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A critical plane approach for life prediction of high cycle fatigue under multiaxial variable amplitude loading(Morel paper 2000)

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Abstract

Morel's method is based on a mesoscopic approach to critical plane type with the choice of plastic deformation as mesoscopic cumulative damage variable. Multiaxial and variable amplitude loading can be analyzed with this method.

Keywords: Fatigue; Multi-axial; High cycle; Damage, Plastic deformation

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Nomenclature

Macroscopic quantities

$\underline{\underline{\Sigma}}$	macroscopic stress tensor
$\underline{\underline{E}}$	macroscopic strain tensor
$\underline{C}$	macroscopic shear stress vector
$\underline{T}$	macroscopic resolved shear stress vector acting on an easy glide direction
T_a	amplitude of the macroscopic resolved shear stress
P	macroscopic hydrostatic stress

Mesoscopic quantities

$\underline{\underline{\sigma}}$	mesoscopic stress tensor
$\underline{\underline{\varepsilon}}$	mesoscopic strain tensor
$\underline{\tau}$	mesoscopic resolved shear stress vector acting on an easy glide direction
γ^p	mesoscopic shear plastic strain
Γ	accumulated plastic mesostrain
T_σ	measure proportional to an upper bound of the plastic mesostrain accumulated on an elementary material pla
T_Σ	maximum value of T_σ
H	phase-difference coefficient

1 Constant amplitude loading

1.1. Local stress estimation in high cycle fatigue

By assuming that only one glide system (defined by a normal vector $\underline{n}$ to a plane and a vector (direction) $\underline{M}$ within this plane) is active for every plastically deforming grain of the metal, Papadopoulos [9] established a macro–meso passage for a glide system activated in a flowing crystal:

$$\underline{\tau} = \underline{T} - \mu\gamma^p \underline{m} \tag{1}$$

where $\underline{\tau}$ and $\underline{T}$ are the mesoscopic and macroscopic resolved shear stresses acting along the slip direction $\underline{m}$ and defined by:

$$\underline{\tau} = (\underline{m} \cdot \underline{\underline{\sigma}} \cdot \underline{n})\underline{m} \tag{2}$$

$$\underline{T} = (\underline{m} \cdot \underline{\underline{\Sigma}} \cdot \underline{n})\underline{m} \tag{3}$$

γ^p is the plastic mesoscopic shear strain.

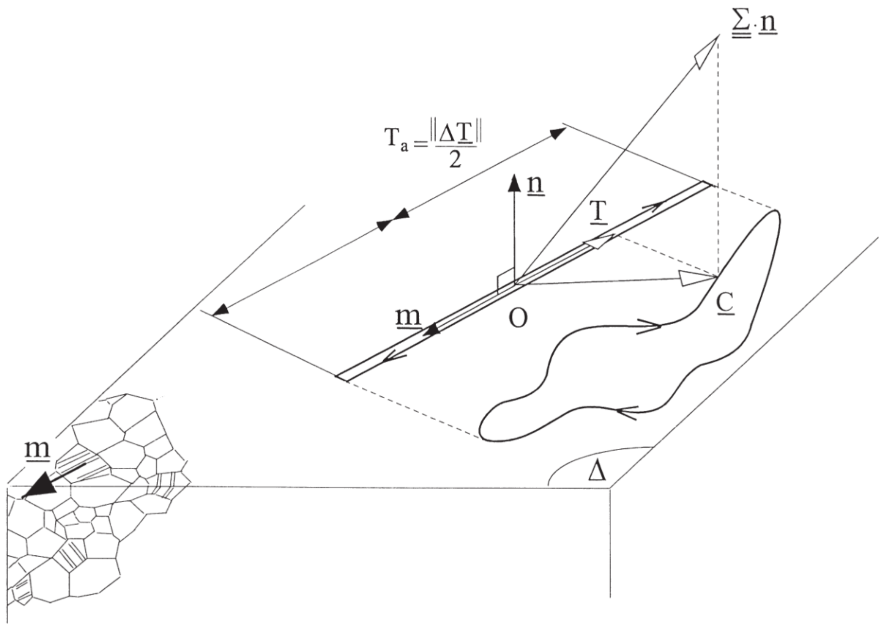

Fig. 1: Path of the macroscopic shear stress $\underline{C}$ acting on a material plane Δ and the corresponding path of the macroscopic resolved shear stress $\underline{T}$ acting on an easy glide direction.

