Assignment 8: Mass Spring system

Mengzhu Wang

February 15, 2024

1 Rope Constructor

Object Rope has a start and an end and includes some nodes. Each node has a mass, which is called a particle. The line segment between the particles is a spring. We get a still rope now.

2 Euler Method

Hooke's law states that the force between the two particles connected by a spring is proportional to distance between them,

$$f_{b\to a} = -k_s \frac{b-a}{\|b-a\|}(\|b-a\| - l)$$

Two particles of a spring are both influenced by the force. Calculate the sum of forces for each particle. There are two methods to estimate position, including explicit method and semi-implicit method.

$$\begin{aligned}
&a = F/m + gravity \\
&v(t+1) = v(t) + a(t) * dt \\
&x(t+1) = x(t) + v(t) * dt \quad // \ \textbf{explicit method} \\
&x(t+1) = x(t) + v(t+1) * dt \quad // \ \textbf{semi-implicit method}
\end{aligned}$$

The rope of explicit method is unsable. We can reduce step to alleviate instability.

3 Verlet Method

The verlet integrator offers greater stability than the much simpler Euler method. It upgrades position as below,

$$x(t+1) = x(t) + [x(t) - x(t+1)] + a(t) * dt * dt$$

4 Damping

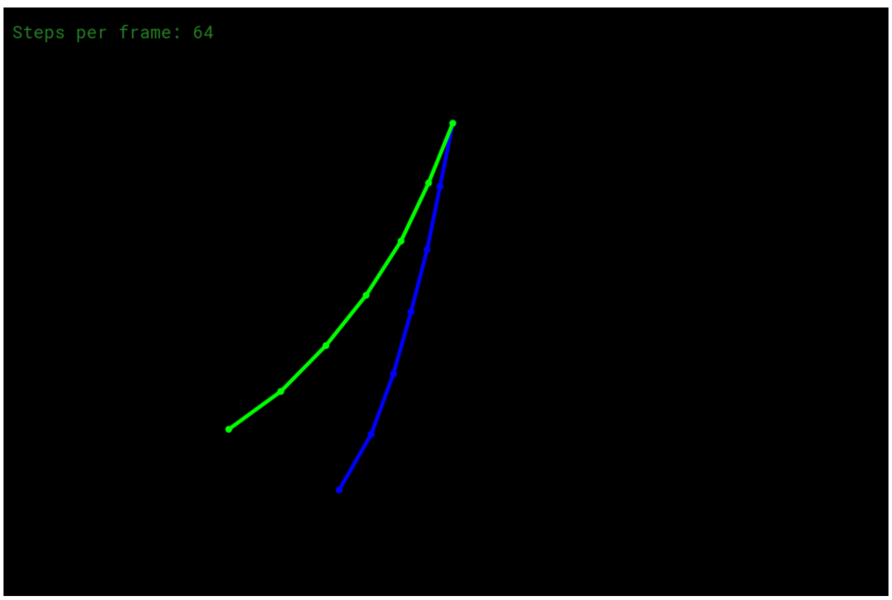
Springs in reality don't jump forever because the kinetic energy decreases due to friction. We can add global damping to simulate it.

For Euler method, global damping force is $f_d = -k_d v$. We can change acceleration equation to

$$a = F/m + gravity - -k_d v/m$$

For verlet method, the position calculation can change to

$$x(t+1) = x(t) + (1 - damping\_factor) * [x(t) - x(t+1)] + a(t) * dt * dt$$


Steps per frame: 64