A new strategy for fatigue analysis in presence of general multiaxial time varying loadings

Keywords : Fatigue ; Energy ; High cycle ; Plasticity ; Mean stress

Ma Zepeng, Patrick Le Tallec

Solid Mechanics Laboratory
Ecole Polytechnique

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Outline

Outline

Outline

Physical consideration

1. We follow the Dang Van paradigm. The structure is elastic at the macroscopic scale.
2. At each material points, there is a stochastic distribution of weak points which will undergo strong plastic yielding.
3. Fatigue function of energy dissipation.

Statistics method

From a microscopic point of view, there is a distribution of weakening scales, namely $s \in [1, \infty)$.

- $1 \leqslant s \leqslant \sigma_y / S_{max} \longrightarrow S_{max} \leqslant \sigma_y / s \longrightarrow$ elastic regime $\longrightarrow$ no energy dissipation.
- $\sigma_y / S_{max} \leqslant s \leqslant \infty \longrightarrow S_{max} \geqslant \sigma_y / s \longrightarrow$ plastic regime $\longrightarrow$ energy dissipation.

We assume the weakening scales have a probability distribution of power law :

$$\boxed{P(s) = Cs^{-\beta}}$$

Statistics method

Weakening scales distribution

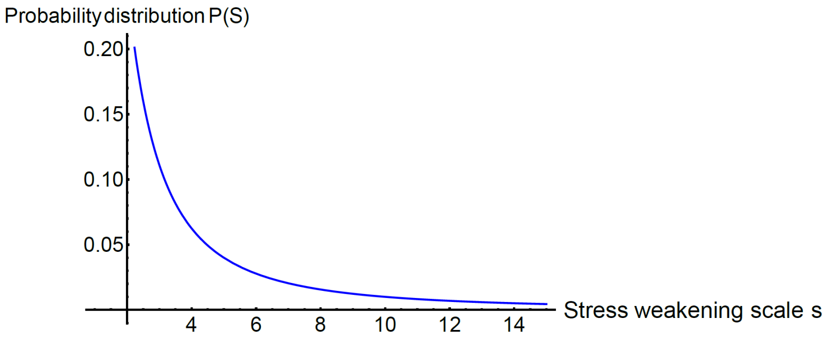


FIGURE : Weakening scales s probability distribution curve

Outline

Mean stress effect

The idea is to consider Maitournam and Krebs' work that the yield limit σ_y is reduced in presence of positive mean stress :

$$f(s) = ||\underline{\underline{S}}(s) - \underline{\underline{b}}(s)|| + (\lambda\Sigma_H - \sigma_y)/s \leqslant 0 \quad (1)$$

with $\underline{\underline{S}}(s)$ denoting the deviatoric part of the stress tensor at microscale, and $\underline{\underline{b}}(s)$ the corresponding backstress at the same scale.

Outline

Description of the mesoscopic stress state

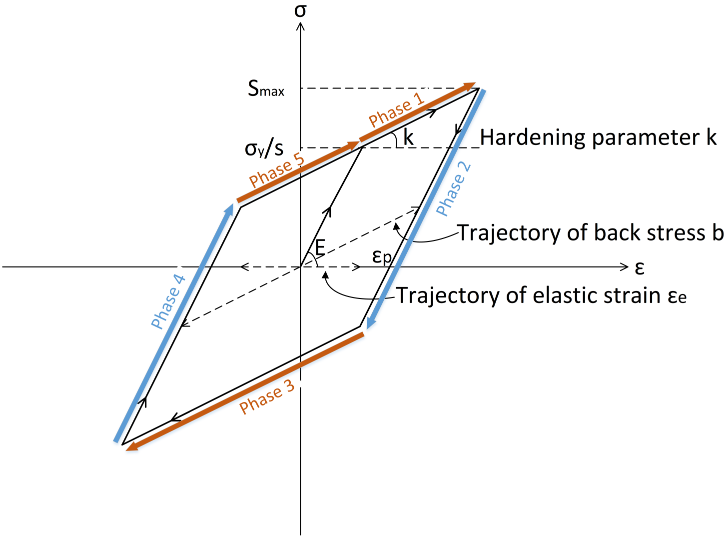
- $\dot{\underline{\underline{S}}}(s, M, t) = dev\dot{\underline{\underline{\Sigma}}}(M, t) - \frac{E}{1+\nu}\dot{\underline{\underline{\varepsilon}}}^p(s, M, t)$, Taylor-Lin scale transition model.
- $\dot{\underline{\underline{b}}}(s, M, t) = \frac{kE}{E-k}\dot{\underline{\underline{\varepsilon}}}^p(s, M, t)$, kinematic hardening model.
- $\dot{\underline{\underline{\varepsilon}}}^p(s, M, t) = \gamma\frac{\partial f(s, M, t)}{\partial \underline{\underline{S}}}$, associated plastic flow rule.