1.2. Initiation of slip in the crystal

The plasticity criterion is determined by Schmid's law with isotropic and kinematic hardening:

$$f(\underline{\tau}, \underline{b}, \tau_y) = (\underline{\tau} - \underline{b}) \cdot (\underline{\tau} - \underline{b}) - \tau_y^2 = 0 \tag{4}$$

where $\underline{b}$ is the kinematical hardening parameter back stress.

In the description of his method, the author draws heavily on the work developed by Papadopoulos including the use of a measure of plastic deformation mesoscopic Cumulative T_σ associated with a particular material plane and modeling the behavior of grain in three distinct phases (hardening, saturation and softening); he considers the cumulative mesoscopic plastic deformation Γ as damage parameter and assumes that the initiation of a fatigue crack occurs when the latter reaches a critical value $D = D_R = \Gamma_R$ (Fig. 2).

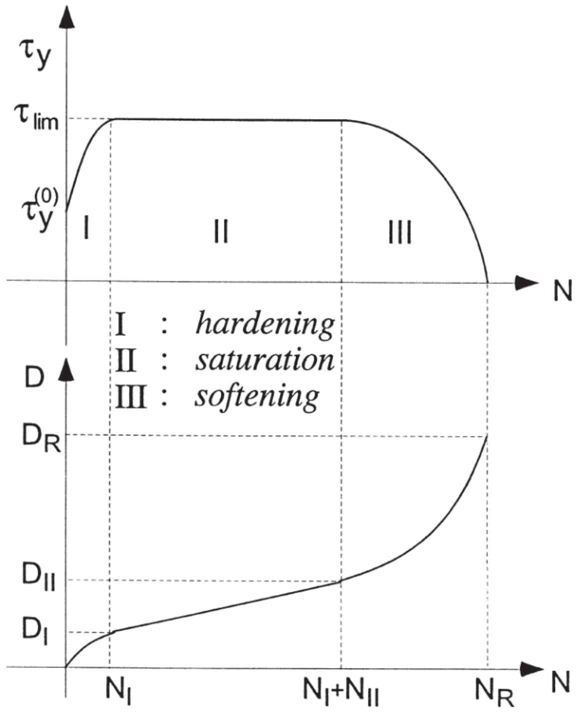

Fig. 2: Yield limit and damage evolutions in the three behavior phases (hardening, saturation and softening) when a cyclic loading is applied.

1.3. Multiaxial endurance criterion

To describe mesoscopic plasticity, Papadopoulos criterion is adopted:

$$\max_{\theta,\varphi}(T_\sigma(\theta,\varphi)) + \alpha P_{max} \leqslant \beta \tag{5}$$

T s is a function of the orientation of a material plane Δ through the angles θ and φ, spherical co-ordinates of the unit normal $\underline{n}$ to the plane Δ. $T_\sigma(\theta,\varphi)$ is estimated by an integration carried out through the whole area of the plane Δ:

$$T_\sigma(\theta,\varphi) = \sqrt{\int_{x=0}^{2\pi} T_a^2(\varphi,\theta,\psi)d\psi} \tag{6}$$

T_a is the amplitude of the macroscopic resolved shear stress acting on a line of the plane Δ directed by $\underline{m}$. This line is determined by the angle ψ from an arbitrary but fixed axis in Δ.

