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Independence of the torsional fatigue limit upon a mean shear stress

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Abstract

The influence of a superimposed mean shear stress on the high-cycle fatigue endurance limit of metals is a question that has been discussed for a long time but general agreement between among researchers has not yet been reached. Though seemingly a question of minor importance, the hypothesis on such effect is a critical issue in the formulation of high-cycle multiaxial fatigue criteria. The available experimental data referring to fatigue tests on steels in which a static shear (torsional) stress is superimposed to cyclic torsion have been reviewed, showing that in most cases the effect of static shear stress is negligible as long as the maximum shear stress does not exceed the static shear yield strength. The limited amount of relevant experimental data found in the literature has led the present authors to perform torsion fatigue tests on a quenched and tempered steel, commonly employed in highly stressed mechanical components, with different levels of superimposed mean shear stress. The statistical analysis of the test results shows that no significant effect of the superimposed static shear stress may be observed.

Keywords: Fatigue limit; Staircase; Torsion; Mean stress effect; Steel; Multiaxial fatigue criteria

1. Introduction

The question on the influence of a mean shear stress upon the fatigue limit in torsion of steels has not yet been answered in a definitive way. However, in the high-cycle fatigue literature, the mean shear stress effect has frequently been neglected. In support of this opinion, the classical paper of Sines [1] is recurrently cited. In this paper Sines reached the conclusion that a superimposed mean static torsion has no effect on the fatigue limit of metals subjected to cyclic torsion.

This consideration was based on the data collected by Smith [2,3], who gathered independent test results on the fatigue limit in torsion of various metals, including steels, aluminium alloys and bronze.

Limiting this analysis to steels, the experimental data available to the scientific community are sometimes in disagreement with Sines conclusion, but it must be pointed out that they refer to tests performed in a time period ranging from the early work of Gough to some tests carried out in the 1970s, mainly by German researchers. This implies that both the test equipment and the steel quality, in terms of macro and microstructure characteristics, have changed during this time period, making comparison difficult.

This is the reason why the authors decided to carry out tests on a 39NiCrMo3 steel, produced with contemporary technologies, employing a modern servo-hydraulic axial-torsional testing machine, to contribute to clarify the problem of the influence of a static shear stress on the fatigue endurance in torsion, concentrating on fatigue lives exceeding one million cycles.

2. Review of relevant results from the literature

A review of the existing experimental data can only be performed on a small quantity of data, there being

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Nomenclature

τ_{aW}	shear stress amplitude at fatigue limit (with superimposed mean shear stress)
τ_m	mean shear stress
τ_W	fully reversed torsion fatigue limit
R_m	tensile strength
σ_Y	tensile yield strength
τ_Y	shear yield strength
σ_W	fully reversed uniaxial fatigue limit
R	loading ratio (R = τ_{min}/τ_{max})
$m_{\tau_{aW}}$	mean (median) value of the torsion fatigue limit
$s_{\tau_{aW}}$	standard deviation of the torsion fatigue limit
F	normal distribution function

very few published results, to the best of the authors' knowledge. They consist of a total of 24 experimental results, as reported in Table 1, where they are ordered according to publication date.

The first data set is extracted from the pioneering work of Gough [4] on multiaxial fatigue, published in final form in 1951, and it refers to the fatigue limit in torsion of solid specimens made of a S65A steel (chemical composition: C = 0.24%; Si = 0.20%; Mn = 0.57%; S = 0.004%; P = 0.015%; Ni = 3.06%; Cr = 1.29%; Mo = 0.54%; Va = 0.25%), with superimposed mean torsion. These results allowed Gough to conclude that the effect of a mean static shear stress on the fatigue limit in torsion is negligible.

The same conclusion can be drawn from the analysis of Smith's data [3], published in 1942, which form the most numerous set of experimental points reported so far in a single paper. They refers to two batches of a SAE 3140 steel (chemical composition: C = 0.37%; Mn = 0.75%; P = 0.017%; S = 0.030%; Si = 0.178%; Ni = 1.33%; Cr = 0.65%), one hot-rolled, the other heat treated, in the form of unnotched solid specimens. The results, in Smith's words, are thus summarised

> '[…] the magnitude of any endurance range of stress is constant and equal to the magnitude of the endur-

Table 1
Experimental data collected from the literature (all values expressed in MPa). Values between parentheses are evaluated according to von Mises yield theory ($\tau_Y = \sigma_Y/\sqrt{3}$)

