# A new strategy for fatigue analysis in presence of general multiaxial time varying loadings

Keywords : Fatigue ; Energy ; High cycle ; Plasticity ; Mean stress

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# Outline

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# Physical consideration

1. We follow the Dang Van paradigm. The structure is elastic at the macroscopic scale.
2. At each material points, there is a stochastic distribution of weak points which will undergo strong plastic yielding, which contribute to energy dissipation without affecting the overall macroscopic stress.
3. Fatigue function of energy dissipation.

# Statistics method

From a microscopic point of view, there is a distribution of weakening scales, namely $s \in [1, \infty)$. $S_{max}$ is the macroscopic stress intensity. $\sigma_y$ is the yield limit before weakening.

- $1 \leqslant s \leqslant \sigma_y / S_{max} \longrightarrow S_{max} \leqslant \sigma_y / s \longrightarrow$ elastic regime $\longrightarrow$ no energy dissipation.
- $\sigma_y / S_{max} \leqslant s \leqslant \infty \longrightarrow S_{max} \geqslant \sigma_y / s \longrightarrow$ plastic regime $\longrightarrow$ energy dissipation.

We assume the weakening scales have a probability distribution of power law :

$$\boxed{P(s) = Cs^{-\beta}}$$

# Statistics method

## Weakening scales distribution

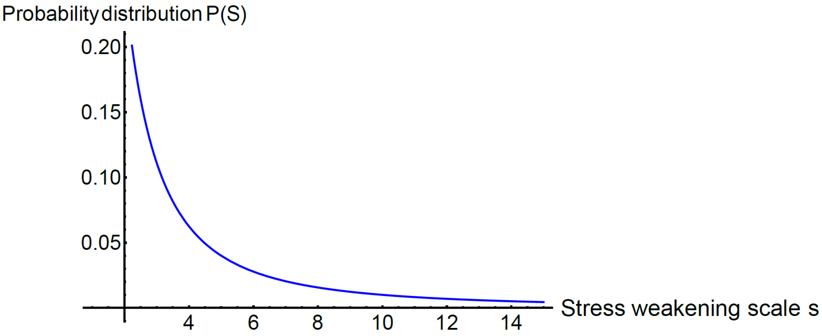


FIGURE : Weakening scales $s$ probability distribution curve

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# Mean stress effect

The idea is to consider Maitournam and Krebs' that the yield limit $\sigma_y$ is reduced in presence of positive mean stress :

$$f(s) = ||\underline{\underline{S}}(s) - \underline{\underline{b}}(s)|| + (\lambda\Sigma_H - \sigma_y)/s \leqslant 0 \tag{1}$$

with $\underline{\underline{S}}(s)$ denoting the deviatoric part of the stress tensor at microscale, and $\underline{\underline{b}}(s)$ the corresponding backstress at the same scale.

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# Description of the mesoscopic stress state

- $\dot{\underline{\underline{S}}}(s, M, t) = dev\dot{\underline{\underline{\Sigma}}}(M, t) - \frac{E}{1+\nu}\dot{\underline{\underline{\varepsilon}}}^p(s, M, t)$, Taylor-Lin scale transition model.
- $\dot{\underline{\underline{b}}}(s, M, t) = \frac{kE}{E-k}\dot{\underline{\underline{\varepsilon}}}^p(s, M, t)$, kinematic hardening model.
- $\dot{\underline{\underline{\varepsilon}}}^p(s, M, t) = \gamma\frac{\partial f(s, M, t)}{\partial \underline{\underline{S}}}$, associated plastic flow rule.

The local dissipated energy rate per volume at weakening scales $s$ is given by :

$$\dot{w}(s, M, t) = (\underline{\underline{S}} - \underline{\underline{b}})(s, M, t) : \dot{\underline{\underline{\varepsilon}}}^p(s, M, t).$$