The local dissipated energy rate per volume at weakening scales s is given by :

$$\dot{w}(s, M, t) = (\underline{\underline{S}} - \underline{\underline{b}})(s, M, t) : \dot{\underline{\underline{\varepsilon}}}^p(s, M, t).$$

Outline

Uniaxial cyclic load



Cyclic load calculation

Energy dissipation at one scale s

- $dW = (S - b)d\varepsilon^p = \dfrac{(E-k)(1+\nu)}{E(E+k\nu)} \dfrac{\sigma_y - \lambda\Sigma_H}{s} \left(S_{max} - \dfrac{\sigma_y - \lambda\Sigma_H}{s} \right)$ (phase 1)
- $dW(phase1) = dW(phase5) = \dfrac{1}{2} dW(phase3).$

Cyclic load calculation

Total dissipated energy W at all scales during one cycle

$$\begin{aligned} W_{cyc} &= 4\int_{(\sigma_y-\lambda\Sigma_H)/S_{max}}^{\infty} dW(s,M,t)P(s)ds \\ &= \frac{4(E-k)(1+\nu)(\beta-1)}{E(E+k\nu)\beta(\beta+1)}\frac{S_{max}^{\beta+1}}{(\sigma_y-\lambda\Sigma_H)^{\beta-1}}. \end{aligned} \tag{2}$$

Damage law

$$D = f\left(\frac{W_{cyc}}{W_F}\right) \tag{3}$$

Outline

Generalized damage accumulation

We combine a damage incremental law with increment of dissipated energy per unit time :

Damage incremental law

$$\delta[1-(1-D)^{\gamma+1}]^{1-\alpha} = \frac{\dot{W}}{W_F}\delta t. \tag{4}$$

W_F is the energy threshold of the material.

Multi-scale dissipated energy

$$\dot{W}(M,t) = \int_{s=1}^{\infty} \dot{w}(s,M,t)P(s)ds = \int_{s=1}^{\infty} \left(\underline{\underline{S}} - \underline{\underline{b}}\right)(s,M,t) : \underline{\underline{\dot{\varepsilon}^p}}(s,M,t)P(s)ds. \tag{5}$$

Failure when D=1.

Outline

Outline

4 Loop on time and scales
- Integration rules for $\dot{W}$ and δD
- Regime determination under multiple scales

Integration rules

- No cycle counting. Only dissipated energy integration.
- Gaussian quadrature rule for scale integration, time implicit for dissipated energy.

Outline

4 Loop on time and scales
- Integration rules for $\dot{W}$ and δD
- Regime determination under multiple scales

Regime determination

The material could be both in elastic and plastic regime at different scales.