Reference	Year	Material	runout	R_m	τ_Y	τ_Y	τ_{aW}	τ_m	τ_{max}	τ_{max}/τ_Y	τ_{aW}/τ_W
Smith	1942	SAE 3140	$> 10^7$	793	510	386	303	0	303	0.79	1.00
Smith	1942	SAE 3140	$> 10^7$	793	510	386	331	110	441	1.14	1.09
Smith	1942	SAE 3140	$> 10^7$	793	510	386	272	272	545	1.41	0.90
Smith	1942	SAE 3140	$> 10^7$	793	510	386	224	431	655	1.70	0.74
Smith	1942	SAE 3140	$> 10^7$	1117	1055	762	386	0	386	0.51	1.00
Smith	1942	SAE 3140	$> 10^7$	1117	1055	762	396	155	552	0.72	1.03
Smith	1942	SAE 3140	$> 10^7$	1117	1055	762	400	400	800	1.05	1.04
Smith	1942	SAE 3140	$> 10^7$	1117	1055	762	283	558	841	1.10	0.73
Gough-Clenshaw	1951	BSS S65A	$> 10^7$	1001	947	715	371	0	371	0.52	1.00
Gough-Clenshaw	1951	BSS S65A	$> 10^7$	1001	947	715	339	170	509	0.71	0.91
Gough-Clenshaw	1951	BSS S65A	$> 10^7$	1001	947	715	343	344	686	0.96	0.93
Chodorowsky	1956	NiCrMo steel	$> 10^7$	848	728	403	283	0	283	0.70	1.00
Chodorowsky	1956	NiCrMo steel	$> 10^7$	848	728	403	273	77	350	0.87	0.96
Chodorowsky	1956	NiCrMo steel	$> 10^7$	848	728	403	244	185	429	1.07	0.86
Chodorowsky	1956	NiCrMo steel	$> 10^7$	848	728	403	236	239	476	1.18	0.83
Issler	1973	St 35	$2\ 10^6$	392	294	(170)	120	0	120	0.70	1.00
Issler	1973	St 35	$2\ 10^6$	392	294	(170)	117	57	174	1.02	0.98
Issler	1973	St 35	$2\ 10^6$	392	294	(170)	103	112	215	1.27	0.86
Lempp	1977	42 CrMo 4 V	$2\ 10^6$	1025	888	(513)	265	0	265	0.52	1.00
Lempp	1977	42 CrMo 4 V	$2\ 10^6$	1025	888	(513)	225	225	450	0.88	0.85
Heidenreich et al.	1984	34 Cr 4	$1.5\ 10^6$	710	550	(318)	204	0	204	0.64	1.00
Heidenreich et al.	1984	34 Cr 4	$1.5\ 10^6$	710	550	(318)	175	175	350	1.10	0.86
JSMS databook	1996	JIS SCM440	$> 10^7$	940	812	(469)	387	0	387	0.82	1.00
JSMS databook	1996	JIS SCM440	$> 10^7$	940	812	(469)	333	333	666	1.42	0.86

ance range of stress for completely reversed cycles of stress, provided that the maximum stress in the range does not exceed the torsional static yield strength of the steel.'

In 1956 Chodorowsky [5] reported torsional fatigue test results on solid and hollow specimens made of a Ni-Cr-Mo steel (chemical composition: C = 0.29%; Si = 0.15%; Mn = 0.66%; S = 0.015%; P = 0.013%; Ni = 2.55%; Cr = 0.58%; Mo = 0.58%). The effect of a static shear stress on the torsional fatigue strength was investigated using hollow specimens and it was concluded that the fatigue strength in torsion is affected by a mean shear stress, with the fatigue limit decreasing linearly with the increase of the mean stress. Nevertheless, as Chodorowsky himself pointed out, in two out of the four test points the maximum shear stress exceeded the torsional static yield stress.

All the above-mentioned data sets were obtained before 1956, date of the last reference. Only in 1973, Issler [6] reported the results of tests conducted on both solid and hollow specimens, made of a St35 low carbon steel (chemical composition: C = 0.125–0.13%; Si = 0.15–0.20%; Mn = 0.4%; P = 0.012–0.015%; S = 0.021–0.024%), subjected to multiaxial stresses. Part of Issler data refers to torsion tests with different mean shear stress values and are reported in Table 1. Being evident only for values of the applied maximum shear stress beyond the torsional yield limit, the effect of static shear stress on the fatigue strength is considered negligible by Issler, whose fatigue criterion reflects this consideration.

The same hydraulic multiaxial testing rig described by Issler was used to obtain the data reported by Lempp [7] in 1977. Hollow specimen made of a 42CrMo4 steel (chemical composition: C = 0.42%; Si = 0.22%; Mn = 0.8%; P = 0.014%; S = 0.029%; Cr = 1.13%; Mo = 0.22%) were subjected to different values of static shear stress and shear stress amplitudes in torsion. The results of Lempp clearly show a detrimental effect of the static shear on the fatigue strength, even for values of τ_{max} below the torsional yield limit.

Two further experimental data were published in 1984 by Heidenreich et al. [8]. These data were obtained with 34Cr4 steel hollow specimens, with a testing equipment similar to that of Issler and Lempp.

The last two experimental data points were extracted from the JSMS databook [9] and they refer to fully reversed torsion and repeated torsion tests on solid specimens made of a JIS SCM440 steel (chemical composition: C = 0.40%; Si = 0.20%; Mn = 0.78%; P = 0.018%; S = 0.018%; Cr = 0.92%; Mo = 0.16%). It is underlined that from the huge amount of fatigue data contained in the JSMS databook (more than 3000 fatigue curves), only two experimental data could be retained in this analysis, witnessing the rarity of recent fatigue test results satisfying the scope of the present authors' analysis. Perhaps, it would be of interest to notice that the experimental campaign, the results of which are presented in this paper, spanned over more than one year involving a large amount of man-months work. This could be a state of affairs that might discourage researchers from undertaking such experimental campaigns and eventually, at least partly, explains the rarity of this kind of test data in the fatigue literature.

