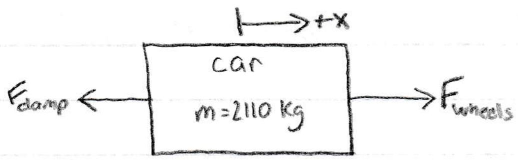
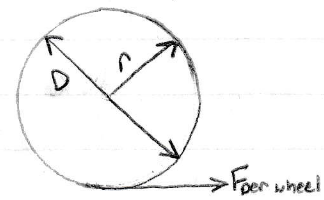
a) Givens: $m = 2110\ \text{Kg}$ $b = 62\ \frac{N\cdot sec}{m}$ $V_0 = 0\ mph = 0\ \frac{m}{s}$

$D = 0.48\ m$ $t_f = 2.4\ sec$ $V_f = 60\ mph = 26.8224\ \frac{m}{s}$

$r = 0.24\ m$





$\overset{+}{\rightarrow} \sum F = ma = F_{wheels} - F_{damp}$

$(F_{per\ wheel})r = T_{max}$

$F_{per\ wheel} = T_{max}/r$

$F_{wheels} = 4F_{per\ wheel} = 4T_{max}/r$

$$ma = \frac{4T_{max}}{r} - F_{damp} = \frac{4T_{max}}{r} - bv$$

$$4\frac{T_{max}}{r} = ma + bv = m\dot{v} + bv$$

$$4T_{max} = mr\dot{v} + brv$$

$\downarrow$ Laplace

$$\frac{4T_{max}}{s} = mr(sV(s) - \cancel{v(0)}^{\,0}) + brV(s) = mrsV(s) + br\,V(s) = (mrs + br)V(s)$$

$$V(s) = \frac{4T_{max}}{s(mrs+br)} = \frac{4T_{max}}{rs(ms+b)} = \frac{4T_{max}}{mrs(s+\frac{b}{m})} * \frac{\frac{b}{m}}{\frac{b}{m}} = \frac{4T_{max}\frac{b}{m}}{brs(s+\frac{b}{m})}$$

$$V(s) = \frac{4T_{max}}{br}\left(\frac{\frac{b}{m}}{s(s+\frac{b}{m})}\right) \Rightarrow \mathcal{L}^{-1}\{V(s)\} = \mathcal{L}^{-1}\left\{\frac{4T_{max}}{br}\left(\frac{\frac{b}{m}}{s(s+\frac{b}{m})}\right)\right\} = \frac{4T_{max}}{br}\mathcal{L}^{-1}\left\{\frac{\frac{b}{m}}{s(s+\frac{b}{m})}\right\}$$

Using the identity that $\mathcal{L}^{-1}\left\{\frac{a}{s(s+a)}\right\} = 1 - e^{-at}$ where $a = \frac{b}{m}$

$$\mathcal{L}^{-1}\{V(s)\} = v(t) = \frac{4T_{max}}{br}\left(1 - e^{-\frac{b}{m}t}\right)$$

I.C.'s

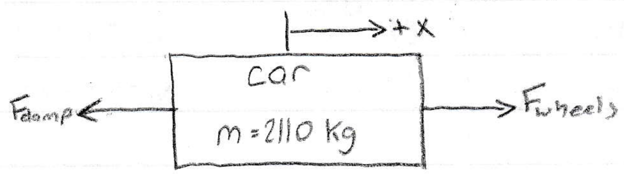
$v(0) = 0\ \frac{m}{s}$; $v(2.4) = 26.8224\ \frac{m}{s}$

$$v(0) = 0 = \frac{4T_{max}}{br}(1 - e^0) = \frac{4T_{max}}{br}(0) = 0 \rightarrow 0 = 0$$

$$v(2.4) = 26.8224 = \frac{4T_{max}}{br}\left(1 - e^{-\frac{b}{m}(2.4)}\right) \rightarrow T_{max} = \frac{26.8224\,br}{4(1-e^{-b/mt})} = \frac{(26.8224\ \frac{m}{s})(62\ \frac{N\cdot sec}{m})(0.24m)}{4(1-e^{-62/2110\,(2.4)})}$$

$$\boxed{T_{max} = 1465\ N\cdot m}$$

b)



$+\sum F = 0$ (since @ steady state / constant velocity of $26.8224 \frac{m}{s}$)

$F_{wheels} - F_{damp} = 0$

$F_{wheels} = F_{damp}$

$F_{wheels} = 4 F_{per\ wheel} = 4 \frac{T_{ss}}{r}$

$F_{damp} = bv$

$4 \frac{T_{ss}}{r} = bv$

$T_{ss} = \frac{1}{4} brv = \frac{1}{4}(62 \frac{N \cdot sec}{m})(0.24\,m)(26.8224 \frac{m}{s})$