Elastic regime :

There we have $\underline{\underline{\dot{\varepsilon}}}^p = 0$, $\underline{\underline{\dot{b}}} = 0$ and $\underline{\underline{\dot{S}}} = dev\underline{\underline{\dot{\Sigma}}}$, so

$$\underline{\underline{\dot{S}}} - \underline{\underline{\dot{b}}} = dev\underline{\underline{\dot{\Sigma}}},$$

yielding

$$\left(\underline{\underline{S}} - \underline{\underline{b}}\right)(t + dt) = \left(\underline{\underline{S}} - \underline{\underline{b}}\right)(t) + dev\underline{\underline{\dot{\Sigma}}}dt := \left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}(t + dt). \tag{6}$$

We are in elastic regime at scale s as long as we satisfy

$$\left(\underline{\underline{S}} - \underline{\underline{b}}\right)(t + dt) \leqslant \left(\sigma_y - \lambda\Sigma_H\right)/s.$$

Regime determination and damage integration

Final expression of energy dissipation during time step dt

$$
\begin{aligned}
W &= \dot{W} dt \\
&= \frac{1}{2} \sum_i \omega_i \dot{w} \left[\left(\frac{x+1}{2} \right)^{\frac{1}{1-\beta}} \right] dt \\
&= \frac{(E-k)(1+\nu)}{2E(E+k\nu)} \sum_i \omega_i \left\langle \left\| \underline{\underline{S}} - \underline{\underline{b}} \right\|_{trial} - \frac{\sigma_y - \lambda \Sigma_H}{\left(\frac{x_i+1}{2} \right)^{\frac{1}{1-\beta}}} \right\rangle \frac{\sigma_y - \lambda \Sigma_H}{\left(\frac{x_i+1}{2} \right)^{\frac{1}{1-\beta}}}.
\end{aligned} \tag{7}
$$

We use 25 Gauss Legendre points in numerical integration.

$$
g_{n+1} = g_n + \frac{\dot{W} dt}{W_F} = g_n + \frac{W}{W_F},
$$

with $D_n = \left[1 - \left(1 - g_n^{\frac{1}{1-\alpha}} \right)^{\frac{1}{\gamma+1}} \right].$

Outline

Outline

5 Application
- One dimensional application to simple cyclic data
- One dimensional application to PSA data
- Multi-dimensional application to PSA data

Material parameters

The test is performed on a sinusoidal axial load $\Sigma = Csin(t)$, giving the deviatoric amplitude $S_{max} = \sqrt{\frac{2}{3}}C$.

Parameters	**Value**
Load	$\Sigma = 5e8sin(t)$ Pa
Young's modulus	$E = 2e11$ Pa
Hardening parameter	$k = 6e8$ Pa
Weakening scales distribution exponent	$\beta = 3$
Hydrostatic pressure sensitivity	$\lambda = 0.5$
Macroscopic yield stress	$\sigma_y = 6.38e8$ Pa
Mean stress	$\Sigma_H = 0$ Pa
Material parameter from Chaboche law(Wohler curve exponent)	$\gamma = 0.5$
Non-linearity of damage accumulation	$\alpha = 0.8$
Initial damage	$D = 0$
Initial time	$t = 0$ s
Dissipated energy to failure per unit volume	$W_F = 3e6$ J
Looping step	1e-4 s

TABLE : Material parameters in a simple cyclic load

Sinusoidal test

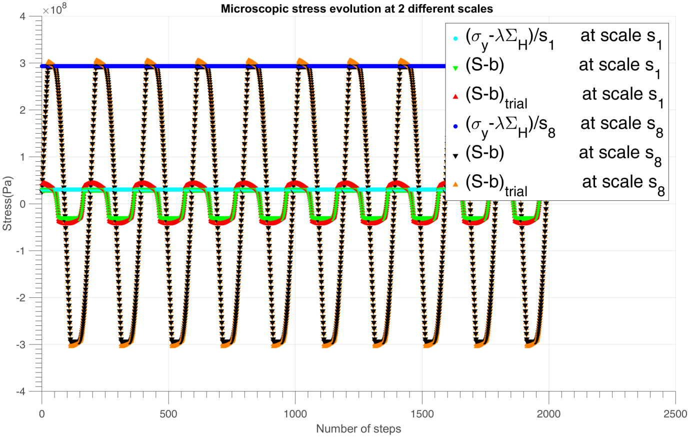


FIGURE : Microscopic $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}$ and $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)$ evolution with time under different weakening scales in sinusoidal load($s_1 = 21.21657929229650$ and $s_8 = 2.176132808422946$)