The total set of experimental points derived from the cited references are reported in Table 1 and in Fig. 1, where the torsion fatigue limit is plotted versus the maximum shear stress. Actually, in Fig. 1 the ordinate is the shear stress amplitude normalised to the fatigue limit in reversed torsion and the abscissa is the maximum shear stress normalised to the torsional yield limit. The total number of experimental points (24) is relatively small, and among them 11, i.e. 46% of all the available points, refer to tests where τ_{max} exceeds the torsional yield limit τ_Y.

3. Effect of mean shear stress in multiaxial fatigue criteria

The theoretical interpretation of the mean shear stress effect on the fatigue strength in torsion has never been proposed in terms of simple formulae involving alternating and mean stresses, such as the linear relationship proposed by Goodman for normal stresses, with the only exception of Yokobori [10], who derived from his criterion the following equation:

$$\frac{\tau_{aW}}{\tau_W} = 1-n\left(\frac{\tau_m}{\tau_Y}\right) \tag{1}$$

where the parameter n depends on the microstructure and the dimensions of defects or inclusions.Though not proposed explicitly, similar relations can be derived from

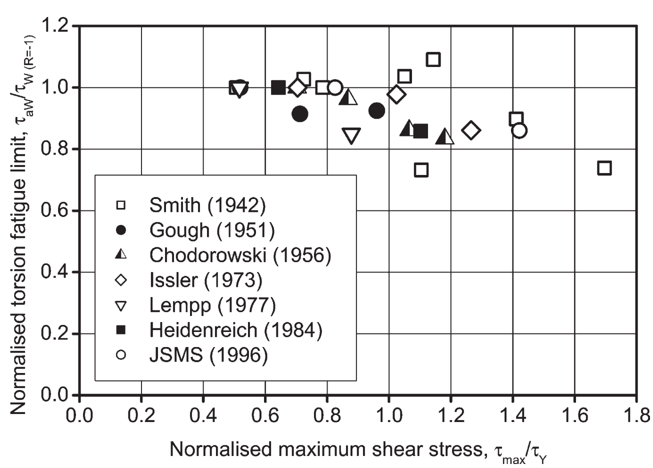

Fig. 1. Data collected form the literature on the variation of the fatigue limit in torsion, with superimposed static shear stress.

the expressions of a number of other fatigue criteria, where the effect of a static shear stress on the fatigue strength in torsion is included in the definition of the equivalent stress.

Before examining these criteria in detail, let us cite some of the most commonly employed criteria, which exclude any effect of τ_m on τ_{aW}. A more detailed analysis and comparison of multiaxial fatigue criteria may be found in Papadopoulos et al. [11]. The first two are Sines [1] and Crossland [12] criteria, which are both expressing the influence of a mean stress state through the hydrostatic stress. Since the hydrostatic stress depends on normal stress components only, any influence of static shear stress is excluded; similarly, the equivalent alternating stress is calculated on the alternating part of the time varying stress tensor components, such as the mean shear stress does not appear in the expression of these criteria.

Another criterion whose formulation includes hydrostatic stress as a limiting factor for the permissible shear stress amplitude is the Dang Van criterion [13], which also requires taking into account the alternating part of the shear stress only. The Dang Van criterion can be included into the group of fatigue criteria based on the critical plane approach, like the criterion proposed by Matake [14], who defined the critical stress quantity as the maximum shear stress vector amplitude, which is a scalar quantity depending on the elementary plane orientation. On the plane experiencing this maximum shear stress amplitude, one finally has to combine this value with the maximum normal stress acting on that plane. Again the critical quantity value is by definition not influenced by the mean shear stress since the only limiting quantity, the normal stress, is absent on the critical plane in pure torsion tests.

Other criteria define critical quantities in such a way that the effect of a static shear stress on the fatigue strength in torsion is explicitly included. The equivalent shear stress intensity theory proposed by Liu [15] belongs to this class, resulting in the following expression for the fatigue strength in case of pure torsion with superimposed static shear stress:

$$\tau_{\mathrm{aW}} = \frac{\sigma_{\mathrm{W}}}{\sqrt{(3a + 2b) + \frac{12}{7} a \cdot m \cdot \tau_{\mathrm{m}}^2}} \tag{2}$$

where a, b and m are material parameters. On the contrary, the criterion proposed by Papadopoulos [16], belonging to the category of integral approaches, is formulated on the hypothesis that τ_m has no effect on τ_{aW}. Another criterion predicting an influence of the static shear stress on τ_{aW} is the one proposed by Nishihara and Kawamoto [17], which in case of torsion can be written:

$$\frac{\tau_{\mathrm{aW}}^2}{\tau_{\mathrm{W}}^2} + \frac{\tau_{\mathrm{m}}^2}{(\mathrm{R_m}/\sqrt{3})} = 1 \tag{3}$$

Additionally, two other criteria based on the critical plane approach may be referred to, where a combination of the amplitude of the shear stress on the plane of maximum alternating shear stress and the normal stress on that plane are chosen as the limit condition with respect to fatigue. In this way, the resulting expression of the criteria applied to non symmetric cyclic torsion includes the effect of static shear stress on the fatigue limit in torsion.

