



Categories of Secondary-Storage Hash Tables

Static Hash Tables (静态哈希表)

- The number of buckets does not change

Dynamic Hash Tables (动态哈希表)

- The number of buckets is allowed to vary so that there is about one block per bucket
- Extensible hash tables (可扩展哈希表)
- Linear hash tables (线性哈希表)

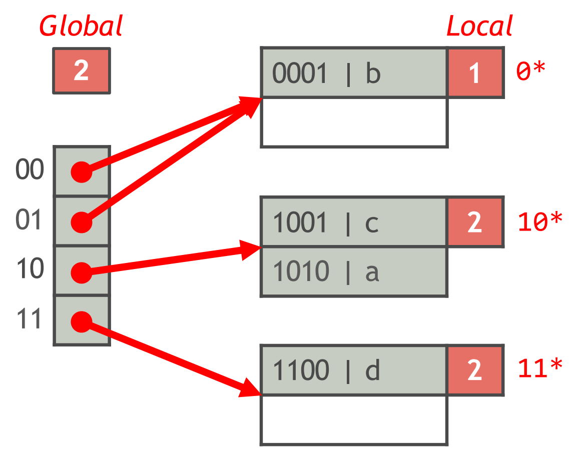
Hash-based Index Structures
Extensible Hash Tables

Extensible Hash Tables (可扩展哈希表)

An extensible hash table is comprised of 2^i buckets

- i is called the global depth
- An index entry with key K belongs to the bucket numbered by the first i bits of $hash(K)$

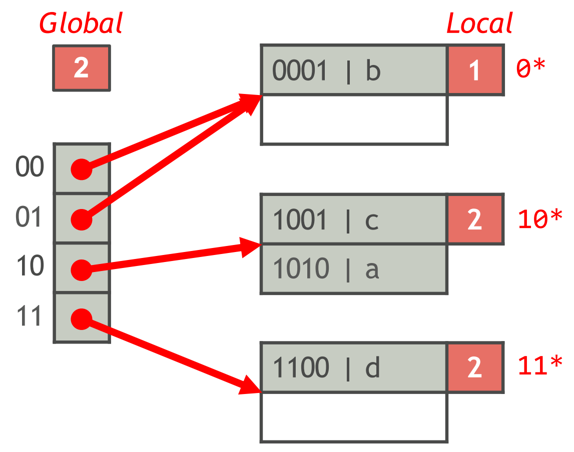
Example:
$hash(a) = 1010, hash(b) = 0001, hash(c) = 1001, hash(d) = 1100$



Extensible Hash Tables (Cont'd)

Every bucket keeps a pointer to a page where the index entries in the bucket are stored

- Several buckets can share a page if all the index entires in those buckets can fit in the page
- Every page records # bits of $hash(K)$ (local depth) used to determine the membership of index entires in this page

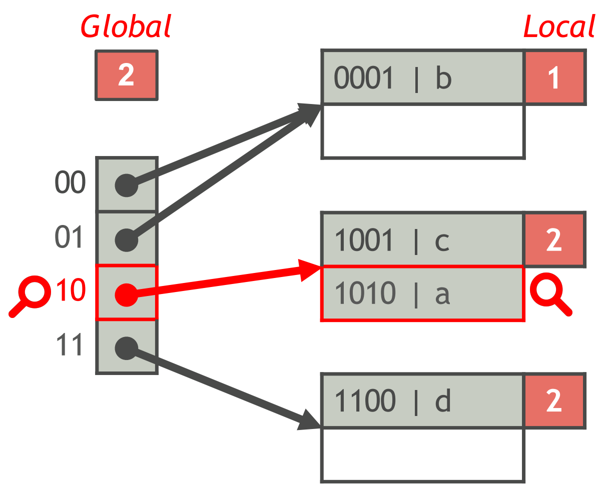


Extensible Hash Table Lookup

Find the index entry with key K

1. Determine the bucket where the entry belongs to
2. Find the entry in the page that the bucket points to

Example: $K = a$, $hash(a) = 1010$

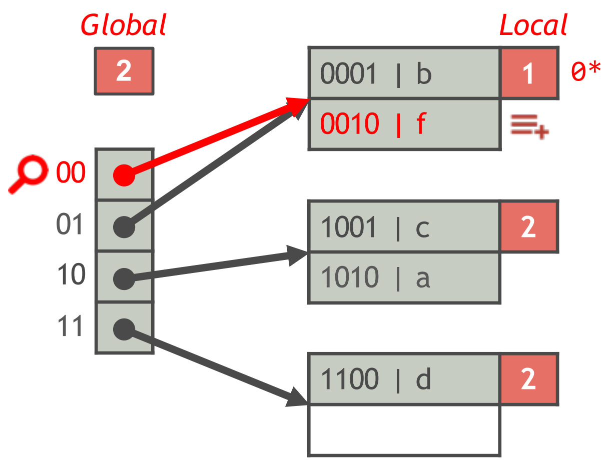


Extensible Hash Table Insert

Insert an index entry with key K

1. Find the page P where the entry is to be inserted
2. If P has enough space, done!
   Otherwise, split P into P and a new page P'

Example: $K = f$, $hash(f) = 0010$

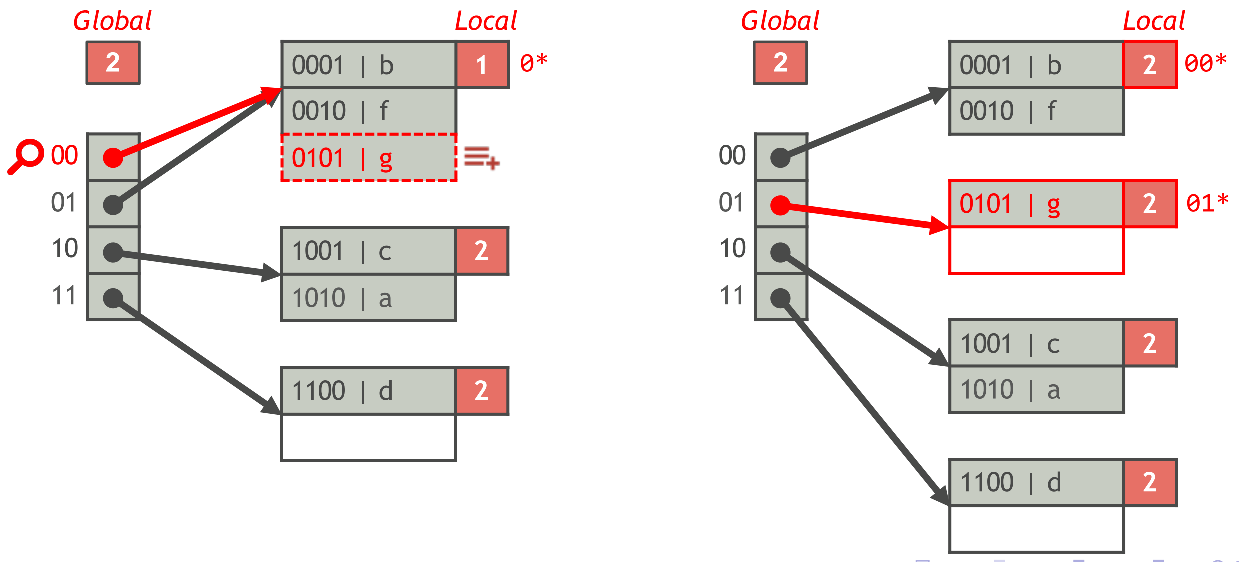


Extensible Hash Table Insert (Cont'd)

If P overflows and the local depth of P is less than the global depth,

1. Increase P's local depth by 1
2. Re-assign some index entries in P to a new bucket page P' (P and P' have the same local depth)

Example: $K = g$, $hash(g) = 0101$

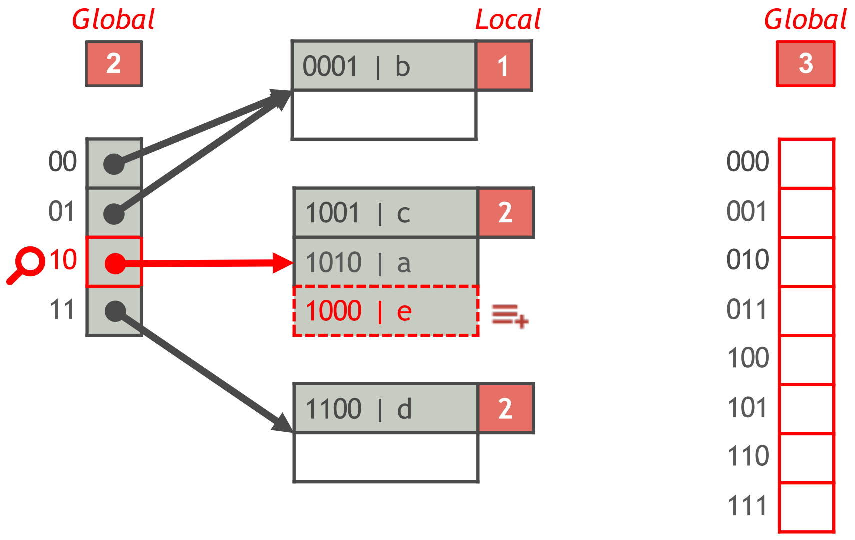


Extensible Hash Table Insert (Cont'd)

If P overflows and the local depth of P is equal to the global depth,

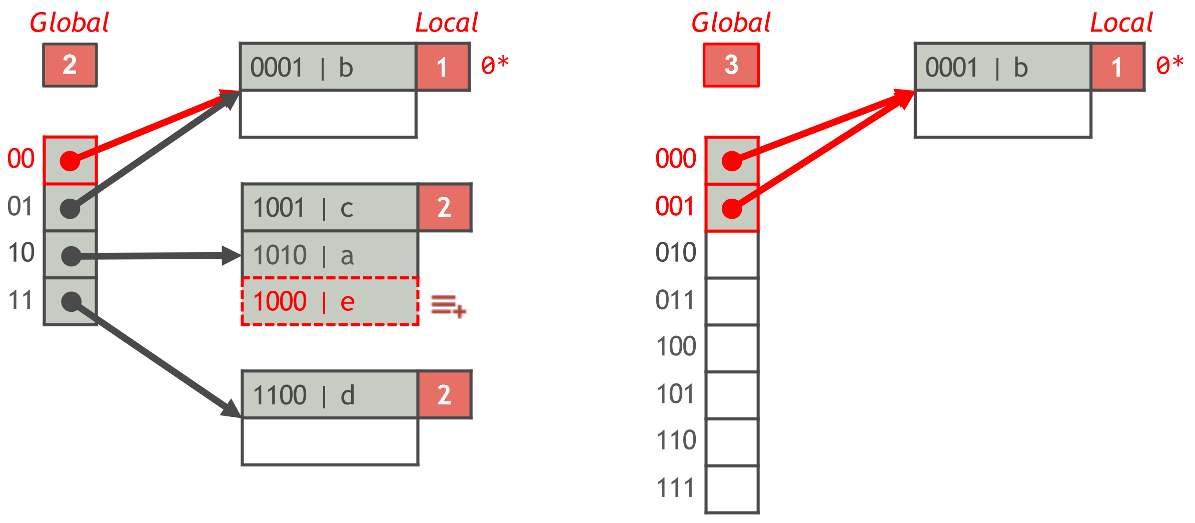
1. Increase the global depth by 1 (double # buckets)
2. Re-organize the buckets; if a page overflows, split it

