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A New Strategy for Fatigue Analysis in Presence of General Multiaxial Time Varying Loadings

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"Science is organized knowledge. Wisdom is organized life."

Immanuel Kant

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Dedicated to the ones who supported me

Nomenclature

Σ_{max}	maximum stress during the loading cycles
S_{max}	maximum deviatoric stress during the loading cycles
σ_m	mean stress
σ_H	hydrostatic stress
σ_{-1}	fatigue limit for fully reversed condition
s_{-1}	tensile fatigue limit for $R = -1$
b	back stress
σ_y	macroscopic yield stress
N	current number of cycles
N_F	number of cycles to failure
$\dot{p}$	accumulated plastic strain rate given as $\sqrt{\frac{2}{3}}\|\|\dot{\varepsilon}_p\|\|$
D	damage variable
σ_a	stress amplitude
σ_u	ultimate tensile stress
$\langle\,\rangle$	Macaulay bracket symbol which keeps the positive value and set negative value to zero
$\dot{w}$	energy dissipation rate at a certain scale
$\dot{W}$	energy dissipation rate at all scales
W_{cyc}	dissipated energy per cycle
$\dot{\varepsilon}_p$	rate of effective plastic strain
$\dot{p}$	accumulated plastic strain rate given as $\sqrt{\frac{2}{3}}\|\|\dot{\varepsilon}_p\|\|$
E	Young's modulus
$k = 500 \sim 800MPa$	hardening parameter
β	weakening scales distribution exponent
γ	material parameter from Chaboche law(Wohler curve exponent)
α	characterizes non-linearity of damage accumulation
a	material parameter from Chaboche law
$\lambda = 0 \sim 3$	hydrostatic pressure sensitivity
$\underline{\underline{S}} = dev\dot{\underline{\underline{\Sigma}}}$	deviatoric part of the stress tensor
$A_{II} = \tau_{oct,a} = \sqrt{\frac{1}{3}J_{2,a}}$	the amplitude of octahedral shear stress
$\dot{\varepsilon}_p$	rate of effective plastic strain
W_0	reference density of damage energy
J_2	The second principal invariant of the stress deviatoric tensor
$\sigma_{VM} = \sqrt{3J_2}$	Von Mises stress

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Handling general loadings

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1.1 Multiscale energy dissipation approach

Fatigue failure is a damage accumulation process in which material property deteriorates continuously under fatigue loading and the damage depends on the size of stress and strain. With the

accumulation of fatigue damage, some accidents occur for these components. Thus, it is important to formulate an accurate method to evaluate the fatigue damage accumulation and effectively predict the fatigue life of these components even when subjected to complex loadings.

The problem is then to define criteria able to predict this endurance limit. The micro-macro approach applied to the field of endurance was born with the work of Van [1973], and since it has been used many times, including by Papadopoulos [2001] to take better account of loading path effects. For simplicity and to avoid too costly identification procedures of fatigue data, criteria are often expressed using two parameters. The first relates generally to a shear stress τ (on a plane or on average over an elementary volume) while the second σ reflects the normal stress effects (mean and amplitude) often through the hydrostatic stress are the most numerous (Crossland [1956], Sines [1959], Morel [1998], Thu [2008]). The normal stress acting on the material plane is sometimes defined from a critical plane(Findley [1959]), or through integration at every plane of an elementary volume(Liu and Zenner [1993]). In particular, a probabilistic approach based on this type of integration is proposed in Thu [2008] .

Other authors use energy based approaches. Ellyin [1974] is one of the first to propose a fatigue criterion based on cyclic shear deformation energy. This approach was taken up and complemented by Lefebvre [1981] and Ellyin et al. [1991] for the case of multiaxial loadings. In France, this approach is reflected in the work of Froustey C. [1992] and then in Palin-Luc [1996] and Banvillet [2001]. In recent years, a new class of criteria coupling mesoplasticity and damage has also emerged. In Lemaitre et al. [1999], for example, the author use the approach introduced by Lemaître and Chaboche [1985] based on the thermodynamics of irreversible processes and the mechanics of continuous media. Models based on plasticity-damage coupling were also proposed in Flacelière [2004],Monchiet [2006]. In the case of fatigue, we usually employ in this damage framework the concept of the loading cycle instead of time to evaluate the evolution of damage and to measure the fatigue lifetime. The equations then depend on the load through globally defined quantities over a cycle, such as amplitude, maximum value, mean value. The growth equation of fatigue damage is therefore taken in the form as described in Chapter **??**:

$$\delta D = f(D)\delta N$$

$$\delta N = f_n \delta t$$

where δt is a time sampling of the history in a given number of time intervals $\delta t_1, \delta t_2 ... \delta t_i, ...$ and f_n is the mean frequency of those cycles during the considered time step.

The problem in these approaches is then to take into account the presence of complex variations of the stress tensor. Heuristic formulations with different multiaxial fatigue criteria have been proposed, but most of them still requires the notion of load cycles. The objective of the present chapter is to contribute to the development of life models that take into account such complex variations while avoiding the notion of load cycle. Our fundamental thought is to assume within a micro-macro approach that the local dissipated energy at small scale contributes to the damage which governs fatigue at failure. We follow the Dang Van paradigm. The structure is elastic at the macroscopic scale. At each material point, there is a stochastic distribution of weak points which will undergo strong plastic yielding, which contributes to energy dissipation and cause damage, without affecting the overall macroscopic stress.

Our model considers a plastic behavior at the mesoscopic scales with a dependence of the yield function not only on the deviatoric part of the stress but also on the hydrostatic part. A kinematic hardening under the assumption of associative plasticity is also introduced.

Instead of using the number of cycles, we will use in addition as in Lemaitre et al. [1999] the concept of nonlinear damage accumulation during the loading history. To approach real life loading history more accurately, non-linear damage accumulation laws are also considered in our model to

take into account the sequencing effect. Fatigue will then be determined from the energy plastically dissipated at all scales during plastic shakedown cycles and from a phenomenological fatigue law linking damage evolution and accumulated mesoscopic plastic dissipation.

The chapter is organized as follows. From section 1.1 to 1.3, we present the existing methods and background of our strategy. In section 1.4 we propose the notion of weakening scales and multiscale yield function and describe the plastic dissipation resulting from this notion. And in section 1.5, based on microscopic plasticity we construct the energy dissipation formula. Section 1.6 gives the proposed nonlinear damage accumulation law and summarizes the full model that we propose. Then we combine the energy dissipation law and nonlinear damage accumulation law with its numerical implementation presented in section 1.7, while section 1.8 is devoted to various validations on typical load histories classically treated in the literature. Parameter identification strategy is presented in section 1.9.

1.2 Kinematic Hardening Models

1.2.1 Linear Kinematic Hardening

A hardening rule is needed in microplasticiy to describe the behavior of the material once it is plastically deformed or yielded. One possible hardening rule is the isotropic rule, which assumes that strain hardening corresponds to an enlargement of the yield surface (i.e. an increase in yield stress) without change of shape or position in the stress space. Another is the kinematic rule, which assumes that strain hardening shifts the yield surface without changing its size or shape. Kinematic hardening

Figure 1.1 – Kinematic hardening: a) uniaxial stress-strain diagram, b) evolution of the yield surface in the biaxial stress plane

rules are necessary,especially for the case of unloading and cyclic loading.In Kinematic Hardening the current loading surface is assumed not to expand but to move as a rigid body within the stress space (Figure 1.1(b)). The use of kinematic hardening is, for example, necessary to model the so-called Bauschinger effect (Bauschinger, 1881). This effect is often observed in metals subjected to cyclic loading. Even if the magnitudes of the yield stress in tension and in compression are initially the same, this is no longer the case when the material is preloaded into the plastic range and then unloaded. For example, after previous yielding in tension, yielding in compression may start at a stress level lower than the initial yield stress (Figure 1.1(a)).

Kinematic hardening leads to a translation of the loading surface, i.e. to a shift of the origin of the initial yield surface. If the initial yield surface is described by a yield function of the form

$$f(\sigma) = F(\sigma) - \sigma_0$$

the shifted surface is obviously described by

$$f(\sigma, \sigma_b) = F(\sigma - \sigma_b) - \sigma_0$$

where σ_b is the so-called backstress that represents the center of the shifted elastic domain and plays the role of a tensorial hardening variable. Now we need a kinematic hardening law that governs the evolution of the back stress. Melan [1938] proposed a law of the form

$$\dot{\sigma}_b = \overline{H}_K \dot{\varepsilon}_p \tag{1.2.1}$$

where $\dot{\varepsilon}_p$ is the rate of effective plastic strain. According to which the rate of the back stress is proportional to the plastic strain rate. It is a macroscopic variable representing the dislocation substructure resistance to deformation. The proportionality factor $\overline{H}_K$ is directly related to the plastic modulus and is derived from a simple monotonic uniaxial curve.The linear hardening law Eq.(1.2.1) is often credited to Prager (1955, 1956); we will call it the MelanPrager hardening rule.

1.2.2 Non-linear Kinematic Hardening

To describe cyclic plasticity, one of the famous model is the non-linear kinematic hardening model formulated by Armstrong and Frederick. It is based on a physical mechanism of strain hardening and dynamic recovery and is capable of simulating the multiaxial Bauschinger effect (movement of the yield surface in the stress space). Therefore, the model has been examined and implemented in commercial software and finite element analysis.

The Armstrong-Frederick model (AF) is a modification of the MelanPrager linear kinematic hardening model. The only modification of this simple model is the "recal" term which changes the evolution law for the symmetric backstress tensor σ_b from a classical linear kinematic hardening law (MelanPrager) to a nonlinear kinematic hardening law. The term is proportional to the current back stress multiplied by the norm of the plastic strain rate. According to the Armstrong-Frederick rule, the evolution of the back stress is governed by the differential equation:

$$\dot{\boldsymbol{\sigma}}_b = \underbrace{\overline{H}_K \dot{\varepsilon}_p}_{lin.kin.hardening} - \underbrace{\gamma \dot{p} \boldsymbol{\sigma}_b}_{recall-term, nonlinear hardening} \tag{1.2.2}$$

where $\dot{p}$ is the accumulated plastic strain rate given as $\sqrt{\frac{2}{3}}||\dot{\varepsilon}_p||$. The constants $\overline{H}_K$ and γ are determined from uniaxial tests. At the onset of yielding, the back stress is still zero and Eq.(1.2.2) gives the same response as the linear hardening law Eq.(1.2.1). As the back stress develops, the additional term becomes activated and slows down the rate at which the back stress grows (i.e. reduces the tangent plastic modulus).

1.3 Mean stress effect in local model

Positive mean stress clearly reduces the fatigue life of the material. In design evaluation of multiaxial fatigue with mean stress, a simplified, conservative relation between mean stress and equivalent alternating stress is necessary. We can improve the model by modifying the yield function σ_y and the localization tensor in order to take mean stress effect into account.

Christensen approach

The yield function that was given by Christensen [2000] integrates measures of damage, as well as intrinsic yield strength and concept of transitions. The derived yield function formalism resulted in the form as:

$$\frac{\alpha K}{\sqrt{3}}\sigma_{kk} + \frac{(1+\alpha)^2}{2}s_{ij}s_{ij} \leqslant \frac{K^2}{(1+\alpha)}. \tag{1.3.1}$$

Here α changes the shape of the yield function, thus it is called the shape parameter.

$$\alpha = \frac{\left|\sigma_{11}^C\right|}{\sigma_{11}^T} - 1.$$

The new parameter K is called the ideal or intrinsic strength which uniformly expands or contrasts the yield function, thus is is called the scale parameter:

$$K = \frac{(\sigma_{11}^C)^2}{\sqrt{3}\sigma_{11}^T}.$$

The intrinsic strength would occur if there were no damage or microstructure disturbance. σ_{11}^C and σ_{11}^T are respectively compressive and tensile yield stress in uniaxial states where $\left|\sigma_{11}^C\right| \geqslant \sigma_{11}^T$(for ductile materials, $\frac{1}{2} \leqslant \frac{\left|\sigma_{11}^C\right|}{\sigma_{11}^T} \leqslant 1$). The yield stress in uniaxial and shear states are given by:

$$\begin{aligned}
\sigma_{11}^C &= \frac{-\sqrt{3}K}{(1+\alpha)} \\
\sigma_{12}^Y &= \frac{K}{(1+\alpha)^{3/2}} \\
\sigma_{11}^T &= \frac{\sqrt{3}K}{(1+\alpha)^2}.
\end{aligned} \tag{1.3.2}$$

At $\alpha = 0$, relations Eq.(1.3.1) and Eq.(1.3.2) show the behavior to be that of purely Mises type. This is taken to be the ideal condition where the intrinsic strength K solely determines the yield strength. As the shape parameter increases beyond the value $\alpha = 1$, the yield function behaves in accordance with a state of increasing crack density or any other physical weakening. The term fracture, as used here for behavior at or near $\alpha = 1$, actually corresponds to fracture mechanics for non-interacting cracks. Beyond this range near $\alpha = 1$ or $\alpha \to \infty$ has simply been called yield or failure. Parameter α could be easily viewed as a damage measure or microstructure parameter since it represents microstructure changes on any scale that causes deviation from the ideal state.

It is concluded a decrease in mean stress σ_{kk} reduces the effective value of α. That is, moving the behavior toward ductile case. Alternatively, increasing the mean stress moves α toward larger values, which is taken to be that of brittle behavior.

The fully expanded form of the yield function Eq.(1.3.1) is:

$$\begin{aligned}
&\frac{\alpha K}{\sqrt{3}}(\sigma_{11}+\sigma_{22}+\sigma_{33}) \\
&+(1+\alpha)^2\left[\frac{(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2}{6} + (\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2)\right] \\
&\leqslant \frac{K^2}{(1+\alpha)}.
\end{aligned} \tag{1.3.3}$$

The most compact form of Eq.(1.3.1) is:

$$\frac{1}{2}s_{ij}s_{ij} \leqslant \eta K^2, \tag{1.3.4}$$

where η is a nondimensional scaling factor on K^2, determined by mean normal stress.

$$\eta = \frac{1 - \dfrac{\alpha(1+\alpha)}{\sqrt{3}K}\sigma_{kk}}{(1+\alpha)^3} < 1$$

The mean stress σ_{kk} has a positive relationship with the shape parameter α. We suppose the material endures transition from ductile to brittle when α reaches 1. That means α has a very similar physical meaning with the damage parameter D.