The first is the Findley's criterion [18], which, for the special case of pure torsion, can be rewritten:

$$\tau_{\mathrm{aW}} = \sqrt{k^2\tau_{\mathrm{m}}^2/(1 + k^2)^2 - (k^2\tau_{\mathrm{m}} - \lambda^2)} - k\tau_{\mathrm{m}}/(1 + k^2) \tag{4}$$

where λ and k are material constants. Findley compared this prediction with Smith data and concluded that if k is sufficiently small, the fatigue strength in shear can be considered almost independent from a static shear component.

The second criterion is the one proposed by Robert [19], who combined the shear stress amplitude with both mean value and amplitude of the normal stress. When applied to torsion, the resulting expression is:

$$\tau_{\mathrm{aW}} = \frac{\tau_{\mathrm{W}}\sqrt{\alpha^2 + 1} - \beta\tau_{\mathrm{m}}\sin 2\varphi}{\cos 2\varphi + \alpha \sin 2\varphi} \tag{5}$$

where α, β are material constants, $2\varphi = \arctan(\alpha + \beta\tau_{\mathrm{m}}/\tau_{\mathrm{a}})$ and τ_a is the alternating applied shear stress. Eq. (5) clearly shows a detrimental influence of a static shear stress on the fatigue strength in torsion.

From this concise analysis of the criteria, it may be observed that the question of the influence of mean shear stress on the fatigue limit in torsion remains at least controversial. In fact, when five of the cited criteria are applied to the case of non symmetric cyclic torsion, it is predicted that the fatigue limit is independent from the superimposed mean shear stress while, in the remaining criteria, a detrimental effect of τ_m is considered.

4. Testing

4.1. Material

The material used for the fatigue experimental campaign is a steel of grade 39NiCrMo3, supplied in quenched and tempered bars of 35 mm diameter. The chemical composition of the material is shown in Table 2. This material has been chosen because it is commonly employed for the production of highly fatigue stressed mechanical parts. Macro and micro hardness measurements have been performed in order to verify the uniformity of the heat treatment. A constant macro-hardness of HRC 26 has been obtained, while a micro-hardness

Table 2
Chemical composition of the 39NiCrMo3 steel

Element	*C*	*Si*	*Mn*	*P*	*S*	*Cr*	*Mo*	*Ni*	*Al*
[%]	0.388	0.255	0.682	0.007	0.004	0.761	0.182	0.794	0.026
	Co	*Cu*	*Nb*	*Ti*	*V*	*W*	*Pb*	*Sn*	*Ca*
	0.007	0.194	<0.001	0.003	0.004	0.008	<0.001	0.012	0.002

Table 3
Monotonic properties of the 39NiCrMo3 steel

Young modulus	E = 206,000 MPa
Tensile strength	R_m= 856 MPa
Tensile yield strength	$R_{p0.2}$= 625 MPa
Elongation at failure	A% = 18.5%
Monotonic strength coefficient	K = 1160 MPa
Monotonic strength exponent	n = 0.0989

of HV 279 has been measured along a diameter from the surface to the core, ensuring that a uniform heat treatment has been applied. Monotonic tests with standard specimens, extracted from the inner part of the bars, have been also carried out. Monotonic properties are summarized in Table 3.

4.2. Specimen

The specimens have been designed so as to fulfil the requirements of maximum torque and precision of the load cell of the testing system. Hourglass shape specimens have been chosen since this geometry gives the possibility to concentrate failures in a limited area of the specimen. Moreover, hourglass specimens provide sufficient stiffness for running tests at the desired loading frequency. Specimen shape and dimensions are shown in Fig. 2.

The specimens have been obtained by turning followed by conventional polishing with progressively finer emery papers. A final average roughness R_a = 0.69 μm has been measured. After polishing, surface residual stresses have been measured in a reduced set taken from the whole batch of the specimens by means of an X-ray diffractometer, revealing that non-uniform compressive residual stresses were present at the material surface. The presence of residual stresses is mainly due to the effect of the technological process adopted in the production of the specimens, and they are introduced into the surface layer of the material even during the final polishing with emery papers. In order to avoid the disturbance of surface residual stresses, that may affect the results of the fatigue tests, all the specimens have been submitted to an electrolytic polishing (EP) treatment, following the ASTM E 1558–99 standard [20]. Special care has been applied to the optimisation of the electrolytic polishing process, in order to avoid the formation of surface pits or non-uniform removal of material. By successive steps of gentle electro-polishing, a thin layer of about 0.04 mm, with an axial extension of the treated area of 15 mm in correspondence of the minimum diameter, has been removed. After electrolytic polishing, the measured residual stresses are negligible and an average surface roughness R_a = 0.64 μm has been achieved.

