A new strategy for fatigue analysis in presence of general multiaxial time varying loadings

Keywords : Fatigue ; Energy ; High cycle ; Plasticity ; Mean stress

Ma Zepeng

Solid Mechanics Laboratory
Ecole Polytechnique

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Physical consideration

1. We follow the Dang Van paradigm. The structure is elastic at the macroscopic scale.
2. At each material points, there is a stochastic distribution of weak points which will undergo strong plastic yielding, which contribute to energy dissipation without affecting the overall macroscopic stress.

Statistics method

From a microscopic point of view, there is a distribution of weakening scales, namely $s \in [1, \infty)$. S_{max} is the macroscopic stress intensity. σ_y is the yield limit before weakening.

- $1 \leqslant s \leqslant \sigma_y / S_{max} \longrightarrow S_{max} \leqslant \sigma_y / s \longrightarrow$ elastic regime $\longrightarrow$ no energy dissipation.
- $\sigma_y / S_{max} \leqslant s \leqslant \infty \longrightarrow S_{max} \geqslant \sigma_y / s \longrightarrow$ plastic regime $\longrightarrow$ energy dissipation.

Which is to say at each scale s there is a weakened yield limit σ_y / s, zero initial plastic strain $\underline{\underline{\varepsilon}}_p$ and zero initial backstress $\underline{\underline{b}}$ at initial time t_0.

We assume the weakening scales have a probability distribution of power law :

$$P(s) = Cs^{-\beta},$$

$$\int_1^{\infty} P(s) ds = \left[\frac{Cs^{1-\beta}}{1-\beta} \right]_1^{\infty} = 0 - \frac{C}{1-\beta} = 1$$

we get $C = \beta - 1$, so

$$\boxed{P(s) = Cs^{-\beta} = (\beta - 1) s^{-\beta}}$$

Statistics method

Weakening scales distribution

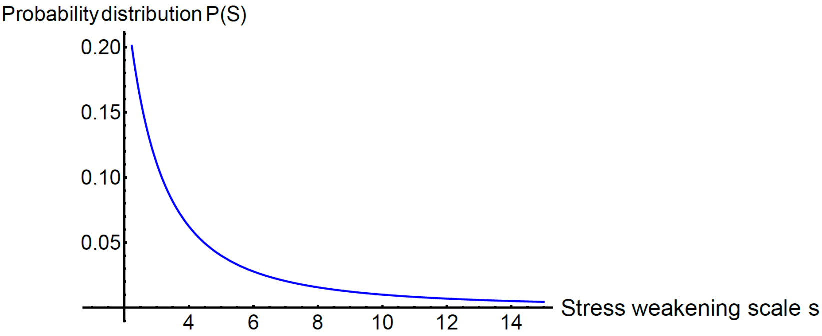

FIGURE : Weakening scales s probability distribution curve

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Mean stress effect

The idea is to consider as in the work of Maitournam and Krebs that the yield limit σ_y can be reduced in presence of positive mean stress. The mesoscopic yield function can therefore be written as :

$$f(s) = ||\underline{\underline{S}}(s) - \underline{\underline{b}}(s)|| + (\lambda\Sigma_H - \sigma_y)/s \leqslant 0 \quad (1)$$

with $\underline{\underline{S}}(s)$ denoting the deviatoric part of the stress tensor at microscale, and $\underline{\underline{b}}(s)$ the corresponding backstress at the same scale.

Outline

1. Weakening scales and yield function
   - The concept of weakening scales
   - Yield function with mean stress effect
   - Local plastic model

Description of the mesoscopic stress state

- $\dot{\underline{\underline{S}}}(s, M, t) = dev\dot{\underline{\underline{\Sigma}}}(M, t) - \frac{E}{1+\nu}\dot{\underline{\underline{\varepsilon}}}^p(s, M, t)$, Taylor-Lin scale transition model.
- $\dot{\underline{\underline{b}}}(s, M, t) = \frac{kE}{E-k}\dot{\underline{\underline{\varepsilon}}}^p(s, M, t)$, kinematic hardening model.
- $\dot{\underline{\underline{\varepsilon}}}^p(s, M, t) = \gamma\frac{\partial f(s, M, t)}{\partial \underline{\underline{S}}}$, the associated plastic flow rule assuming $\gamma = 0$ when $f < 0$(elastic) and $\gamma \geqslant 0$ when $f = 0$(plastic).