The Gerber parabola

Several models are available addressing the influence of tensile mean stress on fatigue life. Among these are the Gerber (Germany, 1874), Goodman (England, 1899), and Soderberg (USA, 1930) models. The modified Goodman criterion is often used as a design criterion because it is more conservative than the Gerber criterion. The use of the Gerber criterion in the determination of member size is generally more computationally intensive and so rather unattractive for many designers.

The effect of mean stress on the fatigue strength is commonly presented in Haigh diagrams as shown in Figure 1.2, where S_a/S_f is plotted against S_m/S_u at fatigue limit. S_a is the fatigue strength at a given life under fully reversed ($S_m = 0$,$R = -1$) conditions. S_u is the ultimate tensile strength. S_f is the reversed fatigue strength in absence of mean stress. The data points thus represent combinations of S_a and S_m giving that life. The results were obtained for small unnotched specimens, tested at various tensile mean stresses. The straight lines are the modified Goodman and the Soderberg lines, and the curved line is the Gerber parabola. These are empirical relationships that are represented by the following equations:

Modified Goodman: $S_a/S_f + S_m/S_u = 1$

Gerber: $S_a/S_f + (S_m/S_u)^2 = 1$

Soderberg: $S_a/S_f + S_m/S_y = 1$

Figure 1.2 – Haigh diagram showing test data points for the effect of mean stress, and the Gerber, modified Goodman and Soderberg relations.

In the following section, we introduce the sale dependent mean stress effect in our model.

1.4 Weakening scales and yield function

1.4.1 The concept of weakening scales

We follow the Dang Van paradigm. The structure is elastic at the macroscopic scale. At each material points, there is a stochastic distribution of weak points which will undergo strong plastic yielding, without contributing to the overall macroscopic stress. From a microscopic point of view, there is a distribution of weakening scales, namely $s \in [1, \infty)$. In order to introduce our concept, lest us imagine that we can measure the macroscopic stress intensity at present time by a given value S_a. Let σ_y be the yield limit before weakening. Then we imagine that for a given scale s:

• either $1 \leqslant s \leqslant \sigma_y/S_a$, then $S_a \leqslant \sigma_y/s$, the material stays in the elastic regime and there is no energy dissipation at this scale.

• or $\sigma_y/S_a \leqslant s \leqslant \infty$, then $S_a \geqslant \sigma_y/s$, the material is in plastic regime at this scale, which evolves through kinematic hardening, say from zero initial plastic strain $\underline{\underline{\varepsilon}}_p(s)$ and zero initial backstress $\underline{\underline{b}}(s)$ at initial time t_0. There is then dissipated energy at scale s contributing to the fatigue limit.

1.4.2 Distribution of weakening scales

We assume the weakening scales have a probability distribution function following a power law:

$$P(s) = Hs^{-\beta} = (\beta - 1)s^{-\beta}, \tag{1.4.1}$$

where β is a material constant. The choice of a power law comes with two reasons: on one hand, this type of distribution corresponds to a scale invariant process, on the other hand it leads in cyclic loading to a prediction of a number of cycles to life limit as a power law function of the stress intensity. More general laws can also be proposed, without changing the spirit of the model.

The probability of weakening scales is shown in Figure 1.3 and Figure 1.4. We can see that smaller β leads to larger probability of weakening for large s.

Figure 1.3 – Weakening scales s probability distribution curve when $\beta = 1.5$

Figure 1.4 – Weakening scales s probability distribution curve when $\beta = 5$

1.4.3 Yield function with mean stress effect

Positive mean stress clearly reduces the fatigue life of the material. In design evaluation of multi-axial fatigue with mean stress, a simplified, conservative relation between mean stress and equivalent alternating stress is necessary. We can improve the model to come by modifying the yield function σ_y and the localization tensor.

Present choice

In the model to come, our idea is to consider as in Maitournam et al. [2011] that the yield limit σ_y can be reduced in presence of positive mean stress. The mesoscopic yield function can therefore be written as:

$$f\left(s\right) = ||\underline{\underline{S}}(s) - \underline{\underline{b}}(s)|| + \left(\lambda \Sigma_H - \sigma_y\right)/s \leqslant 0 \tag{1.4.2}$$

with $\underline{\underline{S}}$ denoting the deviatoric part of the stress tensor at microscale, and $\underline{\underline{b}}(s)$ the corresponding backstress at the same scale. The material remain in elastic regime when $f < 0$ and in plastic regime when $f = 0$. The parameter λ can itself be a function of Σ_H with a different value in traction(λ_+) than in compression(λ_-).

1.4.4 Local plastic model

We can now describe the mesoscopic stress state. The model considers a plastic behavior at the mesoscopic scale. The mesoscopic stress evolution equations are thus:

$$\dot{\underline{\underline{S}}}(s, M, t) = dev\dot{\underline{\underline{\Sigma}}}(M, t) - \frac{E}{1+\nu}\dot{\underline{\underline{\varepsilon}}}^p(s, M, t), \tag{1.4.3}$$

which defines a Taylor-Lin scale transition model with unit localization tensor (Bosia and Constantinescu [2012]). The mesoscopic deviatoric strain rate tensor is thus equal to the macroscopic strain rate tensor $dev\dot{\underline{\underline{\Sigma}}} = \dfrac{1+\nu}{E} dev\dot{\underline{\underline{\varepsilon}}}$ with $dev\underline{\underline{\Sigma}}$ the deviatoric part of the macroscopic stress tensor. It is complemented by

$$\dot{\underline{\underline{b}}}(s, M, t) = \frac{kE}{E-k}\dot{\underline{\underline{\varepsilon}}}^p(s, M, t), \tag{1.4.4}$$

which is our kinematic hardening model, and by

$$\dot{\underline{\underline{\varepsilon}}}^p(s,M,t) = C\frac{\partial f(s,M,t)}{\partial \underline{\underline{S}}}, \tag{1.4.5}$$

which is the associated plastic flow rule assuming $C = 0$ when $f < 0$ and $C \geqslant 0$ when $f = 0$.

Here E denotes the Young's modulus and k the hardening parameter. The local dissipated energy rate per unit volume at weakening scales s is given by the local entropy dissipation:

$$\dot{w}(s,M,t) = (\underline{\underline{S}} - \underline{\underline{b}})(s,M,t) : \dot{\underline{\underline{\varepsilon}}}^p(s,M,t). \tag{1.4.6}$$

1.5 Construction of an energy based fatigue approach

In a preliminary step, we will consider a simple macroscopic loading history $\underline{\underline{\Sigma}}(M,t)$ which is uniaxial along direction $\underline{\underline{s}}_1$, with $\underline{\underline{s}}_1$ a given stress tensor of unit norm. In traction there is

$$\underline{\underline{s}}_1 = \sqrt{\frac{3}{2}}\begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}.$$

Time periodic of deviatoric amplitude S_a, constant mean stress Σ_H and a Von Mises flow rule are taken into account to see if we get a prediction of local failure for a number of cycles N_F varying as $S_a^{-\gamma}$.

In uniaxial cyclic loading, there will be 3 kinds of loading patterns, as is shown in Figure 1.5:

1. Elastic regime, in phase 2 and 4,where we have no plastic flow $\dot{\underline{\underline{\varepsilon}}}^p(s,M,t) = \dot{\underline{\underline{b}}} = 0$, and where the stress is below the yield limit $|\underline{\underline{S}} - \underline{\underline{b}}| < (\sigma_y - \lambda\Sigma_H)/s$, and where we have therefore $\dot{\underline{\underline{S}}} = dev\dot{\underline{\underline{\Sigma}}}$.

2. Plastic regime according to plastic flow rule, with increasing plastic deformation, in phase 5 and 1, where $\dot{\underline{\underline{\varepsilon}}}^p(s,M,t) = \xi\frac{\underline{\underline{S}}(s) - \underline{\underline{b}}(s)}{||\underline{\underline{S}}(s) - \underline{\underline{b}}(s)||} > 0$ with $\xi = \left|dev\dot{\underline{\underline{\Sigma}}}\right|\left(\frac{kE}{E-k} + \frac{E}{1+\nu}\right)^{-1}$ (detailed in annex) , $\underline{\underline{S}} - \underline{\underline{b}} = \underline{\underline{s}}_1(\sigma_y - \lambda\Sigma_H)/s$ and $\dot{\underline{\underline{S}}} - \dot{\underline{\underline{b}}} = 0$.

3. Plastic regime in the other direction, in phase 3, where we now have $\dot{\underline{\underline{\varepsilon}}}^p(s,M,t) < 0$, then $\underline{\underline{S}} - \underline{\underline{b}} = -\underline{\underline{s}}_1(\sigma_y - \lambda\Sigma_H)/s$ and $\dot{\underline{\underline{S}}} - \dot{\underline{\underline{b}}} = 0$.

In phase 1, a direct analysis yields the energy dissipation at scale s:

$$dW = (S-b)d\varepsilon^p = \frac{(E-k)(1+\nu)}{E(E+k\nu)}\frac{(\sigma_y - \lambda\Sigma_H)}{s}\left(S_a - \frac{(\sigma_y - \lambda\Sigma_H)}{s}\right). \tag{1.5.1}$$

A similar analysis yields

$$dW(phase1) = dW(phase5) = \frac{1}{2}dW(phase3).$$

We can then calculate the local dissipated energy W at point M during one cycle by cumulating the input of all sub-scales plastic regime with their probabilities (Zepeng [2017]).

$$\begin{aligned} W_{cyc} &= 4\int_{(\sigma_y-\lambda\Sigma_H)/S_a}^{\infty} dW(s,M,t)P(s)ds \\ &= 4\int_{(\sigma_y-\lambda\Sigma_H)/S_a}^{\infty} \frac{(E-k)(1+\nu)}{E(E+k\nu)}\frac{(\sigma_y-\lambda\Sigma_H)}{s}\left(S_a - \frac{(\sigma_y-\lambda\Sigma_H)}{s}\right)(\beta-1)\,s^{-\beta}ds \\ &= \frac{4(E-k)(1+\nu)(\beta-1)}{E(E+k\nu)\beta(\beta+1)}\frac{S_a^{\beta+1}}{(\sigma_y-\lambda\Sigma_H)^{\beta-1}}. \end{aligned} \tag{1.5.2}$$

Figure 1.5 – Uniaxial load with plastic dissipation

So we have a power law relationship between stress intensity and the dissipated energy per cycle.

$$W_{cyc} = C_1 S_a^{\beta+1}, \tag{1.5.3}$$

with

$$C_1 = f(\lambda, \beta) = \frac{4(E-k)(1+\nu)\,(\beta-1)}{E(E+k\nu)\beta\,(\beta+1)\,(\sigma_y - \lambda\Sigma_H)^{\beta-1}}.$$

If the dissipated energy accumulates until a failure value W_0, we can get directly the number of cycles to failure from Eq.(1.5.3) as:

$$N_F = \frac{W_0}{W_{cyc}} = \frac{W_0}{C_1} S_a^{-\beta-1}. \tag{1.5.4}$$

As for the time to failure in cyclic loading, it will be:

$$T_F = N_F t_{cyc}.$$

From Eq.(1.5.2), we then obtain that in uniaxial cyclic loading the model predicts as expected (Chapter **??**) a power law dependence of the number of cycles to failure in function of S_a. However, experiments shows that the damage or the energy accumulation of a material evolves non-linearly in time and present a load dependent cycle (Chapter **??**). We should introduce below a method to handle such a nonlinearity.

1.6 Nonlinearity of damage accumulation

1.6.1 Energy approach with Chaboche law

The Chaboche law (Lemaitre and Chaboche [1990]) is essentially a damage incremental law for cyclic loads with a deviatoric stress intensity A_{II} and hydrostatic mean part Σ_H, defining the damage increase by:

$$\delta D = \left(1-(1-D)^{\gamma+1}\right)^{\alpha}\left(\frac{A_{II}}{M(\sigma_H)\,(1-D)}\right)^{\gamma}\delta N, \tag{1.6.1}$$

using an effective intensity $A^*_{II} = A_{II}/\,(1-D)$ evolving with damage D. And the mean stress effect is present both in exponential factor α and in denominator $M(\sigma_H)$.

$$\alpha = 1 - a\left\langle\frac{\frac{1}{2}A_{II} - \sigma_{-1}M(\sigma_H)}{\sigma_u - A_{II}}\right\rangle,$$

$$M(\sigma_H) = M_0\,(1-3c\sigma_{H,max})\,.$$

Eq.(1.6.1) writes equivalently:

$$\delta[1-(1-D)^{\gamma+1}]^{1-\alpha} = (1-\alpha)(\gamma+1)\left(\frac{A_{II}}{M(\Sigma_H)}\right)^{\gamma}\delta N = \frac{1}{N_F(\sigma)}\delta N. \tag{1.6.2}$$

Here $N_F(\sigma)$ denotes the number of cycles at intensity σ to failure as obtained by integration of Eq.(1.6.2) from $D=0$ to $D=1$.

Similar to Eq.(1.6.2), we define here the "equivalent damage" $\tilde{D}$(Figure 1.6.3) :

$$\tilde{D} = 1-(1-D)^{\gamma+1}, \tag{1.6.3}$$

with D the damage variable introduced by Chaboche in its model to scale the stress intensity:

$$A_{II} \longrightarrow \frac{A_{II}}{1-D}.$$

We have

- $\tilde{D}=0$ when $D=0$ (undamaged material),
- $\tilde{D}=1$ when $D=1$ (failure of material),

and a nonlinear relation in between as in Figure 1.6:

$$\delta\tilde{D} = (\gamma+1)\,(1-D)^{\gamma}\,\delta D.$$

Change of damage measure $\tilde{D} = 1-(1-D)^{\gamma+1}$ makes the evolution (1.6.1) explicit. It writes

$$\frac{d\tilde{D}}{dN} = c_D\tilde{D}^{\alpha}\left(\frac{A_{II}}{M(\Sigma_H)}\right)^{\gamma}, \tag{1.6.4}$$

yielding after integration from $\tilde{D}=0$ to $\tilde{D}=1$ a number of cycles to fatigue at constant load given by

$$N_F = \frac{1}{(1-\alpha)}\left(\frac{A_{II}}{M(\Sigma_H)}\right)^{-\gamma}.$$

This method as written requires cycle counting which is difficult and technical for complex load histories. In addition, it allows only a limited influence of multiaxiality.