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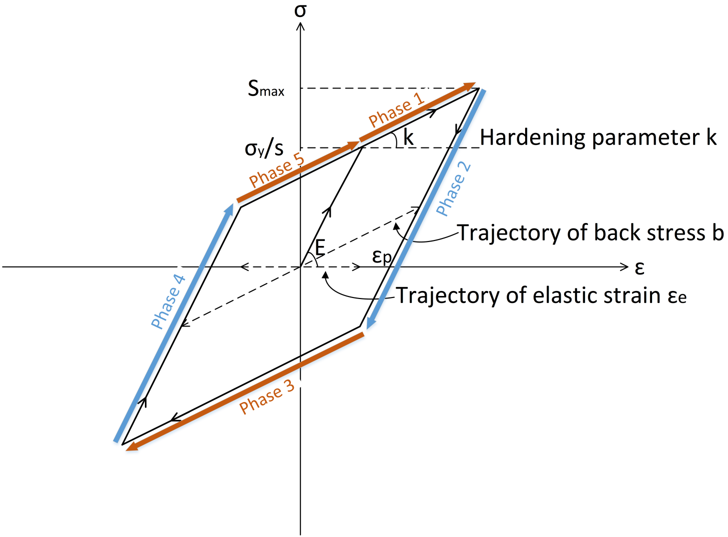
# Uniaxial cyclic load



# Cyclic load calculation

## Energy dissipation at one scale s

- $dW = (S-b)d\varepsilon^p = \frac{(E-k)(1+\nu)}{E(E+k\nu)} \frac{\sigma_y - \lambda\Sigma_H}{s} \left( S_{max} - \frac{\sigma_y - \lambda\Sigma_H}{s} \right)$ (phase 1)
- $dW(phase1) = dW(phase5) = \frac{1}{2} dW(phase3).$

# Cyclic load calculation

## Total dissipated energy $W$ at all scales during one cycle

$$
\begin{aligned}
W_{cyc} &= 4\int_{(\sigma_y-\lambda\Sigma_H)/S_{max}}^{\infty} dW(s,M,t)P(s)ds \\
&= \frac{4(E-k)(1+\nu)\,(\beta-1)}{E(E+k\nu)\beta\,(\beta+1)}\,\frac{S_{max}^{\beta+1}}{(\sigma_y-\lambda\Sigma_H)^{\beta-1}}.
\end{aligned}
\tag{2}
$$

## Damage law

$$
N_{cyc} = f\left(\frac{W_{cyc}}{W_F}\right) \tag{3}
$$

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# Generalized damage accumulation

We combine a damage incremental law with relative increment of dissipated energy per unit time :

## Damage incremental law

$$\delta[1-(1-D)^{\gamma+1}]^{1-\alpha} = \frac{\dot{W}}{W_F}\delta t. \tag{4}$$

$W_F$ is the energy threshold of the material.

## Multi-scale dissipated energy

$$\dot{W}(M,t) = \int_{s=1}^{\infty} \dot{w}(s,M,t)P(s)ds = \int_{s=1}^{\infty} \left(\underline{\underline{S}} - \underline{\underline{b}}\right)(s,M,t) : \underline{\underline{\dot{\varepsilon}^{p}}}(s,M,t)P(s)ds. \tag{5}$$

Failure when D=1.

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4 Loop on time and scales
- Integration rules for $\dot{W}$ and $\delta D$
- Regime determination under multiple scales

# Integration rules

- No cycle counting. Only dissipated energy integration.
- Gaussian quadrature rule for scale integration, time implicit for dissipated energy.

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# Regime determination

The material could be both in elastic and plastic regime at different scales.

## Elastic regime :

There we have $\underline{\underline{\dot{\varepsilon}}}^p = 0$, $\underline{\underline{\dot{b}}} = 0$ and $\underline{\underline{\dot{S}}} = dev\underline{\underline{\dot{\Sigma}}}$, so

$$\underline{\underline{\dot{S}}} - \underline{\underline{\dot{b}}} = dev\underline{\underline{\dot{\Sigma}}},$$

yielding

$$\left(\underline{\underline{S}} - \underline{\underline{b}}\right)(t + dt) = \left(\underline{\underline{S}} - \underline{\underline{b}}\right)(t) + dev\underline{\underline{\dot{\Sigma}}}dt := \left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}(s, t + dt). \tag{6}$$

We are in elastic regime at scale $s$ as long as we satisfy

$$\left(\underline{\underline{S}} - \underline{\underline{b}}\right)(t + dt) \leqslant \left(\sigma_y - \lambda\Sigma_H\right)/s.$$

