Assignment 5: Ray Intersection With Triangle

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1 Ray Generation

Creating a primary ray for each pixel of the frame can be done by tracing a line starting at the camera's origin and passing through the middle of each pixel. To compute the position of a point at the center of a pixel, we need to convert the pixel coordinates which are originally expressed in **raster space** (the point coordinates are expressed in pixels with the coordinates (0,0) being the top-left corner of the frame) to **world space**.

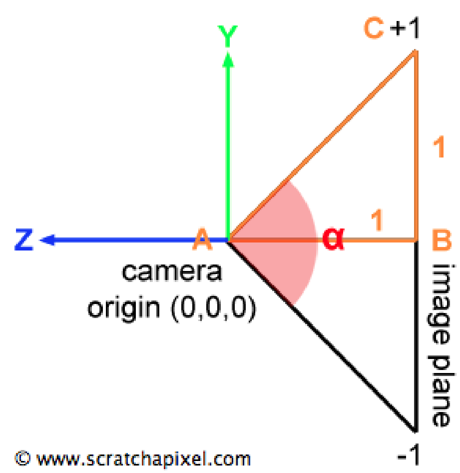
The coordinates of this point are first expressed in ***raster space*** (the pixels coordinate plus an offset of 0.5), then converted to ***NDC (Normalized Device Coordinates) space*** (the coordiantes are remapped to the range [0, 1]), then converted to ***screen space*** (the NDC coordinates are remapped to the [-1, 1]). Applying the final camera-to-world transformation 4×4 matrix transforms the coordinated in screen space to ***world space***.

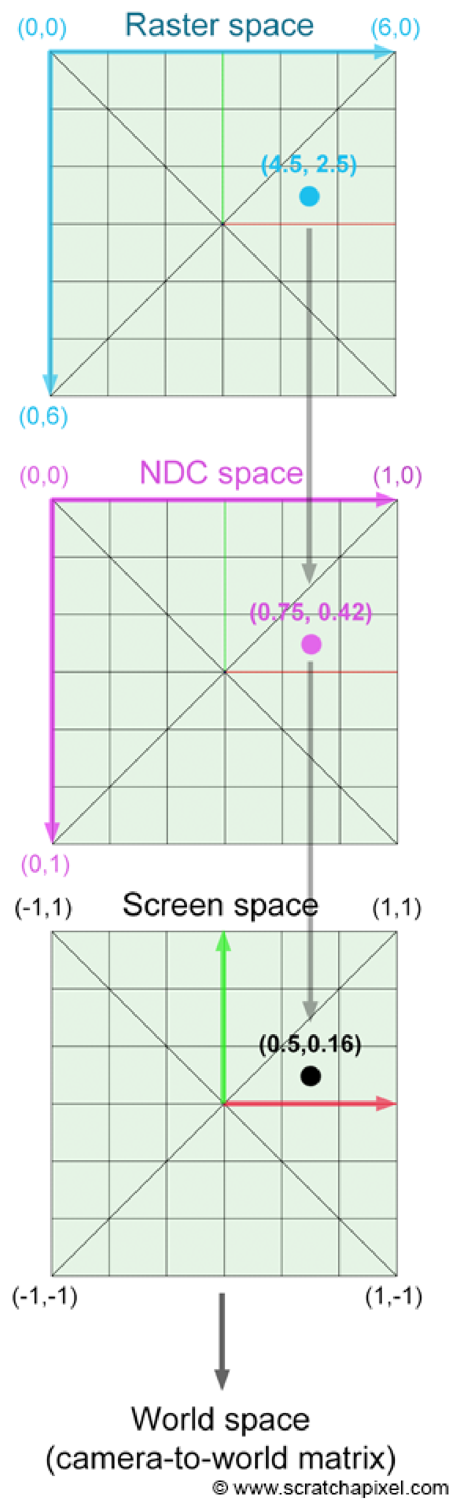
$$\begin{aligned} PixelNDC_x &= \frac{Pixel_x + 0.5}{ImageWidth} \\ PixelNDC_y &= \frac{Pixel_y + 0.5}{ImageHeight} \end{aligned} \tag{1}$$

$$\begin{aligned} PixelScreen_x &= 2 * PixelNDC_x - 1 \\ PixelScreen_y &= 1 - 2 * PixelNDC_y \end{aligned} \tag{2}$$

$$ImageAspectRatio = \frac{ImageWidth}{ImageHeight} \tag{3}$$

$$\begin{aligned} &PixelCamera_x = (2 * PixelScreen_x - 1) * ImageAspectRatio * tan(\frac{\alpha}{2}) \\ &PixelCamera_y = (1 - 2 * PixelScreen_y) * tan(\frac{\alpha}{2}) \\ &P_{cameraSpace} = (PixelCamera_x, PixelCamera_y, -1) \end{aligned} \tag{4}$$




(0,0)
Raster space
(6,0)
(4.5, 2.5)
(0,6)
(0,0)
NDC space
(1,0)
(0.75, 0.42)
(0,1)
(-1,1)
Screen space
(1,1)
(0.5,0.16)
(-1,-1)
(1,-1)
World space
(camera-to-world matrix)
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2 Ray Intersection

Möller Trumbore Algorithm is a faster approach to solve ray intersection with plane, giving barycentric coordinate directly. The ray equation is equal to triangle equation

$$\vec{\mathbf{O}} + t\vec{\mathbf{D}} = (1 - b_1 - b_2)\vec{\mathbf{P}}_0 + b_1\vec{\mathbf{P}}_1 + b_2\vec{\mathbf{P}}_2 \tag{5}$$

Solve for intersection, we can get

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{\vec{\mathbf{S}}_1 \cdot \vec{\mathbf{E}}_1} \begin{bmatrix} \vec{\mathbf{S}}_2 \cdot \vec{\mathbf{E}}_2 \\ \vec{\mathbf{S}}_1 \cdot \vec{\mathbf{S}} \\ \vec{\mathbf{S}}_2 \cdot \vec{\mathbf{D}} \end{bmatrix} \tag{6}$$

where

$$\begin{aligned} \vec{\mathbf{E}}_1 &= \vec{\mathbf{P}}_1 - \vec{\mathbf{P}}_0 \\ \vec{\mathbf{E}}_2 &= \vec{\mathbf{P}}_2 - \vec{\mathbf{P}}_0 \\ \vec{\mathbf{S}} &= \vec{\mathbf{O}} - \vec{\mathbf{P}}_0 \\ \vec{\mathbf{S}} &= \vec{\mathbf{D}} \times \vec{\mathbf{E}}_2 \\ \vec{\mathbf{S}}_2 &= \vec{\mathbf{S}} \times \vec{\mathbf{E}}_1 \end{aligned} \tag{7}$$

$1 - b_1 - b_2, b_1, b_2$ are barycentric coordinates. If these parameters are non-negative, the intersection is inside the triangle.