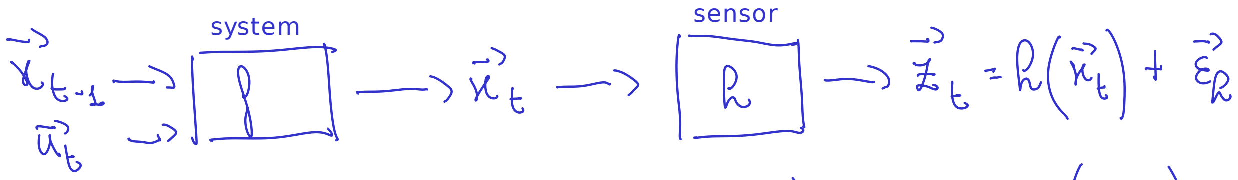
FILTERING / STATE ESTIMATION

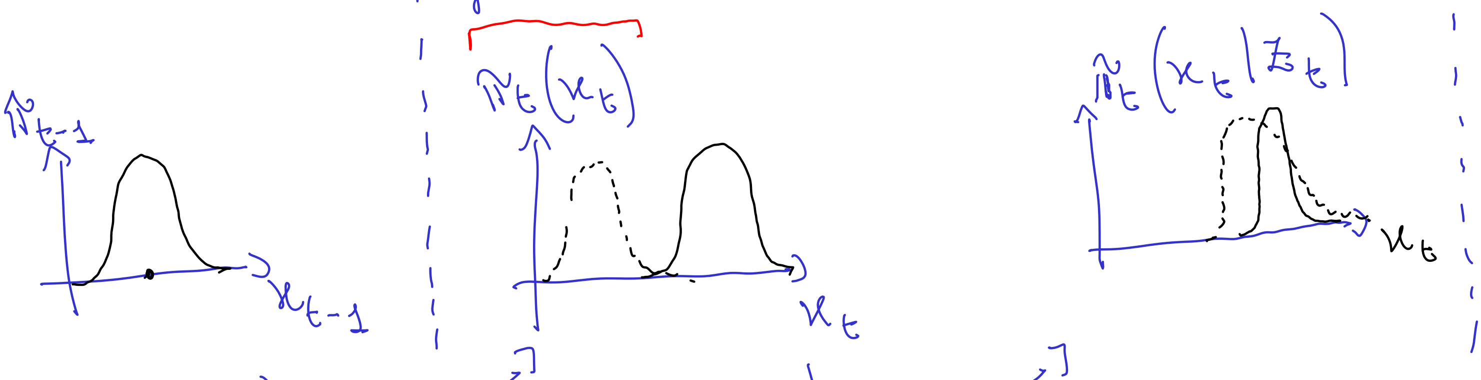


$f(t, x_{t-1}, u_t)$ = system dynamics

$h(t, x_t)$ = measurement model, $\quad \vec{\varepsilon}_h \sim N(0, R)$

$$\vec{x}_t = f(t, x_{t-1}, u_t) + \vec{\varepsilon}_f$$

$$\vec{\varepsilon}_f \sim N(0, Q)$$



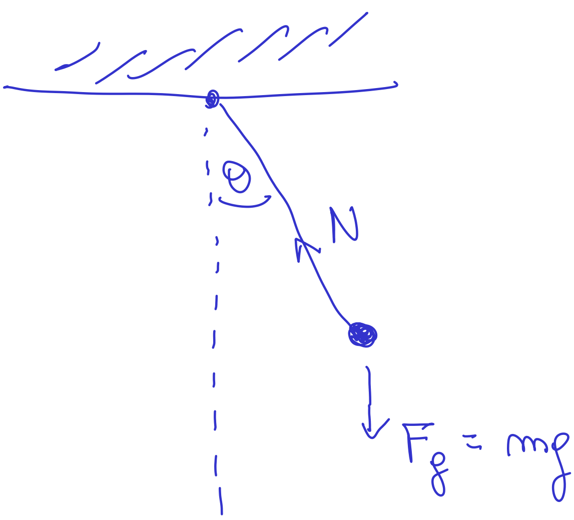
1° PREDICT

2° UPDATE with z_t

$$\tilde{\pi}_t(x_t | z_t) = \frac{L(z_t | x_t) \cdot \tilde{\pi}_t(x_t)}{N}$$

Classic Bayesian update rule

EXAMPLE



$$u_t = 0 \quad \forall t$$

$$\vec{x}_t = \begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix}$$

$$\vec{x}_t = f(t, \vec{x}_{t-1}) = ?$$

$$\vec{z}_t = h(\vec{x}_t) = \vec{x}_{1,t} = \theta_t$$

→ Implement an EKF to estimate $\vec{x}_t = \vec{x}(t) = \begin{pmatrix} \theta(t) \\ \dot{\theta}(t) \end{pmatrix}$

$$\boxed{\vec{x}_t = f(t, \vec{x}_{t-1})}$$

Discrete State-space representation of
a general dynamical system (with no input control)

How can we represent the pendulum dynamics in discrete state-space representation?

$$\frac{d\vec{x}(t)}{dt} = \begin{pmatrix} \frac{d\theta(t)}{dt} \\ \frac{d\dot{\theta}(t)}{dt} \end{pmatrix} = \begin{pmatrix} \dot{\theta}(t) \\ -\frac{g}{l}\sin(\theta(t)) \end{pmatrix}$$

Continuous dynamics
of the pendulum

$$\begin{cases} \frac{dx_1(t)}{dt} = x_2(t) \\ \frac{dx_2(t)}{dt} = -\frac{g}{l}\sin(\theta(t)) \end{cases}$$

$$\vec{x}(t) = \underbrace{\vec{x}(t-1) + \Delta t \cdot \left.\frac{d\vec{x}}{dt}\right|_{t-1}}_{f(t, \vec{x}_{t-1})}$$

Discretized dynamics
with Euler method

$$\begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} = \begin{pmatrix} x_{1,t-1} \\ x_{2,t-1} \end{pmatrix} + \Delta t \cdot \begin{pmatrix} x_{2,t-1} \\ -\frac{g}{l}\sin(x_{1,t-1}) \end{pmatrix}$$

$$f(t, x_{t-1}) = f(x_{t-1}) =$$

time-invariant in our case

$$= \begin{pmatrix} f_1(x_{t-1}) \\ f_2(x_{t-1}) \end{pmatrix}$$

With implicit time dependence:

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{pmatrix} = \begin{pmatrix} x_1 + \Delta t\, x_2 \\ x_2 - \Delta t \cdot \frac{g}{\ell} \sin(x_1) \end{pmatrix}$$

$$f : \mathbb{R}^2 \to \mathbb{R}^2$$

$$J_f = \begin{pmatrix} 1 & \Delta t \\ -\Delta t \cdot \frac{g}{\ell} \cdot \cos(x_1) & 1 \end{pmatrix}$$

Jacobian of dynamics model

$$z_t = h(x_t) = x_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} =$$

$$J_h = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Jacobian of measurement model