Now in our model we use the same growth rule as in Chaboche in cyclic load regime, but replace stress intensity by multiscale dissipated energy in Eq.(1.6.4), which removes cycle counting.

The evolution (1.6.4) then writes

$$\frac{d\tilde{D}}{dt} = \tilde{D}^{\alpha}\dot{W}/W_0,$$

or in a differential form

$$d\tilde{D} = \tilde{D}^{\alpha}\frac{dW}{W_0} = \tilde{D}^{\alpha}\frac{W_{cyc}dN}{W_0}. \tag{1.6.5}$$

Figure 1.6 – The relation between $\tilde{D}$ and D when $\gamma = 2$

The replacement of dW by $W_{cyc}dN$ is only introduced to handle cyclic loadings if they occur. The number of cycles to failure in constant loading case, obtained by integrating $\tilde{D}$ from $\tilde{D}_0$ to 1 is then:

$$N_F = \frac{W_0}{(1-\alpha)\,W_{cyc}}\left(1 - D_0^{1-\alpha}\right).$$

With initial damage $D_0 = 0$, we finally get with our proposed expression Eq.(1.5.3) of cyclic energy dissipation:

$$N_F = \frac{W_0}{(1-\alpha)\,W_{cyc}} = \frac{W_0}{(1-\alpha)C_1} S_a^{-\beta-1}. \tag{1.6.6}$$

From Eq.(1.6.6), we see $(-\beta - 1) = -\gamma$ is related to the slops in S-N curve and that $\dfrac{W_0}{(1-\alpha)C_1}$ defines the number of cycles to failure.

1.6.2 Sequence effect

Experiments show fatigue tests started with high stress then change to low stress has less fatigue life than the combination of high stress life proportion plus the low one. This phenomenon of sequence effect is load history dependent, so we need a stress induced parameter to describe it.

This is done in Chaboche with three ingredients:

1. a damage sensitive effective stress:

$$\sigma_D^{eff} = J_2(\underline{\underline{\Sigma}})/(1-D) = A_{II}/(1-D);$$

2. a $(\sigma_D^{eff})^\gamma$ controlled law for damage growth

$$\frac{dD}{dN} = c_\gamma \tilde{D}^\alpha (\sigma_D^{eff})^\gamma;$$

3. a load dependence of exponent α (from 1 at zero load to 0 at large loads). In Chaboche model, the proposition of α is
$$\alpha = 1 - a \left\langle \frac{\sigma_{eq} - \sigma_{fatigue}}{\sigma_u - \sigma_{eq}} \right\rangle \tag{1.6.7}$$
in order to recover the proper high-low sequencing effect.

Many fatigue damage accumulation models are based on the two level loading experiments which is one of the basic random loading analysis. To facilitate the validation and interpretation of an α dependence on stress we will also use two-stress level loading, the specimen is firstly loaded at stress Σ_1 for T_1 cycles and then at stress Σ_2 for T_2 cycles until failure. We can then observe if the experimental results are satisfactory.

During a loading time T_1, we cycle from $\tilde{D} = 0$ to $\tilde{D} = \tilde{D}_1$. By integrating Eq.(1.6.5) of our proposed approach, we get:
$$\left(1 - \tilde{D}_1\right)^{1-\alpha_1} = \frac{T_1}{T_{F1}}, \tag{1.6.8}$$
with T_{F1} the time to failure with this loading.

Then we cycle from $\tilde{D} = \tilde{D}_1$ to failure $\tilde{D} = 1$, which yields
$$1 - \left(1 - \tilde{D}_1\right)^{1-\alpha_2} = \frac{T_2}{T_{F2}}. \tag{1.6.9}$$

From Eq.(1.6.8) and Eq.(1.6.9), after elimination of $(1 - D_1)$ we get:
$$\frac{T_2}{T_{F2}} = 1 - \left(\frac{T_1}{T_{F1}}\right)^{\eta}, \tag{1.6.10}$$
with
$$\eta = \frac{1 - \alpha_2}{1 - \alpha_1}. \tag{1.6.11}$$

In the case of high-low loading sequence we have $\Sigma_1 > \Sigma_2$, which gives $\alpha_1 < \alpha_2$, so it comes to:
$$\eta = \frac{1 - \alpha_2}{1 - \alpha_1} < 1 \implies \frac{T_2}{T_{F2}} = 1 - \left(\frac{T_1}{T_{F1}}\right)^{\eta} < 1 - \frac{T_1}{T_{F1}} \implies \frac{T_1}{T_{F1}} + \frac{T_2}{T_{F2}} < 1.$$

The α dependence on stress intensity does therefore predict a sequencing effect where a low loading sequence following a high one will reduce the life of the structure if α decreases when the load increases.

To get the same effect in our construction, we propose here to introduce s_{min}, which is the minimum scale that experiences plastic dissipation thus causes energy loss:
$$s_{min} = \frac{(\sigma_y - \lambda \Sigma_H)}{S_a}. \tag{1.6.12}$$

We propose a load dependent α through s_{min}. Possible choice of α is expressed as Eq.(1.6.13):
$$\alpha = 1 - a \left(\frac{\frac{1}{s_{min}}}{1 - \frac{1}{s_{min}}} \right). \tag{1.6.13}$$

There is no notion of fatigue limit in our model, $\sigma_{faigue} = 0$. The intensity of loading
$$\frac{\sigma_{eq} - \sigma_{fatigue}}{\sigma_u - \sigma_{eq}} = \frac{1}{\frac{\sigma_u}{\sigma_{eq}} - 1}$$
is measured by
$$\left(\frac{\frac{1}{s_{min}}}{1 - \frac{1}{s_{min}}} \right) = (s_{min} - 1)^{-1}.$$

This means that we measure the distance of load to ultimate failure by local variable s_{min} through

$$\frac{\sigma_u}{\sigma_{eq}} - 1 \longrightarrow (s_{min} - 1)$$

We can see from Figure 1.7 that with our proposition, cycling 1 for fifty percent of its failure time leaves a reserve before failure to cycling 2 of much less than fifty percent. To conclude, the cumulative damage under high-low loading sequence, as we deduced, has the addition of partial lives less than unit. Similarly, the cumulative damage under low-high loading sequence has addition of partial lives more than 1:

$$\frac{T_1}{T_{F1}} + \frac{T_2}{T_{F2}} > 1.$$

The curve in both cases is depicted in Figure 1.7. For constant two-level stress loading, $\alpha_1 = \alpha_2$, the

Figure 1.7 – High to low and low to high loading sequence comparison in 4-point bending($F_{low} = 5000N, F_{high} = 30000N, Radius = 0.2m, \Sigma_u = 1.67E8$), with the proposed damage accumulation law (Eq.(1.6.5)) induced equation Eq.(1.6.10) and Chaboche type α (Eq.(1.6.7)) containing Crossland criterion

Chaboche law returns to the Miner rule when $F_{low} = F_{high}$ where:

$$\frac{T_1}{T_{F1}} + \frac{T_2}{T_{F2}} = 1.$$

1.6.3 The final model

In summary, our damage based fatigue life criterion using a damage evolution governed by a multiscale plastic energy dissipation, has four ingredients.

- a scale dependent yield limit
$$\frac{1}{s}(\sigma_y - \lambda \Sigma_H^{macro})$$
associated to a microscopic plastic evolution governed by the standard plastic evolution laws Eq.(1.4.3) - Eq.(1.4.5);

- a multiscale plastic energy dissipation obtained by summing plastic dissipation across our power law scale distribution
$$\dot{W}(M,t) = \int_{s=1}^{\infty} \left(\underline{\underline{S}} - \underline{\underline{b}}\right)(s,M,t) : \underline{\underline{\dot{\varepsilon}}}^p(s,M,t) s^{-\beta} ds; \tag{1.6.14}$$

- a load intensity sequencing effect that we have represented by the formula :
$$1 - \alpha = a(s_{min} - 1)^{-f}; \tag{1.6.15}$$

- a exponential damage evolution law with load dependent exponent α given by the above formula and coefficient $\dot{W}/W_0$ governed by the multiscale plastic dissipation rate:
$$\frac{d\tilde{D}}{dt} = \tilde{D}^{\alpha} \dot{W}/W_0. \tag{1.6.16}$$

In this model we have five independent coefficients in addition to the construction of the local plastic model Eq.(1.4.3) - Eq.(1.4.5) :

1. reference density of damage energy : W_0 (in MPa)
2. mean stress effect coefficient : λ
3. slope of SN curve : $\beta + 1$
4. sensitivity to load intensity a.
5. exponent in the load sequence effect f.

1.7 Numerical strategy

1.7.1 Scale discretization

We now need to propose a practical implementation strategy for our final model of section 1.6.3.Our first approach takes one cycle as unit time. We compute analytically by Eq.(1.5.2) the energy dissipation at each scale during this cycle. The method is valid for simple loading history and which includes the integration on all weakening scales. The damage $\tilde{D}$ is then accumulated after each cycle by numerical integration of Eq.(1.6.16) until we get to the fatigue limit of $\tilde{D} = 1$.

However, there are certain limitations of this method. Firstly we need a load history decomposition in cycles. Secondly in real life the perfect close loop cycle is hardly applicable. Finally, we need to approximate α by its average value per cycle which is computed numerically and can not be done analytically.

Thus we propose in a more general method which can be integrated by a step by step strategy. We compute numerically the dissipation at different scales using an implicit Euler time integration of the constitutive laws of section 1.7.2. After which we make a numerical integration on different scales. Then we can update the damage and go to next time step.

Instead of doing the scale integration directly which can be difficult for complex loading, the Gaussian Quadrature rule with Legendre points is used to give the value of local dissipated energy rate. To use the Gaussian quadrature rule the limit range of integral must be from -1 to 1, while the total dissipated energy is expressed by integrating all the weakening scale s ranging from 1 to infinity with their occurrence probabilities:

$$\dot{W} = \int_1^\infty \dot{w}(s)(\beta - 1)(s)^{-\beta} ds.$$

To change the limit range of integral from $[1, \infty]$ to $[1, 0]$ we take as new integration variable $u(s) = s^{1-\beta}$. Therefore the dissipated energy summed on all scales is:

$$\begin{aligned} \dot{W} &= \int_1^\infty \dot{w}(s)(\beta - 1)(s)^{-\beta} ds \\ &= \int_0^1 \dot{w}\left(u^{\frac{1}{1-\beta}}\right) du \\ &= \frac{1}{2}\int_{-1}^1 \dot{w}\left[\left(\frac{x+1}{2}\right)^{\frac{1}{1-\beta}}\right] dx \end{aligned} \tag{1.7.1}$$

given $u = \dfrac{x+1}{2}$. So the dissipated energy rate integrated over all scales takes the form of Eq.(1.7.2):

$$\dot{W} = \frac{1}{2}\int_{-1}^1 \dot{w}\left[\left(\frac{x+1}{2}\right)^{\frac{1}{1-\beta}}, t\right] dx \approx \frac{1}{2}\sum_i \omega_i \dot{w}\left[\left(\frac{x_i+1}{2}\right)^{\frac{1}{1-\beta}}, t\right], \tag{1.7.2}$$

where ω_i and x_i are respectively the weights and nodes of the Gauss Legendre integration rule used for the numerical integration. In this work, we used a 64 points Gaussian Legendre integration rule (Leg) with $s_i = \left(\dfrac{x_i+1}{2}\right)^{\frac{1}{1-\beta}}$ being the associated scale.

1.7.2 Calculation of local plastic dissipation

The material could be both in elastic and plastic regime depending on the considered scale. To be more elaborate, we reuse the fundamental equations in different regimes. At scale s, we have a dissipation rate given by:

$$\dot{w}(s) = \left(\underline{\underline{S}} - \underline{\underline{b}}\right) : \dot{\underline{\underline{\varepsilon}}}^p,$$

which differs between plastic and elastic regime.

Elastic regime:

There we have plastic strain rate $\dot{\underline{\underline{\varepsilon}}}^p = 0$, back stress rate $\dot{\underline{\underline{b}}} = 0$ and deviatoric stress rate $\dot{\underline{\underline{S}}} = dev\dot{\underline{\underline{\Sigma}}}$, leading to

$$\dot{\underline{\underline{S}}} - \dot{\underline{\underline{b}}} = dev\dot{\underline{\underline{\Sigma}}},$$

meaning

$$\left(\underline{\underline{S}} - \underline{\underline{b}}\right)(t+dt) = \left(\underline{\underline{S}} - \underline{\underline{b}}\right)(t) + dev\dot{\underline{\underline{\Sigma}}}dt.$$

At each time step we define a trial stress:

$$\left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial} := \left(\underline{\underline{S}} - \underline{\underline{b}}\right)(t+dt). \tag{1.7.3}$$

We are in elastic regime at scale s as long as we satisfy

$$\left|\left|\underline{\underline{S}} - \underline{\underline{b}}\right|\right|_{trial} \leqslant \left(\sigma_y - \lambda\Sigma_H\right)/s. \tag{1.7.4}$$

Plastic regime:

When we leave elastic regime at scale s, i.e. when the above inequality Eq.(1.7.4) is violated, we have:

$$\begin{cases} \dot{\underline{\underline{\varepsilon}}}^p = \xi\dfrac{\underline{\underline{S}} - \underline{\underline{b}}}{\left|\left|\underline{\underline{S}} - \underline{\underline{b}}\right|\right|}, \xi > 0, & \text{plastic flow,} \quad (1.7.5) \\ \left|\left|\underline{\underline{S}} - \underline{\underline{b}}\right|\right| = \left(\sigma_y - \lambda\Sigma_H\right)/s, & \text{yield limit,} \quad (1.7.6) \\ \left(\underline{\underline{S}} - \underline{\underline{b}}\right) : \left(\dot{\underline{\underline{S}}} - \dot{\underline{\underline{b}}}\right) = 0, & \text{yield limit time invariance,} \quad (1.7.7) \\ \dot{\underline{\underline{b}}} = \dfrac{kE}{E-k}\dot{\underline{\underline{\varepsilon}}}^p, & \text{kinematic hardening rule,} \quad (1.7.8) \\ \dot{\underline{\underline{S}}} = dev\dot{\underline{\underline{\Sigma}}} - \dfrac{E}{1+\nu}\dot{\underline{\underline{\varepsilon}}}^p, & \text{localisation rule.} \quad (1.7.9) \end{cases}$$

In all cases, we get by integrating Eq.1.7.9, Eq.1.7.8 with the use of Eq.1.7.5.

$$\left(\underline{\underline{S}} - \underline{\underline{b}}\right)(s, t+dt) = \frac{\left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}(s, t+dt)}{1+\eta}, \tag{1.7.10}$$

and because of the yield condition Eq.1.7.6, we have

$$\eta = max\left\{\underbrace{0}_{elastic\ regime}, \underbrace{\frac{\left|\left|\underline{\underline{S}} - \underline{\underline{b}}\right|\right|_{trial}}{\left(\sigma_y - \lambda\Sigma_H\right)/s} - 1}_{plastic\ regime\ when\ this\ number\ is\ positive}\right\},$$

That is to say, when the structure is in elastic regime at time t and scale s, we have $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)(s,t) = \left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}(s,t)$. Otherwise, if the norm of $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}(s,t)$ is greater than the local yield limit $\left(\sigma_y - \lambda\Sigma_H\right)/s$, $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)(s,t)$ will be projected on the yield limit.