# Regime determination and damage integration

## Final expression of energy dissipation during time step dt

$$
\begin{aligned}
W &= \dot{W} dt \\
&= \frac{1}{2} \sum_i \omega_i \dot{w} \left[ \left( \frac{x+1}{2} \right)^{\frac{1}{1-\beta}} \right] dt \\
&= \frac{(E-k)(1+\nu)}{2E(E+k\nu)} \sum_i \omega_i \left\langle \left\| \underline{\underline{S}} - \underline{\underline{b}} \right\|_{trial} - \frac{\sigma_y - \lambda \Sigma_H}{\left( \frac{x_i+1}{2} \right)^{\frac{1}{1-\beta}}} \right\rangle \frac{\sigma_y - \lambda \Sigma_H}{\left( \frac{x_i+1}{2} \right)^{\frac{1}{1-\beta}}}.
\end{aligned} \tag{7}
$$

$$
g_{n+1} = g_n + \frac{\dot{W} dt}{W_F} = g_n + \frac{W}{W_F},
$$

with $D_n = \left[ 1 - \left( 1 - g_n^{\frac{1}{1-\alpha}} \right)^{\frac{1}{\gamma+1}} \right].$

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# Material parameters

The test is performed on a sinusoidal axial load $\Sigma = Csin(t)$, giving the deviatoric amplitude $S_{max} = \sqrt{\frac{2}{3}}C$.

| **Parameters** | **Value** |
|---|---|
| Load | $\Sigma = 5e8sin(t)$ Pa |
| Young's modulus | $E = 2e11$ Pa |
| Hardening parameter | $k = 6e8$ Pa |
| Weakening scales distribution exponent | $\beta = 3$ |
| Hydrostatic pressure sensitivity | $\lambda = 0.5$ |
| Macroscopic yield stress | $\sigma_y = 6.38e8$ Pa |
| Mean stress | $\Sigma_H = 0$ Pa |
| Material parameter from Chaboche law(Wohler curve exponent) | $\gamma = 0.5$ |
| Non-linearity of damage accumulation | $\alpha = 0.8$ |
| Initial damage | $D = 0$ |
| Initial time | $t = 0$ s |
| Dissipated energy to failure per unit volume | $W_F = 3e6$ J |
| Looping step | 1e-4 s |

TABLE : Material parameters in a simple cyclic load

# Sinusoidal test

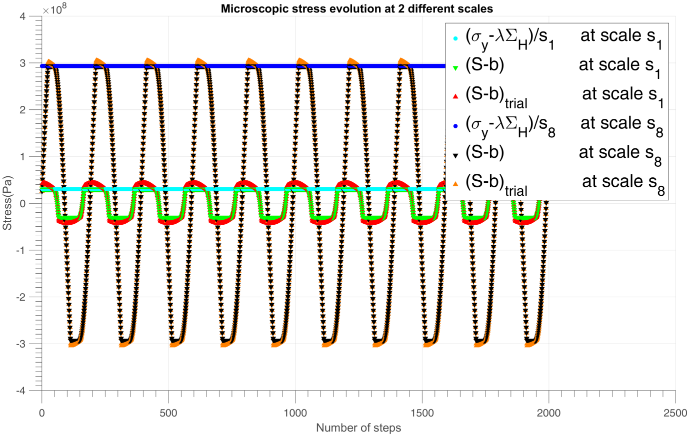


FIGURE : Microscopic $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}$ and $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)$ evolution with time under different weakening scales in sinusoidal load($s_1 = 21.21657929229650$ and $s_8 = 2.176132808422946$)

# Sinusoidal test

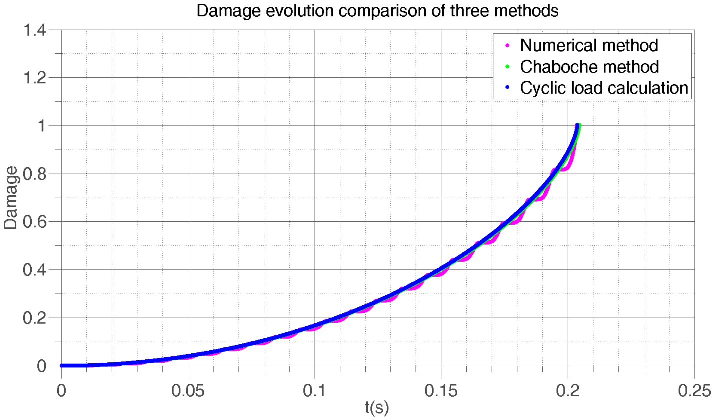


FIGURE : Damage evolution with time under sinusoidal load with two different methods

# Convergence test

Because the step by step damage accumulation grows in a power law, so the amplitude of difference grows with time. However, the difference between the two methods swing around 0 so we could consider the numerical method converges in cyclic load calculation method.