Example: $K = e$, $hash(e) = 1000$



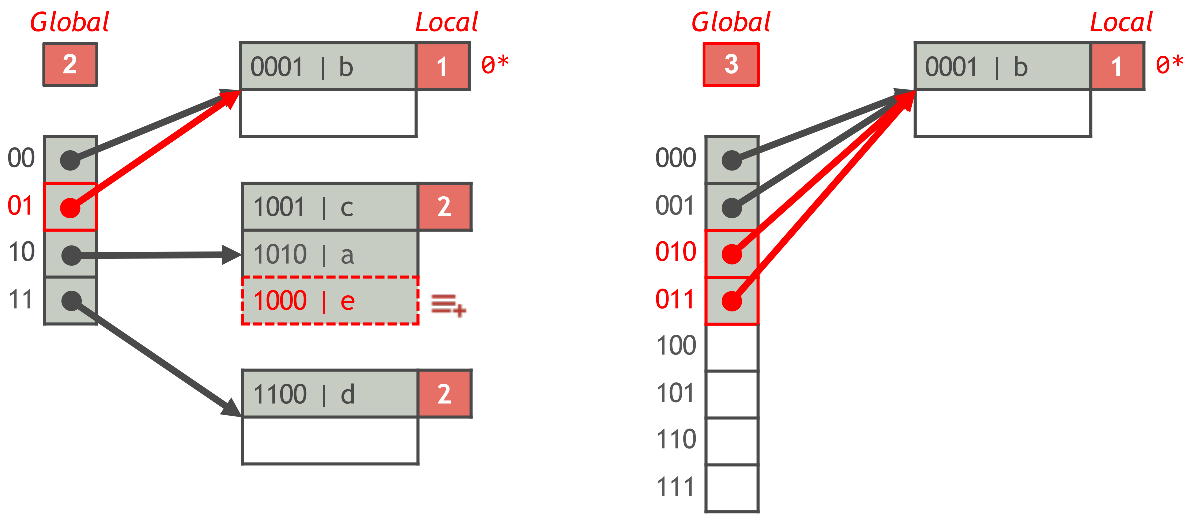
Extensible Hash Table Insert: Example

Example: $K = e,\ hash(e) = 1000$



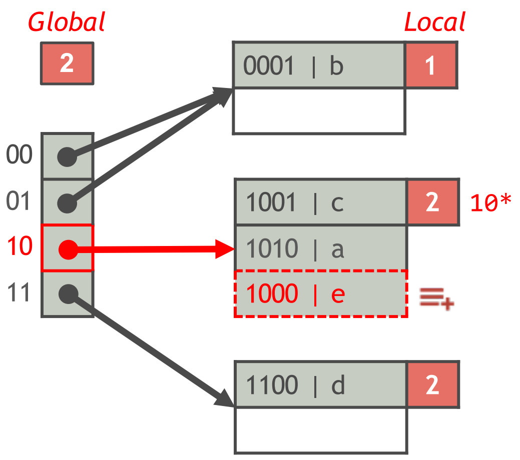
Extensible Hash Table Insert: Example

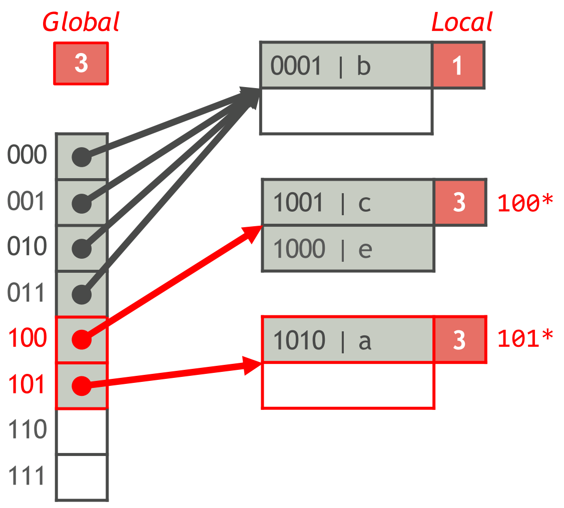
Example: $K = e,\ hash(e) = 1000$



Extensible Hash Table Insert: Example

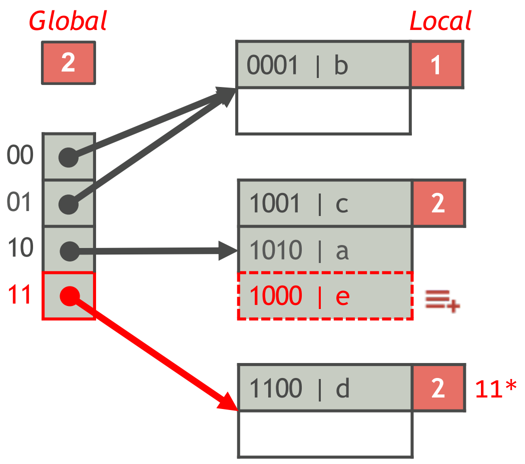
Example: $K = e$, $hash(e) = 1000$

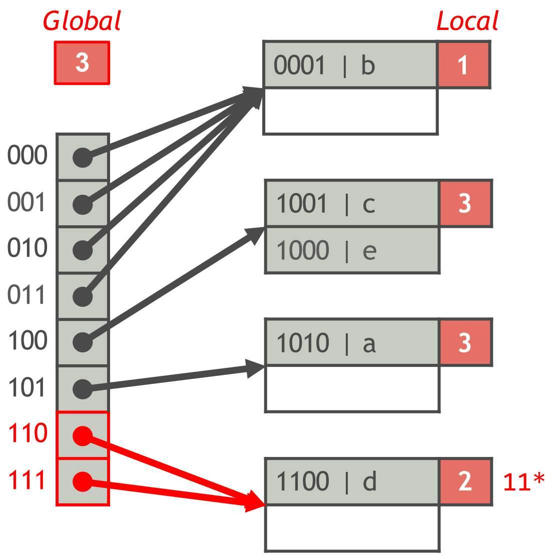




Extensible Hash Table Insert: Example

Example: $K = e$, $hash(e) = 1000$



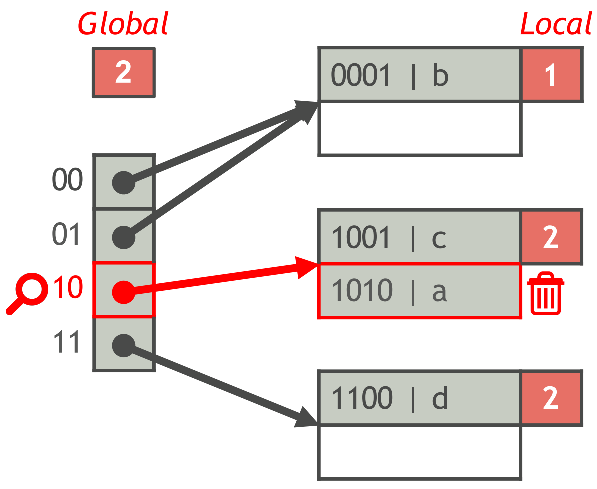


Extensible Hash Table Delete

Delete the index entry with key K

1. Find the page where the entry belongs to
2. Delete the entry from the page

Example: $K = a$, $hash(a) = 1010$



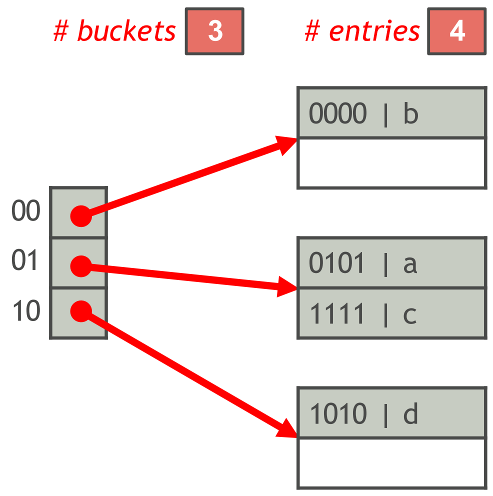
Hash-based Index Structures
Linear Hash Tables

Linear Hash Tables (线性哈希表)

A linear hash table is comprised of n buckets

- Every bucket keeps a pointer to a linked list of pages holding the index entries in the bucket
- Suppose each page can hold at most b index entries. The linear hash table stores at most θbn entries, where $0 < \theta < 1$ is a threshold

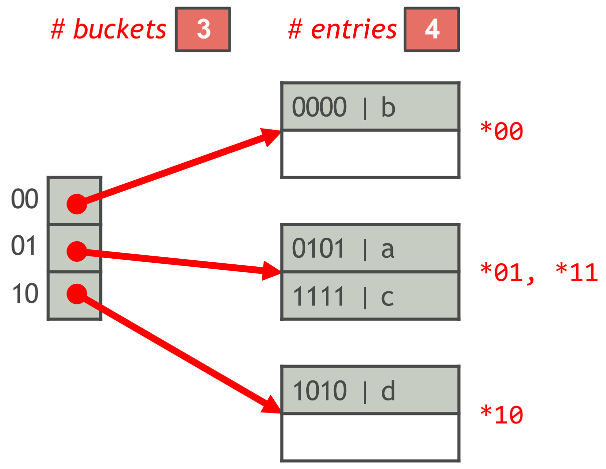
Example: $b = 2$, $\theta = 0.85$



Hashing Scheme

- The buckets are numbered from 0 to $n-1$
- Let $m = 2^{\lfloor \log_2 n \rfloor}$, so $m \leq n < 2m$
- If $hash(K) \mod 2m < n$, index entry with key K belongs to bucket $hash(K) \mod 2m$; Otherwise, it belongs to bucket $hash(K) \mod m$