The material parameters α and β can be related to the fatigue limits of two standard fatigue tests, for example fully reversed tension-compression, s, and fully reversed torsion,t:

$$\alpha = \sqrt{\pi}\frac{t - s/2}{s/3}, \beta = \sqrt{\pi}t \tag{7}$$

Hereafter, the maximum value of T_σ will be denoted as T_Σ:

$$T_\Sigma = \max_{\theta,\varphi}(T_\sigma(\theta,\varphi)) \tag{8}$$

1.4. Limit loading estimation

The estimation of the yield limit in the saturation phase τ_s (defining the cyclic behavior of the crystal) is carried out by the definition of a limit loading.

$$T_{\Sigma lim} + \alpha P_{maxlim} = \beta \tag{9}$$

$$T_{\Sigma lim} = \frac{-\alpha P_m + \beta}{\alpha + \frac{T_\Sigma}{P_a}} \cdot \frac{T_\Sigma}{P_a} \tag{10}$$

Now we need to establish τ_{lim}. The corresponding actual amplitude of the shear stress, defined as half of the longest chord of the closed curve, is denoted as C_A.

$$H = \frac{T_\Sigma}{C_A} = \frac{T_{\Sigma lim}}{\tau_{lim}} \Rightarrow \tau_{lim} = \frac{T_{\Sigma lim}}{H} \tag{11}$$

where H constitutes a 'phase-difference coefficient'. It is worth mentioning that the more open the elliptic path, the higher the coefficient H.

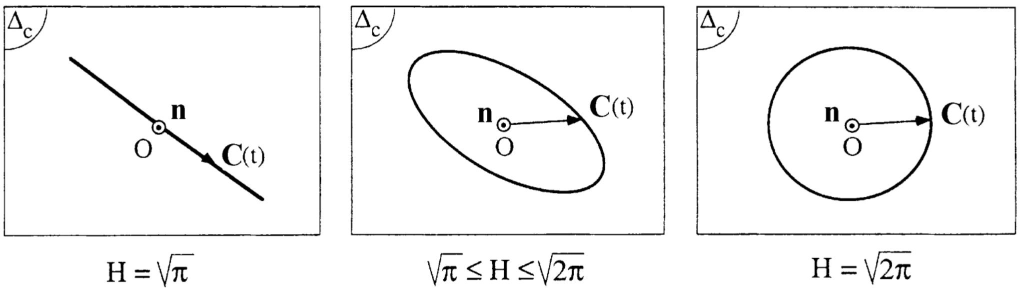

Fig. 3: Different paths and corresponding values of the phase-difference coefficient H.

For a proportional loading, H is equal to $\sqrt{\pi}$. In the case of a particular circular path, H reaches the maximum value $\sqrt{2\pi}$(Fig. 3).

1.5. Number of cycles to failure

Once the accumulated plastic mesostrain Γ along the particular gliding system reaches a critical value Γ_R, these grains are said to be broken. An analytical expression of the number of cycles to initiation (S–N curve) can be achieved:

$$\Gamma = \Gamma_R \Rightarrow N_F = pln\left(\frac{C_A}{C_A - \tau_{lim}}\right) + q\left(\frac{\tau_{lim}}{C_A - \tau_{lim}}\right) - \frac{r}{C_A} \tag{12}$$

where p, q and r are functions of the hardening parameters of the three phases defined above.

From Eq.(10) and Eq.(11) we can find the yield point τ_s of the crystal in the saturation phase as a function of the amplitude P_a and the mean value P_m of the hydrostatic pressure, the phase difference of coefficient H and two material related parameters α and β :

$$\tau_{lim} = \tau_s = \frac{-\alpha P_m + \beta}{\alpha \frac{P_a}{C_A} + H} \tag{13}$$

In the last relation Eq.(12), the detrimental effect of out-of-phase loading is introduced through τ_{lim}. As the coefficient H increases, τ_{lim} decreases, therefore N_F decreases which leads to more accumulated damage. The identification of the model parameters requires two endurance limits and a single S–N curve (parameters p, q and r).

