Nonlinear control and aerospace applications - Lab session 3

The plant we consider is a Chua circuit described by the following state equations:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \alpha(x_2 - x_1 - \rho(x_1)) \\ x_1 - x_2 + x_3 + u \\ -\beta x_2 - R x_3 \end{bmatrix} \qquad (1)
$$
$$
y = x_1
$$

where $R = 0.1$, $\alpha = 10.4$, $\beta = 16.5$ and $\rho(x_1) = -1.16x_1 + 0.041x_1^3$.

Exercise 1
The main goal of this exercise is to design a sliding mode controller ensuring an accurate asymptotic tracking of constant reference signals.

1. Following the feedback linearization methodology, design the 'feedback linearization' and 'state transformation' blocks in Figure 1. The resulting general control law is

$$
u = \frac{1}{b(x)} (v - a(x)) .
$$

2. Implement in Simulink the red blocks in Figure 1.

3. Define the sliding surface function

$$
\begin{aligned}
\mathbf{s}(x,t) &\doteq \tilde{y}^{(\gamma-1)} + k_\gamma \, \tilde{y}^{(\gamma-2)} + \ldots + k_2 \, \tilde{y} \\
&= \tilde{\mu}_\gamma + k_\gamma \, \tilde{\mu}_{\gamma-1} + \ldots + k_2 \, \tilde{\mu}_1 .
\end{aligned}
$$

   where $\tilde{\mu} = (\tilde{\mu}_1, \ldots, \tilde{\mu}_\gamma) \doteq (\tilde{y}, \dot{\tilde{y}}, \ldots, \tilde{y}^{(\gamma-1)})$, $\tilde{y} \doteq r - y$ is the tracking error, γ is the system relative degree and the parameters $k_i \in \mathbb{R}$ are chosen so that all the roots of the polynomial $P(\lambda) = \lambda^{(\gamma-1)} + k_\gamma \, \lambda^{(\gamma-2)} + \ldots + k_2$ have a negative real part.

4. Implement in Simulink the sliding mode control law

$$
v = r^{(\gamma)} + k_\gamma \, \tilde{\mu}_\gamma + \ldots + k_2 \, \tilde{\mu}_2 + k_1 \, \sigma(\eta \mathbf{s})
$$

   where k_1 and η are parameters to tune (this law corresponds to the blue block in Figure 1). A possible tuning procedure is the following:

   (a) Start with $\sigma(\eta \mathbf{s}) = \text{sign}(\mathbf{s})$ and tune k_1 only.

   (b) In the case of chattering, set $\sigma(\eta \mathbf{s}) = \tanh(\eta \mathbf{s})$ and tune η (and, if necessary, k_1).

   (c) To improve the tracking performance, the parameters k_i, $i = 2, \ldots, \gamma$, can be re-designed considering that the closed-loop dynamics near the sliding surface is determined by the roots of $P(\lambda)$.

5. Test in simulation the closed-loop system, considering constant reference signals of different amplitudes. For each simulation, generate a plot comparing the reference with the resulting output and another plot showing the command input u.

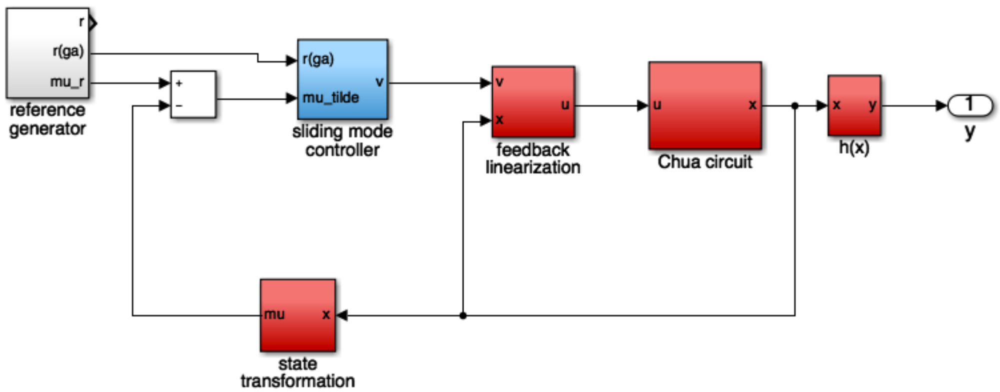

Figure 1: Chua circuit control scheme.

Exercise 2

The main goal of this exercise is to design a sliding mode controller ensuring an accurate asymptotic tracking of constant reference signals despite the presence of model uncertainties. Another goal is comparison with a pure feedback linearization controller.

1. Repeat Exercise 1, using for control design the approximated nonlinearity $\hat{\rho}(x_1) = -1.194x_1 + 0.0422x_1^3$. Note that the Chua circuit to control is in any case defined by (1), with the true nonlinearity $\rho(x_1) = -1.16x_1 + 0.041x_1^3$.

2. Repeat Step 1 of Exercise 2, using a pure feedback linearization control law

$$v = r^{(\gamma)} + k_\gamma \, \tilde{\mu}_\gamma + \ldots + k_2 \, \tilde{\mu}_2 + k_1 \, \tilde{\mu}_1$$

   where the parameters $k_i \in \mathbb{R}$ are chosen so that all the roots of the polynomial $Q(\lambda) = \lambda^{(\gamma)} + k_\gamma \, \lambda^{(\gamma-1)} + \ldots + k_1$ have negative real part.