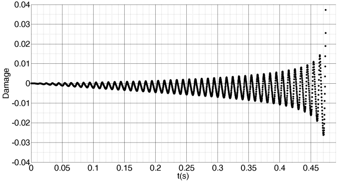


FIGURE : Difference between cyclic load calculation and numerical method as function of time(time step=1/5000s)

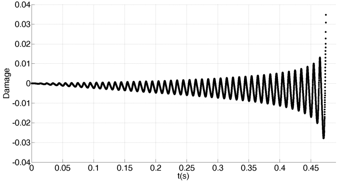


FIGURE : Difference between cyclic load calculation and numerical method as function of time(time step=1/15000s)

# Outline

5 Application
- One dimensional application to simple cyclic data
- One dimensional application to PSA data
- Multi-dimensional application to PSA data

# One dimensional application

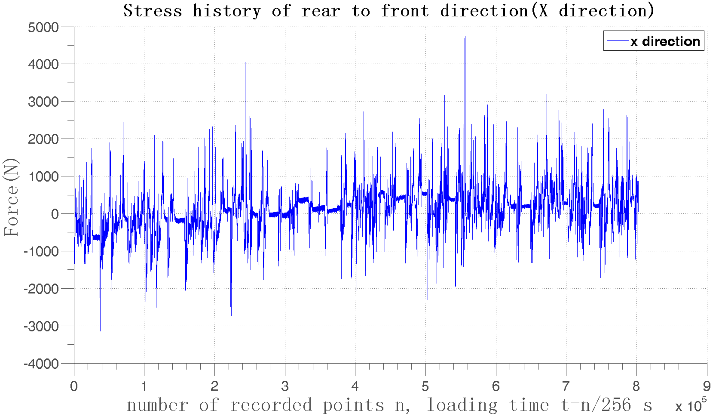


FIGURE : Loading history of X direction, force vs the record index n, with 256 sample recorded per second

# One dimensional application

## Numerical methods :

- The sample recording rate is 256 per second. In order to accumulate damage using very small steps, we have created 10 additional points between every 2 recorded points by linear interpolation.
- The force on wheel is firstly considered as under uniaxial loading $F_x$. Here we temporally set $\Sigma_x = F_x/A$ where $A = \dfrac{1}{6e5}m^2$ is the area of force, and $W_F = 3e6J$.

# One dimensional application

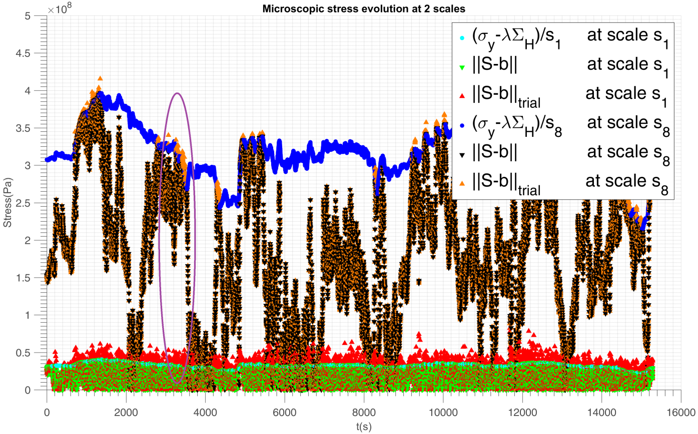


FIGURE : $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|_{trial}$ and $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|$ evolution with time under different weakening scales in PSA load history