Example:
$hash(a) = 0101, hash(b) = 0000, hash(c) = 1111, hash(d) = 1010$

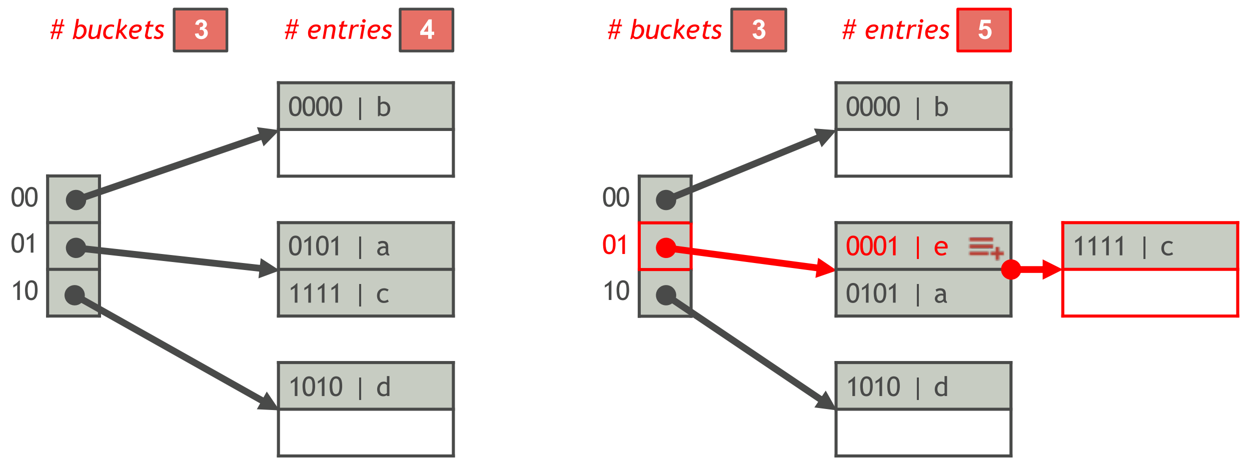


Linear Hash Table Insert

Insert an index entry with key K

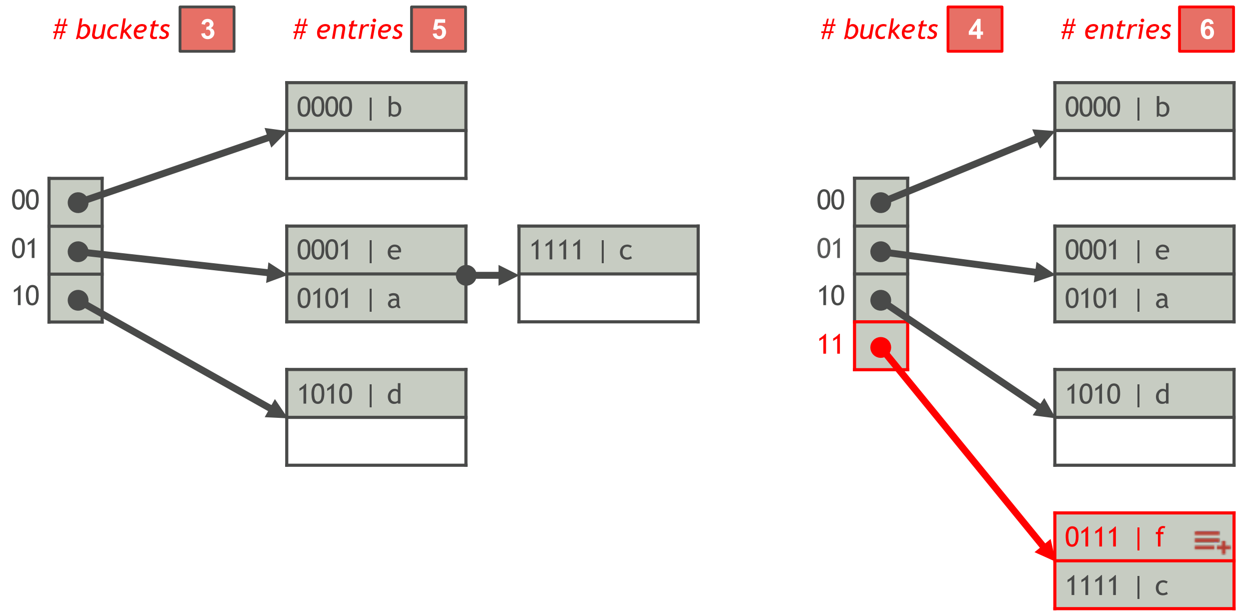
1. Insert the entry into the bucket B where it belongs to
2. Increase # entries by 1
3. If # entries $\leq \theta bn$, done!
   Otherwise, increase # buckets by 1 and redistribute the entries in B

Example: $hash(e) = 0001$, $\theta = 0.85$



Linear Hash Table Insert (Cont'd)

Example: $hash(f) = 0111$, $\theta = 0.85$



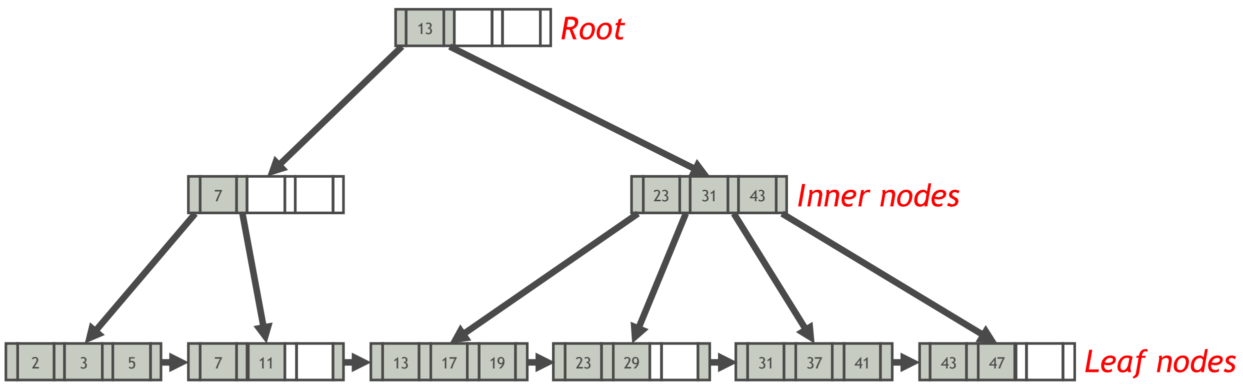
Tree-based Index Structures

Tree-based Index Structures
B+ Trees

B+ Trees

A B+ tree is an M-way search tree with the following properties:

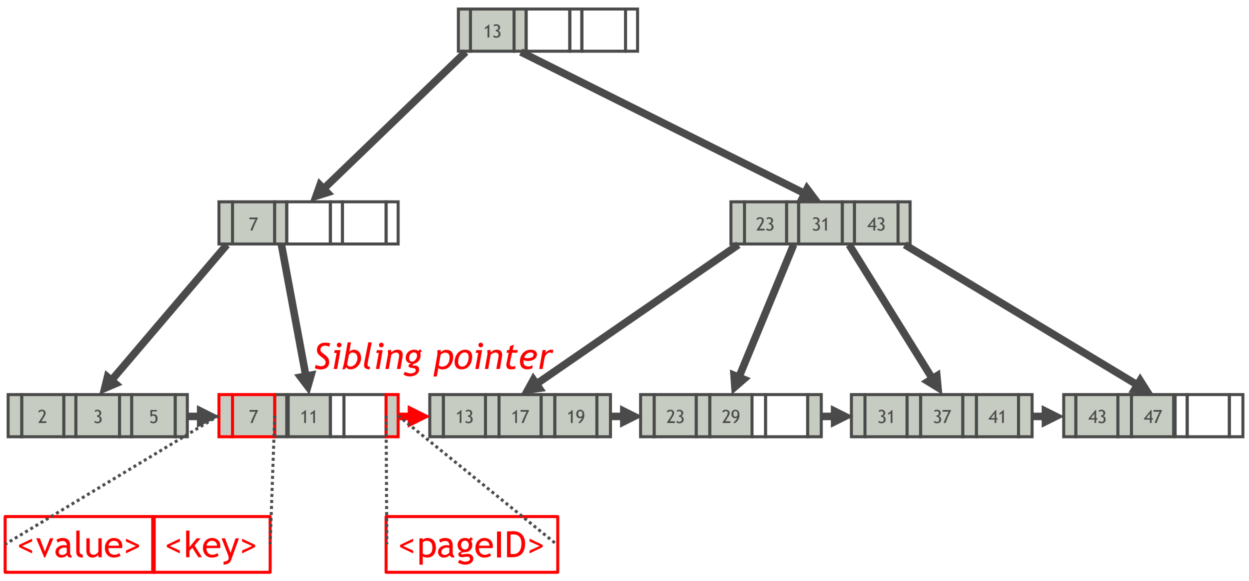
- It is perfectly balanced (i.e., every leaf node is at the same depth)
- Every node other than the root is at least half-full
  $M/2 - 1 \leq \#keys \leq M - 1$
- Every inner node with k keys has $k + 1$ non-null children
- Every node fits a page



B+ Tree Leaf Nodes

Every leaf node is comprised of an array of index entries (key/value pairs) and a pointer to its right sibling

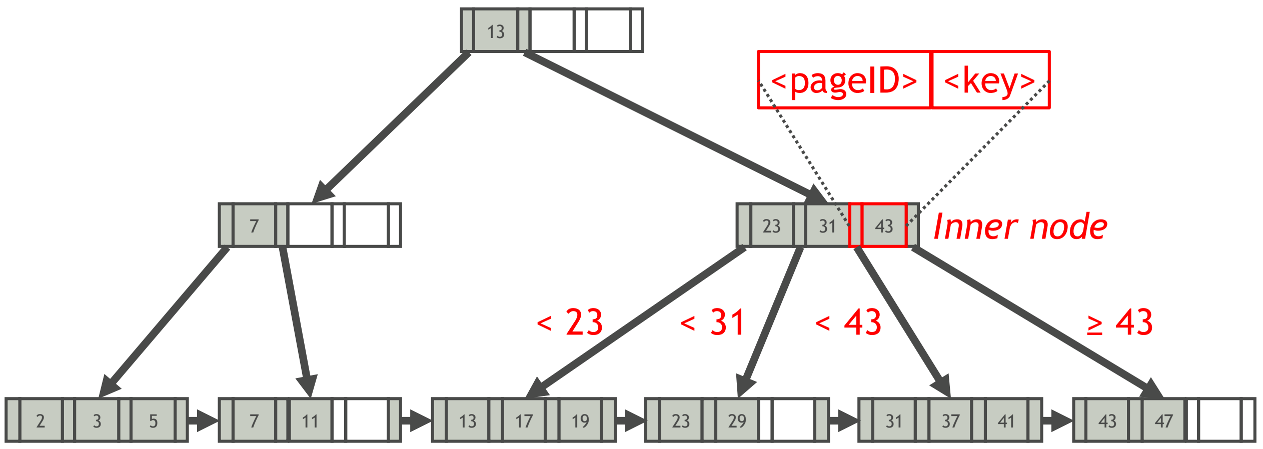
- The index entry array is (usually) kept in sorted key order