Sinusoidal test

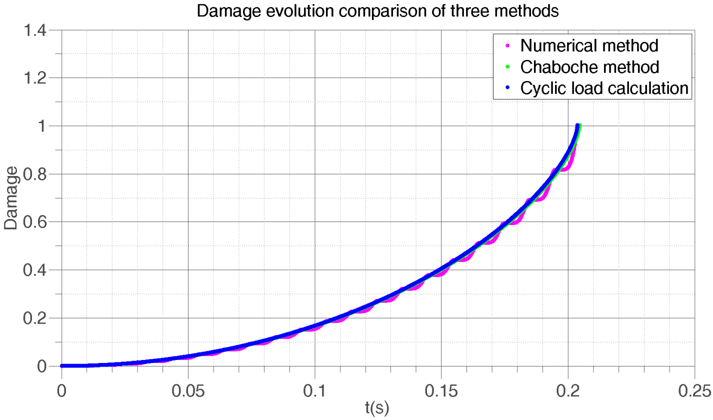


FIGURE : Damage evolution with time under sinusoidal load with two different methods

Outline

One dimensional application

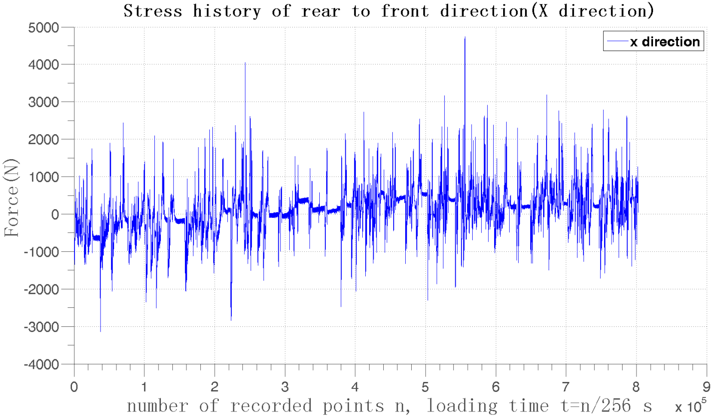


FIGURE : Loading history of X direction, force vs the record index n, with 256 sample recorded per second

One dimensional application

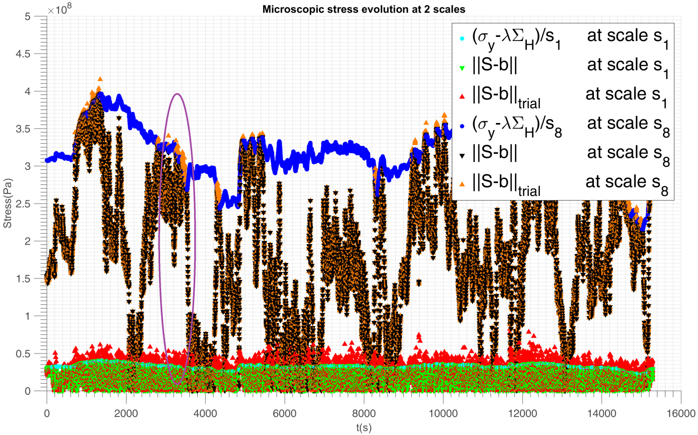


FIGURE : $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|_{trial}$ and $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|$ evolution with time under different weakening scales in PSA load history

