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ME 3360

Hoelzle 9:10-10:05 a.m.

Project Part 1

Part (a)

Simplifying assumptions and rationale:

1. **The only forces acting on the car is the damping force acting on the car to slow it down and the force from the torque acting on the wheels that propels the car forward.** The damping coefficient provided accounts for any resistances and frictions that may dissipate energy from the car, grouping all forces that would act against the car into one. The car itself would only be propelled forward by its engine spinning its tires.

Another assumption included in this then is that the car is travelling on level ground for the duration of its travel. This way the normal force and the car's weight balance each other and the weight does not impact the acceleration of the car. The car is stated to be on an incline in part (e), so for this part where no incline is mentioned, it can be assumed that the road is level.

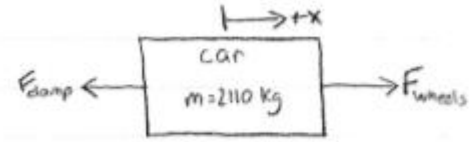
2. **Each of the four wheels contributes equally to moving the car forward.** With the car being all-wheel drive, all four of the wheels will be driven by the engine and provide force to move the car forward. There are also no conditions mentioned that might affect the performance of one individual wheel (i.e. road conditions like potholes or ice).

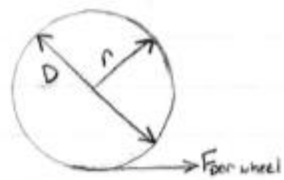
3. **A constant torque can be provided from 0-60 mph.** Assuming this means the torque/force from the torque can be treated as a step response, simplifying the analysis of the Laplace Transform. The actual torque profile of the car is relatively constant, especially in the 0-40 mph range, so the benefits in simplifying the analysis outweigh the small loss in accuracy to the actual torque profile.

a) Givens: $m = 2110\ \text{Kg}$ $\quad b = 62\ \frac{\text{N}\cdot\text{sec}}{\text{m}}$ $\quad V_0 = 0\ \text{mph} = 0\ \frac{\text{m}}{\text{s}}$

$D = 0.48\ \text{m}$ $\quad t_f = 2.4\ \text{sec}$ $\quad V_f = 60\ \text{mph} = 26.8224\ \frac{\text{m}}{\text{s}}$

$r = 0.24\ \text{m}$





$\overset{+}{\rightarrow} \sum F = ma = F_{wheels} - F_{damp}$

$(F_{per\ wheel})\, r = T_{max}$

$F_{per\ wheel} = T_{max}/r$

$F_{wheels} = 4F_{per\ wheel} = 4T_{max}/r$

$ma = \frac{4T_{max}}{r} - F_{damp} = \frac{4T_{max}}{r} - bv$

$4\frac{T_{max}}{r} = ma + bv = m\dot{v} + bv$

$4T_{max} = mr\dot{v} + brv$

$\downarrow$ Laplace

$\frac{4T_{max}}{s} = mr(sV(s) - \cancel{v(0)}^{0}) + brV(s) = mrsV(s) + brV(s) = (mrs + br)V(s)$

$V(s) = \frac{4T_{max}}{s(mrs+br)} = \frac{4T_{max}}{rs(ms+b)} = \frac{4T_{max}}{mrs(s+\frac{b}{m})} * \frac{\frac{b}{m}}{\frac{b}{m}} = \frac{4T_{max}\frac{b}{m}}{brs(s+\frac{b}{m})}$

$V(s) = \frac{4T_{max}}{br}\left(\frac{\frac{b}{m}}{s(s+\frac{b}{m})}\right) \Rightarrow \mathcal{L}^{-1}\{V(s)\} = \mathcal{L}^{-1}\left\{\frac{4T_{max}}{br}\left(\frac{\frac{b}{m}}{s(s+\frac{b}{m})}\right)\right\} = \frac{4T_{max}}{br}\mathcal{L}^{-1}\left\{\frac{\frac{b}{m}}{s(s+\frac{b}{m})}\right\}$

Using the identity that $\mathcal{L}^{-1}\left\{\frac{a}{s(s+a)}\right\} = 1 - e^{-at}$ where $a = \frac{b}{m}$

$\mathcal{L}^{-1}\{V(s)\} = v(t) = \frac{4T_{max}}{br}\left(1 - e^{-\frac{b}{m}t}\right)$

<u>I.C.'s</u>

$v(0) = 0\ \frac{\text{m}}{\text{s}}$; $v(2.4) = 26.8224\ \frac{\text{m}}{\text{s}}$