Knowing the distinction between elastic and plastic regime under multiple scales, we compute the general expression of the dissipated energy rate at scale s.

$$\dot{w}(s) = \left(\underline{\underline{S}} - \underline{\underline{b}}\right) : \dot{\underline{\underline{\varepsilon}}}^p = \gamma\frac{\left(\sigma_y - \lambda\Sigma_H\right)}{s}. \tag{1.7.11}$$

From Eq.(**??**) and Eq.(**??**) in annex, we get:

$$
\begin{aligned}
E\gamma dt &= \left\langle \left|\left|\underline{\underline{S}} - \underline{\underline{b}}\right|\right|_{trial} - \frac{(\sigma_y - \lambda\Sigma_H)}{s} \right\rangle / \left(\frac{1}{1+\nu} + \frac{k}{E-k} \right) \\
&= \left\langle \left|\left|\underline{\underline{S}} - \underline{\underline{b}}\right|\right|_{trial} - \frac{(\sigma_y - \lambda\Sigma_H)}{s} \right\rangle \frac{(E-k)(1+\nu)}{(E+k\nu)},
\end{aligned}
\tag{1.7.12}
$$

where $\langle\ \rangle$ is Macaulay bracket symbol defined as $\langle m \rangle = 0$ if $m \leqslant 0$, otherwise $\langle m \rangle = m$. Thus the dissipated energy rate only depends on the evolution of the variable $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)$, which must therefore be mentioned during the whole cycle under study.

We replace γ deduced from Eq.(1.7.12) in Eq.(1.7.11) to give the expression of local energy dissipation rate at scale s:

$$
\dot{w}(s)dt = \frac{(E-k)(1+\nu)}{E(E+k\nu)} \left\langle \left|\left|\underline{\underline{S}} - \underline{\underline{b}}\right|\right|_{trial} - \frac{(\sigma_y - \lambda\Sigma_H)}{s} \right\rangle \frac{(\sigma_y - \lambda\Sigma_H)}{s}. \tag{1.7.13}
$$

With Eq.(1.7.2), the final expression of energy dissipation W during time step dt writes:

$$
\begin{aligned}
W &= \dot{W} dt \\
&= \frac{1}{2} \sum_i \omega_i \dot{w} \left[\left(\frac{x+1}{2} \right)^{\frac{1}{1-\beta}} \right] dt \\
&= \frac{(E-k)(1+\nu)}{2E(E+k\nu)} \sum_i \omega_i \left\langle \left|\left|\underline{\underline{S}} - \underline{\underline{b}}\right|\right|_{trial} - \frac{(\sigma_y - \lambda\Sigma_H)}{\left(\frac{x_i+1}{2} \right)^{\frac{1}{1-\beta}}} \right\rangle \frac{(\sigma_y - \lambda\Sigma_H)}{\left(\frac{x_i+1}{2} \right)^{\frac{1}{1-\beta}}}.
\end{aligned}
\tag{1.7.14}
$$

The mean stress effect term in Chaboche model is $s_{-1}\left(1 - 3\frac{\sigma_H}{\sigma_u}\right)$, where the fatigue limit at zero mean stress s_{-1} is reduced in the presence of σ_H. In our model, the yield limit decreases with positive mean stress. Because of the presence of the term $\lambda\Sigma_H$ which will be positive when Σ_H is positive. In Eq.1.7.14, the coefficient λ can change values if Σ_H changes sign (see section 1.4.3).

1.7.3 Damage integration algorithm

Numerically the change of α is extremely nonlinear with time. From Figure 1.8 we can see the mean value of α depends on loading pattern.

Because of the possible large variations in time of α, the evolution problem in damage is very nonlinear and thus, one needs to develop and validate an improved numerical time integration strategy at least for two specific cases: the constant amplitude case and the random load case.

We propose in constant amplitude load with very small evolution of damage per cycle to numerically calculate W_{cyc} and the mean values of α through one cycle or several cycles(out-of-phase condition) and apply the result to life prediction by using Eq.(1.6.6):

$$
N_F = \frac{W_0}{(1-\alpha_m)\, W_{cyc}}, \tag{1.7.15}
$$

which is obtained by direct integration of our damage law assuming time uniform dissipation in one cycle and frozen α. In this way the numerical cost is not as high as for the numerical implementation of all the loading points in random loading case. Because of the symmetrical shape of the evolution of α, numerically the mean value of α does not strongly depend on the number of steps per cycle. In the verification process we have compared $100 \sim 1000$ time steps per cycle (Figure 1.10).

Figure 1.8 – The evolution of α in bending and torsion with $\Sigma_y = 1080MPa$, $\Sigma_{bending} = \Sigma_{torsion} = 500MPa$, $a = 0.3$ and 200 time steps in one cycle

For complex cyclic load cases, the idea is to accurately compute the history of plastic dissipation during one cycle (multiscale calculation with time refinement), and to use this precomputed result in the time integration of the scalar damage evolution law with a time stepping which is adapted to the time variation of α.

Figure 1.9 – Comparing numerical strategy with optimal time steps in one cycle with the old one, in this way the number of steps in unit cycle is reduced from 400 to 35, meaning a cost reduction factor of 0.0875 (with $\Delta\alpha = 0.05$, $a = 0.4$, $\lambda = 0.1$, $\Sigma_y = 230MPa$, $\Sigma_{bending} = 225MPa$)

If α varies significantly, we use optimal time step numerical method to update D during one time step. In more details, let us suppose that our first few cycles calculation with fine time steps Δt has produced a sequence $\alpha(i)$ and $W_{cyc}(i)$ of exponent and dissipation at time $i\Delta t$. We then construct an adaptive time stepping strategy with variable time steps $\Delta t_{ref}(j)$, reference exponent $\alpha_{ref}(j)$ and

dissipated energy $W_{ref}(j)$ by regrouping together adjacent time steps $\Delta t(i)$ with similar exponents $\alpha(i)$. This sequence is incremented as follows.

For $t_{ref}(j)$ and $\alpha_{ref}(j) = \alpha(t_{ref}(j))$ given, we set

$$\Delta t_{ref}(j) = \sum_{\substack{t(i) \geqslant t_{ref}(j) \\ \left\|\alpha(i) - \alpha_{ref}(j)\right\| \leqslant \Delta\alpha}} \Delta t, \quad t_{ref}(j+1) = t_{ref}(j) + \Delta t_{ref}(j).$$

The same goes for the dissipated energy:

$$\Delta W_{ref}(j) = \sum_{\substack{t(i) \geqslant t_{ref}(j) \\ \left\|\alpha(i) - \alpha_{ref}(j)\right\| \leqslant \Delta\alpha}} W_{cyc}(i), \quad W_{ref}(j+1) = W_{ref}(j) + \Delta W_{ref}(j).$$

We finally use these new time steps with corresponding α_{ref} and W_{ref} to update the damage by looping on all the following cycles with the new optimal time steps j and cycles N, and updating damage in each cycle by:

$$D = D + D^{\alpha_{ref}(j)} \frac{\Delta W_{ref}(j)}{W_0}, \tag{1.7.16}$$

with values $\alpha_{ref}(j)$ and $\Delta W_{ref}(j)$ precomputed in the first few cycles. This strategy is validated in Figure 1.9.

Complexity analysis

The optimal time step method clearly reduces the numerical cost. Typically, we assume the material has fatigue life of 1×10^6 cycles to failure and we implement 1000 time steps in unit cycle. The reduction factor of points in unit cycle for example as in Figure 1.9 equals $35/400 = 0.0875$. We can then compare the cost between full numerical strategy and the new one.

Full numerical strategy:	1000 time steps $\times$ 64 scales $\times$ 1×10^6 cycles
Optimal cyclic strategy:	1000 time steps $\times$ 64 scales $\times$ 5 cycles until stabilization $+ 1 \times 10^6$ cycles $\times$ (1000 time steps $\times$ reduction factor)
Ratio between optimal and full:	$\approx \dfrac{reduction\ factor}{64\ scales} = \dfrac{1}{731}$

The same strategy is applied to random loading situations which are made of repeated sequence of random loads:

- calculation of dissipated energy and exponent on one sequence;
- time coarsening;
- repeated integration of damage through the different sequences using Eq.1.7.16.

In such a strategy, the extra cost of introducing multiple scales in the calculation becomes negligible as compared to the time integration of damage in the evolution process.

(a) 30 time steps in unit cycle($\beta = 1.1$, $a = 0.001$).

(b) 100 time steps in unit cycle($\beta = 1.1$, $a = 0.001$).

(c) Relative error $\left(\frac{NF_{num} - NF_{analytical}}{NF_{analytical}}\right)$ with 30 time steps in unit cycle ($\beta = 1.1$, $a = 0.001$)

(d) Relative error $\left(\frac{NF_{num} - NF_{analytical}}{NF_{analytical}}\right)$ with 100 time steps in unit cycle ($\beta = 1.1$, $a = 0.001$)

Figure 1.10 – S-N curve of bending test on 30NCD16 steel using numerical and analytical method (Eq.(1.7.15)) with different time steps. Data are those of table.1.1

Altogether, we have three numerical approaches for integrating damage.

1. Numerical results with varying α(equally divide time in unit cycle), and do the scale integration all along the fatigue life time, which can be of high numerical cost

$$\delta D = D^{\alpha} \frac{\dot{W}}{W_0} \delta t.$$

This method only serves for qualification purposes.

2. Numerical results with optimal time steps(equally divide αin unit cycle), here we have $\Delta\alpha = 0.01$, to reduce time steps needed, after several cycles adaptation we iterate using the recorded scalar values of α_{ref}, W_{ref} and t_{ref} to decide fatigue life time.

$$\delta D = D^{\alpha_{ref}} \frac{W_{ref}}{W_0} \delta t_{ref}.$$

3. Analytical results after integration of D (with mean alpha from numerical strategy)

$$N_F = \frac{W_0}{(1-\alpha_m) W_{cyc}},$$

which is derived from the differential equation

$$\delta D = D^{\alpha_m} \frac{W_{cyc}}{W_0} \delta N.$$

Although method 2 is much more numerically efficient than the original numerical method 1, it is still not cheap in the experimental fitting process. We need the analytical formula method 3 to do the fitting process. To validate the feasibility, we now compare only the analytical(method 3) one and optimal time steps(method 2) one. The results are shown in Figure 1.11 and Figure 1.12, the relation ship between relative error and $\Delta\alpha$ with 2000 time steps in unit cycle is shown in Figure 1.13 and Figure 1.14.

Now we can conclude that with more time steps in unit cycle, we get closer results with the original numerical method(method 2) in HCF regime. With smaller $\Delta\alpha$ value, we get less relative error between numerical(method 3) and analytical(method 1) results. This indicates that in constant amplitude cyclic loading, with moderate values of β it is feasible to use the analytical formula, given α_m is calculated using sufficient large time steps and small $\Delta\alpha$ in the first several cycles.

Figure 1.11 – S-N curve using analytical and numerical results with optimal time steps methods ($\beta = 1.1$, $a = 0.01$, $\Delta\alpha = 2E-5$ in unit cycle), yielding 200 full time steps reduced to 197 optimal time steps

Figure 1.12 – S-N curve using analytical and numerical results with optimal time steps methods ($\beta = 1.1$, $a = 0.01$, $\Delta\alpha = 1E-5$ in unit cycle), yielding 200 full time steps reduced to 199 optimal time steps

Figure 1.13 – Relative error $\left(\frac{NF_{opt} - NF_{analytical}}{NF_{analytical}}\right)$ between analytical and numerical results with optimal time steps methods ($\beta = 1.1$, $a = 0.01$, $\Delta\alpha = 2 \times 10^{-5}$ in unit cycle)

Figure 1.14 – Relative error $\left(\frac{NF_{opt} - NF_{analytical}}{NF_{analytical}}\right)$ between analytical and numerical results with optimal time steps methods ($\beta = 1.1$, $a = 0.01$, $\Delta\alpha = 1 \times 10^{-5}$ in unit cycle)

1.8 Validation on recovery tests

1.8.1 Recovery of Chaboche law on cyclic loading

The test is first performed on a sinusoidal uniaxial load $\Sigma_{11}(t) = Asin(t)$, giving a deviatoric amplitude$S_a(t) = \sqrt{J_{2,a}(dev\underline{\underline{\Sigma(t)}})}$ so that $S_a(t) = \left\| \sqrt{\frac{1}{3}}\Sigma_{11}(t) \right\|$. We use parameters in Table.1.1 to recover the classic Chaboche law in cyclic loading.

Parameters	Value
Young's modulus	$E = 191$ GPa
Hardening parameter	$k = 1$ GPa
Weakening scales distribution exponent	$\beta = 1.5$
Hydrostatic pressure sensitivity	$\lambda_{+-} = 0.6$
Macroscopic yield stress	$\sigma_y = 1080$ MPa
Sequencing effect sensitivity	$a = 0.1$
Dissipated energy to failure per unit volume	$W_0 = 5$ MJ(MPa)

Table 1.1 – Material parameters in a simple cyclic load

We use matlab to numerically realize our method. The plot of $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|_{trial}$ and $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|$ during the first cycles at two different scales($s_3 = 1.21$ and $s_{10} = 1.13$) are shown in Figure 1.15 and Figure 1.16. We use here a fixed value of $\lambda(\lambda_+ = \lambda_-)$, thus the local yield limit is reduced in traction and increased in compression.