4.3. Testing equipment

All the tests have been performed by employing a MTS multiaxial tension/torsion servo-hydraulic testing system, with a capacity of 250 kN axial force and of 2500 Nm torque. The testing machine is equipped with a load train alignment device, in order to ensure proper clamping of the specimens and to avoid secondary bending effects. All the tests have been performed at a constant frequency of 30 Hz.

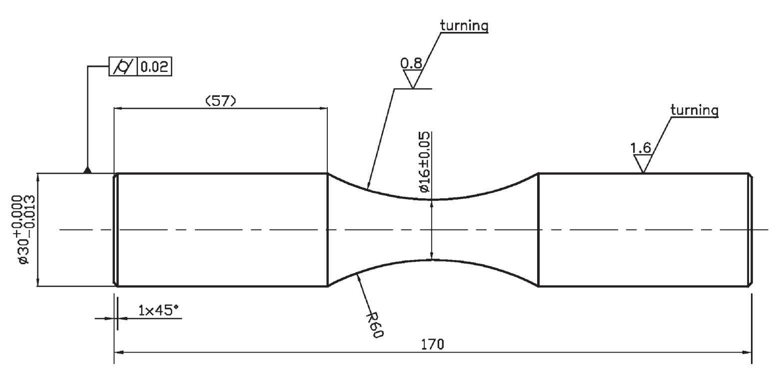

Fig. 2. Specimen shape and dimensions.

4.4. Test procedure

Thirty nine specimens have been tested. The specimens have been divided into three sets of 13 specimens each: the first 13 specimens have been tested in pure alternating shear stress while, for the other two sets, constant levels of superimposed mean shear stress have been applied. The values of the superimposed mean shear stress has been carefully chosen, so that in none of the experiments did the maximum shear stress exceed nor even reach the torsional yield stress of the material. During the fatigue tests, the axial force was kept under control and set equal to zero, in order to ensure that pure shear stress was applied to the specimen. For each value of the superimposed mean shear stress the fatigue limit was assessed by adopting the staircase method: a constant stress increment of 15 MPa have been used, resulting from the evaluation of uncertainties of the testing equipment and procedure. The run-out has been fixed at $5 \cdot 10^6$ cycles, while an increment of 10% in angular rotation has been adopted as the test termination criterion. All tests results are reported in Table 4, where they have been ordered according to the staircase sequence.

5. Test results

5.1. Fracture observations

For all the fatigue tests conducted applying a cyclic shear stress just above the fatigue limit, a uniform cracking behaviour in the failed specimens has been observed. The specimens do not reveal any presence of macroscopic crack until the very end of the test. Then, in a few cycles, a small crack is nucleated on one of the planes of maximum shear and it is able to propagate in Mode II up to a length of several hundred microns. Then crack branching occurs but only one of the cracks makes the specimens fail, propagating on one of the planes of maximum normal stress, as already observed by Marquis [21]. Typical crack patterns observed at the surface are shown in Fig. 3. In this figure, the longitudinal axis of the specimen is coincident with the vertical axis of the pictures. It may be noted that the nucleation of the first small crack occurs on one of the planes of maximum shear stress amplitude irrespective of the amount of the superimposed mean shear stress, while only a small difference in the orientation of the final fracture surface has been observed.

5.2. Statistical analysis and interpretation of results

The test results have been analysed according to the staircase method: the value of the fatigue limit and the corresponding standard deviation calculated for each value of the superimposed mean stress are shown in Table 5. However, it must be pointed out that the value of the standard deviation derived from the staircase formula does not satisfy the significance test formula (e.g., see Braam [22]) for any of the present experiment sets. In fact, the calculated standard deviation is lower than the difference between successive stress levels in the staircase procedure, fixed at 15 MPa. Moreover, the number of specimens employed for each of the three sets of experiments does not allow computing the standard deviation with the required accuracy. This causes difficulties from a statistical point of view in assessing the significance of the differences between the experimentally determined fatigue limits in torsion resulting from the three sets of experiments (τ_m = 0 MPa, τ_m = 45 MPa, τ_m = 90 MPa).

In order to circumvent these difficulties, which are known to arise when adopting the staircase method (see Little [23]), a more detailed statistical analysis of the experimental results has been performed. The Maximum

Table 4
Test results on the 39NiCrMo3 steel. N_f is the number of cycles at failure, corresponding to the 10% increment of angular rotation