# One dimensional application

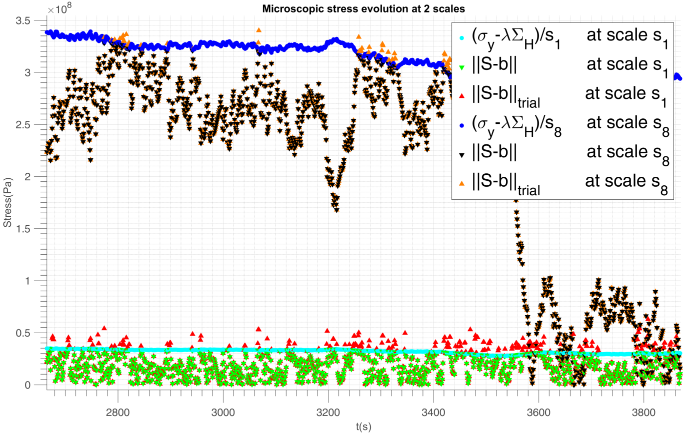


FIGURE : Circled area magnification where there is more $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|_{trial} > \sigma_y$(plasticity) at scale $s_1$ than at $s_8$

# One dimensional application

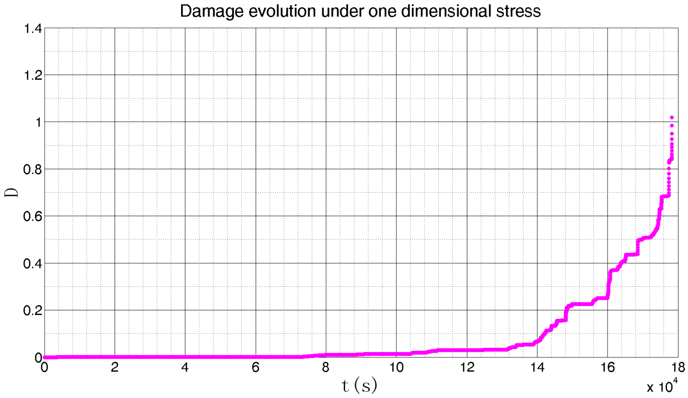


FIGURE : Damage evolution with time at one dimension PSA load history

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5 Application
- One dimensional application to simple cyclic data
- One dimensional application to PSA data
- Multi-dimensional application to PSA data

# Multi-dimensional application

We now consider a situation where we have force recorded measured in 3 different directions as shown in FIGURE 10.

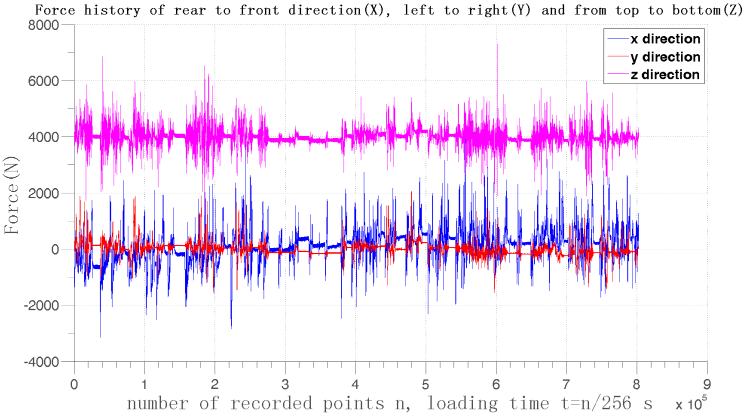



FIGURE : Loading history of 3 different directions

# Multi-dimensional application

In real case, the vertical force $F_z$ is much larger than the axial and horizontal forces $F_x$ and $F_y$. However, in order to investigate large domains of interest, we first scale the axial and horizontal forces to reach comparable impact and transform them in principal stresses $c_x \dfrac{F_x}{A}$ applied along the stress principle vector $\underline{e}_\alpha$(respectively $\underline{e}_\beta$) that we choose randomly. We therefore consider the following macroscopic stress tensor :

$$\underline{\underline{\Sigma}} = \frac{F_z(t)}{A}\underline{e}_1 \otimes \underline{e}_1 + c_x\frac{F_x(t)}{A}\underline{e}_\alpha \otimes \underline{e}_\alpha + c_y\frac{F_y(t)}{A}\underline{e}_\beta \otimes \underline{e}_\beta \tag{8}$$

where $\underline{e}_\alpha$ and $\underline{e}_\beta$ are principal vectors whose spherical coordinate are $\theta_x$, $\varphi_x$, $\theta_y$ and $\varphi_y$ respectively :