One dimensional application

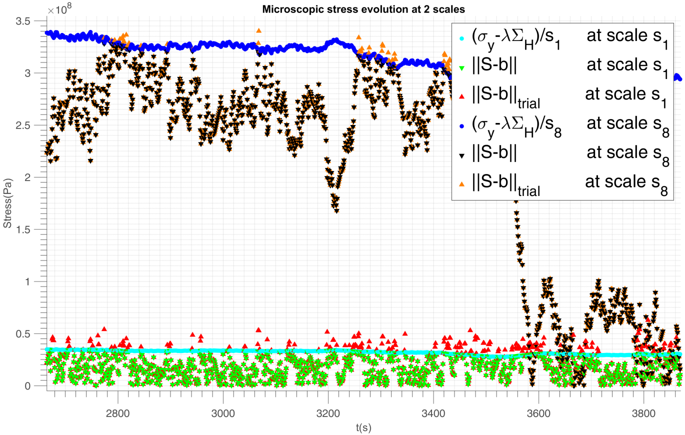


FIGURE : Circled area magnification where there is more $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|_{trial} > \sigma_y$(plasticity) at scale s_1 than at s_8

One dimensional application

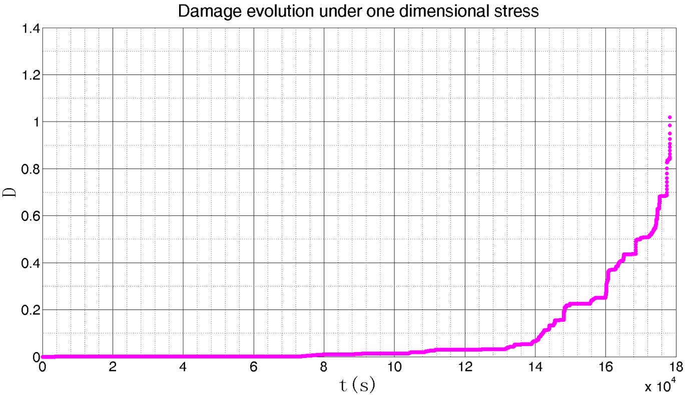


FIGURE : Damage evolution with time at one dimension PSA load history

Outline

Multi-dimensional application

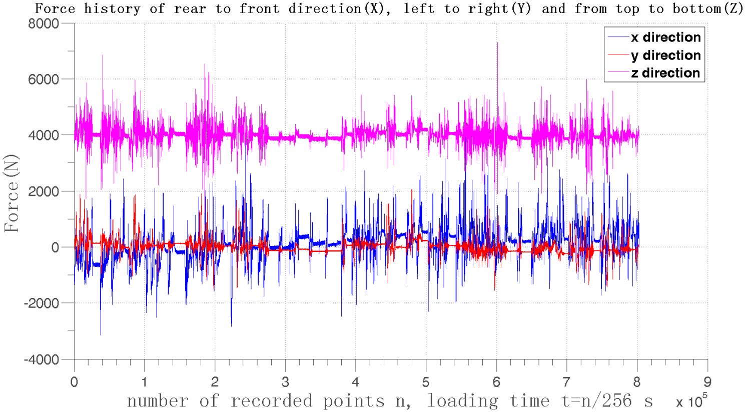


FIGURE : Loading history of 3 different directions

Multi-dimensional application

In real case, the vertical force F_z is much larger than the axial and horizontal forces F_x and F_y. However, in order to investigate large domains of interest, we first scale the axial and horizontal forces to reach comparable impact and transform them in principal stresses $c_x \frac{F_x}{A}$ applied along the stress principle vector $\underline{e}_\alpha$(respectively $\underline{e}_\beta$) that we choose randomly. We therefore consider the following macroscopic stress tensor :