The local dissipated energy rate per volume at weakening scales s is given by the local entropy dissipation :

$$\dot{w}(s, M, t) = (\underline{\underline{S}} - \underline{\underline{b}})(s, M, t) : \dot{\underline{\underline{\varepsilon}}}^p(s, M, t).$$

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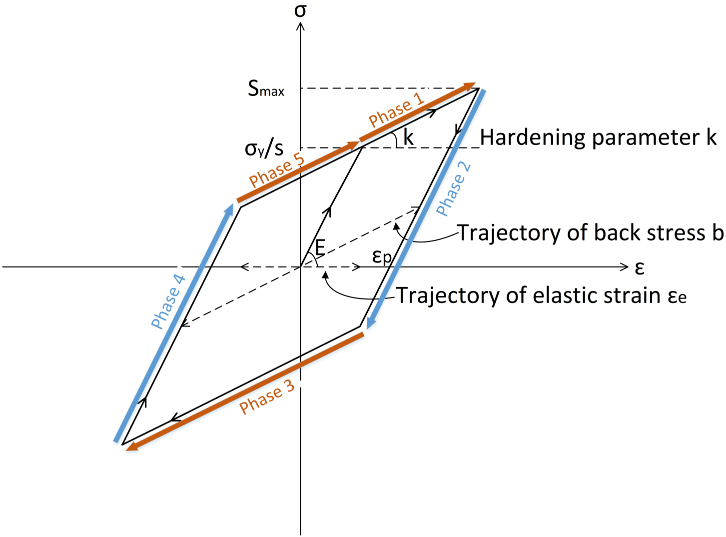
Uniaxial cyclic load



Cyclic load calculation

Energy dissipation at one scale s

- $dW = (S-b)d\varepsilon^p = \dfrac{(E-k)(1+\nu)}{E(E+k\nu)} \dfrac{\sigma_y - \lambda\Sigma_H}{s} \left(S_{max} - \dfrac{\sigma_y - \lambda\Sigma_H}{s} \right)$ (phase 1)
- $dW(phase1) = dW(phase5) = \dfrac{1}{2} dW(phase3).$

Total dissipated energy W at all scales during one cycle

$$
\begin{aligned}
W_{cyc} &= 4 \int_{(\sigma_y - \lambda\Sigma_H)/S_{max}}^{\infty} dW(s, M, t) P(s) ds \\
&= \frac{4(E-k)(1+\nu)\,(\beta-1)}{E(E+k\nu)\beta\,(\beta+1)} \frac{S_{max}^{\beta+1}}{(\sigma_y - \lambda\Sigma_H)^{\beta-1}}.
\end{aligned} \tag{2}
$$

Where k is the hardening modulus.

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Generalized damage accumulation

We combine a damage incremental law with relative increment of dissipated energy per unit time :

Damage incremental law

$$\delta[1-(1-D)^{\gamma+1}]^{1-\alpha} = \frac{\dot{W}}{W_F}\delta t. \quad (3)$$

W_F is the energy threshold of the material.

Generalized damage accumulation

$$\dot{W}(M,t) = \int_{s=1}^{\infty} \dot{w}(s,M,t)P(s)ds = \int_{s=1}^{\infty} \left(\underline{\underline{S}} - \underline{\underline{b}}\right)(s,M,t) : \underline{\underline{\dot{\varepsilon}^p}}(s,M,t)P(s)ds. \quad (4)$$

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4 Loop on time and scales
- Integration rules for $\dot{W}$ and δD
- Regime determination under multiple scales

Integration rules

- There are certain limitations of cycle counting method. Firstly we need a load history decomposition in cycles. Secondly in real life the perfect close loop cycle is hardly applicable.
- We propose in a more general method which can be integrated using a step by step strategy.
- Instead of doing the scale integration directly which can be difficult for complex loading, the Gaussian Quadrature rule with Legendre points is used to give the value of local dissipated energy rate.