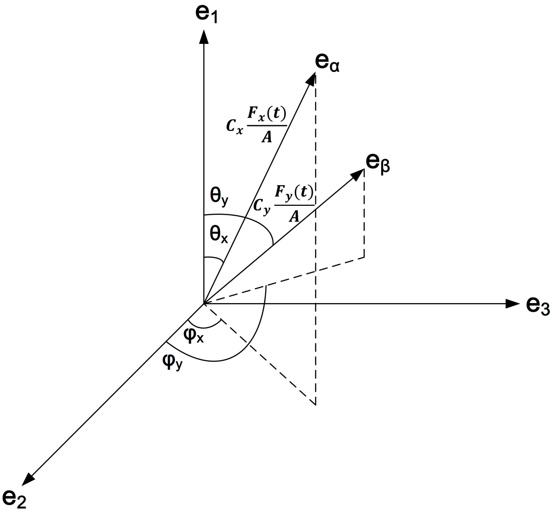
$$\underline{e}_\alpha = cos\theta_x\underline{e}_1 + sin\theta_x cos\varphi_x\underline{e}_2 + sin\theta_x sin\varphi_x\underline{e}_3,$$

$$\underline{e}_\beta = cos\theta_y\underline{e}_1 + sin\theta_y cos\varphi_y\underline{e}_2 + sin\theta_y sin\varphi_y\underline{e}_3.$$

| **Parameter** | A($m^2$) | $c_x$ | $c_y$ | $\theta_x$ | $\varphi_x$ | $\theta_y$ | $\varphi_y$ |
|---|---|---|---|---|---|---|---|
| **Value** | 1/6e4 | 10 | 60 | 0.5 | 0.3 | 0.6 | 0.4 |

TABLE : The structural data in 3D analysis

# Multi-dimensional application



# Multi-dimensional application

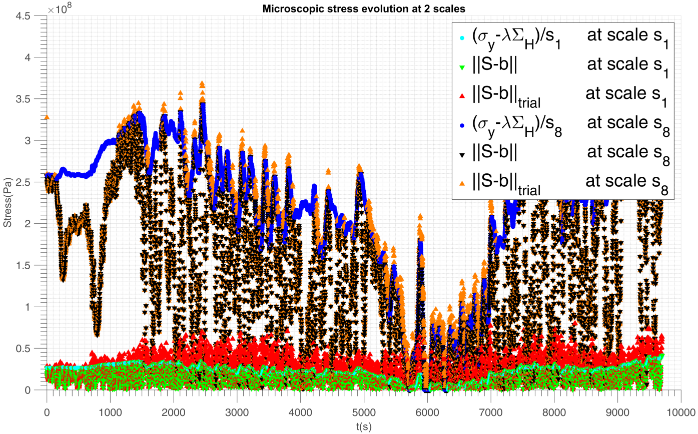



FIGURE : $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|_{trial}$ and $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|$ evolution with time under different weakening scales in PSA load history

# Multi-dimensional application

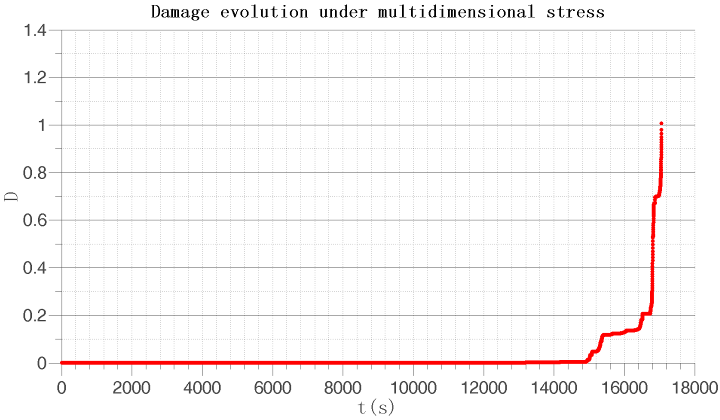



FIGURE : Damage evolution under multidimensional stress

# Discussion

## Discussion

1. We work on the stress tensor directly in 3D analysis in stead of using the multidimensional equivalent stress.
2. The energy based fatigue approach takes into account impurities and hardness in the material and is applicable to any type of micro plasticity law and multiaxial load geometry.
3. The small step-by-step strategy does not ignore small fluctuations in load history and the big stress effect is magnified which reflects the real situation.

*Thanks for your attention*!