B+ Tree Inner Nodes

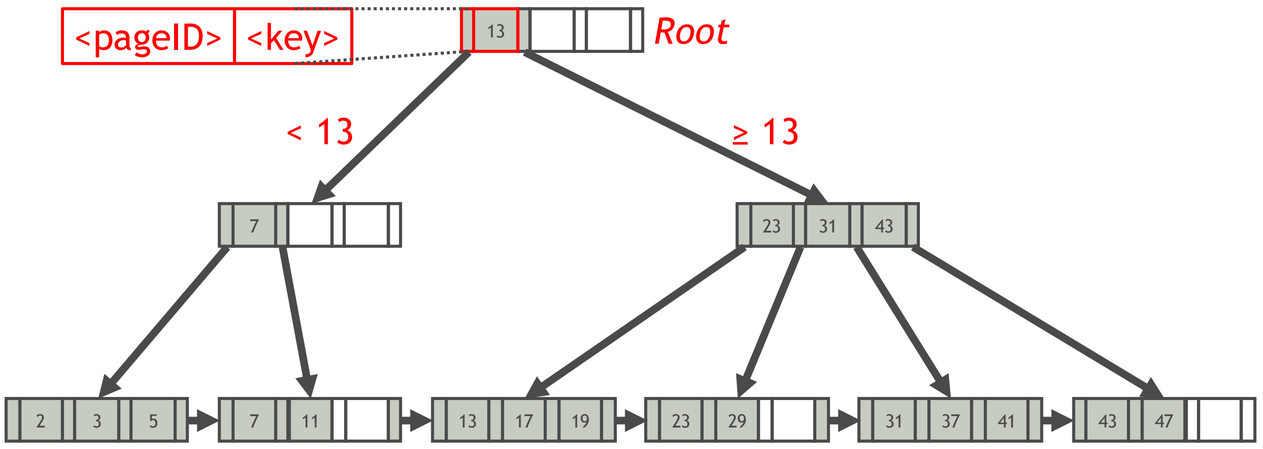
Every inner node is comprised of an array of keys and an array of pointers to its children

- The keys are derived from the attribute(s) that the index is based on
- The arrays are (usually) kept in sorted key order



B+ Tree Root Node

The root contains at least one key

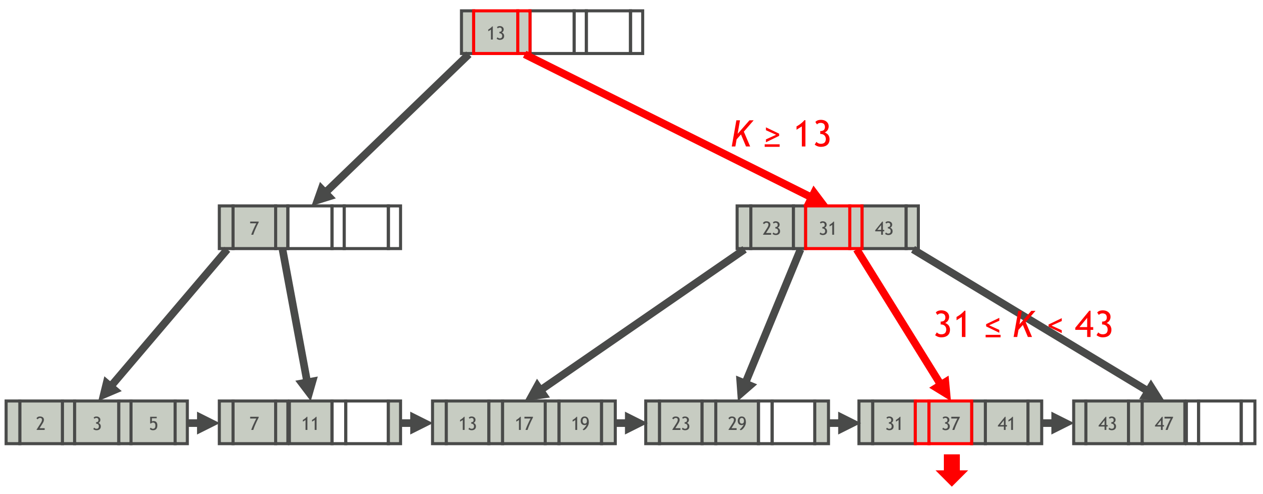


B+ Tree Lookup

Find the index entry with key K

1. Find the leaf node where K belongs to by following the direction of the keys in the inner nodes
2. Find the entry with key K in the leaf node

Example: $K = 37$

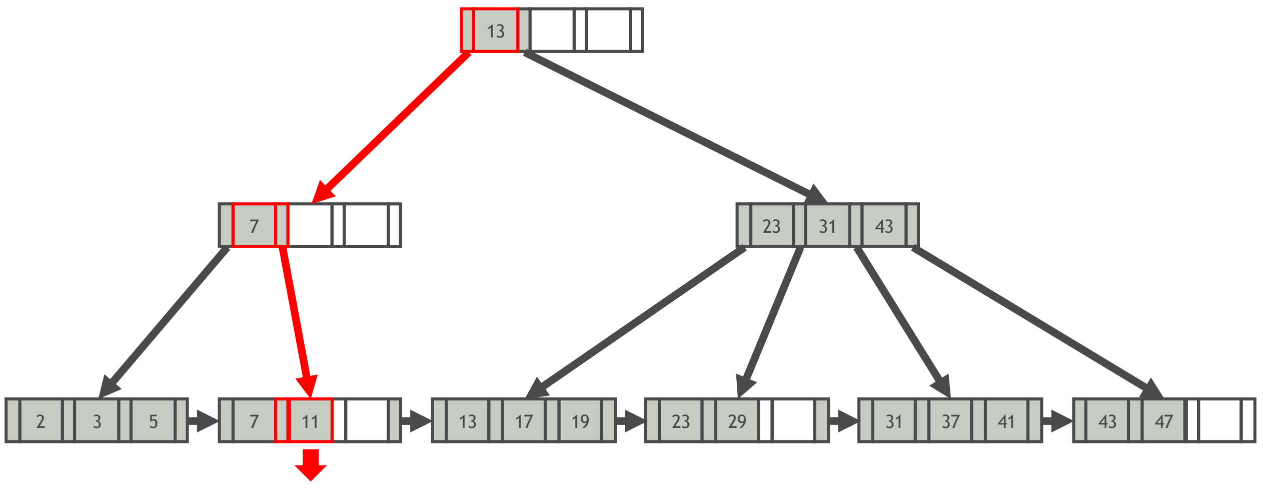


B+ Tree Range Query

Find the index entries with keys $K \in [L, U]$

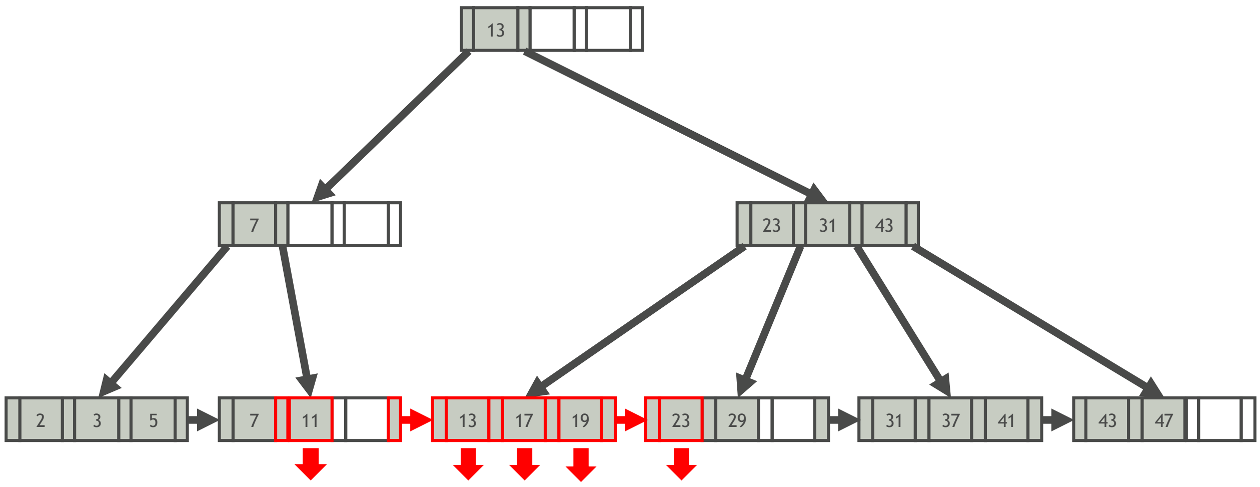
1. Find the first index entry E with the smallest key $\geq L$
2. Scan the contiguous index entries with keys $\leq U$ to the right of E

Example: $K \in [10, 25]$



B+ Tree Range Query (Cont'd)

Example: $K \in [10, 25]$



B+ Tree Insert

Insert an index entry with key K

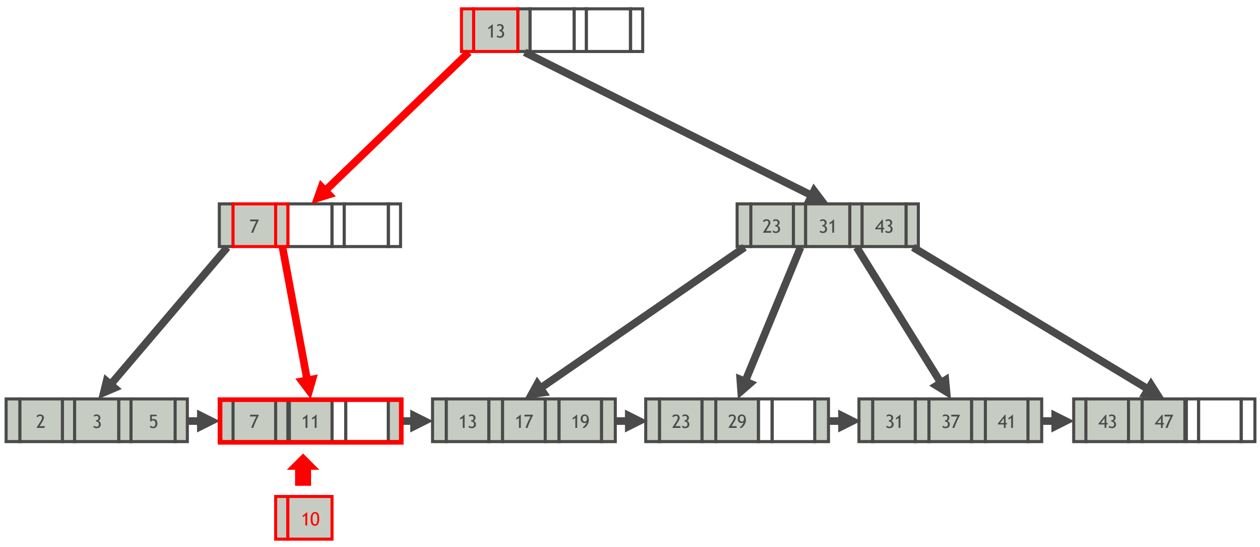
1. Find the correct leaf node L where the entry is to be inserted
2. Put the entry into L in sorted key order
3. If L has enough space, done!
   Otherwise, split the keys in L into L and a new node L_2
   1. Redistribute the entries evenly, copy up the middle key
   2. Insert an index entry pointing to L_2 into the parent of L

