Nonlinear control and aerospace applications - Lab session 5

Exercise 1

Consider two reference frames F1=$\{O, \mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3\}$ and F2=$\{O, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$, where $\mathbf{b}_k = \mathbf{T}_{323}\mathbf{i}_k$, $k = 1, 2, 3$, and $\mathbf{T}_{323}$ is a proper Euler rotation matrix with angles $\psi = \pi/3$, $\theta = -\pi/6$, $\phi = \pi/4$. Let the following vector be given:

$$\mathbf{r} = -\mathbf{b}_1 + 4\mathbf{b}_2 + 2\mathbf{b}_3.$$

1. Compute the components of $\mathbf{r}$ in F1.
2. Supposing to be an observer in F2, plot the following vectors:
   (a) the vector $\mathbf{r}$;
   (b) the rotated vector $\mathbf{r}' = \mathbf{T}_{323}\mathbf{r}$, where $\mathbf{T}_{323}$ is the cosine matrix associated with the above rotation.
3. Compute the norms of $\mathbf{r}$ and $\mathbf{r}'$.
4. Derive the general expression of the matrix $\mathbf{T}_{323}$ as a function of ψ, θ, ϕ (Matlab symbolic toolbox).
5. Compute the axis of the rotation produced by $\mathbf{T}_{323}$.

Exercise 2

Consider a reference frame F=$\{O, \mathbf{i}, \mathbf{j}, \mathbf{k}\}$.

1. Plot the vector $\mathbf{r} = 0\mathbf{i} + 0\mathbf{j} + 2\mathbf{k}$ and the following rotated vectors:
   (a) $\mathbf{r}_1 = \mathbf{T}_1(\pi/2)\mathbf{r}$;
   (b) $\mathbf{r}_2 = \mathbf{T}_3(\pi/2)\mathbf{T}_1(\pi/2)\mathbf{r}$;
   (c) $\mathbf{r}_3 = \mathbf{T}_3(\pi/2)\mathbf{r}$;
   (d) $\mathbf{r}_2 = \mathbf{T}_1(\pi/2)\mathbf{T}_3(\pi/2)\mathbf{r}$.
2. By means of graphical considerations, verify that $\mathbf{T}_3(\pi/2)\mathbf{T}_1(\pi/2)$ is an extrinsic rotation $x - z$ or an intrinsic rotation $z - x'$, where x' denotes the rotated x axis.
3. By means of graphical considerations, verify that $\mathbf{T}_1(\pi/2)\mathbf{T}_3(\pi/2)$ is an extrinsic rotation $z - x$ or an intrinsic rotation $x - z'$, where z' denotes the rotated z axis.
4. For each of the 4 rotations above, compute the eigenvalues and the axis of rotation.

Exercise 3

Repeat Steps 1-3 of Exercise 1 using quaternions instead of rotation matrices.

Exercise 4

Let the quaternions $\mathfrak{q} = \begin{pmatrix} 0.866, & 0.4319, & 0.216, & 0.1296 \end{pmatrix}$ and $\mathfrak{p} = \begin{pmatrix} 0.9659 & 0.183 & 0 & 0.183 \end{pmatrix}$ be given. Compute:

1. The reciprocals $\mathfrak{q}^{-1}$ and $\mathfrak{p}^{-1}$.
2. The dot products $\mathfrak{q} \cdot \mathfrak{p}$ and $\mathfrak{p} \cdot \mathfrak{q}$.
3. The quaternion products $\mathfrak{q} \otimes \mathfrak{p}$ and $\mathfrak{p} \otimes \mathfrak{q}$.
4. The quaternion products $\mathfrak{I} \otimes \mathfrak{p}$ and $\mathfrak{p} \otimes \mathfrak{I}$, where $\mathfrak{I}$ is the identity element.

Exercise 5

1. Compute the quaternion corresponding to the DCM
$$\mathbf{T} = \begin{bmatrix} 0.2276 & -0.9354 & 0.2706 \\ 0.7571 & -0.004773 & -0.6533 \\ 0.6124 & 0.3536 & 0.7071 \end{bmatrix}.$$
2. Consider the quaternion
$$q = \begin{pmatrix} 0.588, & 0.03378, & -0.2566, & 0.7663 \end{pmatrix}.$$
   Compute the corresponding DCM and Euler angles 313.