Integration rules

The dissipated energy summed on all scales is :

$$
\begin{aligned}
\dot{W} &= \int_1^{\infty} \dot{w}(s)(\beta - 1)(s)^{-\beta} ds \\
&= \frac{1}{2}\int_{-1}^{1} \dot{w}\left[\left(\frac{x+1}{2}\right)^{\frac{1}{1-\beta}}\right] dx \\
&\approx \frac{1}{2}\sum_i \omega_i d\dot{w}\left[\left(\frac{x_i+1}{2}\right)^{\frac{1}{1-\beta}}\right],
\end{aligned}
\tag{5}
$$

where ω_i and x_i are respectively the weights and nodes of the Gauss Legerndre integration rule used for the numerical integration. In this work, we used 25 points.

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4 Loop on time and scales

- Integration rules for $\dot{W}$ and δD
- Regime determination under multiple scales

Regime determination

The material could be both in elastic and plastic regime under different scales.

Elastic regime :

There we have $\dot{\underline{\underline{\varepsilon}}}^p = 0$, $\dot{\underline{\underline{b}}} = 0$ and $\dot{\underline{\underline{S}}} = dev\dot{\underline{\underline{\Sigma}}}$, so

$$\dot{\underline{\underline{S}}} - \dot{\underline{\underline{b}}} = dev\dot{\underline{\underline{\Sigma}}},$$

yielding

$$\left(\underline{\underline{S}} - \underline{\underline{b}}\right)(t + dt) = \left(\underline{\underline{S}} - \underline{\underline{b}}\right)(t) + dev\dot{\underline{\underline{\Sigma}}}dt := \left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}(s, t + dt). \tag{6}$$

We are in elastic regime at scale s as long as we satisfy

$$\left(\underline{\underline{S}} - \underline{\underline{b}}\right)(t + dt) \leqslant (\sigma_y - \lambda\Sigma_H)/s.$$

Regime determination

Plastic regime :

$$
\begin{cases}
\dot{\underline{\underline{\varepsilon}}}^p = \gamma \dfrac{\underline{\underline{S}} - \underline{\underline{b}}}{\left|\left|\underline{\underline{S}} - \underline{\underline{b}}\right|\right|}, \gamma > 0, & \text{plastic flow,} \quad (7) \\
\left|\left|\underline{\underline{S}} - \underline{\underline{b}}\right|\right| = \left(\sigma_y - \lambda \Sigma_H\right)/s, & \text{yield limit,} \quad (8) \\
\left(\underline{\underline{S}} - \underline{\underline{b}}\right) : \left(\dot{\underline{\underline{S}}} - \dot{\underline{\underline{b}}}\right) = 0, & \text{yield limit time invariance,} \quad (9) \\
\dot{\underline{\underline{b}}} = \dfrac{kE}{E-k} \dot{\underline{\underline{\varepsilon}}}^p, & \text{kinematic hardening rule,} \quad (10) \\
\dot{\underline{\underline{S}}} = dev \dot{\underline{\underline{\Sigma}}} - \dfrac{E}{1+\nu} \dot{\underline{\underline{\varepsilon}}}^p, & \text{localisation rule.} \quad (11)
\end{cases}
$$

Regime determination

In all cases, we get in plastic regime :

$$\left(\underline{\underline{S}} - \underline{\underline{b}}\right)(s, t+dt) = \frac{\left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}(s, t+dt)}{1+\eta}, \tag{12}$$

$$\eta = max\left\{ \underbrace{0}_{elastic\ regime}, \underbrace{\frac{\left|\left|\underline{\underline{S}} - \underline{\underline{b}}\right|\right|_{trial}}{\left(\sigma_y - \lambda\Sigma_H\right)/s} - 1}_{plastic\ regime\ when\ this\ number\ is\ positive} \right\}.$$

That is to say, when the structure is in elastic regime at time t and scale s, we have $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)(s,t) = \left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}(s,t)$. Otherwise, if the norm of $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}(s,t)$ is greater than the local yield limit $\left(\sigma_y - \lambda\Sigma_H\right)/s$, $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)(s,t)$ will be projected on the yield limit.