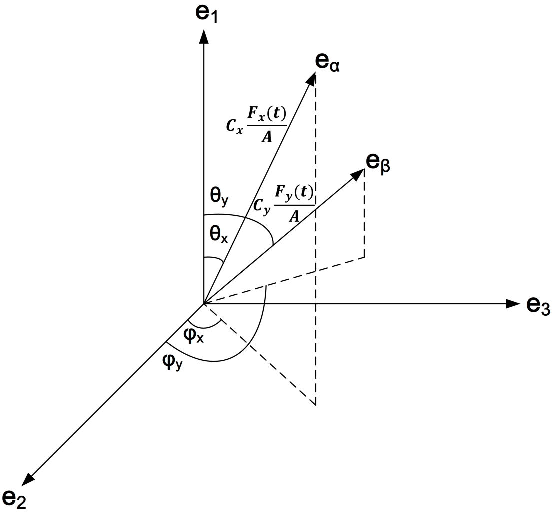
$$\underline{\underline{\Sigma}} = \frac{F_z(t)}{A} \underline{e}_1 \otimes \underline{e}_1 + c_x \frac{F_x(t)}{A} \underline{e}_\alpha \otimes \underline{e}_\alpha + c_y \frac{F_y(t)}{A} \underline{e}_\beta \otimes \underline{e}_\beta \tag{8}$$

where $\underline{e}_\alpha$ and $\underline{e}_\beta$ are principal vectors whose spherical coordinate are θ_x, φ_x, θ_y and φ_y respectively :

$$\underline{e}_\alpha = cos\theta_x \underline{e}_1 + sin\theta_x cos\varphi_x \underline{e}_2 + sin\theta_x sin\varphi_x \underline{e}_3,$$

$$\underline{e}_\beta = cos\theta_y \underline{e}_1 + sin\theta_y cos\varphi_y \underline{e}_2 + sin\theta_y sin\varphi_y \underline{e}_3.$$

Multi-dimensional application



Multi-dimensional application

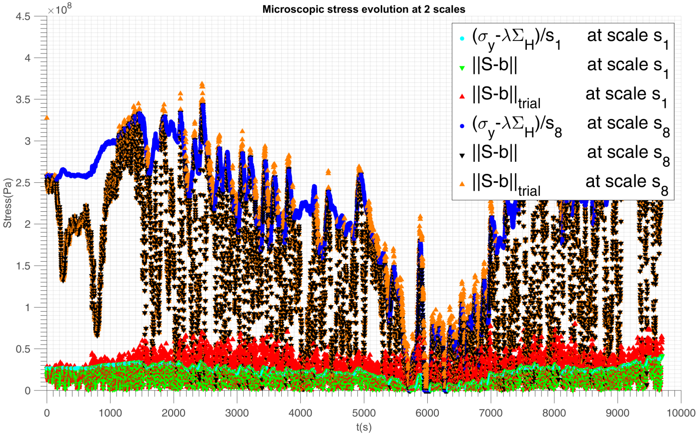


FIGURE : $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|_{trial}$ and $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|$ evolution with time under different weakening scales in PSA load history

Multi-dimensional application

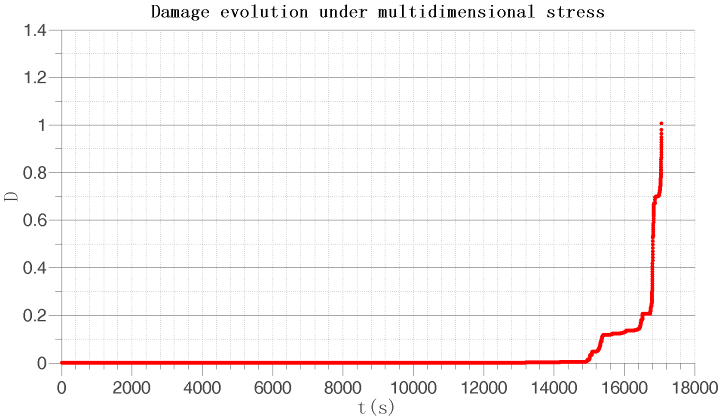


FIGURE : Damage evolution under multidimensional stress

Results and discussion

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1. We get rid of cycle counting method which for complex loading is hardly applicable.
2. The small step-by-step strategy does not ignore small fluctuations in load history and the big stress effect is magnified which reflects the real situation.
3. The energy based fatigue approach takes into account impurities and hardness in the material and is applicable to any type of micro plasticity law and multiaxial load geometry.

Thanks for your attention!