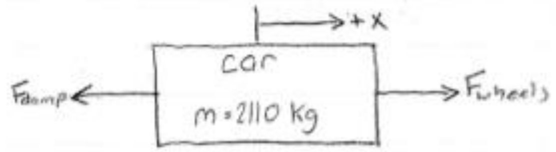
$v(0) = 0 = \frac{4T_{max}}{br}(1 - e^{0}) = \frac{4T_{max}}{br}(0) = 0 \rightarrow 0 = 0$

$v(2.4) = 26.8224 = \frac{4T_{max}}{br}\left(1 - e^{-\frac{b}{m}(2.4)}\right) \rightarrow T_{max} = \frac{26.8224\, br}{4(1 - e^{-b/m\, t})} = \frac{(26.8224\ \frac{\text{m}}{\text{s}})(62\ \frac{\text{N}\cdot\text{sec}}{\text{m}})(0.24\text{m})}{4(1 - e^{-62/2110\,(2.4)})}$

$\boxed{T_{max} = 1465\ \text{N}\cdot\text{m}}$

Part (b)

b)



$+\sum F = 0$ (since @ steady state / constant velocity of $26.8224 \frac{m}{s}$)

$F_{wheels} - F_{damp} = 0$

$F_{wheels} = F_{damp}$

$$F_{wheels} = 4 F_{per\ wheel} = 4 \frac{T_{ss}}{r}$$

$$F_{damp} = bv$$

$4 \frac{T_{ss}}{r} = bv$

$T_{ss} = \frac{1}{4} brv = \frac{1}{4}\left(62 \frac{N \cdot sec}{m}\right)(0.24\ m)\left(26.8224 \frac{m}{s}\right)$

$\boxed{T_{ss} = 99.8\ N \cdot m}$

Part (c)

c) $T(t) = T_{max}\,u(t) - (T_{max} - T_{ss})\,u(t-2.4)$

$$T(s) = \frac{T_{max}}{s} - \frac{(T_{max} - T_{ss})}{s} e^{-2.4s}$$

*Using same set up as part (a)...

$$\frac{4T(t)}{r} = m\dot{v} + bv$$

$$4T(t) = mr\dot{v} + brv$$

$$4T(s) = (mrs + br)\,V(s)$$

$$V(s) = \frac{4T(s)}{mrs+br} = \frac{4}{mrs+br}\left(\frac{T_{max}}{s} - \frac{(T_{max}-T_{ss})}{s} e^{-2.4s}\right)$$

$$V(s) = \left(\frac{4T_{max}}{mrs(s+\frac{b}{m})} - \frac{4(T_{max}-T_{ss})}{mrs(s+\frac{b}{m})} e^{-2.4s}\right) * \frac{\frac{b}{m}}{\frac{b}{m}}$$

$$V(s) = \underbrace{\frac{4T_{max}}{br}\,\frac{\frac{b}{m}}{s(s+\frac{b}{m})}}_{\text{solved in part (a)}} - \underbrace{\frac{4(T_{max}-T_{ss})}{br}\,\frac{\frac{b}{m}}{s(s+\frac{b}{m})}}_{\text{same form as part (a) but w/ } (T_{max}-T_{ss}) \text{ in place of } T_{max} \text{ \& time shift of } (t-2.4)} e^{-2.4s}$$

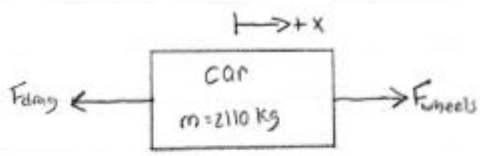
$$\therefore \boxed{v(t) = \frac{4T_{max}}{br}\left(1 - e^{-\frac{b}{m}t}\right) - \frac{4(T_{max}-T_{ss})}{br}\left(1 - e^{-\frac{b}{m}t}\right)u(t-2.4) \quad \text{for } t \geq 0}$$

With values plugged in:

$$\boxed{v(t) = 393.9\left(1 - e^{-\frac{62}{2110}t}\right) - 367.1\left(1 - e^{-\frac{62}{2110}t}\right)u(t-2.4) \quad \text{for } t \geq 0}$$

Part (d)

d)