Regime determination

Final expression of energy dissipation during time step dt

$$
\begin{aligned}
W &= \dot{W} dt \\
&= \frac{1}{2} \sum_i \omega_i \dot{w} \left[\left(\frac{x+1}{2} \right)^{\frac{1}{1-\beta}} \right] dt \\
&= \frac{(E-k)(1+\nu)}{2E(E+k\nu)} \sum_i \omega_i \left\langle \left\| \underline{\underline{S}} - \underline{\underline{b}} \right\|_{trial} - \frac{\sigma_y - \lambda \Sigma_H}{\left(\frac{x_i+1}{2} \right)^{\frac{1}{1-\beta}}} \right\rangle \frac{\sigma_y - \lambda \Sigma_H}{\left(\frac{x_i+1}{2} \right)^{\frac{1}{1-\beta}}}.
\end{aligned}
\tag{13}
$$

$$
g_{n+1} = g_n + \frac{\dot{W} dt}{W_F} = g_n + \frac{W}{W_F},
$$

with $D_n = \left[1 - \left(1 - g_n^{\frac{1}{1-\alpha}} \right)^{\frac{1}{\gamma+1}} \right].$

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5 Application
- One dimensional application to simple cyclic data
- One dimensional application to PSA data
- Multi-dimensional application to PSA data

Material parameters

The test is performed on a sinusoidal axial load $\Sigma = Csin(t)$, giving the deviatoric amplitude $S_{max} = \sqrt{\frac{2}{3}}C$.

Parameters	**Value**
Load	$\Sigma = 5e8sin(t)$ Pa
Young's modulus	$E = 2e11$ Pa
Hardening parameter	$k = 6e8$ Pa
Weakening scales distribution exponent	$\beta = 3$
Hydrostatic pressure sensitivity	$\lambda = 0.5$
Macroscopic yield stress	$\sigma_y = 6.38e8$ Pa
Mean stress	$\Sigma_H = 0$ Pa
Material parameter from Chaboche law(Wohler curve exponent)	$\gamma = 0.5$
Non-linearity of damage accumulation	$\alpha = 0.8$
Initial damage	$D = 0$
Initial time	$t = 0$ s
Dissipated energy to failure per unit volume	$W_F = 3e6$ J
Looping step	1 s

TABLE : Material parameters in a simple cyclic load

Sinusoidal test

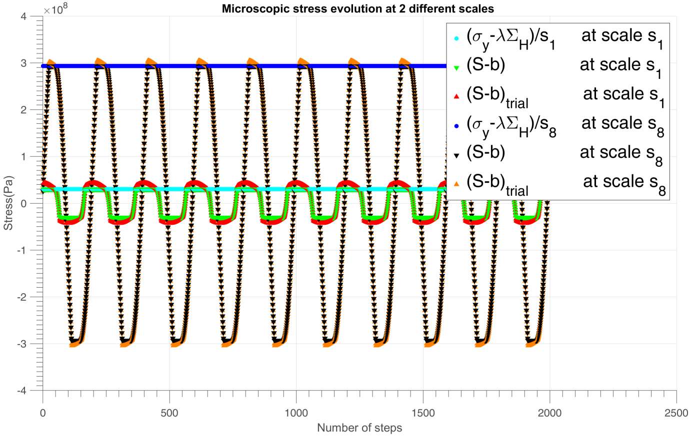

FIGURE : Microscopic $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)_{trial}$ and $\left(\underline{\underline{S}} - \underline{\underline{b}}\right)$ evolution with time under different weakening scales in sinusoidal load($s_1 = 21.21657929229650$ and $s_8 = 2.176132808422946$)

Sinusoidal test

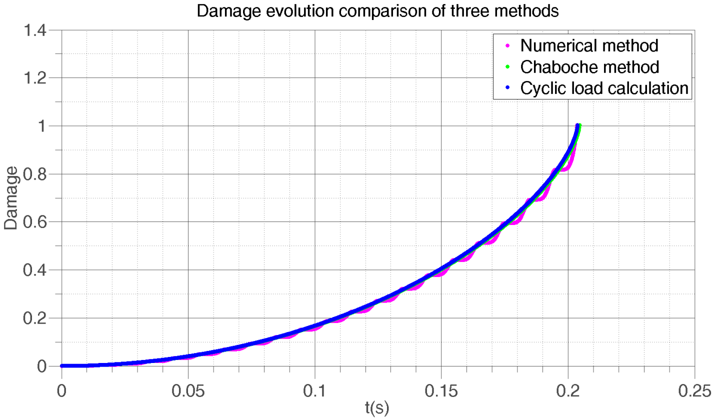

FIGURE : Damage evolution with time under sinusoidal load with two different methods

Convergence test

Because the step by step damage accumulation grows in a power law, so the amplitude of difference grows with time. However, the difference between the two methods swing around 0 so we could consider the numerical method converges in cyclic load calculation method.