$\boxed{T_{ss} = 99.8\ N \cdot m}$

c) $T(t) = T_{max}\, u(t) - (T_{max} - T_{ss})\, u(t-2.4)$

$$T(s) = \frac{T_{max}}{s} - \frac{(T_{max} - T_{ss})}{s} e^{-2.4s}$$

*Using same set up as part (a)...

$$\frac{4T(t)}{r} = m\dot{v} + bv$$

$$4T(t) = mr\dot{v} + brv$$

$$4T(s) = (mrs + br)\, V(s)$$

$$V(s) = \frac{4T(s)}{mrs + br} = \frac{4}{mrs + br}\left(\frac{T_{max}}{s} - \frac{(T_{max} - T_{ss})}{s} e^{-2.4s}\right)$$

$$V(s) = \left(\frac{4T_{max}}{mrs(s + \frac{b}{m})} - \frac{4(T_{max} - T_{ss})}{mrs(s + \frac{b}{m})} e^{-2.4s}\right) * \frac{\frac{b}{m}}{\frac{b}{m}}$$

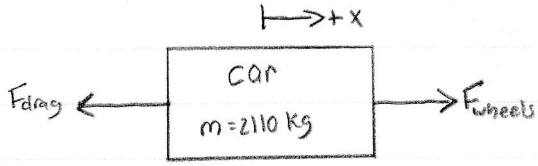
$$V(s) = \underbrace{\frac{4T_{max}}{br} \frac{\frac{b}{m}}{s(s + \frac{b}{m})}}_{\text{solved in part (a)}} - \underbrace{\frac{4(T_{max} - T_{ss})}{br} \frac{\frac{b}{m}}{s(s + \frac{b}{m})}}_{\text{same form as part (a) but w/ } (T_{max} - T_{ss}) \text{ in place of } T_{max} \text{ \& time shift of } (t-2.4)} e^{-2.4s}$$

$$\therefore \boxed{v(t) = \frac{4T_{max}}{br}\left(1 - e^{-\frac{b}{m}t}\right) - \frac{4(T_{max} - T_{ss})}{br}\left(1 - e^{-\frac{b}{m}t}\right) u(t-2.4) \quad \text{for } t \geq 0}$$

With values plugged in:

$$\boxed{v(t) = 393.9\left(1 - e^{-\frac{62}{2110}t}\right) - 367.1\left(1 - e^{-\frac{62}{2110}t}\right) u(t-2.4) \quad \text{for } t \geq 0}$$

d)



$$\xrightarrow{+} \sum F_x = ma = F_{wheels} - F_{drag}$$

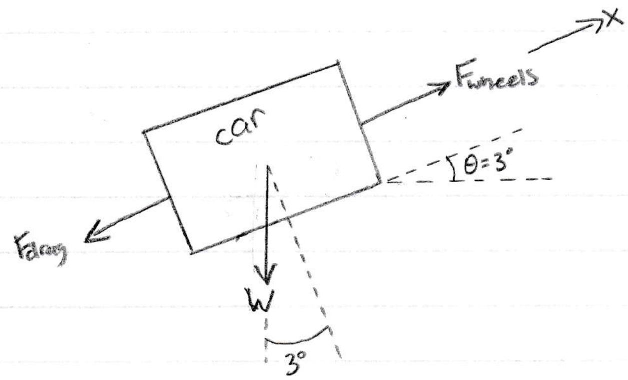
$$ma = F_{wheels} - bv$$

$$m\dot{v} = F_{wheels} - bv$$

$$\dot{v} = \frac{(F_{wheels} - bv)}{m} \longrightarrow \text{for Simulink model}$$

$$F_{wheels} = \begin{cases} 4\frac{T_{max}}{r} & \text{for } 0 \le t < 2.4 \\ 4\frac{T_{ss}}{r} & \text{for } t \ge 2.4 \end{cases}$$

e)



$\xrightarrow{+} \sum F_x = ma = F_{wheels} - F_{drag} - W \sin\theta \, u(t-10)$

$m\dot{v} = F_{wheels} - mg\sin\theta \, u(t-10) - bv$

$m\dot{v} = F_{combined} - bv$

$\dot{v} = \dfrac{F_{combined} - bv}{m}$ → Simulink model

$$F_{combined} = \begin{cases} 4 \frac{T_{max}}{r} & \text{for } 0 \le t < 2.4 \\ 4 \frac{T_{ss}}{r} & \text{for } 2.4 \le t < 10 \\ 4 \frac{T_{ss}}{r} - mg\sin\theta & \text{for } t \ge 10 \end{cases}$$