To split an inner node,

1. Redistribute the entries evenly
2. Push up the middle key

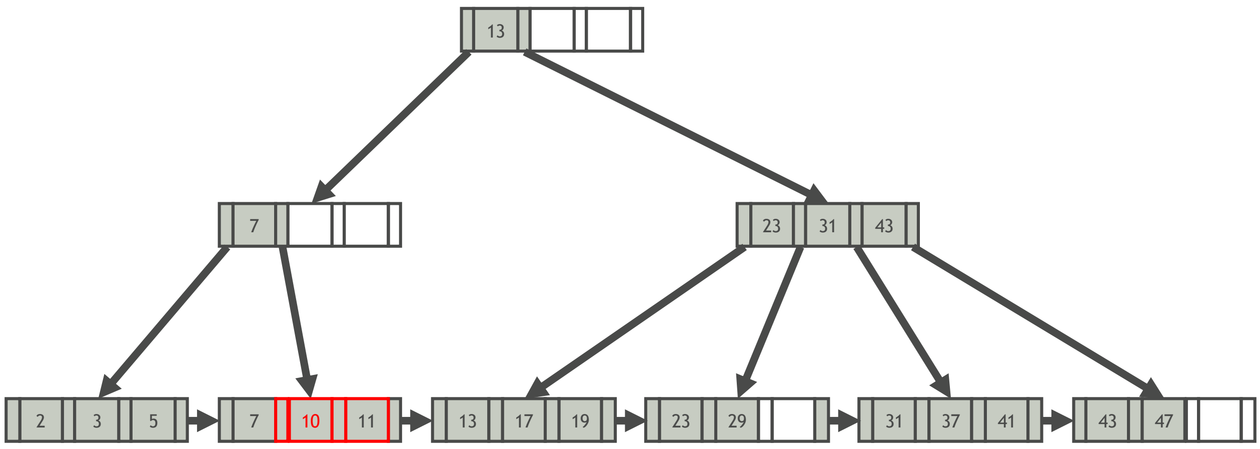
B+ Tree Insert: Example 1 (w/o Node Split)

Example: $K = 10$



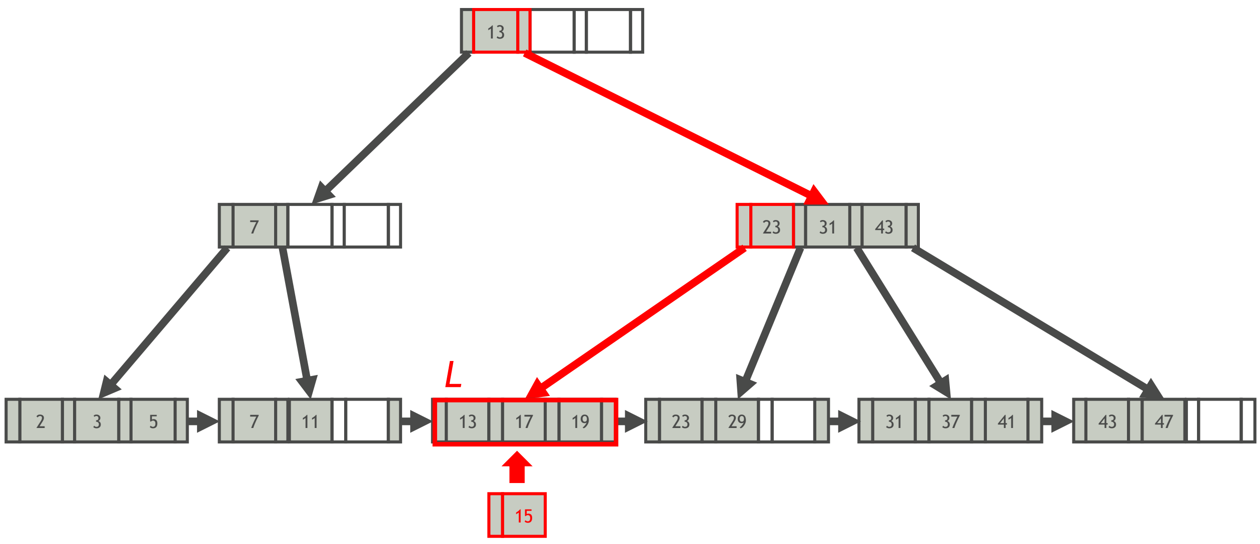
B+ Tree Insert: Example 1 (w/o Node Split)

Example: $K = 10$



B+ Tree Insert: Example 2 (w/ Node Split)

Example: $K = 15$



B+ Tree Insert: Example 2 (w/ Node Split)

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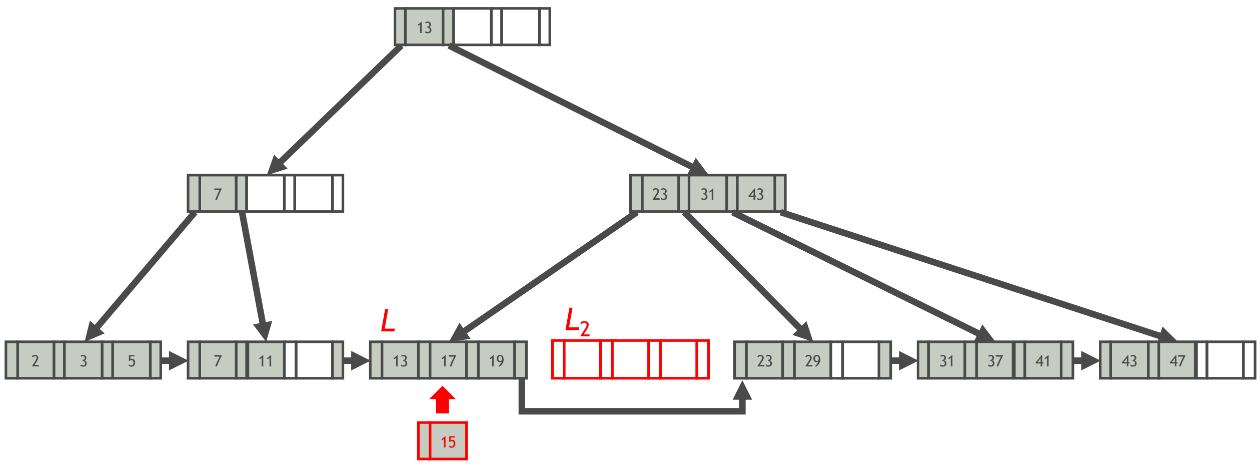

B+ Tree Insert: Example 2 (w/ Node Split)

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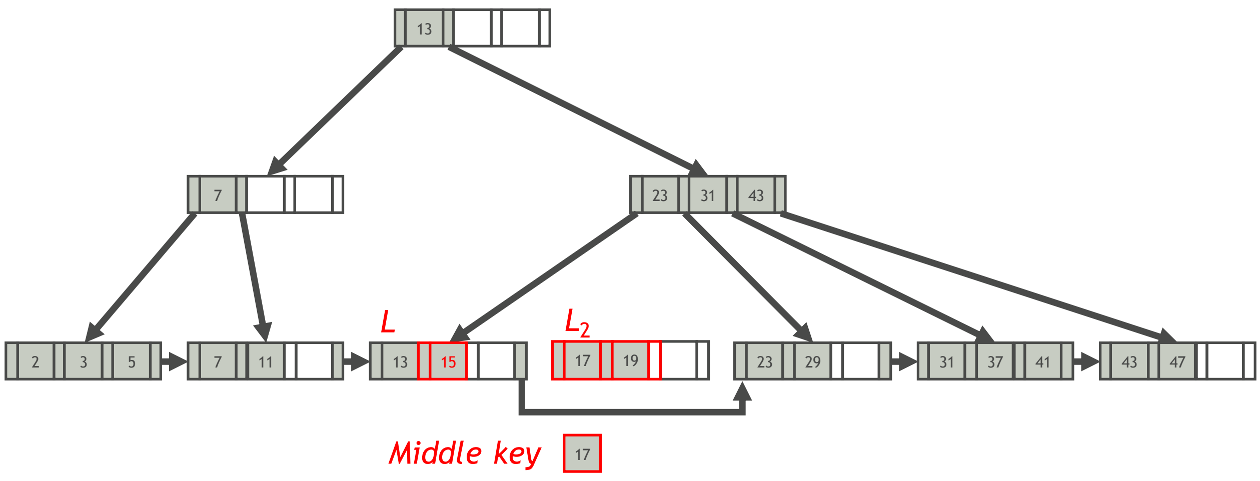

B+ Tree Insert: Example 2 (w/ Node Split)

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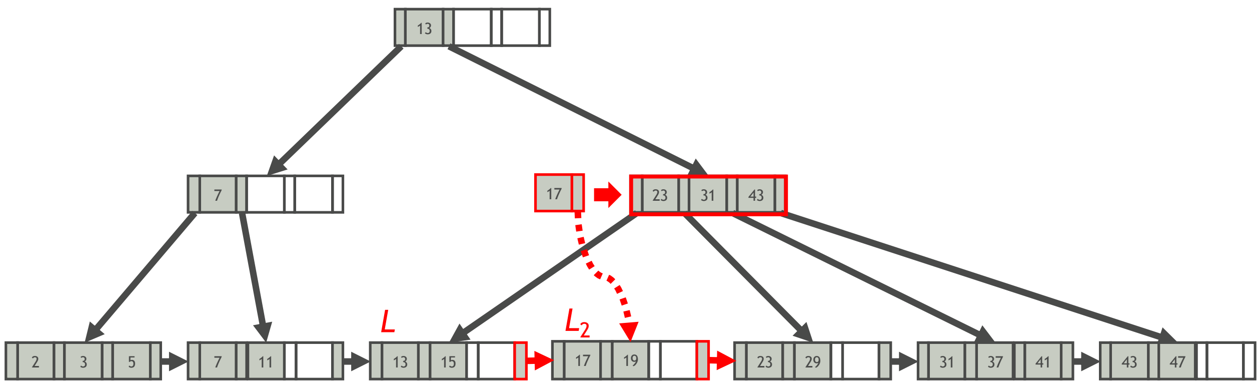

B+ Tree Insert: Example 2 (w/ Node Split)

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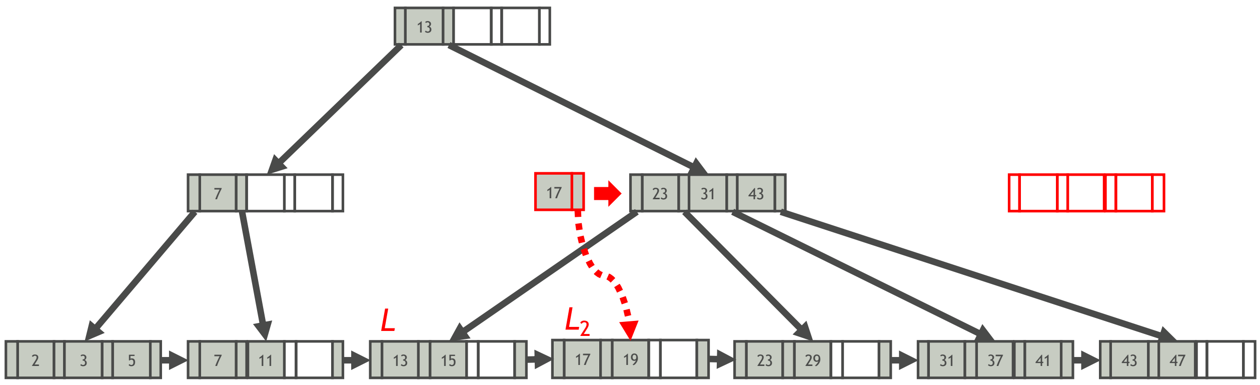