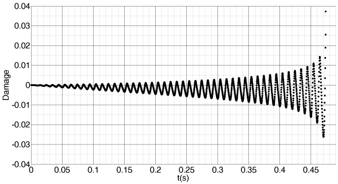

FIGURE : Difference between cyclic load calculation and numerical method as function of time(time step=1/5000s)

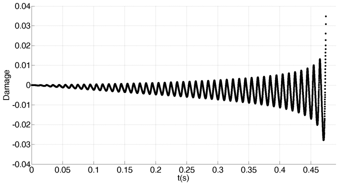

FIGURE : Difference between cyclic load calculation and numerical method as function of time(time step=1/15000s)

Outline

5 Application

- One dimensional application to simple cyclic data
- One dimensional application to PSA data
- Multi-dimensional application to PSA data

One dimensional application

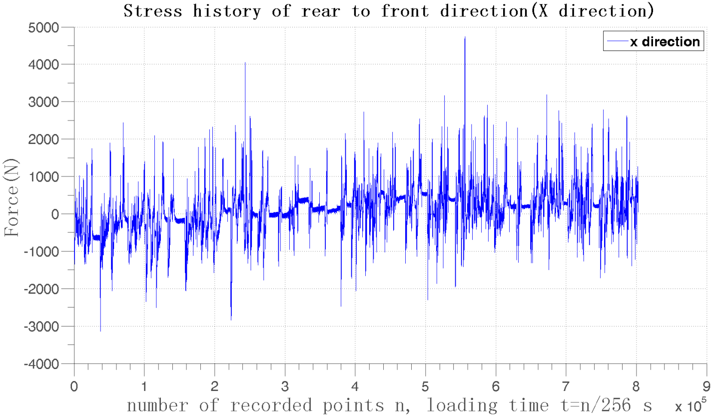

FIGURE : Loading history of X direction, force vs the record index n, with 256 sample recorded per second

One dimensional application

Numerical methods :

- The sample recording rate is 256 per second. In order to accumulate damage using very small steps, we have created 10 additional points between every 2 recorded points by linear interpolation. So the sample rate is $256 * 10$ per second.
- The force on wheel is firstly considered as under uniaxial loading F_x. Here we temporally set $\Sigma_x = F_x/A$ where $A = \frac{1}{6e5} m^2$ is the area of force, and $W_F = 3e6J$.

One dimensional application

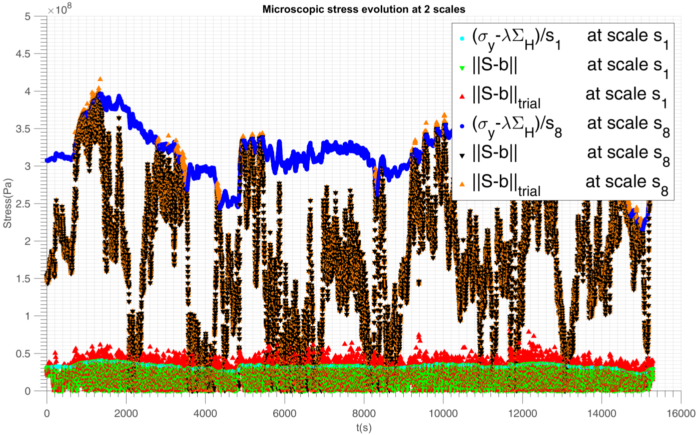

FIGURE : $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|_{trial}$ and $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|$ evolution with time under different weakening scales in PSA load history