Figure 1.15 – Microscopic $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}$ and $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)$ evolution with time under different weakening scales($s_3 = 1.21$ and $s_{10} = 1.13$) in sinusoidal load with zero mean stress

The time history of dissipated energy is depicted in Figure 1.17. We scale S_a in the plot to see more clearly the relation between energy dissipation and stress intensity. The choice of α does not affect W; it only concerns damage accumulation rate. Smaller α causes faster accumulation.

The "jump" in energy evolution is due to activation of new scales while in-between two scales the dissipated energy follows the stress increment at each time step. In other words, because in our

Figure 1.16 – Microscopic $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}$ and $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)$ evolution with time under different weakening scales($s_3 = 1.21$ and $s_{10} = 1.13$) in sinusoidal load with mean stress=300 MPa

method the dissipated energy W(Figure 1.17), sums energy dissipation at all scales, any additional violation of $\|S - b\|_{trial}$ at local yield limit(Figure 1.15) introduces an additional dissipation.

Figure 1.17 – Validation of dissipated energy in all scales with analytical(method 3) and numerical method(method 1) with $\beta = 1.1$,$\Sigma = 0.85\sigma_y$. The time evolution of α does not play a role in the dissipation calculation which is normal since α does not enter in the dissipation calculation

We take the mean value of α during all the iteration process of numerical method as α_m. The energy and damage accumulation is shown in Figure 1.19 and Figure 1.22. Here we give 100 time steps in one cycle to see the relative difference between changing α and α_m, also $\dot{W}$ and $W_{cyc}/stepnumber$

Figure 1.18 – Validation of dissipated energy in all scales with analytical and numerical method(enlargement of Figure 1.17)

Figure 1.19 – Dissipated energy accumulation through time with different methods, there are 100 time steps in unit cycle

method. The more time steps we give, the more precision we get. The relative difference between analytical energy loss and numerical one is shown in Figure 1.21 from which we conclude that the three methods converge in terms of elastic energy dissipation, but due to nonlinear effects the damage evolution does not have the same history per cycle. The frozen α delays damage, the varying α increases damage during the phase of strong loading. The difference has a significant impact if damage

Figure 1.20 – Dissipated energy accumulation through time with of 3 methods(enlargement of Figure 1.19)

Figure 1.21 – Relative difference $\frac{W_{analytical} - W_{numerical}}{W_{analytical}}$ between analytical energy loss and numerical one with α varying with time of Figure 1.19

occurs with very few cycles. It will not when computing on a large number of cycles.

The cyclic load calculation is only valid for very simple such as proportional loading in fatigue. However, the convergence of the two methods is based on the small value of β(close to 1), in case of large values of β(typically around 5), the numerical strategy gives shorter life than the analytical one due to extreme non-linearity in the energy dissipation history per cycle. The relative error is around

Figure 1.22 – Damage evolution with time under sinusoidal load with different methods, there are 100 time steps in unit cycle($\beta = 1.1$,$\Sigma = 0.85\sigma_y$)

Figure 1.23 – Damage evolution with time under sinusoidal load with two different methods(enlargement of Figure 1.22)

20% as shown in Figure 1.25 and Figure 1.26. Nevertheless the analytical formula can still be used as a comparison group to verify the numerical results. And in the identification process we need the analytical form to fit the $S - N$ curve of a certain material. The outcome is satisfactory. Hence, to be

Figure 1.24 – Relative difference $\frac{D_{analytical} - D_{numerical}}{D_{analytical}}$ evolution with time of Figure 1.22

more general for any loading history, we adopt the numerical method after identification of β.

Figure 1.25 – Validation of dissipated energy in all scales with analytical and numerical method with $\beta = 5$, $\Sigma = 0.4\sigma_y$

Figure 1.26 – Validation of dissipated energy in all scales with analytical and numerical method with $\beta = 5$, $\Sigma = 0.8\sigma_y$

Figure 1.27 – Damage evolution with time under sinusoidal load with $\beta = 5$, $\Sigma = 0.4\sigma_y$. In such a severe loading and with extreme non-linearity, the simple Chaboche like formula with frozen α departs from the outcome of the full numerical model

Figure 1.28 – Damage evolution with time under sinusoidal load with $\beta = 5$, $\Sigma = 0.8\sigma_y$

1.8.2 Numerical recovery of sequence effect

We adopt the parameter α to take into account the sequence effect. The high-low loading sequence clearly reduces the fatigue life, as depicted in Figure 1.29. In order to cover this phenomenon, we let α change with time($\alpha = 1 - a\,(s_{min}(t) - 1)^{-f}$). Here a is the sequence effect sensitivity. According to Eq.(1.6.12), we have:

$$s_{min}(t) = \frac{\Sigma_y - \lambda \Sigma_H(t)}{S_a(t)},$$

which is the minimum weakening scale that activates energy loss. We use a general law for α of the type $\alpha = \alpha(s_{min})$ with the idea that for us s_{min} is a measure of present intensity of macroscopic stress. It is therefore a mechanical based stress norm. The impact of this construction of α can be seen on Figure 1.30, on a test case specifically built to illustrate such a sequence effect.

Figure 1.29 – Two level sequence effect. By comparing the vertical figures we can see high stress gives high $(1-\alpha)$ value which causes fast damage accumulation speed. The evolution of $(1-\alpha)$ is highly nonlinear and follows the value of stress at each time step.

Major damage effect

To see the influence of sequence effect factor of α, we first fix α for all tests to see the results. When α is fixed, it becomes denominator in the final expression of N_F (Eq.(1.6.6)) and has the same impact as W_0. We find that the fatigue life of random loading is widely dispersed (Figure 1.30). In this case we need to use $\alpha = f(s_{min})$ which evolves with time to make large stress intensity bring more damage.

Figure 1.30 – Fatigue life of random loading dispersion

After comparison with the experimental data we find out that large stresses cause much more damage than the smaller ones. It is necessary to include this major stress induced damage to our stress intensity parameter α. With the new α used in Eq.(1.6.15) compared to Eq.(1.6.13) we are able to calibrate our model better with the experimental results by using

$$\alpha = 1 - a\left(\frac{\frac{1}{s_{min}}}{1-\frac{1}{s_{min}}}\right)^f. \tag{1.8.1}$$

With $f = 1.1$, we use the power to magnify large stress impact and minify lower stress damage. The demonstration of major damage effect using magnification power is depicted in Figure 1.31. With larger value of power, the sequence effect is more significant(bigger dispersion between high-low and low-high sequence).

The larger value of S_a causes more damage in the presence of the power β, leading to faster increase of

$$(1-\alpha) = a\left(\frac{\frac{1}{s_{min}}}{1-\frac{1}{s_{min}}}\right)^f = a(s_{min}-1)^{-f}.$$

which causes faster damage accumulation. We can also see this effect in Figure 1.32.

To assess large stress correctly we define the larger stress intensity as the value of stress which makes expression in the bracket in the second term of α greater than 1($\frac{1}{s_{min}(t)-1} > 1$), then we use power β to magnify this term. In this way the damage is accelerated for large stresses. The deviatoric

Figure 1.31 – Major damage effect using different magnification power of Eq.(1.8.1) on sequence effect. Here high stress is 1MPa and low stress is 0.8MPa. We see that using a large power f in Eq.(1.8.1) induces a stronger sequence effect.

stress S_a, above which the damage is magnified, is determined from:

$$\alpha(t) = 1 - a\left(\frac{1}{s_{min}(t) - 1}\right)^{1.1},$$

$$s_{min}(t) = \frac{\Sigma_y - \lambda\Sigma_H(t)}{S_a(t)} < 2,$$

$$S_{large}(t) > \frac{\Sigma_y - \lambda\Sigma_H(t)}{2}.$$

The major damage effect can be seen in Figure 1.32, which occurs when S is more than half the macroscopic yield stress of the material.

Figure 1.32 – (1-α) term which stands for the load intensity evolution, both with and without the magnification power f

1.9 Identification strategy

In our tests we keep $f = 1.1$(justified in Chapter 2 by Figure 2.4 and Figure 2.5). The positive hydrostatic stress and negative one have different effect on the yield limit. It is necessary to adopt 2 parameters to describe this behavior. So we divide the hydrostatic sensitivity λ into 2 parts. λ_+ and λ_-. In the analytical formula, the amplitude of the stress intensity is adopted and the average value of tension hydrostatic stress(+) and compressive hydrostatic stress(-) is introduced.

For a good lifetime prediction, it is necessary to first identify the appropriate parameters of the model. For this purpose, we use the analytical formula Eq.(1.7.15) obtained in uniaxial cyclic loading case. With distinction of the hydrostatic stress in the presence of non-zero mean stress and since in fully reversed uniaxial loading we spend an equal time in compression and in traction, Eq.(1.5.2) now writes:

$$W_{cyc} = \frac{2(E-k)(1+\nu)\,(\beta-1)}{E(E+k\nu)\beta\,(\beta+1)} \left[\frac{S_a^{\beta+1}}{\left(\sigma_y - \lambda_+ \overline{\Sigma}_{H+}(t)\right)^{\beta-1}} + \frac{S_a^{\beta+1}}{\left(\sigma_y - \lambda_- \overline{\Sigma}_{H-}(t)\right)^{\beta-1}} \right]. \tag{1.9.1}$$

$$N_{Fnum} = \frac{W_0}{(1-\alpha)} \frac{E(E+k\nu)\beta\,(\beta+1)}{2(E-k)(1+\nu)\,(\beta-1)} \frac{1}{\dfrac{S_a^{\beta+1}}{\left(\sigma_y - \lambda_+ \overline{\Sigma}_{H+}(t)\right)^{\beta-1}} + \dfrac{S_a^{\beta+1}}{\left(\sigma_y - \lambda_- \overline{\Sigma}_{H-}(t)\right)^{\beta-1}}}. \tag{1.9.2}$$

To use our analytical model Eq.(1.9.2) to fit the experiments, we employ matlab Least-Squares (model fitting) algorithm.

The best fitted parameters in uniaxial cyclic loading case are deduced from the minimization of the sum of square of difference between uniaxial numerical and experimental results:

$$\min_{\beta,\lambda,W_0} \left\{ \sum_i \left(N_{Fnum} - N_{Fexp}\right)^2 \right\} \tag{1.9.3}$$

Assume there are i sets of experimental data. To clarify the identification process, let us separate the parameters into:

- Experimental data: $S_{a(i)}$, $\Sigma_{H(i)}$, $N_{Fexp(i)}$, σ_y, E, k, ν.
- Material constants: β, λ_{+-}, W_0 (to be fitted)
- Input data from experimental data: $S_{max(i)}$, $\Sigma_{H(i)}$
- Input data from experimental data and parameter constants: $\alpha_{(i)}$
- Output data: $N_{Fnum(i)}$

In this process, σ_y, E, k and ν are given elastoplastic material constants. For each test (i), the load parameters are maximum amplitude $S_{a(i)}$, mean hydrostatic stress $\overline{\Sigma}_{H(i)}$, and the experimental number of cycles to failure is given by $N_{Fexp(i)}$.

The exponent $\alpha_{(i)}$ is cycle average obtained by

$$\alpha_{(i)} = mean \left[1 - a \left(\frac{1}{\dfrac{\Sigma_y - \lambda_{+-}\Sigma_H(t)}{S_a(t)} - 1} \right)^{1.1} \right] (i). \tag{1.9.4}$$

The parameters to be calibrated are W_0, β and λ. Since the exponent $\alpha_{(i)}$ depends on β and λ, we proceed iteratively by:

1. We first identify the S-N curve slope β and the energy scale W_0 using the analytical formula with torsion tests because there is no λ_{+-} impact in this kind of loading. We start from an initial guess β from which we can deduce $\alpha_{(i)}$ by numerical calculation of Eq.1.9.4 and identify β and W_0 by least squares. Because our analytical formula is not derivable in all ranges, when the identified value of β or W_0 get stuck in local minimum value, we regenerate a random β or W_0 in their range so as to get the global least square value.

2. Then, the parameter λ_+ are identified from numerical bending tests and we keep $\lambda_- = 0$. The final parameters correspond to the λ leading to the lowest identification error in β and W_0. This strategy handles the nonlinearity in β and is well adapted to the low sensitivity in λ.

The analytical formula Eq.(1.9.2) with mean stress effect converges with the numerical method very well in the case of small β and λ_{+-}.

Parameter sensitivity analysis

The parameters we introduced during the deduction need to be calibrated. The source of the parameter identification are listed in Table.1.2. We perform a sensitivity analysis to see the influence of each parameter by comparing the results obtained respectively for the reference value, an upper bound and a lower bound of each parameter.

Parameters	**Strategy**
Hydrostatic pressure sensitivity λ_+	hydrostatic stress sensitivity (identified)
Non-linearity of damage accumulation a	amplification factor of load intensity (guessed)
Weakening scales distribution exponent β	to be calibrated (identified)
Dissipated energy to failure per defect W_0	energy scaling (identified)

Table 1.2 – Parameters concerned

We analyze the sensitivity of parameters separately as in Table.1.3(uniaxial) and Table.2.4(random loading). The parameter β has more influence on the random loading case because it acts not only as the S-N curve slope but also the power magnification factor of large stress intensity. The λ has little influence because both tests are conducted on very small or zero mean stress load history.

In Miner's law the parameter α is zero, the maximum value is below 1. For $\alpha = 1$ the damage accumulation line becomes flat and there will be unlimited lifetime. To keep α in the range of $[0, 1]$ where in random amplitude tests there is $S_a = 163.3MPa$; we set the sensitivity of load intensity a to a maximum value of 0.29 to keep α positive.

The weakening scale distribution exponent(also the slope of S-N curve of the material) β ranges from 1 to 5. The hydrostatic pressure sensitivity λ is from positive mean stress test, which has the range of $0 \sim 0.8$. In constant amplitude cyclic loading, the dissipated energy to failure per defect W_0(in MPa) is related to fatigue lifetime of the material.