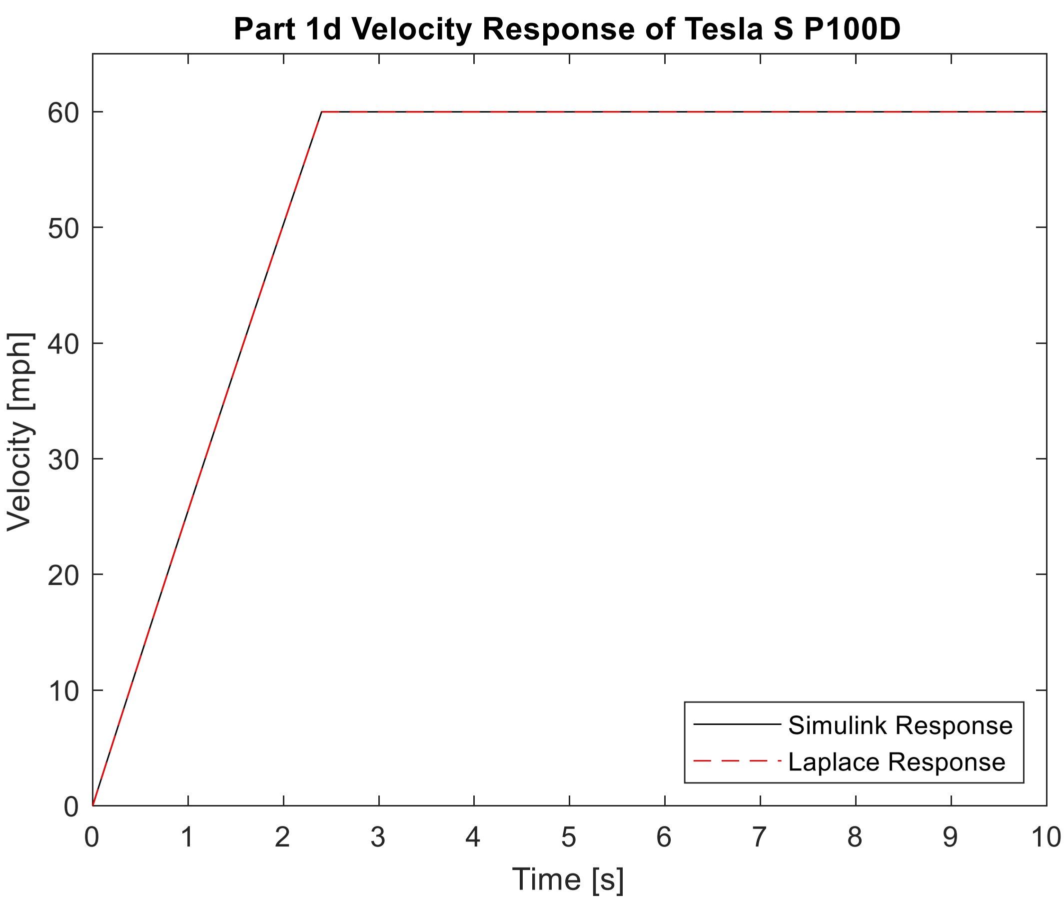
$$\overset{+}{\rightarrow} \sum F_x = ma = F_{wheels} - F_{drag}$$

$$ma = F_{wheels} - bv$$

$$m\dot{v} = F_{wheels} - bv$$

$$\dot{v} = \frac{(F_{wheels} - bv)}{m} \longrightarrow \text{for Simulink model}$$

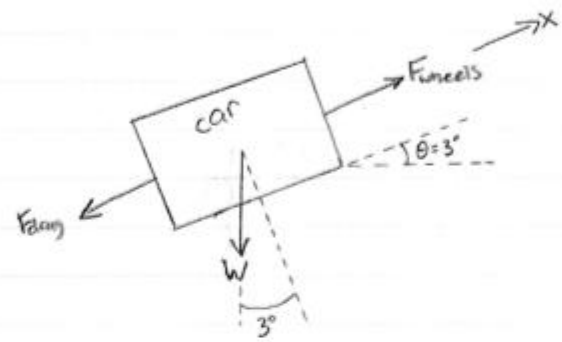
$$F_{wheels} = \begin{cases} 4\frac{T_{max}}{r} & \text{for } 0 \le t < 2.4 \\ 4\frac{T_{ss}}{r} & \text{for } t \ge 2.4 \end{cases}$$



See Appendix A for the Simulink diagram and code used to generate the plot above.