One dimensional application

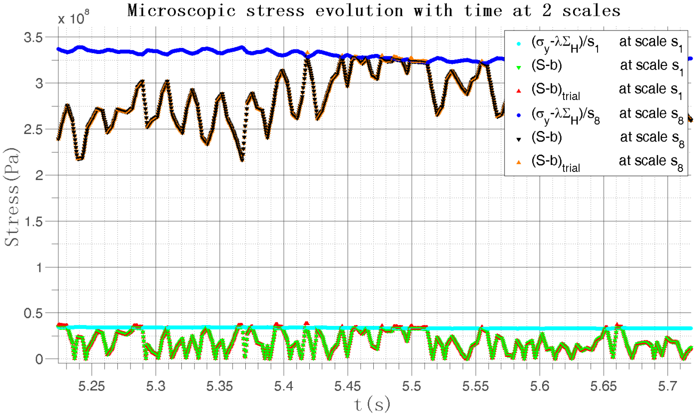

FIGURE : Circled area magnification where there is more $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|_{trial} > \sigma_y$(plasticity) at scale s_1 than at s_8

One dimensional application

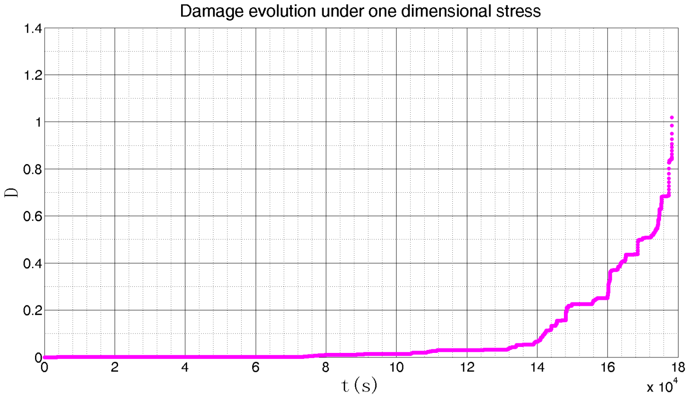

FIGURE : Damage evolution with time at one dimension PSA load history

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Multi-dimensional application

We now consider a situation where we have force recorded measured in 3 different directions as shown in FIGURE 10.

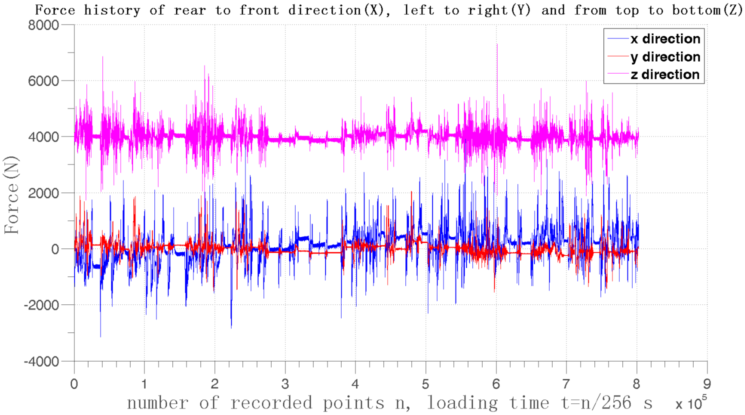


FIGURE : Loading history of 3 different directions

Multi-dimensional application

In real case, the vertical force F_z is much larger than the axial and horizontal forces F_x and F_y. However, in order to investigate large domains of interest, we first scale the axial and horizontal forces to reach comparable impact and transform them in principal stresses $c_x \frac{F_x}{A}$ applied along the stress principle vector $\underline{e}_\alpha$(respectively $\underline{e}_\beta$) that we choose randomly. We therefore consider the following macroscopic stress tensor :

$$\underline{\underline{\Sigma}} = \frac{F_z(t)}{A} \underline{e}_1 \otimes \underline{e}_1 + c_x \frac{F_x(t)}{A} \underline{e}_\alpha \otimes \underline{e}_\alpha + c_y \frac{F_y(t)}{A} \underline{e}_\beta \otimes \underline{e}_\beta \tag{14}$$

where $\underline{e}_\alpha$ and $\underline{e}_\beta$ are principal vectors whose spherical coordinate are θ_x, φ_x, θ_y and φ_y respectively :