τ_m =0 MPa		τ_m =45 MPa		τ_m = 90 MPa	
τ_a	N_f	τ_a	N_f	τ_a	N_f
285	697,000	255	$> 5\ 10^6$	255	2,290,000
270	2,410,000	270	2,516,000	240	$> 5\ 10^6$
255	$> 5\ 10^6$	255	$> 5\ 10^6$	255	$> 5\ 10^6$
270	1,076,000	270	1,397,000	270	655,000
255	$> 5\ 10^6$	255	3,536,000	255	2,526,000
270	1,720,000	240	$> 5\ 10^6$	240	$> 5\ 10^6$
255	$> 5\ 10^6$	255	$> 5\ 10^6$	255	2,948,000
270	1,263,000	270	724,000	240	$> 5\ 10^6$
255	$> 5\ 10^6$	255	$> 5\ 10^6$	255	$> 5\ 10^6$
270	$> 5\ 10^6$	270	3,554,000	270	1,059,000
285	289,000	255	$> 5\ 10^6$	255	3,850,000
270	1,584,000	270	2,051,000	240	$> 5\ 10^6$
255	$> 5\ 10^6$	255	$> 5\ 10^6$	255	3,209,000

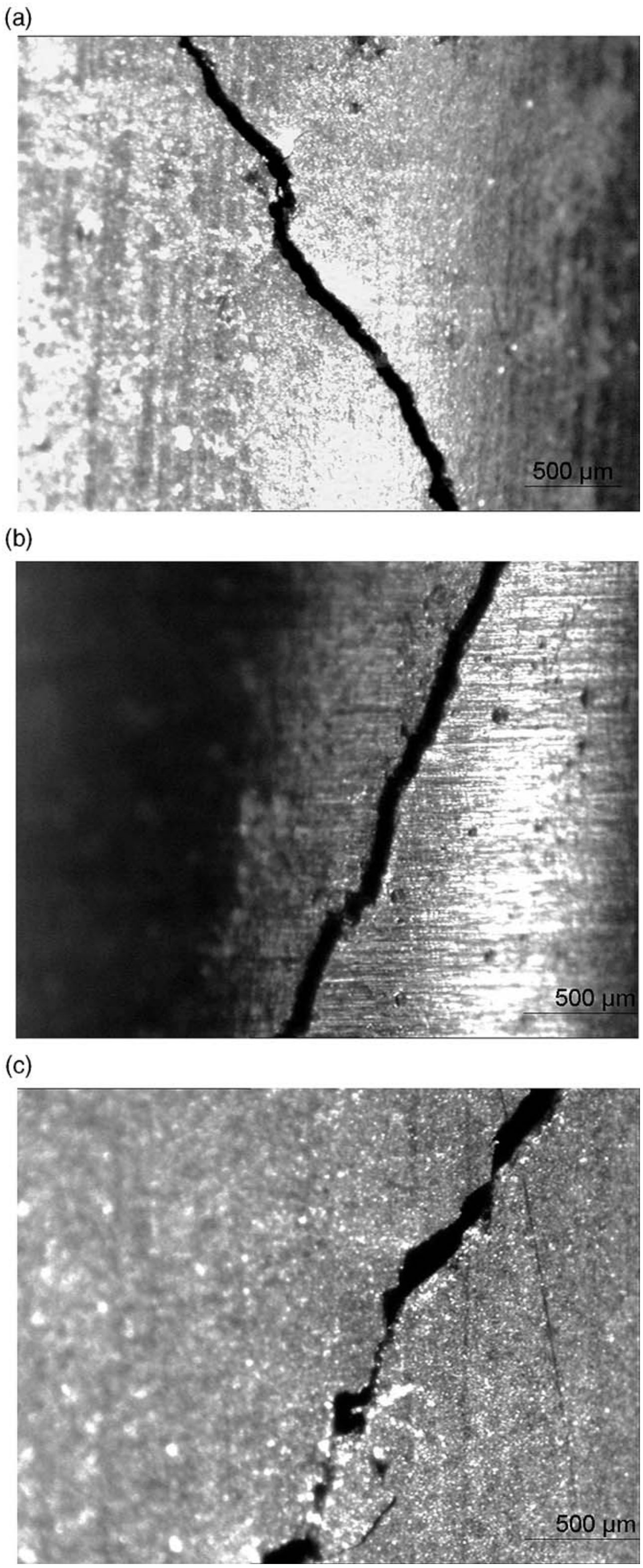

Fig. 3. Surface crack patterns in the failed specimens: a) τ_m= 0 MPa, b) τ_m= 45 MPa, c) τ_m= 90 MPa (the longitudinal axis corresponds to the vertical axis of the pictures).

Log-Likelihood estimation may be applied to the staircase test results. It is assumed that the fatigue limit values follow a normal distribution and the test results of the three sets of experiments are considered as a single set in which the mean value of the torsion fatigue limit ($m_{\tau_{aW}}$) is assumed to be linearly dependent on the applied mean shear stress, while the standard deviation ($s_{\tau_{aW}}$) is supposed to be constant. Details of this procedure are given in the Appendix.

By applying the Likelihood Ratio method (LR test), the dependence of the torsion fatigue limit upon the superimposed mean shear stress may be assessed. When a 95% confidence region is chosen and the standard deviation is estimated from the Maximum Likelihood, only a slight dependence of the torsion fatigue limit on the superimposed mean torsion is found. However, since it is well known that the standard deviation $s_{\tau_{aW}}$ is underestimated when a small number of specimen is employed [23], if the standard deviation is increased to about 5% of the fatigue limit [24], the LR test demonstrates the independence of the fatigue limit upon a superimposed mean torsion.

