Nick Carcione, Andrew MacLean

ME 3360

Hoelzle 9:10-10:05 a.m.

Project Part 3

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1. $e_{final} = \pm\frac{1}{2}$ mph $= \pm 0.2235$ m/s

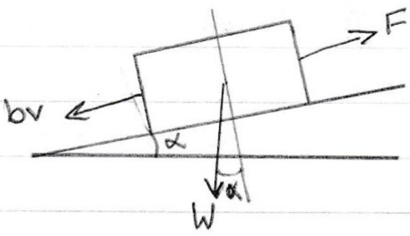
   %OS < 2%

   $T_s < 10$ sec

2. a) Choose $\bar{v}_{el} = 60$ mph $= 26.8224$ m/s

   $\bar{\alpha} = 0°$

   b)



$$m\ddot{x} = -b\dot{x} + F - mg\sin\alpha \longrightarrow F = m\ddot{x} + b\dot{x} + mg\sin\alpha \longrightarrow \frac{2\Delta T}{D} = m\ddot{x} + b\dot{x} + mg\sin\alpha$$

$$\boxed{\frac{4\Delta N K_b i_f(t)\, l\, i_a(t)\, r\cos(\theta(t))}{D} = m\dot{v}_{el} + b v_{el} + mg\sin\alpha} \quad -(1)$$

$$\boxed{v(t) = R_f i_f(t) + L_f \dot{i}_f(t)} \quad -(2)$$

$$v(t) = R_a i_a(t) + L_a \dot{i}_a(t) + E_b(t) = R_a i_a(t) + L_a \dot{i}_a(t) + N B(t) 2rl\dot{\theta}(t)\cos(\theta(t)) + 2Nrl\sin(\theta(t))\frac{dB(t)}{dt}$$

$$\boxed{v(t) = R_a i_a(t) + L_a \dot{i}_a(t) + N K_b i_f(t) 2rl\dot{\theta}(t)\cos(\theta(t)) + 2Nrl\sin(\theta(t)) K_b \dot{i}_f(t)} \quad -(3)$$

Finding $\bar{i}_a, \bar{i}_f, \bar{v}$:

$$\frac{4\Delta N K_b \bar{i}_f l\, \bar{i}_a r\cancelto{1}{\cos(\bar{\theta})}}{D} = \cancelto{0}{m\dot{\bar{v}}_{el}} + b\bar{v}_{el} + \cancelto{0}{mg\sin\bar{\alpha}} \longrightarrow \boxed{\frac{4\Delta N K_b l r \bar{i}_f \bar{i}_a}{D} = b\bar{v}_{el}} \quad -(4)$$

$$\bar{v} = R_f \bar{i}_f + \cancel{L_f \dot{\bar{i}}_f} \longrightarrow \boxed{\bar{v} = R_f \bar{i}_f} \quad -(5)$$

Note: $\theta(t) = \frac{2\Delta x(t)}{D} \longrightarrow \dot{\theta}(t) = \frac{2\Delta v_{el}}{D} \longrightarrow \dot{\bar{\theta}} = \frac{2\Delta \bar{v}_{el}}{D}$

$$\bar{v} = R_a \bar{i}_a + \cancelto{0}{L_a \dot{\bar{i}}_a} + N K_b \bar{i}_f 2rl \frac{2\Delta}{D}\bar{v}_{el}\cancelto{1}{\cos(\bar{\theta})} + \cancelto{0}{2Nrl\sin(\bar{\theta}) K_b \dot{\bar{i}}_f}$$

$$\hookrightarrow \boxed{\bar{v} = R_a \bar{i}_a + \frac{4\Delta}{D} N K_b r l \bar{i}_f \bar{v}_{el}} \quad -(6)$$

Plug (4), (5), (6) into vpasolve: $\boxed{\begin{aligned}\bar{i}_a &= 811.4\text{ A}\\ \bar{i}_f &= 17.20\text{ A}\\ \bar{v} &= 111.8\text{ V}\end{aligned}}$

Define:

$$i_f = \bar{i}_f + \hat{i}_f \qquad\qquad V_{e1} = \bar{V}_{e1} + \hat{V}_{e1}$$

$$\dot{i}_f = \dot{\hat{i}}_f \quad (\dot{\bar{i}}_f = 0) \qquad\qquad \dot{V}_{e1} = \dot{\hat{V}}_{e1} \quad (\dot{\bar{V}}_{e1} = 0)$$

$$i_a = \bar{i}_a + \hat{i}_a \qquad\qquad \alpha = \hat{\alpha} \quad (\bar{\alpha} = 0)$$

$$\dot{i}_a = \dot{\hat{i}}_a \quad (\dot{\bar{i}}_a = 0) \qquad\qquad V = \bar{V} + \hat{V}$$

$$\theta = \hat{\theta} \quad (\bar{\theta} = 0)$$

Linearizing:

$$(1) - \frac{4\Delta N K_b r l}{D}\left(\bar{i}_a \cos(\bar{\theta})\hat{i}_f + \bar{i}_f \cos(\bar{\theta})\hat{i}_a - \bar{i}_f \bar{i}_a \sin(\bar{\theta})\hat{\theta}\right) = b\hat{V}_{e1} + m\dot{\hat{V}}_{e1} + mg\cos\bar{\alpha}(\hat{\alpha})$$