B+ Tree Insert: Example 2 (w/ Node Split)

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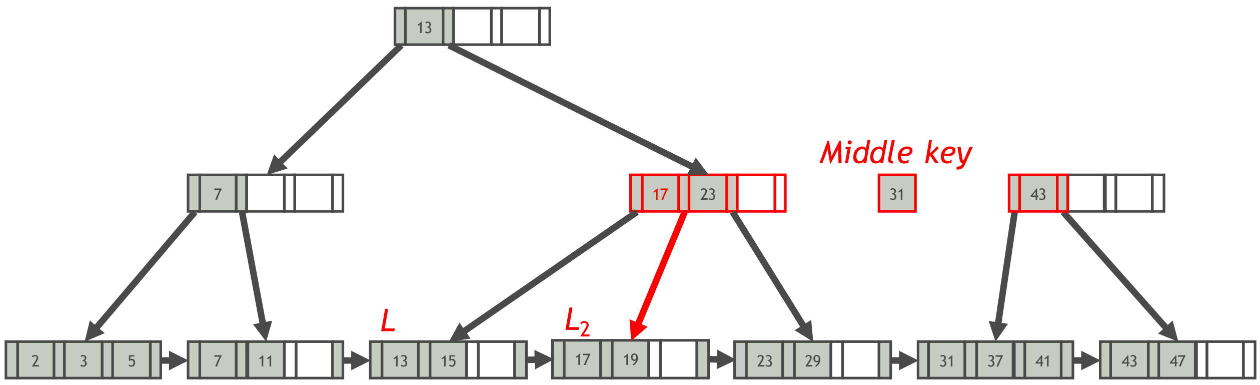

B+ Tree Insert: Example 2 (w/ Node Split)

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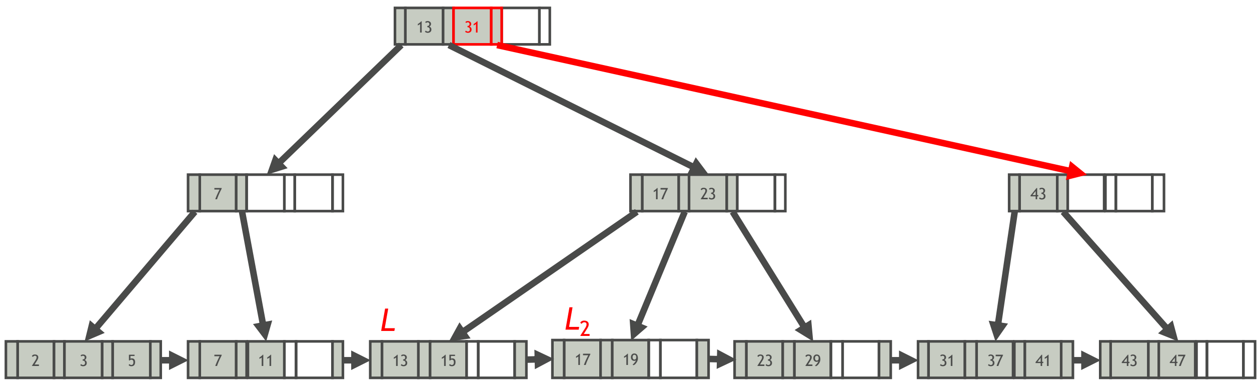


B+ Tree Delete

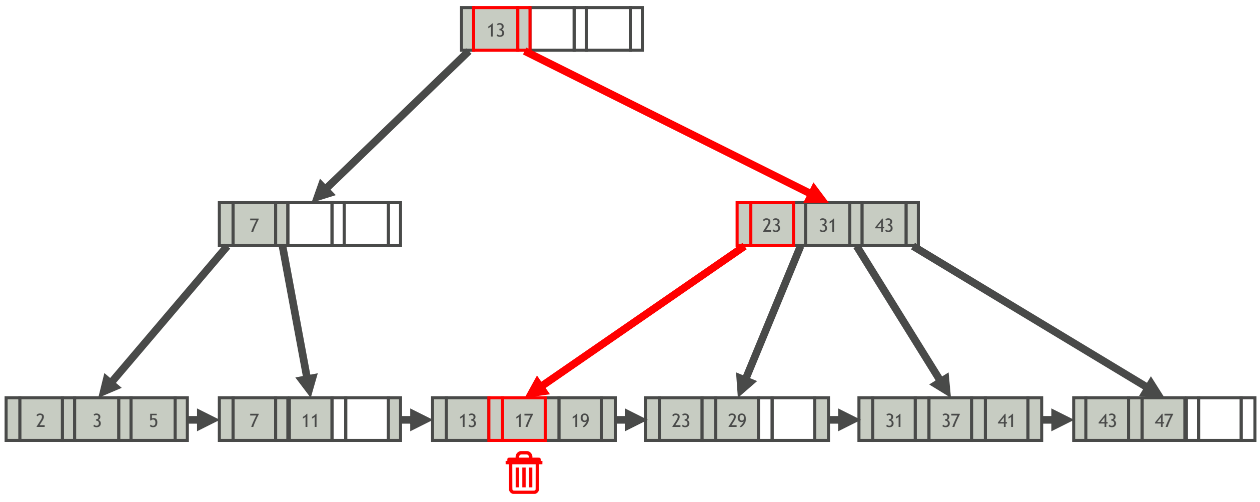
Delete an index entry with key K

1. Find the leaf node L where the entry belongs to
2. Remove the entry from L
3. If L is at least half-full, done!
   Otherwise,
   1. Try to redistribute, borrowing from sibling
   2. If redistribution fails, merge L and its sibling

If merge occurred, must delete entry pointing to L or the sibling from the parent of L

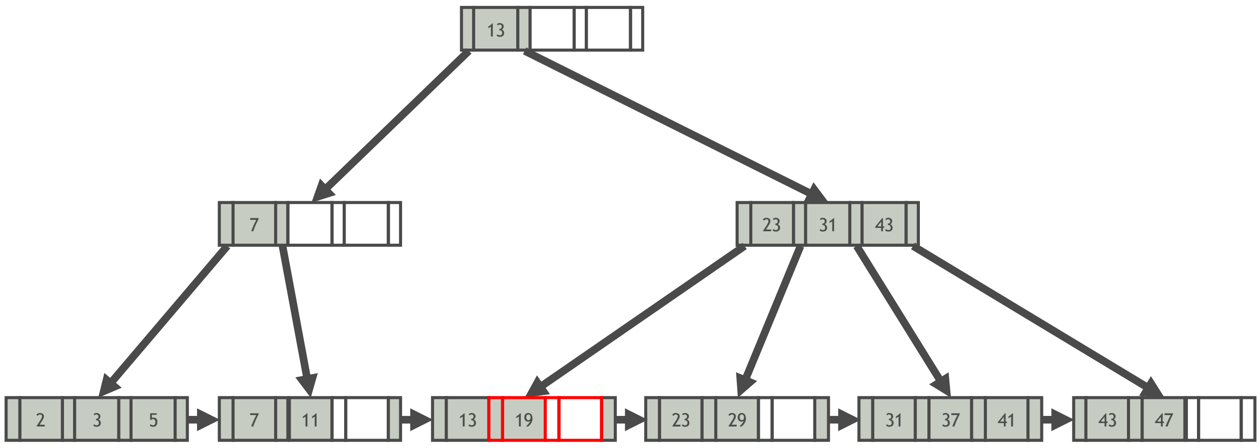
B+ Tree Delete: Example 1 (w/o Node Underflow)

Example: $K = 17$



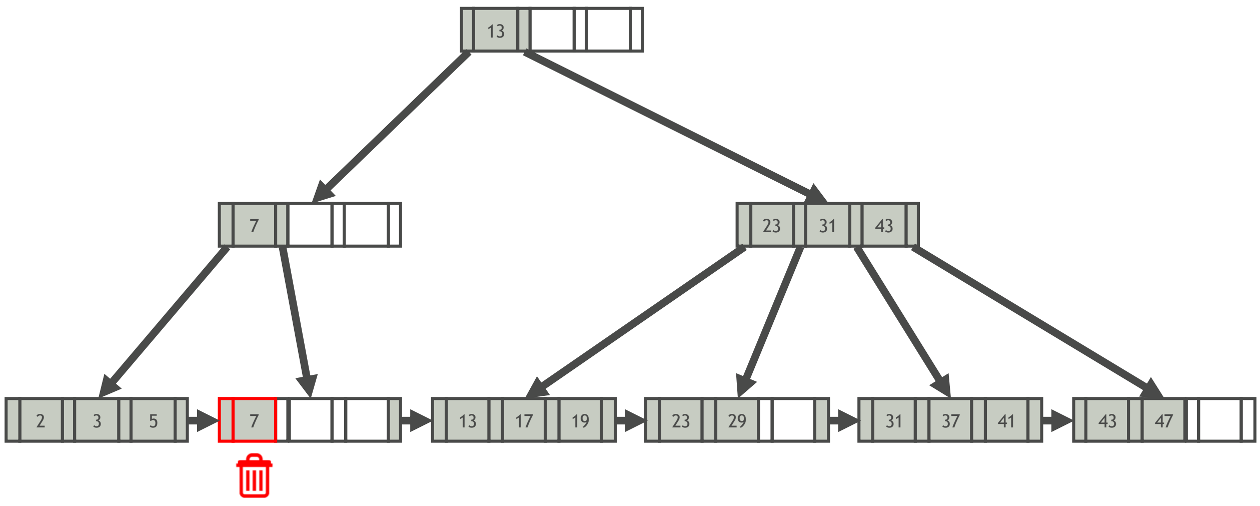
B+ Tree Delete: Example 1 (w/o Node Underflow)

Example: $K = 17$



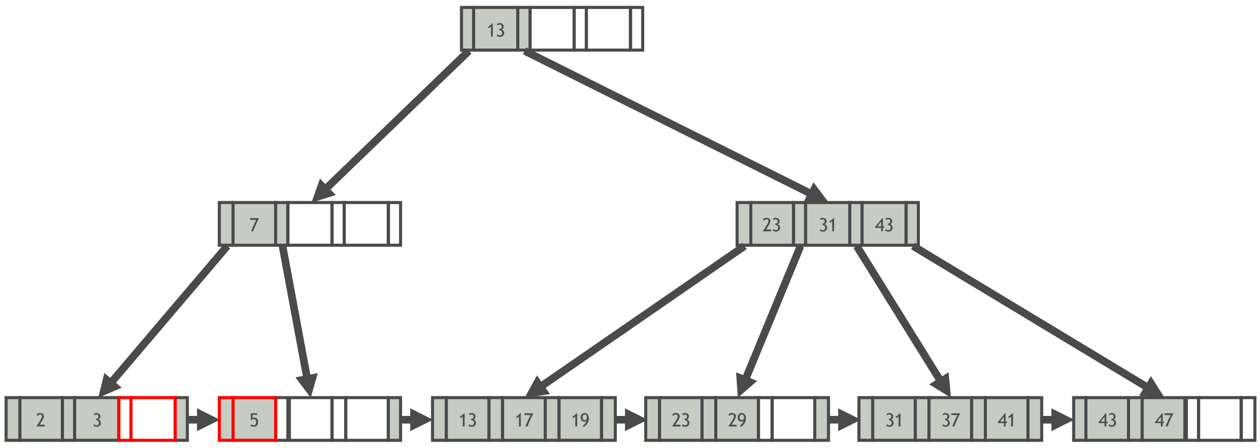
B+ Tree Delete: Example 2 (Key Redistribution)