6. Discussion

The diagram of Fig. 1 is redrawn adding the values resulting from the experiments conducted on the 39NiCrMo3 steel for comparison purposes, Fig. 4. The data points refer to the fatigue limit obtained by the staircase formula, Table 5. From the diagram of Fig. 4 the slight influence of the superimposed static shear stress can be further appreciated. In comparison to the data collected from the literature, i.e. for the same ratio τ_{max}/τ_Y, the experiments presented in this paper show a smaller reduction of the fatigue limit in torsion, in particular for the data point with τ_{max} nearest to τ_Y.

The comparison between the three sets of experimental results made only by taking into account the median value of the fatigue limit (i.e. the value obtained from the staircase formula), shows a slight reduction of the fatigue limit due to the superimposed static shear stress. But the performed statistical analysis gives a more detailed answer: when the statistical distribution of the fatigue strength is also taken into account, the detrimental effect of the superimposed shear stress cannot be established.

If the mean stress sensitivity definition:

$$M = \frac{\tau_{aW(R=-1)} - \tau_{aW(R=0)}}{\tau_{aW(R=0)}} \tag{6}$$

is applied to the experimental results presented in this paper, in the case of τ_m = 45 MPa, a mean stress sensitivity M = 0.019 is obtained, while in the case of τ_m = 90 MPa, M = 0.050. If these figures are compared with the mean stress sensitivity diagram given by Zenner

Table 5
Staircase torsion fatigue limit and standard deviation

Torsion fatigue limit	τ_m =0 MPa	τ_m =45 MPa	τ_m = 90 MPa
Mean [MPa]	265.0	260.0	252.5
Standard deviation [MPa]	4.08	4.08	6.11

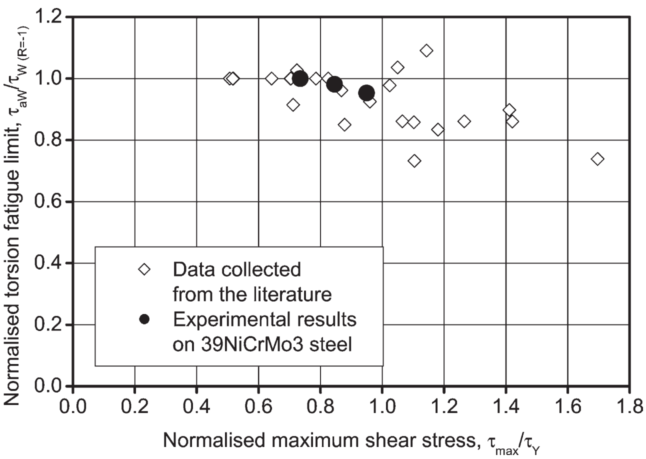

Fig. 4. Variation of the fatigue limit in torsion. Data from experiments on the 39NiCrMo3 steel are compared with those collected from the literature, see Fig. 1.

[25], for materials having an equivalent tensile strength R_m, it may be clearly observed that these two data points lie well below the torsion mean stress sensitivity line given in Ref. [25]. Had the present authors' fatigue experiments been conducted with a loading ratio $R = 0$, the value of the maximum applied shear stress would have exceeded the yield limit in shear and a pronounced effect of the superimposed shear stress might have been expected.

7. Conclusions

The data collected from the literature on the influence of the superimposed mean shear stress τ_m on the fatigue limit in torsion τ_{aW} for steels do not permit to give a definitive answer to this question.

The experimental results presented in this paper have given the possibility to establish that only a slight influence is present on a quenched and tempered steel (39NiCrMo3), produced with contemporary technologies and performed with modern testing equipment.

In the authors' opinion, from a design point of view, the independence of the torsional fatigue limit upon a mean shear stress may be stated. Therefore, the multiaxial fatigue criteria that are based on the hypothesis of the independence of the fatigue limit upon a superimposed mean shear stress, i.e. the range of criteria that are expressing the influence of mean stress state through the hydrostatic stress, receive an enhanced support.

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Appendix A

In order to evaluate from a statistical point of view the significance of the differences in fatigue limit due to the superimposed mean stresses, the Maximum Log-Likelihood Evaluation is applied to the staircase test results. First, one assumes that the fatigue limit is described by the normal distribution. The normal distribution of the fatigue limit is known once the mean value, $m_{\tau_{aW}}$, and the standard deviation $s_{\tau_{aW}}$ are given. The distribution of the fatigue limit is given by the vector ϑ:

$$\vartheta = \begin{bmatrix} m_{\tau_{aW}} \\ s_{\tau_{aW}} \end{bmatrix} = \begin{bmatrix} A + B(\tau_m - \bar{\tau}_m) \\ C \end{bmatrix} \tag{7}$$

