Assignment 1: Rotation and Projection

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1 Model Transformation

1.1 How to rotate around x, y, z axis

Create the model matrix for rotating the triangle around the X/Y/Z axis

$$R_X(\alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\alpha & -sin\alpha & 0 \\ 0 & sin\alpha & cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

$$R_y(\alpha) = \begin{pmatrix} cos\alpha & 0 & sin\alpha & 0 \\ 0 & 1 & 0 & 0 \\ -sin\alpha & 0 & cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

$$R_z(\alpha) = \begin{pmatrix} cos\alpha & -sin\alpha & 0 & 0 \\ sin\alpha & cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

1.2 How to rotate around any axis passing through the origin

Rodrigues' Rotation Formula
rotation by angle α around axis n

$$\mathbf{R}(\mathbf{n}, \alpha) = cos(\alpha)\mathbf{I} + (1 - cos(\alpha))\mathbf{n}\mathbf{n}^T + sin(\alpha) \underbrace{\begin{pmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{pmatrix}}_{\mathbf{N}} \tag{4}$$

2 View Transformation

$M_{view} = R_{view}T_{view}$
- Transform objects together with the camera
- Until camera's at the origin, up at Y, look at -Z

translate e to origin

$$T_{view} = \begin{pmatrix} 1 & 0 & 0 & -x_e \\ 0 & 1 & 0 & -y_e \\ 0 & 0 & 1 & -z_e \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{5}$$

rotate g to -Z, t to Y, (g x t) to X

$$R_{view} = \begin{pmatrix} x_{\hat{g}\times\hat{t}} & y_{\hat{g}\times\hat{t}} & z_{\hat{g}\times\hat{t}} & 0 \\ x_t & y_t & z_t & 0 \\ x_{-g} & y_{-g} & z_{-g} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{6}$$

3 Projection Transformation

Create the projection matrix for the given parameters

3.1 How to convert from fovY and aspect to l, r, b, t

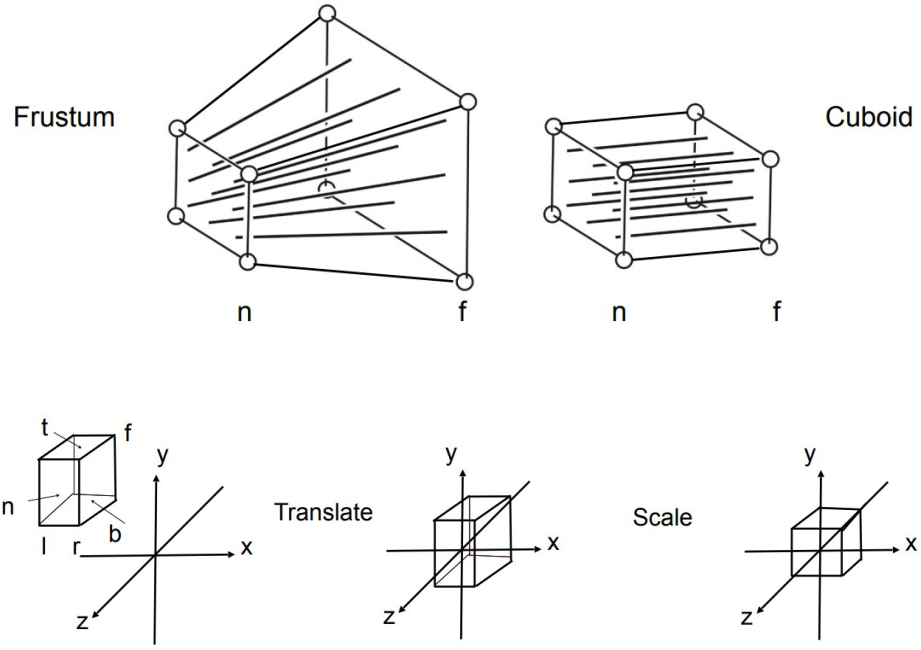
assume symmetry l=-r, b=-t

$$t = |n| tan \frac{fovY}{2} \tag{7}$$

$$r = t \times aspect \tag{8}$$

3.2 How to do perspective projection

$M_{persp} = M_{ortho} M_{persp \to ortho}$
- First squish the frustum into a cuboid (n→n, f→f) ($M_{persp \to ortho}$)
- Do orthographic projection (M_{ortho})



orthographic projection: translate (center to origin) first, then scale (length/width/height to 2)

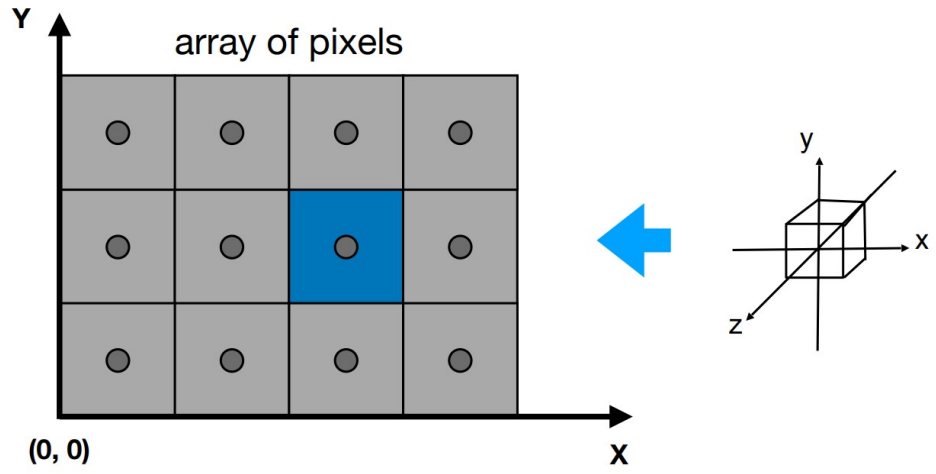
$$M_{ortho} = \begin{pmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{n-f} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -\frac{r+l}{2} \\ 0 & 1 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 1 & -\frac{n+f}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{9}$$

squish the frustum into a cuboid

$$M_{persp \to ortho} = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -nf \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{10}$$

4 Viewport Transformation

canonical cube to screen
- irrelevant to z
- transform in xy plane: $[-1,1]^2$ to [0, width] $\times$ [0, height]



$$M_{viewport} = \begin{pmatrix} \frac{width}{2} & 0 & 0 & \frac{width}{2} \\ 0 & \frac{height}{2} & 0 & \frac{height}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{11}$$

5 Summary

- Model transformation (placing objects)
- View transformation (placing camera)
- Projection transformation
  - Orthographic projection (cuboid to canonical cube $[-1,1]^3$)
  - Perspective projection (frustum to canonical cube)
- Viewport transformation (canonical cube to screen)