Part (e)

e)



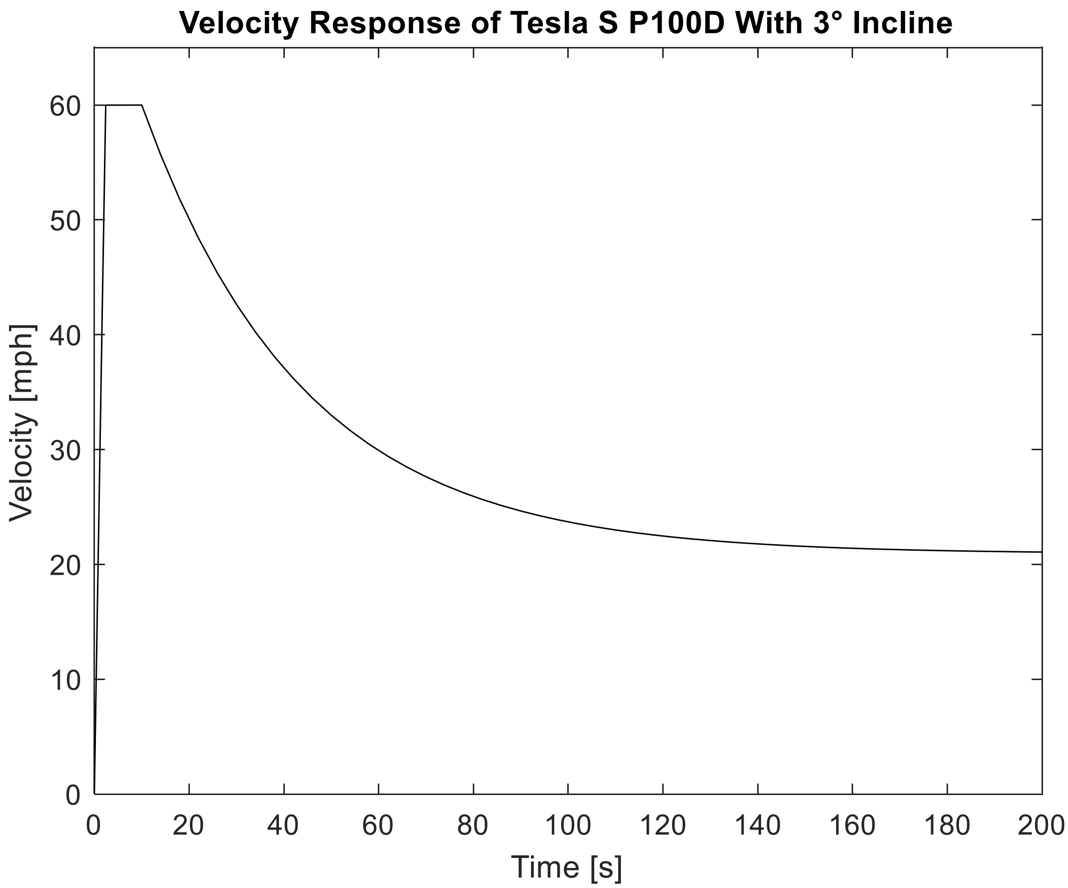
$$\xrightarrow{+} \sum F_x = ma = F_{wheels} - F_{drag} - W\sin\theta\, u(t-10)$$

$$m\dot{v} = F_{wheels} - mg\sin\theta\, u(t-10) - bv$$

$$m\dot{v} = F_{combined} - bv$$

$$\dot{v} = \frac{F_{combined} - bv}{m} \quad \rightarrow \text{Simulink model}$$

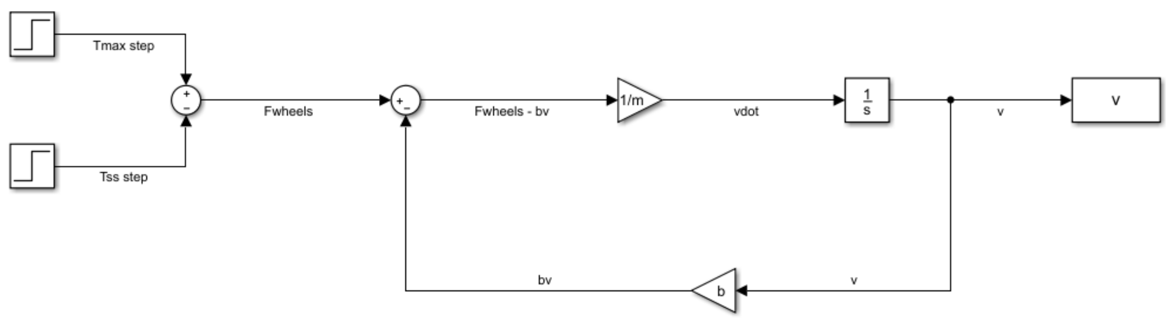
$$F_{combined} = \begin{cases} 4\frac{T_{max}}{r} & \text{for } 0 \le t < 2.4 \\ 4\frac{T_{ss}}{r} & \text{for } 2.4 \le t < 10 \\ 4\frac{T_{ss}}{r} - mg\sin\theta & \text{for } t \ge 10 \end{cases}$$



As can be seen in the graph, the speed of the car steadily decreases to just above 20 mph (~21.1 mph) when it begins going up the 3º incline. See Appendix B for the Simulink diagram and code used to generate the plot above.