Example: $K = 7$



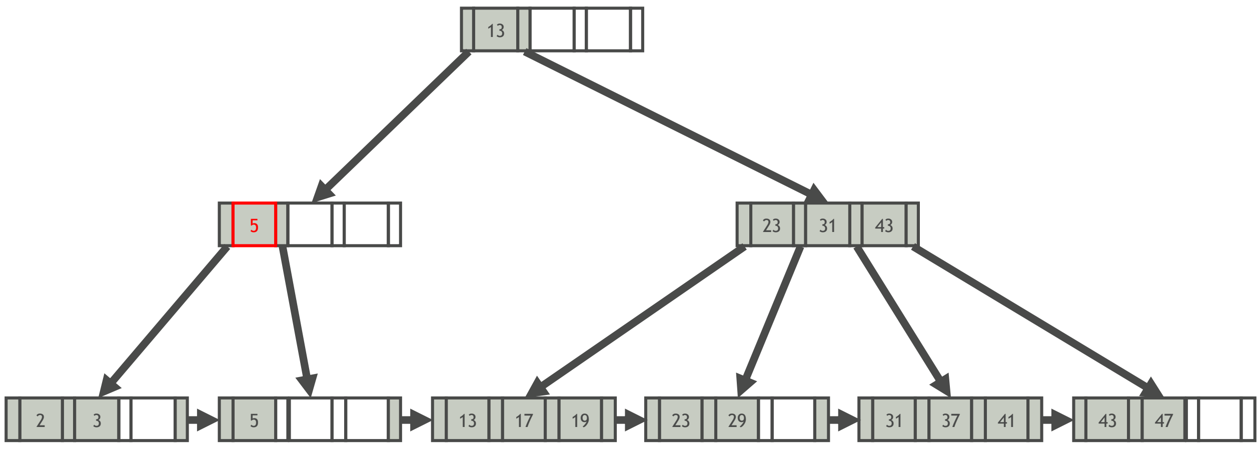
B+ Tree Delete: Example 2 (Key Redistribution)

Example: $K = 7$



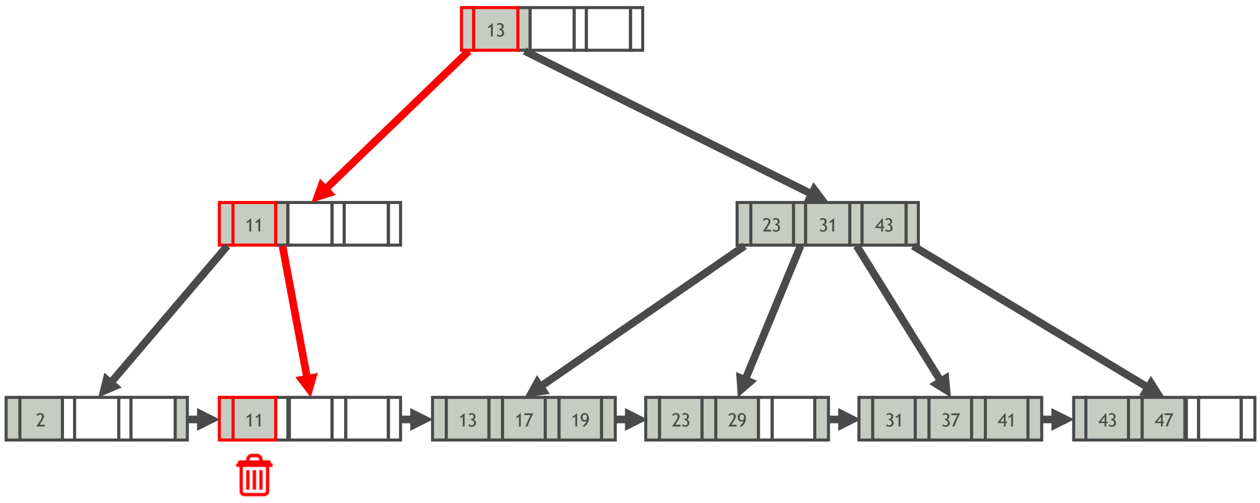
B+ Tree Delete: Example 2 (Key Redistribution)

Example: $K = 7$



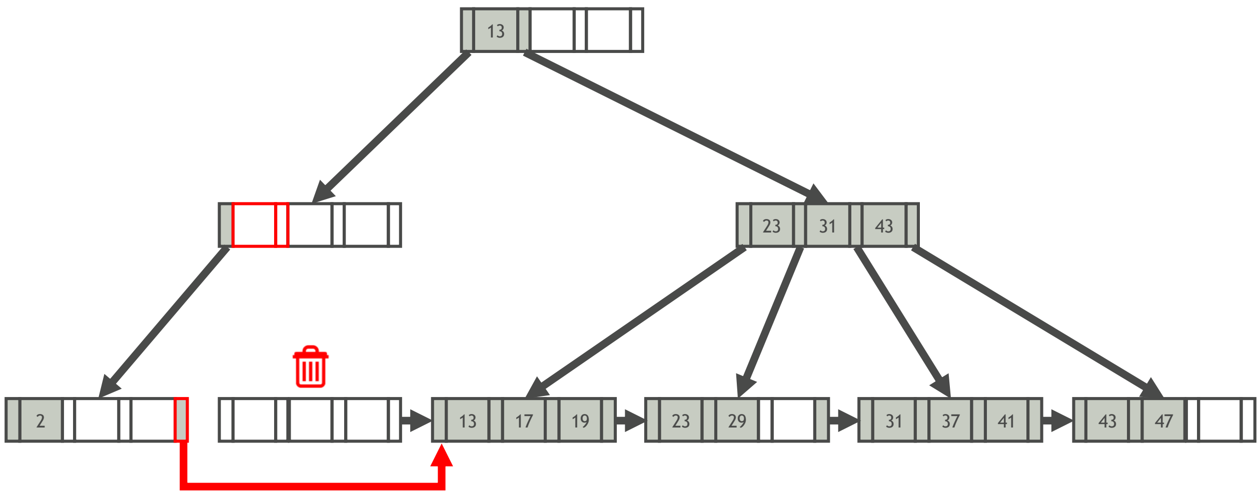
B+ Tree Delete: Example 3 (w/ Node Merge)

Example: $K = 11$



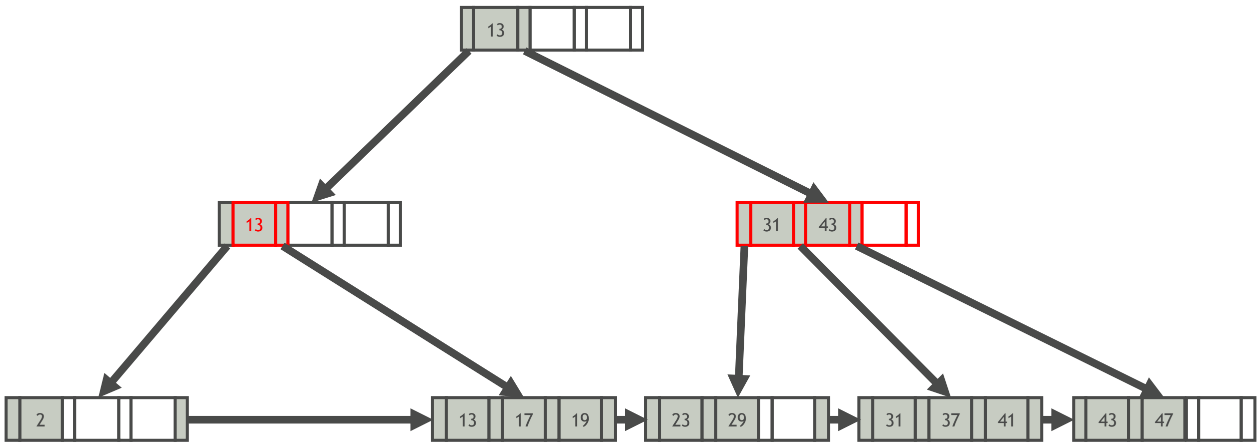
B+ Tree Delete: Example 3 (w/ Node Merge)

Example: $K = 11$



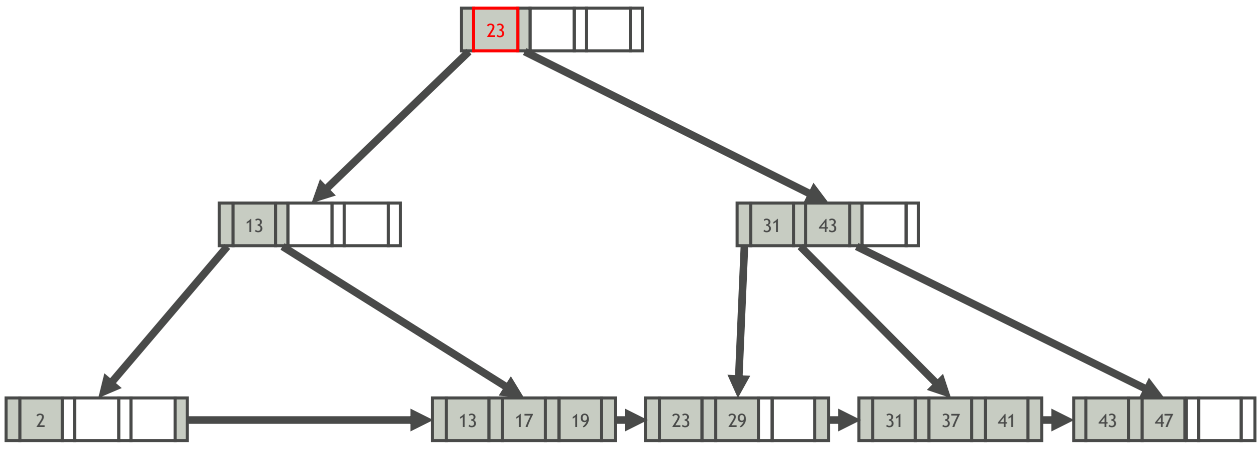
B+ Tree Delete: Example 3 (w/ Node Merge)

Example: $K = 11$



B+ Tree Delete: Example 3 (w/ Node Merge)