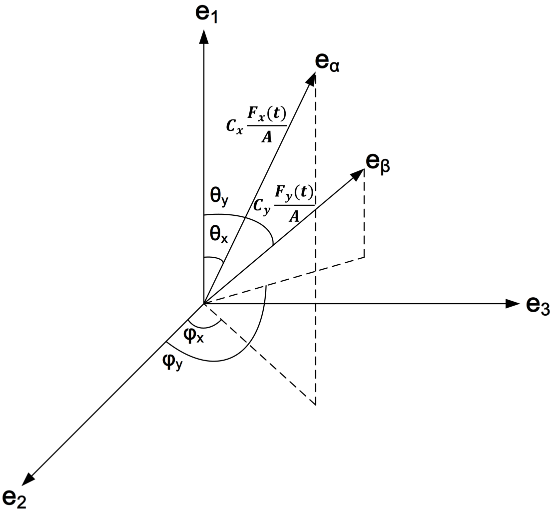
$$\underline{e}_\alpha = cos\theta_x \underline{e}_1 + sin\theta_x cos\varphi_x \underline{e}_2 + sin\theta_x sin\varphi_x \underline{e}_3,$$

$$\underline{e}_\beta = cos\theta_y \underline{e}_1 + sin\theta_y cos\varphi_y \underline{e}_2 + sin\theta_y sin\varphi_y \underline{e}_3.$$

Parameter	A(m^2)	c_x	c_y	θ_x	φ_x	θ_y	φ_y
Value	1/6e4	10	60	0.5	0.3	0.6	0.4

TABLE : The structural data in 3D analysis

Multi-dimensional application



Multi-dimensional application

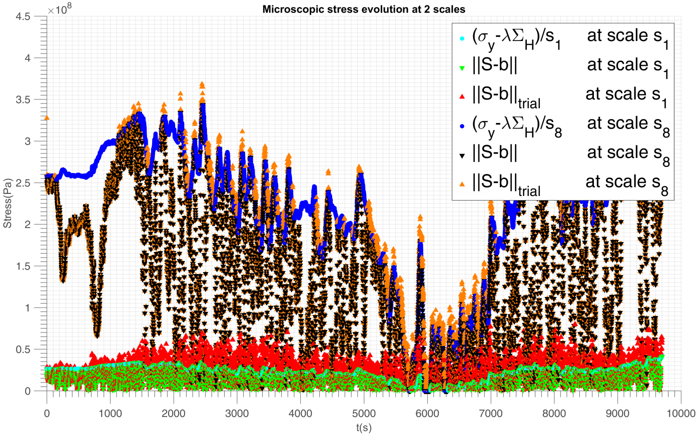


FIGURE : $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|_{trial}$ and $\left\|\underline{\underline{S}} - \underline{\underline{b}}\right\|$ evolution with time under different weakening scales in PSA load history

Multi-dimensional application

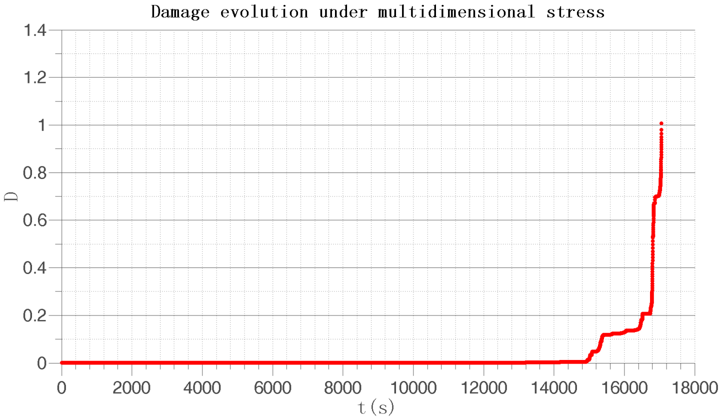


FIGURE : Damage evolution under multidimensional stress

Discussion

Discussion

1. We work on the stress tensor directly in 3D analysis in stead of using the multidimensional equivalent stress.
2. The energy based fatigue approach takes into account impurities and hardness in the material and is applicable to any type of micro plasticity law and multiaxial load geometry.
3. The small step-by-step strategy does not ignore small fluctuations in load history and the big stress effect is magnified which reflects the real situation.

Thanks for your attention!