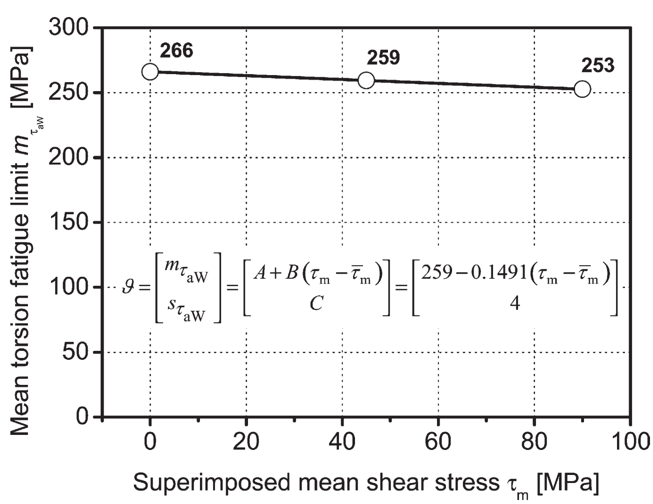

Fig. 5. Estimation of the fatigue limit and standard deviation from Maximum Likelihood Method.

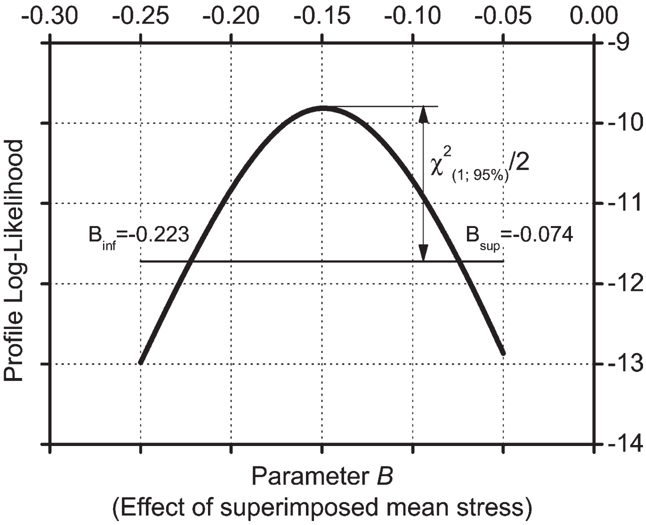

Fig. 6. Profile plot for Likelihood parameter B.

According to Eq. (7), the fatigue limit in torsion is forced to be a linear function of the superimposed mean shear stress. Then, for every stress level τ_i, all the staircase test results are considered together and the parameters A, B, C are chosen so that the Log-Likelihood $\mathfrak{L}$is maximised:

$$\mathfrak{L} = \ln[\Pi_{\tau_i} F(\tau_i;\vartheta)^{p_{\tau_i}}[1-F(\tau_i;\vartheta)]^{q_{\tau_i}}] \quad (8)$$

where F is the cumulative distribution function and p_{τ_i} and q_{τ_i} are the number of failures and runouts at a given stress amplitude τ_i, respectively. In this way, the three values of the fatigue limit and the corresponding standard deviation are assessed (Fig. 5). Fig. 6 gives the profile log-likelihood plot for the model parameter B that represents the effect of the superimposed mean shear stress. The plot indicates the 95% likelihood-ratio-based

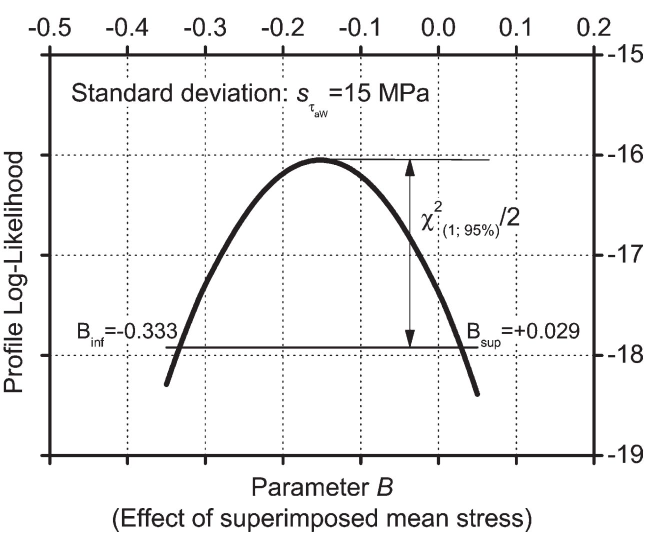

Fig. 7. Profile plot for Likelihood parameter B when standard deviation $s_{\tau_{aW}}$ = 15 MPa.

confidence interval for the parameter B. It must be noted that the confidence interval excludes 0 (see Pascual [26]), showing that, though very light, a dependence of the superimposed mean shear stress cannot be excluded. If the standard deviation is kept fixed in the maximum log-likelihood estimation and its value is set to 15 MPa, in the profile log-likelihood plot for the model parameter B the confidence interval includes 0 (Fig. 7). In this case, when the standard deviation is not underestimated, as it happens when a limited number of samples are employed, even the statistical analysis shows that no effect of the superimposed mean shear stress on the torsion fatigue limit is observed.

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