Nonlinear control and aerospace applications - Lab session 1

Consider a Chua circuit described by the following state equation:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = f(x,u) = \begin{bmatrix} \alpha(x_2 - x_1 - \rho(x_1)) \\ x_1 - x_2 + x_3 + u \\ -\beta x_2 - Rx_3 \end{bmatrix}, \quad \rho(x_1) = \frac{2\,x_1^3}{7} - \frac{8\,x_1}{7} \tag{1}$$

where $\alpha = 10$, $\beta = 100/7$ and $R = 0$.

Exercise 1

1. Find the equilibrium points $\bar{x}$ of the system (1), corresponding to the constant input $u(t) = \bar{u} = 0$, $\forall t$.

2. For each equilibrium point found in Step 1, linearize the system (1) around this point. Linearization can be performed computing the matrices

$$A \doteq \left.\frac{\partial f}{\partial x}\right|_{(\bar{x},\bar{u})}, \quad B \doteq \left.\frac{\partial f}{\partial u}\right|_{(\bar{x},\bar{u})}$$

   where $\partial f/\partial x$ and $\partial f/\partial u$ are the Jacobians of f with respect to x and u, respectively. A and B are the matrices of the linearized system around the given equilibrium point.

3. Considering the linearized systems of Step 2, study the stability of the equilibrium points found in Step 1.

Exercise 2

1. Implement the system (1) in Simulink.

2. Perform a simulation of duration $T_{fin} = 5\,s$, with $u(t) = 0$, $\forall t$, and $x(0) = x_0$, where x_0 is close (but not equal) to one of the equilibrium points found at Step 1 of Exercise 1. Generate a figure with a 3D Plot of the simulated trajectory. On the same figure, plot also the equilibrium points. Generate another figure, showing the state signals. Provide an interpretation of the obtained results, considering also the stability analysis carried out in Exercise 1.

3. Repeat Step 2 with a different initial state $x(0) = x_{p0}$, where x_{p0} is close (but not equal) to x_0. Add the obtained trajectory plot to the first figure of Step 2. Add the obtained state signals plot to the second figure of Step 2.

4. Repeat Steps 2 and 3 several times, considering larger simulation times T_{fin}. Discuss the possibility of predicting the future behavior of the system state, providing indications on the time horizon over which predictions can be considered reliable (note that this time horizon depends on the initial conditions).

Exercise 3

1. Let A and B be the matrices of the system linearized around the equilibrium point $\bar{u} = 0$, $\bar{x} = (0,0,0)$. On the basis of these matrices, design a linear controller, finalized at stabilizing the same equilibrium point.

2. Apply the controller to the system (1) implemented in Simulink.

3. For the controlled system, repeat Step 2 of Exercise 2 several times, considering different initial conditions and a simulation time $T_{fin} \geq 30\,s$. Discuss the obtained results.