2 Variable amplitude loads

To determine the life time in the case of multiaxial variable amplitude loads, the author performs the following steps:

1. Identification of the critical plane Δ_c by maximizing the standard deviation of the macroscopic resolved shear stress in different directions. The author introduces a new parameter $T_{\sigma rms}$:

$$T_{\sigma rms}(\theta,\varphi) = \sqrt{\int_{\psi=0}^{2\pi} T_{rms}^2(\theta,\varphi,\psi)d\psi} \tag{14}$$

where $T_{rms}(\theta,\varphi,\psi)$ is the standard deviation of the macroscopic resolved shear stress.

2. Regarding the estimation of the phase difference coefficient H, it seems reasonable when the phase shift is not explicitly defined (or cannot be estimated) to use the most conservative value $\sqrt{2\pi}$.

3. On each direction (m) of the critical plane Δ_c:

(a) determination of the macroscopic changes in hydrostatic pressure $P(t)$ and the macroscopic resolved shear stress $T_a(t)$;

(b) evaluation of the saturation point by averaging the mesoscopic resolved shear stress thresholds calculated over the whole sequence, $\tau_s = (\tau_{lim})_{mean}$, τ_{lim} is calculated with Eq.(13);

(c) estimation of the cumulative mesoscopic plastic strain Γ by following the evolution of the mesoscopic yield stress σ_y according to the three phases (hardening, saturation and softening);

(d) calculating the number of sequences (associated with m direction) necessary to achieve Γ_R;

4. For complex multiaxial loadings, the author assumes that the hardening and softening phases are small compared to the saturation phase[1]. In such a case, the yield limit remains constant for the whole lifetime and damage accumulation is purely linear. The analytical expression of the S–N curve(Eq.(12)) becomes:

$$\Gamma = \Gamma_R \;\Rightarrow\; N_F = q\left(\frac{\tau_{lim}}{C_A - \tau_{lim}}\right) \tag{15}$$

where

$$q = \frac{c+\mu}{4}\frac{1}{l}$$

The final linear summation of individual damage from different critical planes would then lead to damage assessment:

$$\sum_i \frac{\Gamma^{(i)}}{\Gamma_R^{(i)}} = 1 \tag{16}$$

3 Experimental verification

3.1. In case of constant amplitude test

The author [2] consider the example of an out-of-phase bending–torsion test on a high strength steel (30NCD16). The endurance limits of this material in reversed bending and torsion are, respectively, $f = 680MPa$ and $t = 426MPa$. The multiaxial sinusoidal loading is characterized by the amplitudes $\Sigma_{11a} = 600MPa$, $\Sigma_{12a} = 335MPa$ (no mean stresses) and the phase difference $\beta_{12} = 90°$.

The maximum value of T_σ (denoted as T_Σ) can be deduced numerically. For this loading, we find $T_\Sigma = 697MPa$. On the critical material plane (where T_Σ is reached), C_A is estimated to be $282MPa$. The phase difference coefficient H is then simply deduced: $H = T_\Sigma/C_A = 2.47$.

Besides noting that $P_m = 0MPa$, $P_a = 200MPa$ and $a = 0.67$, $b = 775MPa$, $T_{\Sigma lim}$ is readily computed with the help Eq.(13): $T_{\Sigma lim} = 633MPa$. Finally, $\tau_{lim} = T_{\Sigma lim}/H = 256MPa$. Once p, q and r have been identified from a S–N curve with the least squares line method, $C_A$and τ_{lim} can be introduced into Eq. (12) and the number of cycles to initiation can be finally calculated, i.e. $N_F = 2\times10^5$ cycles.

3.2. In case of variable amplitude test

According to the previous endurance data and the definition of the generalized fatigue limit (for bending $\tau_{lim} = f/2$ and for torsion $\tau_{lim} = t$), one can estimate the parameter $q = 20800$.