Constant amplitude sensitivity test with $f(\beta) = \beta$							
	Ref	**Min**	**Max**	**Ref_n**	**Min_n**	**Max_n**	**Sensitivity**
β	1.1	1.05	1.50	414233	783723	243300	-3.19
λ_+	0.1	0.05	0.50	414233	449598	443376	0.00
W_0	3.27e8	1.00e8	5.00e8	414233	137498	687209	1.08
a	0.1	0.05	0.15	414233	672869	324754	-0.84

Table 1.3 – Example of parameters sensitivity at cyclic loading of BATCH_A_02 on AW-6106 T6 aluminum (table.2.2)

2

Numerical implementation and validation

2.1 Experimental verification

2.1.1 Introduction

The aim of this chapter is to validate the predictive model proposed. This consists in simulating tests available in the literature to determine the lifetime at initiation of crack by the application of the model and to compare these with the experimental lifetimes. The validation of the model involves a wide variety of metallic materials. The loads tested are of two types: cyclic loading of multiaxial stress of constant amplitude and repeated sequences of uniaxial stresses of variable amplitudes. The fatigue data of the materials used and the loads tested are taken from laboratory experiments or the literature.

2.1.2 Random amplitude 1D tests from Cetim on AW-6106 T6 aluminum

What makes automobile fatigue so difficult to predict is that, unlike standard tests done in a laboratory, an automobile's structure has to endure a complex, mostly random, set of static as well as cyclical stresses when in service. For example in Figure 2.1 which could represent load data from testing or measurement, extracting the cyclic information can be challenging.

As we mentioned before, the mean value of α depends on the loading pattern(sinusoidal, linear division points between max and min stresses in unit cycle,...), but our optimal time step numerical strategy is not loading pattern dependent because it equally divides the range of α during the load history, which means the variation amplitude of stress intensity. So in random loading case with only recorded maximum and minimum load history, we first divide linearly between every 2 recorded points into 100 time steps, and then perform numerical tests with optimal time step method.

The first tests are performed on aluminum batches, the characteristics of the sample are shown in Tab.2.1.

There are 12 validated uniaxial fatigue tests on the AW-6106 T6 aluminum sample, in which 2 are of constant amplitude load case and 10 involve random load case. The cyclic stress of test number 1(ep01) and test number 2(ep02) are respectively $131.9MPa$ and $97.0MPa$. We first identify the parameters from these two tests.

Figure 2.1 – Complex loading of a car suspension arm (data from PSA tests)

Figure 2.2 – Specimen geometry for fatigue tests of AW-6106 T6 aluminum(sample given by PSA)

Parameters	**Value**
Young's modulus	$E = 72$ GPa
Hardening parameter	$k = 8.5$ MPa
Macroscopic yield stress	$\sigma_y = 230$ MPa
Thickness	$e = 2.9mm$
Width	$l = 9.95mm$

Table 2.1 – Material parameters of AW-6106 T6 aluminum

The detailed tests information are shown in Tab.2.2. There are 27000($\pm 2.4\%$) recorded points per repetition.

We assume the material parameters like Young's modulus E, hardening parameter k, hydrostatic pressure sensitivity λ_+(for $\overline{\Sigma}_H = 0$ in all cases), macroscopic yield stress σ_y and sequence effect sensitivity a are known. We first identify the weakening scales distribution β, and dissipated energy to failure W_0 from cyclic tests ep01 and ep02. Then fit the major damage effect parameter f to see if

Figure 2.3 – Random loading history on BATCH_A_06 of AW-6106 T6 aluminum (see Tab.2.2)

Specimen	Fmax (kN)	Σ_{max} in the block	Number of repetition	Number of points
BATCH_A_01	3.375			99892
BATCH_A_02	2.475			414298
BATCH_A_04	nom	225.88	95	2500000
BATCH_A_05	nom	225.88	156	4105263
BATCH_A_06	nom	225.88	145	3815789
BATCH_A_07	nom	225.88	90	2368421
BATCH_A_08	nom	225.88	194	5105263
BATCH_A_09	nom	225.88	197	5184211
BATCH_A_10	nom x 0,9	203.292	515	13552632
BATCH_A_11	nom x 0,9	203.292	385	10131579
BATCH_A_12	nom x 0,9	203.292	424	11157895
BATCH_A_13	nom x 0,9	203.292	409	10763158
BATCH_B_01	nom	225.88	121	3184211
BATCH_B_02	nom x 0,8	180.704	380	10000000
BATCH_B_03	nom x 0,8	180.704	380	10000000
BATCH_B_04	nom x 0,9	203.292	406	10684211
BATCH_B_05	nom x 0,9	203.292	454	11947368
BATCH_B_06	nom x 0,9	203.292	518	13631579
BATCH_B_07	nom x 0,9	203.292	553	14552632
BATCH_B_08	nom x 0,9	203.292	612	16105263
BATCH_B_09	nom	225.88	253	6657895
BATCH_B_10	nom	225.88	196	5157895
BATCH_B_11	nom	225.88	178	4684211
BATCH_B_12	nom	225.88	123	3236842

Table 2.2 – Fatigue tests result on AW-6106 T6 aluminum, test data provided by CETIM

our assumption is correct or need to be changed.

The numerical fitting process show that the damage is caused mainly by large stresses (see later). The definition of major stress which affects the value of α now needs to be specified according to the material. To take into account this effect we first find out the proportion stress above a certain value in the repetition signal of random loading, as shown in Tab.2.3. Here ep_a and ep_b are the same material. Since the samples were extracted from aluminum profiles of industrial products, the two batches correspond to two different times of sampling in the production. The variation is supposed to be representative of the regular tolerances you might have in the production. ep_a_01 and ep_a_02 are constant amplitude loading which helps identify the power of weakening scale distribution β. ep_a_03 is low cycle fatigue data. ep_b_02 and ep_b_03 have infinite life time. The data in the table are grabbed from random signal high cycle fatigue loading history.

Stress(MPa)>	**70**	**90**	**110**	**130**	**150**	**170**	**190**
S_a**(MPa)>**	**57.15**	**73.48**	**89.81**	**106.14**	**122.47**	**138.80**	**155.13**
ep_a_04		1.962%	0.904%	0.077%	0.037%	0.018%	0.007%
ep_a_05		1.604%	0.784%	0.044%	0.030%	0.007%	0.007%
ep_a_06		1.645%	0.784%	0.045%	0.030%	0.007%	0.007%
ep_a_07		1.632%	0.788%	0.048%	0.029%	0.007%	0.007%
ep_a_08		1.644%	0.787%	0.048%	0.037%	0.007%	0.007%
ep_a_09		1.655%	0.800%	0.048%	0.037%	0.007%	0.007%
ep_a_10		0.768%	0.134%	0.007%	0.000%	0.000%	0.000%
ep_a_11		0.772%	0.145%	0.007%	0.000%	0.000%	0.000%
ep_a_12		0.779%	0.133%	0.011%	0.000%	0.000%	0.000%
ep_a_13		0.775%	0.141%	0.007%	0.000%	0.000%	0.000%
ep_b_01	4.739%	1.737%	0.840%	0.224%	0.049%	0.034%	0.004%
ep_b_04	1.999%	0.745%	0.156%	0.034%	0.004%	0.000%	0.000%
ep_b_05	2.010%	0.749%	0.148%	0.034%	0.008%	0.000%	0.000%
ep_b_06	1.999%	0.790%	0.118%	0.034%	0.008%	0.000%	0.000%
ep_b_07	2.029%	0.756%	0.152%	0.034%	0.008%	0.000%	0.000%
ep_b_08	1.999%	0.737%	0.137%	0.034%	0.008%	0.000%	0.000%
ep_b_09	4.663%	1.687%	0.798%	0.205%	0.049%	0.034%	0.004%
ep_b_10	4.712%	1.744%	0.809%	0.224%	0.046%	0.034%	0.004%
ep_b_11	4.636%	1.664%	0.790%	0.209%	0.049%	0.034%	0.004%
ep_b_12	0.775%	0.141%	0.007%	0.000%	0.000%	0.000%	0.000%

Table 2.3 – Proportion of stress(MPa) above different threshold with Σ_y=230MPa, test data provided by CETIM on AW-6106 T6 aluminum.

Random amplitude sensitivity test with $f(\beta) = \beta$							
	Ref	**Min**	**Max**	**Ref_n**	**Min_n**	**Max_n**	**Sensitivity**
β	1.1	1.05	1.50	4220452	7469257	1799585	-3.28
λ	0.1	0.05	0.50	4220452	4566335	2175991	-0.13
W_0	3.27e8	1.00e8	5.00e8	4220452	1321761	6420810	0.99
a	0.1	0.05	0.15	4220452	7156622	2827894	-1.03

Table 2.4 – Parameters sensitivity at random loading of ep05 on AW-6106 T6 aluminum

From Tab.2.5 and Tab.2.6 we can see f has positive correlation with β in high cycle fatigue which

is the regime we focus on. However, very large value of f may ignore the small stress variations in the loading history, which goes against our assumption that small stresses also contribute to material damage. So we give $f = 1.1$ in high cycle random loading case to minimize the relative error. The

Constant amplitude sensitivity test with $f(\beta) \neq \beta$							
	Ref	**Min**	**Max**	**Ref_n**	**Min_n**	**Max_n**	**Sensitivity**
β	1.1	1.05	1.50	414233	797377	213682	-3.44
λ	0.1	0.05	0.50	414233	449598	443376	0.00
W_0	3.27e8	1.00e8	5.00e8	414233	137498	687209	1.08
a	0.1	0.05	0.15	414233	672869	324754	-0.84
$f(\beta)$	1.1	1.05	1.5	414233	441661	511644	0.41

Table 2.5 – Parameters sensitivity at cyclic loading of ep02 on AW-6106 T6 aluminum

Random amplitude sensitivity test with $f(\beta) \neq \beta$							
	Ref	**Min**	**Max**	**Ref_n**	**Min_n**	**Max_n**	**Sensitivity**
β	1.1	1.05	1.50	4220452	7254554	2472791	-2.77
λ	0.1	0.05	0.50	4220452	4566335	2175991	-0.13
W_0	3.27e8	1.00e8	5.00e8	4220452	1321761	6420810	0.99
a	0.1	0.05	0.15	4220452	7156622	2827894	-1.03
$f(\beta)$	1.1	1.05	1.5	4220452	4341560	3052299	-0.75

Table 2.6 – Parameters sensitivity at random loading of ep05 on AW-6106 T6 aluminum

sensitivity of parameters is calculated by dividing the percentage of variation of number of points to failure with respect to the reference number of points to failure, by the percentage of variation of parameter with respect to the reference parameter, as shown in Eq.(2.1.1).

$$sensitivity = \frac{(Max_n - Min_n)/Ref_n}{(Max - Min)/Ref}. \tag{2.1.1}$$

After the fitting process, the reference parameters value we use are in Tab.2.7.

Constant α	W_0**(MPa)**	$\lambda_+ = \lambda_-$	β	α	f
	326.9	0.1	1.1	0.7	1.1
Changing α	W_0**(MPa)**	$\lambda_+ = \lambda_-$	β	a	f
	326.9	0.1	1.1	0.1	1.1

Table 2.7 – The parameters in 1D cyclic and random loading on AW-6106 T6 aluminum fatigue tests by Cetim

The best fitted results with constant α are shown in Figure 2.4. The dispersion is relatively large. In conclusion, we are not able to predict the random stress amplitude fatigue life with fixed α, because random stresses not only cause different energy dissipations, but also show a distinctive load sequence effect. So we have to update the value of α at each time step.

We can find that the numerical results are satisfactory when we introduce a major damage influence through the construction of a load dependent α. The dispersion figure with distinction of major damage is depicted in Figure 2.5. Here it is necessary to control the parameter a to make sure $\alpha > 0$ in the most severe situation.

Figure 2.4 – Comparison between experimental and numerical results of 1D cyclic and random loading on aluminum fatigue tests by CETIM with constant α from the first row of Tab.2.7

Figure 2.5 – Comparison between experimental and numerical results of 1D cyclic and random loading on aluminum fatigue tests by Cetim with load dependent α. Coefficients data are given in the second row of Tab.2.7

2.2 Experimental validation of the model on aluminum 6082 T6

2.2.1 Presentation of aluminum 6082 T6

The material tested is aluminum 6082 T6, used by Susmel and Petrone [2003] to validate their method of lifetime prediction. The mechanical properties of this material are summarized in Tab.2.8.

E[GPa]	σ_y[MPa]	σ_u[MPa]	ν
69.4	298	343	0.33

Table 2.8 – Mechanical and dynamic characteristics of aluminum 6082 T6 (Susmel and Petrone [2003])

Specimens of aluminum 6082 T6

The specimens were made from the drawn bars (diameter 30 mm) and the geometrical shape of which is given in Figure 2.6. They are successively polished with $6 - \mu m$ diamond compounds until a good mirror-like finish is obtained.

Figure 2.6 – Specimen geometry for fatigue tests of aluminum 6082 T6 (dimension in millimeters)**??**

2.2.2 Fatigue tests on aluminum 6082 T6

The simulated tests are purely alternate and summarized in Tables 2.9 and 2.10. They consist of simple tests in bending, torsion and bending-torsion in phase and out-phase for two cases of biaxial stress ratio, $\lambda = \tau_{xy,a}/\sigma_{x,a}$ ($\lambda > 1$ and $\lambda < 1$). The expected lifetimes range from 10^4 to 1.5×10^6 cycles.

In the Tables 2.9 and 2.10, $\sigma_{x,a}$ is the normal stress amplitude, $\tau_{xy,a}$ is the torsion amplitude, λ is the biaxial stress ratio, δ is the phase shift between the components of applied stresses and $N_{f,5\%}$ represents the number of cycles at break, defined by a 5% decrease in flexural or torsional stiffness.

It is interesting to note that a reduced amount of plasticity was measured by strain gauges in the PC10T2, PC14T2 and P36BT11 tests (Susmel and Petrone [2003]). They are therefore located in the field of oligocyclic fatigue. Therefore, they are not simulated as we only deal with the field of polycyclic fatigue3.