Appendix A

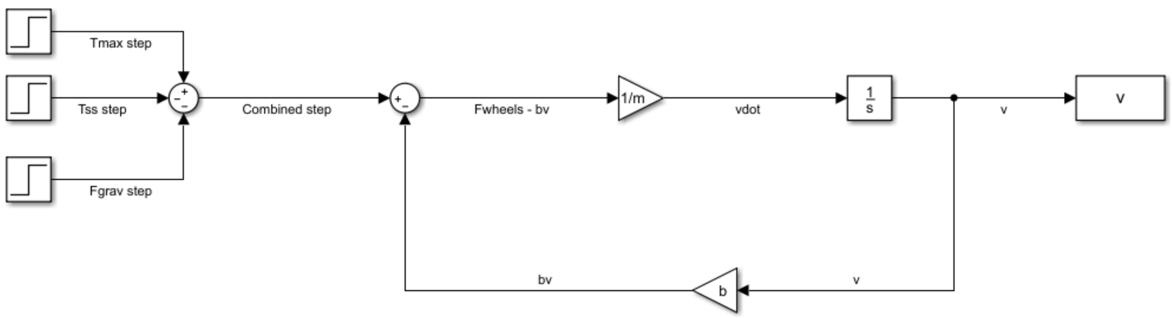
Simulink Diagram and Code for Part (d)




```
clear;
clc;
close all;
syms t s;

%Values given in problem statement or calculated in parts (a) & (b)
m = 2110;
b = 62;
d = 0.48;
r = d / 2;
Tmax = 1465;
Tss = 99.8;
%Conversion constant for converting m/s back to original mph goal
ms_to_mph = 2.23694;

%Calculating the force provided by the max and steady state torques
Fwheels_max = 4 * Tmax / r;
Fwheels_ss = 4 * Tss / r;

%Execute the simulink diagram
sim Part1_d_simu
%Plot the response received from simulink, plotting the velocity as mph
%instead of m/s.  Label and title the graph
plot(v.Time, v.Data*2.23694,'k');
xlabel('Time [s]');
ylabel('Velocity [mph]');
ylim([0 65]);
title('Part 1d Velocity Response of Tesla S P100D');

%V_s is the Laplace function for the velocity found in part (c)
V_s = (4/(m*r*s+b*r)) * (Tmax/s - ((Tmax-Tss)/s)*exp(-2.4*s));
%Inverse Laplace the Laplace funciton and convert it into a function
vel = ilaplace(V_s);
vel_t = matlabFunction(vel);
%Create a vector of time elements for plotting the Laplace response
time = 0:0.01:10;
hold on;
%Plot the Laplace response on top of the simulink response
plot(time, vel_t(time)*ms_to_mph,'r--');
legend('Simulink Response', 'Laplace Response', 'Location', 'southeast');
```

Appendix B

Simulink Diagram and Code for Part (e)




```
clear;
clc;
close all;
syms t s;

%Values given in problem statement or calculated in parts (a) & (b)
m = 2110;
b = 62;
d = 0.48;
r = d / 2;
theta = 3;
g = 9.81;
Tmax = 1465;
Tss = 99.8;
%Conversion constant for converting m/s back to original mph goal
ms_to_mph = 2.23694;

%Calculating the force provided by the max and steady state torques
Fwheels_max = 4 * Tmax / r;
Fwheels_ss = 4 * Tss / r;

%Component of the cars weight acting down the slope
Fgrav = m * g *sind(theta);

%Execute the simulink diagram
sim Part1_e_simu

%Plot the response received from simulink, plotting the velocity as mph
%instead of m/s.  Label and title the graph
plot(v.Time, v.Data*2.23694,'k');
xlabel('Time [s]');
ylabel('Velocity [mph]');
ylim([0 65]);
title(['Velocity Response of Tesla S P100D With 3' char(176) ' Incline']);
```