Example: $K = 11$



Key Compression

- The number of disk I/Os to retrieve a data entry in a B+ tree = the height of the tree $\approx \log_{fan\_out}(\#\text{ of data entries})$
- The fan-out (扇出) of the tree is the number of index entries fit on a page, which is determined by the size of index entries
- The size of an index entry depends primarily on the size of the search key value
- Search key values are very long $\Longrightarrow$ the fan-out is low $\Longrightarrow$ the tree is high $\Longrightarrow$ the query time is long

Prefix Compression (前缀压缩)

- Sorted keys in the same leaf node are likely to have the same prefix
- Instead of storing the entire key each time, extract common prefix and store only unique suffix for each key

Microphone	Microsoft	Microwave

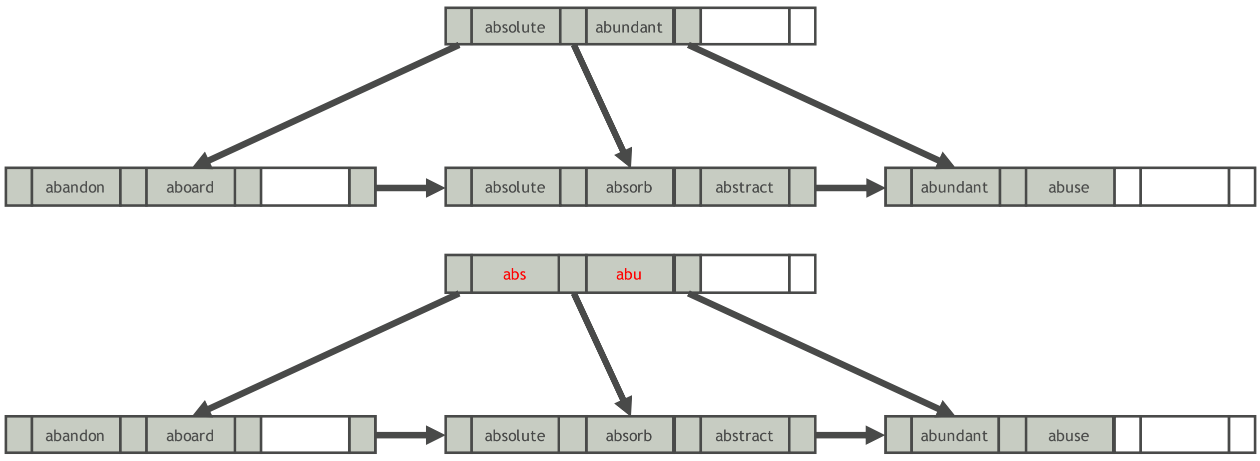
$\downarrow$ Prefix compression

Prefix: Micro

phone	soft	wave

Suffix Truncation (后缀截断)

- The keys in the inner nodes are only used to direct traffic
- We need not store the keys in their entirety in inner nodes
- Store a minimum prefix that is needed to correctly route probes



Bulk Loading (批量加载)

Creating a B+ tree on an existing set of index entries

Top-Down Approach

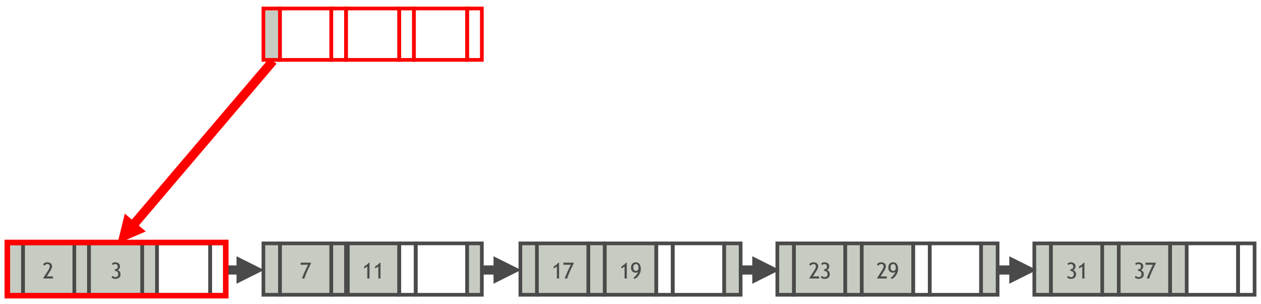
- Insert the index entries one at a time
- Expensive, because each entry requires to start from the root and go down to the appropriate leaf node

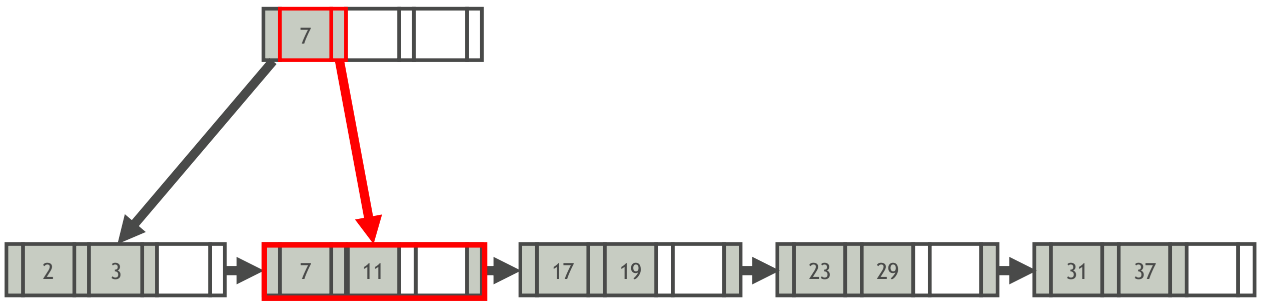
Bottom-Up Approach

1. Sort the index entries according to the search key
2. Allocate an empty inner node as the root and insert a pointer to the first page of sorted entries into it
3. Entries for the leaf pages are always inserted into the right-most inner node just above the leaf level. When that page fills up, it is split

Bulk Loading: Example

Sorted keys: 2, 3, 7, 11, 17, 19, 23, 29, 31, 37





Bulk Loading: Example

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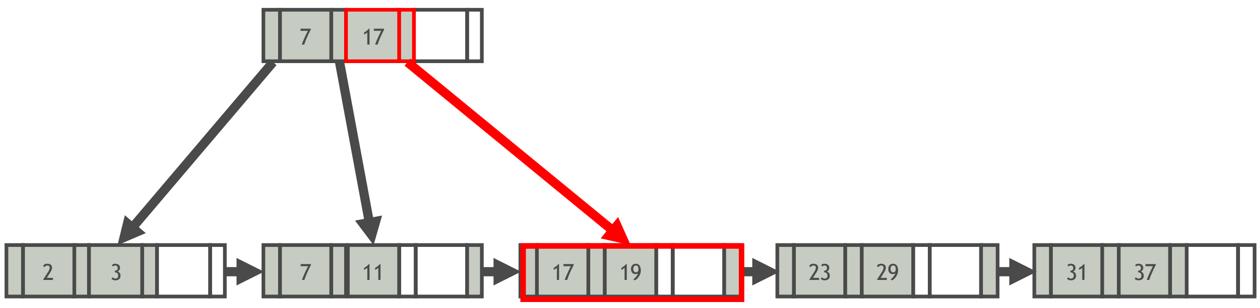

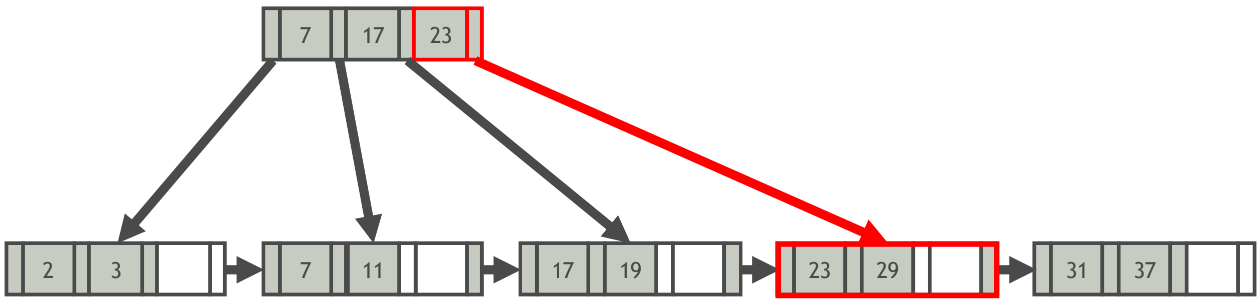

Bulk Loading: Example

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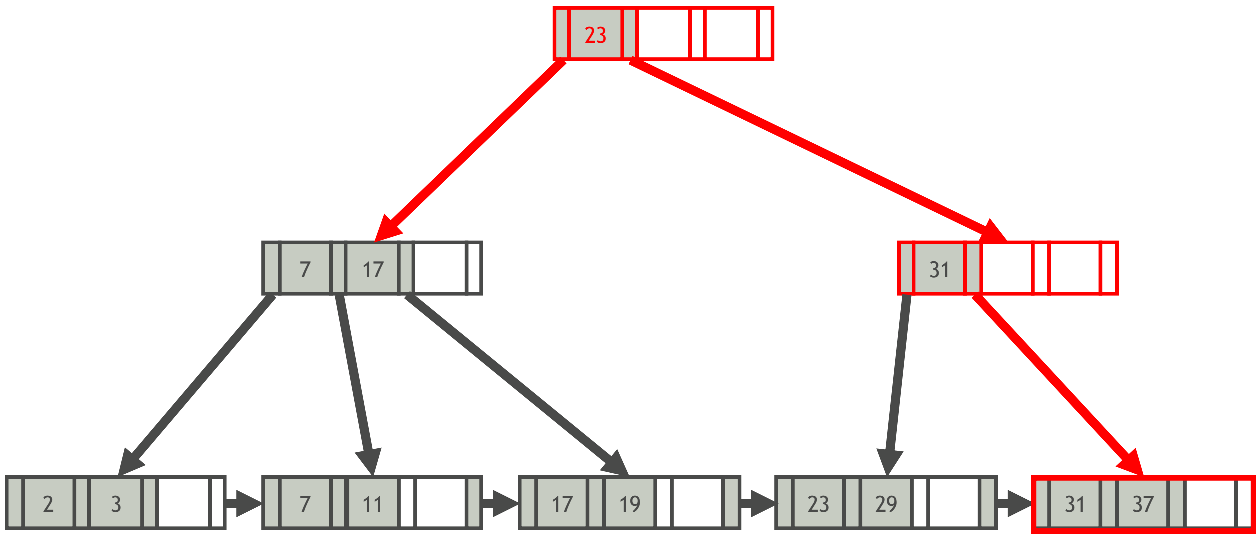

Summary

1. Hash-based Index Structures
   - Extensible Hash Tables
   - Linear Hash Tables
2. Tree-based Index Structures
   - B+ Trees

Q&A

1. 当B+树进行删除操作时，若一个节点不足半满，是优先向左兄弟借，还是优先向右兄弟借呢?
   答: 都可以，取决于B+树的具体实现方法。