Batch $N°$	$\sigma_{x,a}$[MPa]	$\tau_{xy,a}$[MPa]	λ	$\delta[°]$	$N_{f,5\%}$[Cycles]
P1B1	190	0	0	0	160000
P2B2	180	0	0	0	248518
P3B3	164	0	0	0	444411
P4B4	144	0	0	0	1069220
P5B5	224	0	0	0	56285
P6B4	145	0	0	0	1238325
P7B1	187	0	0	0	200480
P8B3	161	0	0	0	423590
PC9T1	0	117	∞	0	534032
PC10T2	0	155	∞	0	26987
PC11T3	0	127	∞	0	76665
PC12T3	0	127	∞	0	132295
PC13T1	0	117	∞	0	203535
PC14T2	0	155	∞	0	16195
PC15T4	0	106	∞	0	>1.1E6
PC16T4	0	104	∞	0	565150

Table 2.9 – Simple bending and torsion tests (R = -1), data from Susmel and Petrone [2003]

2.2.3 Identification of model parameters on aluminum 6082 T6

Once the average coefficient α_m is fixed in constant amplitude cyclic loading,it has the same influence as W_0. The parameters remain to calibrate are λ_+ on the mean stress sensitivity which makes a distinction between bending and torsion, and the exponent β on the slope of $S - N$ curve. The identification strategy is as described in Section 1.9.

In Figure 2.9a and Figure 2.9b the diagonal represents a good correlation between the experimental and predicted lifetimes. The line segments on either side of the diagonal correspond to a fatigue lifetime error of a factor of two. The parameters of the 6082 T6 aluminum model are given in Tab.2.11.

Non-proportional Hardening

Non-proportional hardening is used to describe loading paths where the principal strain axes rotate during cyclic loading. The simplest example would be a bar subjected to alternating cycles of tension and torsion loading. Between the tension and torsion cycles the principal axis would rotate $45°$. Out-of-phase loading is a special case of non-proportional loading and is used to denote cyclic loading histories with sinusoidal or triangular waveforms and a phase difference between the loads. (EFA)

Materials show additional cyclic hardening during this type of loading that is not found in uniaxial or any proportional loading path. Here is an example for $90°$ out-of-phase tension-torsion loading. (Figure 2.7)

The $90°$ out-of-phase loading path has been found to produce the largest degree of non-proportional hardening. The magnitude of the additional hardening observed for this loading path as compared to that observed in uniaxial or proportional loading is highly dependent on the microstructure and the ease with which slip systems develop in a material. A non-proportional effective stress, $S_{a90} = \sqrt{\sigma_{11}^2 + \tau_{12}^2}$, can be introduced which is defined as the equivalent stress under $90°$ out-of-phase loading at high plastic strains in the flat portion of the stress-strain curve. This term reflects the maximum degree of additional hardening that might occur for a given material.

The model predictive results for the periodic loads of constant amplitude with a radial path are in

Batch $N°$	$\sigma_{x,a}$[MPa]	$\tau_{xy,a}$[MPa]	λ	$\delta[°]$	$N_{f,5\%}$[Cycles]
P17BT1	57	100	1.75	0	266435
P18BT2	51	84	1.65	0	1119254
P19BT2	51	84	1.65	0	1416225
P20BT3	71	118	1.66	0	83000
P21BT3	70	118	1.69	0	75695
P22BT1	59	99	1.68	0	630325
P23BT4	132	97	0.73	0	157210
P24BT4	132	99	0.75	0	126470
P25BT5	144	107	0.74	0	35450
P26BT5	149	105	0.7	0	68465
P27BT6	122	90	0.74	0	252658
P28BT7	116	83	0.72	0	316149
P30BT8	148	66	0.45	90	278836
P31BT9	152	47	0.31	90	465010
P32BT8	149	68	0.46	90	118965
P33BT9	155	72	0.46	90	447525
P34BT10	190	105	0.55	90	47940
P35BT10	189	106	0.56	90	30995
P36BT11	79	129	1.63	90	23080
P37BT12	69	110	1.59	90	202807
P38BT13	68	99	1.46	90	262980
P39BT13	68	99	1.46	90	398615
P41BT15	79	116	1.47	90	46045

Table 2.10 – In-phase and out-of-phase bending-torsion tests (R = -1), data from Susmel and Petrone [2003]

β	λ_+	λ_-	W_0	a	f
5.126	0.9	0	1E8 Pa	0.4	1.1

Table 2.11 – Parameter identification of AL6082T6 steel

good agreement with the durations of experimental life. For the latter condition, 90 deg out-of-phase loading was also investigated(Figure 2.9c). These tests indicated a dramatic change in the number of cycles to failure , N_F , as a result of out-of-phase loading. The influence of the plastic strain path on life is thus clearly demonstrated. It is shown that the total strain energy density, $W_t = W_e + +W_p$ (Ellyin et al. [1991]) , correlates with both the in-phase and out-of-phase cyclic tests, and therefore is a proper damage parameter to be used for life predictions. Our microplasticity model does not take this strain path effect into account and we get inaccurate results on this test.

Figure 2.7 – The loading path and loading waveform of multiaxial corrosion fatigue (stress control) (a) loading path of proportional and non-proportional, (b) loading waveform of proportional loading, (c) loading waveform of non-proportion (Huang et al. [2017])

Figure 2.8 – The 90° out-of-phase loading path has been found to produce the largest degree of non-proportional hardening. (EFA)

(a) Bending and torsion tests on 6082 T6 aluminum(R=-1)

(b) Bending-torsion tests on 6082 T6 aluminum(R=-1)

(c) Bending-torsion 90° out of phase tests on 6082 T6 aluminum(R=-1)

Figure 2.9 – Calibration on on 6082 T6 aluminum (Susmel and Petrone [2003]). Comparison between experimental results and our model used with coefficients given in Tab.2.11. We obtain a good correlation in bending and torsion tests. The out of phase test are not satisfactory in these batches: P32BT8, P41BT15, P36BT11.

2.3 Experimental validation of the model on 30NCD16 steel

2.3.1 Presentation of steel 30NCD16

Tests with blocks of loading from database are compared to our model predictions. The material for testing is steel 30NCD16. The mechanical characteristics relating to each lot were determined by Dubar (DUBAR [1992]) by effecting monotonic tensile test batch. He eventually define “average material" one who has characteristics listed in Tab.2.12:

$\sigma_{y0.02\%}$**[MPa]**	$\sigma_{y0.2\%}$**[MPa]**	σ_u**[MPa]**	σ_{-1}**[MPa]**	τ_{-1}**[MPa]**	E**[GPa]**
895	1080	1200	690	428	191

Table 2.12 – Mechanical and dynamic characteristics of 30NCD16 steel DUBAR [1992]

2.3.2 Fatigue tests performed by Dubar on steel 30 NCD 16

Tests carried out under simple bending and torsional stresses are grouped together in Tab.2.13 and 2.14.

Bending Tests (R=-1)	N [Cycles]	$\sigma_{x,m}$ [MPa]	$\sigma_{x,a}$ [MPa]
1	51000	0	820
2	80000	0	795
3	90000	0	790
4	95000	0	785
5	100000	0	780
6	120000	0	765
7	140000	0	752
8	200000	0	725
9	210000	0	720
10	230000	0	715
11	250000	0	708

Table 2.13 – $30NCD16$ steel fully reversed bending tests DUBAR [1992]

The results of combined bending-torsion tests in phase with or without mean stress $\sigma_{x,m}$ are given in the Tab.2.14 and Tab.2.15:

2.3.3 Identification of model parameters for steel 30 NCD 16

The identification of the parameters consists in minimizing the relative difference between the experimental lifetimes and calculated ones for purely alternating bending tests (R = -1). Following the identification strategy of section 1.9, it is clearly indicated in Figure 2.10a by obtaining a good correlation between these different lifetimes

Torsion Tests (R=-1)	N [Cycles]	$\tau_{xy,a}$ [MPa]
16	51000	527
17	80000	505
18	90000	500
19	95000	497
20	100000	495
21	120000	482
22	140000	470
23	200000	450
24	210000	446
25	230000	445
26	250000	440

Table 2.14 – $30NCD16$ steel fully reversed torsion tests (DUBAR [1992])

Bending Tests (R=-1)	N [Cycles]	$\sigma_{x,m}$ [MPa]	$\sigma_{x,a}$ [MPa]	$\tau_{xy,a}$ [MPa]
27	80000	0	600	335
28	200000	0	548	306
29	120000	290	0	460
30	120000	450	0	460
31	250000	450	0	430
32	95000	450	490	285
33	120000	290	500	290

Table 2.15 – $30NCD16$ steel bending-torsion tests (DUBAR [1992])

β	λ_+	λ_-	W_0	a	f
5.3	0.55	0	4.97E8 Pa	0.4	1.1

Table 2.16 – Parameter identification of 30NCD16 steel

(a) Bending and torsion tests on 30NCD16(R=-1)

(b) Bending-torsion tests with mean stress on 30NCD16

Figure 2.10 – Calibration on 30NCD16 (DUBAR [1992]). We can observe that the bending-torsion experimental values are largely dispersed

(a) Bending and torsion tests on 30NCD16(R=-1)

(b) Bending-torsion tests with mean stress on 30NCD16

Figure 2.11 – Calibration on 30NCD16 (DUBAR [1992]). In figure (c) data 29(same N_F with data30 but with smaller $\sigma_{x,m}$) and 32(2-D with large mean stress) from Tab.2.15 are more dispersed. The numerical tests are carried out using the coefficients of Tab.2.16

2.4 Experimental validation of the model on SM45C steel

2.4.1 Presentation of steel SM45C

This is a structural steel widespread use for the crankshafts and the structural components. The chemical composition and mechanical properties of this material is given in Tab. 2.17 and Tab. 2.18.

C	**Mn**	**P**	**S**	**Si**	**Ni**	**Cr**	**Cu**
0.42	0.73	0.02	0.012	0.28	0.14	0.18	0.13

Table 2.17 – Chemical composition of SM45C steel

σ_y **[MPa]**	σ_u **[MPa]**	**E [GPa]**	**G [GPa]**	ν	**A**
638	824	213	82.5	0.29	22

Table 2.18 – Mechanical and dynamic characteristics of SM45C steel

E: Young's modulus,
G: Shear modulus,
ν: Poisson ratio,
A: Elongation at break.

2.4.2 Fatigue tests performed by Dubar on steel SM45C

Preliminary fatigue tests in purely alternating torsion and purely alternating flexion were performed by Lee [2013]. These two types of tests were carried out with test pieces of the same geometric shape. In addition, the author had performed moderate stress bending fatigue tests to study its effect on the lifetime of SM45C steel. All uniaxial fatigue tests performed by Lee [2013] are illustrated in Figure 2.12. This figure shows a reduction in bending life of SM45C steel in the presence of a positive mean stress. Crack initiation was detected when the stiffness of the specimen or specimen used was reduced by 10%.

Figure 2.12 – Fatigue curves on SM45C steel by Lee [2013]

Preliminary fatigue tests were carried out under fully reversed bending and torsion separately.

Bending Tests R=-1	N [Cycles]	$\sigma_{x,a}$ [MPa]	$\sigma_{x,m}$ [MPa]
1	17520	632.1	0
2	33991	590.1	0
3	52427	552.0	0
4	91077	529.5	0
5	156882	506.5	0
6	222261	489.8	0
7	446115	466.7	0
8	822487	463.8	0
9	1279414	459.2	0
10	1453321	463.8	0
11	2440360	454.0	0
12	3428115	455.2	0
13	6880791	450.0	0
14	6213809	437.3	0
15	9342857	441.9	0
16	7240667	424.1	0
1	43043	541.5	196
2	55523	511.5	196
3	74725	514.4	196
4	75362	493.6	196
5	110407	490.7	196
6	146090	471.1	196
7	194951	455.5	196
8	212218	452.0	196
9	297990	430.1	196
10	440286	409.3	196
11	678727	407.6	196
12	597603	386.8	196

Table 2.19 – SM45C steel fully reversed bending tests(extracted from Lee [2013])

Torsion Tests R=-1	N [Cycles]	$\tau_{xy,a}$ [MPa]	$\sigma_{x,m}$ [MPa]
1	27957	404.1	0
2	47749	394.9	0
3	76194	375.3	0
4	100000	363.1	0
5	162305	354.5	0
6	182807	345.8	0
7	296705	338.3	0
8	575636	331.4	0
9	822487	329.1	0
10	2203806	322.2	0

Table 2.20 – SM45 steel fully reversed torsion tests(extracted from Lee [2013])

Group	N [Cycles]	τ_a [MPa]	σ_a [MPa]	σ_m [MPa]
A	29.9E3	282	449	0
	35.7E3	334	354	0
	50E3	223	485	0
	73.8E3	309	357	0
	106E3	217	449	0
	106E3	285	370	0
	112E3	199	449	0
	131E3	194	457	0
	333E3	252	354	0
	431E3	154	437	0
B	53E3	215	441	196
	59.2E3	309	286	196
	70.1E3	155	464	196
	86.3E3	136	473	196
	89.9E3	334	173	196
	92.1E3	209	403	196
	102E3	177	437	196
	135E3	321	167	196
	351E3	179	357	196
	394E3	274	182	196

Table 2.21 – Effect of mean bending stress on out-of-phase(90°) fatigue of SM45C steel (Lee [2013])

2.4.3 Identification of model parameters for steel SM45C

We use the same identification strategy as described in Section 1.9. The fitted curve using experimental data in Tab.2.21 and data with mean stress effect is shown in Figure 2.14b. The tests on SM45C steel have illustrated that the mean bending stress has an influence on both uniaxial and multiaxial fatigue life.

Although the uniaxial experimental data we extracted from Lee's curve (Lee [2013]) of SM45C steel are slightly dispersed, we can find our model quite satisfactory in the case of SM45C steel. As for multiaxial 90 degree out of phase, fully reversed bending-torsion fatigue tests, our model is able to evaluate the cycles to failure.