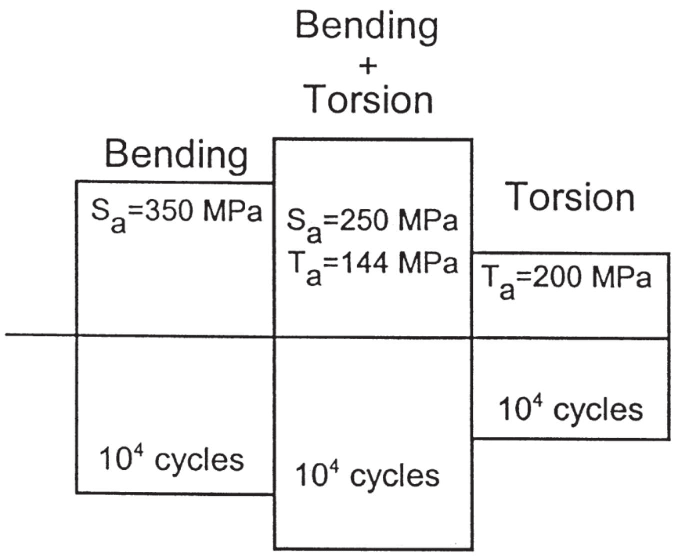

Fig. 4: block sequence tests (bending/bending+torsion/torsion) performed on a mild steel XC18.

Let us consider now a block sequence composed of 10^4 cycles of bending ($\Sigma_a = 350MPa$) followed by 10^4 cycles of combined in-phase bending–torsion ($\Sigma_a, T_a = 250MPa, 144MPa$) followed by 10^4 cycles of torsion ($T_a = 200MPa$). This sequence is repeated until the initiation of a crack. The mean lifetime is found to be $N = 1.73\times10^5$.

The three generalized fatigue limits relative to the three blocks are estimated according to Eq.(13):

$$\tau_{lim}^{bending} = 155MPa$$

$$\tau_{lim}^{torsion} = 179MPa$$

$$\tau_{lim}^{bend+tors} = 157MPa$$

These three values and the parameter q are enough to accumulate the damage in the three blocks using Eq.(15):

$$\frac{\Gamma^{(bending)}}{\Gamma_R^{(bending)}} + \frac{\Gamma^{(bend+tors)}}{\Gamma_R^{(bend+tors)}} + \frac{\Gamma^{(torsion)}}{\Gamma_R^{(torsion)}} = 1$$

The corresponding number of cycles to initiation is: $N_{prediction} < 1.5\times10^5$, that is to say five successive applications of the sequence. This prediction, close to the experimental result $N = 1.73\times10^5$, is a conservative one.

It is important to note that if only one critical plane (either from bending, torsion or bending+torsion loading) is used for damage accumulation, one-third of the damage would be calculated, resulting in a nonconservative prediction.

4 Discussion

Morel's method is promising in its description aspect of limited endurance fatigue phenomenon, through the choice of the mesoscopic plastic deformation. By using cumulative plasticity, a fatigue mechanisms occurring at the mesoscopic scale takes into account the main factors affecting the lifetime cycle fatigue (hydrostatic pressure and influence of phase shift).

However, at the present stage, it does not completely meet the demand of a predictive tool. Indeed, it is a relatively complicated method (search critical plane Δ_c and accumulated damage in each direction in the plan); its application for multiaxial variable amplitude fatigue loads application data that are still not available (an S-N curve, two endurance limits and a particular damage accumulation test). In addition, it is limited to soft materials. On the other hand, it is not completely free of counting method because its author uses the counting of the extrema of the evolution to get the macroscopic resolved shear stress $T(t)$ and the corresponding amplitude P_a and mean values P_m of the hydrostatic stress in each direction (m) in Δ_c. With one purpose of calculating the limit of mesoscopic elasticity $\tau_l im$ in the saturation phase of the crystal. Again, this makes it difficult and daunting task.

References

[1] F. Morel. A critical plane approach for life prediction of high cycle fatigue under multiaxial variable amplitude loading. *International Journal of Fatigue*, 22(2):101 – 119, 2000.

[2] F Morel. A fatigue life prediction method based on a mesoscopic approach in constant amplitude multiaxial loading. *Fatigue and Fracture of Engineering Materials and Structures*, 21(3):241–256, 1998.