$\beta(LCF/HCF)$	λ_+	λ_-	$W_0(LCF/HCF)$	a	f
6/15.8	1.4	0	2.6E8/5.6E5 Pa	0.1	1.1

Table 2.22 – Parameter identification of SM45C steel

(a) Bending and torsion test on SM45C steel(R=-1)

(b) Bending test with mean stress on SM45C steel ($\sigma_m = 196MPa$)

(c) Bending-torsion 90 degree out-of-phase tests on SM45C steel

(d) Bending-torsion 90 degree out-of-phase tests with mean stress on SM45C steel

Figure 2.13 – Calibration on SM45C

(a) Bending and torsion test on SM45C steel (R=-1)

(b) Bending test with mean stress on SM45C steel ($\sigma_m = 196MPa$)

(c) Bending-torsion 90 degree out-of-phase tests on SM45C steel

(d) Bending-torsion 90 degree out-of-phase tests with mean stress on SM45C steel

Figure 2.14 – Calibration on SM45C. The numerical tests are carried out using the coefficients of Tab.2.22

2.5 Experimental validation of the model on 10 HNAP steel

2.5.1 Presentation of the material

Fatigue tests were performed on the HNAP steel. It is a very low carbon steel which resembles the 10 CN 6. In Tab.2.23, its chemical composition is given: The mechanical properties of this steel

C	Mn	Si	P	S	Cr	Cu	Ni	Fe
0.12%	0.71%	0.41%	0.08%	0.03%	0.81%	0.30%	0.50%	the rest

Table 2.23 – Chemical composition of 10 HNAP steel, data from Bedkowski [1994]

are given in Tab.2.24:

$Re_{0.2\%}$	R_m	A	ν	E
418 MPa	566 Mpa	32%	0.29	215000 Mpa

Table 2.24 – Mechanical characteristics of steel 10 HNAP, data from Bedkowski [1994]

where

$Re_{0.2\%}$: elastic limit at 0.2% of plastic deformation,
R_m : maximum tensile strength,
A : elongation at break,
ν : Poisson's coefficient,
E : Young's modulus.

2.5.2 Description of fatigue tests on 10 HNAP steel

The Macha team performed a large number of fatigue tests on the HNAP steel. Thus, it performed not only simple tensile compression and torsion tests (R = -1) in order to establish the corresponding Wöhler curves but also tests under variable loading on cylindrical specimens of the same material (ACHTELIC [1994]). VIDAL carried out tensile tests on this material for various mean stress values. It has established the Wöhler curve in repeated traction in order to validate on this steel the method of Robert whose use requires three Wöhler curves in symmetrical alternating traction, symmetrical alternating torsion and repetitive traction.

Wöhler curve in tension-compression

The model chosen by Macha and recovered by Jabbado [2006] for the tensile-compression Wöhler curve is that of Basquin:

$$lnN = 68.3619.82ln\left(\sigma_{-1}\right), \tag{2.5.1}$$

Wöhler curve in symmetrical alternating torsion

The symmetric alternating torsion Wöhler curve was recovered by Jabbado [2006] using following equation:

$$lnN = 21.550.0385\tau_{-1}. \tag{2.5.2}$$

Tensile fatigue tests for various mean stress values

VIDAL carried out tensile tests on HNAP steel for various values of mean stress. The results are summarized in Tab.2.25. They allowed us to plot the Wöhler curves for different values of the mean stress σ_m.

N_F	$\Sigma_{xx,a}$	σ_m	$\Sigma_{xx,a}$	σ_m	$\Sigma_{xx,a}$	σ_m	$\Sigma_{xx,a}$	σ_m
1.00E+05	311.30	75	224.42	150	257.98	225	251.82	300
2.00E+05	289.36	75	208.40	150	242.47	225	224.42	300
3.00E+05	276.53	75	197.03	150	233.40	225	208.40	300
4.00E+05	267.43	75	188.21	150	226.96	225	197.03	300
5.00E+05	260.37	75	181.00	150	221.97	225	188.21	300
6.00E+05	254.60	75	174.91	150	217.89	225	181.00	300
7.00E+05	249.72	75	169.63	150	214.44	225	174.91	300
8.00E+05	245.49	75	164.97	150	211.46	225	169.63	300
9.00E+05	241.77	75	160.81	150	208.82	225	164.97	300
1.00E+06	238.43	75	233.29	150	206.47	225	160.81	300

Table 2.25 – Experimental results of tensile tests for various values of σ_m, data from VIDAL

Fatigue testing under variable loading

Random multiaxial loading fatigue tests were performed on cylindrical HNAP steel specimens (ACHTELIC [1994]). The load considered is proportional and results from a combination of bending and torsion. The random signal is stationary and has a normal distribution as a probability distribution. Tests of this type have been analyzed and simulated by Carpinteri et al. (Carpinteri et al. [2003]). They were provided to us in the form of tests carried out on the HNAP steel for two values of the angle α': $\alpha' = \pi/8$ and $\alpha' = \pi/4$. α' is the angle made by the resultant moment M with the bending moment M_B (see Figure 2.15). The angle is kept constant during each individual test.

Figure 2.15 – Bending-torsion fatigue tests on cylindrical specimens (Carpinteri et al. [2003])

The stationary random loading sequence contains 49152 values recorded by a time interval of 0.00375 seconds (frequency = 266.67 Hz). It is shown in Figure 2.16. Its total duration is 184.32 seconds. This sequence is multiplied by load coefficients corresponding to bending $f(\sigma_{xx})$ and torsion $f(\tau_{xy})$ in order to obtain random multiaxial loading sequences. As the signal is stationary, the breaking life is determined in terms of number of sequences with break N_{Sq}. Knowing N_{Sq} and the total time in seconds of the sequence studied, it is easy to express the lifetime of the piece in seconds. The results of fatigue tests under variable loads are summarized in Tab.2.26 and Tab.2.27 as a function of angle α' and ratio r; $r = f(\tau_{xy})/f(\sigma_{xx})$.

1st type of tests: $\alpha' = \pi/8$ and $r = f(\tau_{xy})/f(\sigma_{xx}) = 0.2$.

2nd type of tests: $\alpha' = \pi/4$ and $r = f(\tau_{xy})/f(\sigma_{xx}) = 0.5$.

In Figure 2.17, an example of a random multiaxial loading sequence is given.

Figure 2.16 – Bending-torsion fatigue tests on cylindrical specimens (Carpinteri et al. [2003])

N^o	$f(\sigma_{xx})$	$f(\tau_{xy})$	r	$T_{exp}(s)$
1	5.7084	1.1822	0.2	16843.2
2	5.2917	1.0959	0.2	17780.1
3	4.8337	1.0010	0.2	24416.5
4	5.2674	1.0909	0.2	24858.2
5	5.4534	1.1294	0.2	26518.3
6	5.2002	1.0769	0.2	36162.3
7	4.7944	0.9929	0.2	47600.4
8	4.3862	0.9084	0.2	57993.9
9	4.6241	0.9576	0.2	60428
10	4.0194	0.8324	0.2	73373.3
11	4.0127	0.8310	0.2	87609.1
12	4.2292	0.8758	0.2	89185.2
13	3.9213	0.8121	0.2	106900
14	3.7731	0.7814	0.2	117358
15	4.1148	0.8521	0.2	118902
16	3.6150	0.7486	0.2	132448
17	3.3135	0.6862	0.2	170571
18	4.1298	0.8553	0.2	178215
19	3.4761	0.7199	0.2	225288
20	3.3430	0.6923	0.2	352635
21	3.0135	0.6241	0.2	355720

Table 2.26 – Fatigue results under variable loads for $\alpha' = \pi/8$ and $r = f(\tau_{xy})/f(\sigma_{xx}) = 0.2$

2.5.3 Identification of model parameters of 10HNAP steel

As the previous tests, we identify the slope β in pure torsion tests without the mean stress effect. Then fit λ_+ and W_0 with bending tests($R = -1$). The parameters of the HNAP steel model can be

N^o	$f(\sigma_{xx})$	$f(\tau_{xy})$	r	$T_{exp}(s)$
1	4.2519	2.126	0.5	15379.4
2	4.0567	2.0284	0.5	21465.7
3	3.8982	1.9491	0.5	25350.4
4	3.7823	1.8912	0.5	45949
5	3.5963	1.7982	0.5	62434.8
6	3.4497	1.7249	0.5	75225.7
7	2.9423	1.4712	0.5	115009
8	2.8814	1.4407	0.5	136794
9	2.3299	1.165	0.5	203365
10	2.8399	1.42	0.5	221370
11	2.8493	1.4247	0.5	244757
12	2.2542	1.1271	0.5	251723
13	2.3651	1.1826	0.5	288080
14	2.4215	1.2108	0.5	405444

Table 2.27 – Fatigue results under variable loads for $\alpha' = \pi/4$ and $r = f(\tau_{xy})/f(\sigma_{xx}) = 0.5$

Figure 2.17 – Multiaxial random loading sequence

identified by referring to Tab.2.28. They are grouped in Tab.2.28:

β	λ_+	λ_-	W_0	a	f
5.3	1.7	0	6.48E8 Pa	0.4	1.1

Table 2.28 – Model parameters for 10HNAP steel

The constant amplitude loading tests corresponds to number of cycles to failure in the range of $1E5 \sim 1E6$. However, in random loading case, the total reversals to failure are $2E6 \sim 5E7$ (calculated from Tab.2.26 and 2.27). These two cases belongs to different mechanisms, so their sensitivity of sequence effect and mean stress effect can be different as shown in Tab.2.28 and 2.29.

β	λ_+	λ_-	W_0	a	f
5.3	0.3	0	2.2E8 Pa	0.001	1.1

Table 2.29 – Model parameters for 10HNAP steel (random loading)

2.5.4 Simulation of fatigue tests performed on 10HNAP steel

The constant amplitude unidimensional tests data are show in Tab.2.30 and 2.31.

N^o	N_F	$\Sigma_{xx,a}$
1	1.00E+05	326.69
2	2.00E+05	304.42
3	3.00E+05	292.11
4	4.00E+05	283.68
5	5.00E+05	277.30
6	6.00E+05	272.20
7	7.00E+05	267.96
8	8.00E+05	264.34
9	9.00E+05	261.19
10	1.00E+06	258.41

Table 2.30 – Constant amplitude bending tests performed on 10HNAP steel, data from VIDAL

N^o	N_F	$\Sigma_{xy,a}$
1	1.00E+05	260.70
2	2.00E+05	242.70
3	3.00E+05	232.17
4	4.00E+05	224.70
5	5.00E+05	218.90
6	6.00E+05	214.16
7	7.00E+05	210.16
8	8.00E+05	206.69
9	9.00E+05	203.63
10	1.00E+06	200.90

Table 2.31 – Constant amplitude torsion tests performed on 10HNAP steel, data from VIDAL

After determining the parameters of the 10HNAP steel model for each of the batches tested, the number of priming cycles can be obtained by directly applying equation (1.7.15) for the proportional periodic loads of constant amplitude and for multiaxial loadings of variable amplitude.

In Figure 2.22 and Figure 2.23, we give the prediction results of the torsion tests used to identify β and W_0 of the model, then we use the bending with various mean stress to get the parameter λ_+. These are to be taken with caution because of the effect of the gradient because the specimens stressed in tension and in torsion are not of the same nature.

The prediction results of the tensile tests for various values of the mean stress σ_m are summarized in Figure 2.23. These results correlate well with the experimental lifetimes.

The tests of multiaxial loadings of variable amplitude are plotted in Figure 2.24 and Figure 2.25 as a function of the angle α_M and the ratio r. In these figures, the prediction results of the proposed model and that presented by Carpinteri et al. [2003]. For the first type of tests ($\alpha_M = \pi/8$ and r =

Figure 2.18 – S_a of bending tests with mean stress on 10HNAP

0.2), and the second type of tests ($\alpha_M = \pi/4$ and r = 0.5), the predictions of Carpinteri et al. [2003] are both good.

Figure 2.19 – $Hydro_{+-}$ of bending tests with mean stress on 10HNAP

Figure 2.20 – Bending and torsion test on 10HNAP steel(R=-1). Data are presented in Tab.2.30 and 2.31. The torsion best fit and the bending numerical results(optimal time step of method 2 deduced in Chapter 1) are obtained with the coefficients of Tab.2.28

Figure 2.21 – Wöhler tensile curves for various mean stress values. Data are presented in Tab.2.25 and results are obtained with the coefficients of Tab.2.28

Figure 2.22 – Calibration on 10HNAP steel, bending and torsion tests on 10HNAP(R=-1). Data are presented in Tab.2.30 and 2.31 and results obtained with the coefficients of Tab.2.28

Figure 2.23 – Calibration on 10HNAP steel, bending tests with various mean stress on 10HNAP, data from Tab.2.25 and results obtained with the coefficients of Tab.2.28

Figure 2.24 – Random bending-torsion 2D tests on 10HNAP, data from Tab.2.26 and Jabbado [2006]. Results are obtained with the coefficients of Tab.2.29 ($\alpha_M = \pi/8$ and r = 0.2)

Figure 2.25 – Random bending-torsion 2D tests on 10HNAP, data from Tab.2.27 and Jabbado [2006]. Results are obtained with the coefficients of Tab.2.29 ($\alpha_M = \pi/4$ and r = 0.5)

2.6 Conclusions

We work on the stress tensor directly in 3D analysis in stead of using the multidimensional equivalent stress. The strategy can be made more complex by introducing a local space averaging process in the calculation of the local damage, and by taking more general plastic flows. The energy based fatigue approach takes into account impurities and hardness in the material and is applicable to any type of micro plasticity law and multiaxial load geometry. The time implicit strategy gets rid of cycle counting which is hardly applicable to complex loading, big fluctuation is magnified which reflects the real situation.

There are several advantages and drawbacks of our proposed model. The time implicit method does not take the unit of cycle so as to avoid cycle counting and relevant methods such as rain-flow filter. The possibility to handle different S-N curves corresponding to various materials and load conditions via changing the parameters. We also have the random loading suitability with nonlinear damage accumulation. The drawback is this strategy requires a scale by scale analysis which can be complicated for very high cycle fatigue. However, as introduced above, we can use the optimal time step method to calculate precisely the representative loading history sequence and use scalar integration for the rest of fatigue life. In this way the numerical cost can be dramatically reduced without losing the precision.

Since our method is based on the Dang Van paradigm, to deal with mean stress effect and multiaxial loads we only have the parameter λ_{+-}, which is insufficient to fit the experiments. In fact, our microplasticity model is probably too crude to handle situations where there is a clear strain path effect.

Also, we need more experimental data and comparison with other results form the literature.

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