$$\boxed{\frac{4\Delta N K_b r l}{D}\left(\bar{i}_a \hat{i}_f + \bar{i}_f \hat{i}_a\right) = m\dot{\hat{V}}_{e1} + b\hat{V}_{e1} + mg\hat{\alpha}}$$

$$(2) - \boxed{R_f \hat{i}_f + L_f \dot{\hat{i}}_f = \hat{V}}$$

$$(3) - \hat{V} = R_a \hat{i}_a + L_a \dot{\hat{i}}_a + N K_b 2 r l\left(\dot{\bar{\theta}}\cos(\bar{\theta})\hat{i}_f - \cancelto{0}{\bar{i}_f \dot{\bar{\theta}}} \sin(\bar{\theta})\hat{\theta} + \bar{i}_f \cos(\bar{\theta})\dot{\hat{\theta}}\right)$$

$$+ 2NrlK_b\left(\cancelto{0}{\dot{\bar{i}}_f \cos(\bar{\theta})}\dot{\hat{\theta}} + \cancelto{0}{\sin(\bar{\theta})\dot{\hat{i}}_f}\right)$$

$$\hat{V} = R_a \hat{i}_a + L_a \dot{\hat{i}}_a + N K_b 2 r l\left(\frac{2\Delta}{D}\bar{V}_{e1}\hat{i}_f + \frac{2\Delta}{D}\bar{i}_f \hat{V}_{e1}\right)$$

$$\boxed{\hat{V} = R_a \hat{i}_a + L_a \dot{\hat{i}}_a + \frac{4\Delta}{D} N K_b r l \bar{i}_f \hat{V}_{e1} + \frac{4\Delta}{D} N K_b r l \bar{V}_{e1} \hat{i}_f}$$

3. Laplace linearized eqns.

$$V(s) = (L_f s + R_f) I_f(s)$$

$$V(s) = (L_a s + R_a) I_a(s) + \left(\frac{4\Delta}{D} N K_b r l \bar{i}_f\right) V_{e1}(s) + \left(\frac{4\Delta}{D} N K_b r l \bar{V}_{e1}\right) I_f(s)$$

$$\left(\frac{4\Delta}{D} N K_b r l \bar{i}_a\right) I_f(s) + \left(\frac{4\Delta}{D} N K_b r l \bar{i}_f\right) I_a(s) = (ms + b) V_{e1}(s) + mg\alpha(s)$$

$$A\left\{\begin{bmatrix} L_f s + R_f & 0 & 0 \\ \frac{4\Delta}{D} N K_b l r \bar{V}_{e1} & L_a s + R_a & \frac{4\Delta}{D} N K_b r l \bar{i}_f \\ \frac{4\Delta}{D} N K_b r l \bar{i}_a & \frac{4\Delta}{D} N K_b r l \bar{i}_f & -(ms+b) \end{bmatrix}\right. \begin{bmatrix} I_f(s) \\ I_a(s) \\ V_{e1}(s) \end{bmatrix} = \begin{bmatrix} V(s) \\ V(s) \\ mg\alpha(s) \end{bmatrix}$$

Cramer's Rule for $V_{e1}(s)$

$$V_{e1}(s) = \frac{\det\left(\begin{bmatrix} L_f s + R_f & 0 & V(s) \\ \frac{4\Delta}{D} N K_b r l \bar{V}_{e1} & L_a s + R_a & V(s) \\ \frac{4\Delta}{D} N K_b r l \bar{i}_a & \frac{4\Delta}{D} N K_b r l \bar{i}_f & mg\alpha(s) \end{bmatrix}\right)}{\det([A])}$$

$$V_{el}(s) = \frac{(L_f s + R_f)\left[(L_a s + R_a)(mg\alpha(s)) - V(s)\left(\frac{4\Delta}{D} N K_b r \ell \bar{i}_f\right)\right] - 0 + V(s)\left[\left(\frac{4\Delta}{D} N K_b r \ell\right)^2 \bar{V}_{el}\bar{i}_f - \frac{4\Delta}{D} N K_b r \ell \bar{i}_a\right]}{(L_f s + R_f)\left[-(L_a s + R_a)(ms+b) - \left(\frac{4\Delta}{D} N K_b r \ell \bar{i}_f\right)^2\right] - 0 + 0}$$

$$V_{el}(s) = \frac{(R_f + L_f s)(R_a + L_a s) mg\alpha(s) - \left[(R_f + L_f s)\frac{4\Delta}{D} N K_b r \ell \bar{i}_f - (R_a + L_a s)\frac{4\Delta}{D} N K_b r \ell \bar{i}_a + \left(\frac{4\Delta}{D} N K_b r \ell\right)^2 \bar{V}_{el}\bar{i}_f\right] V(s)}{-(R_f + L_f s)\left[(R_a + L_a s)(ms+b) + \frac{16\Delta^2}{D^2} N^2 K_b^2 r^2 \ell^2 \bar{i}_f^2\right]}$$

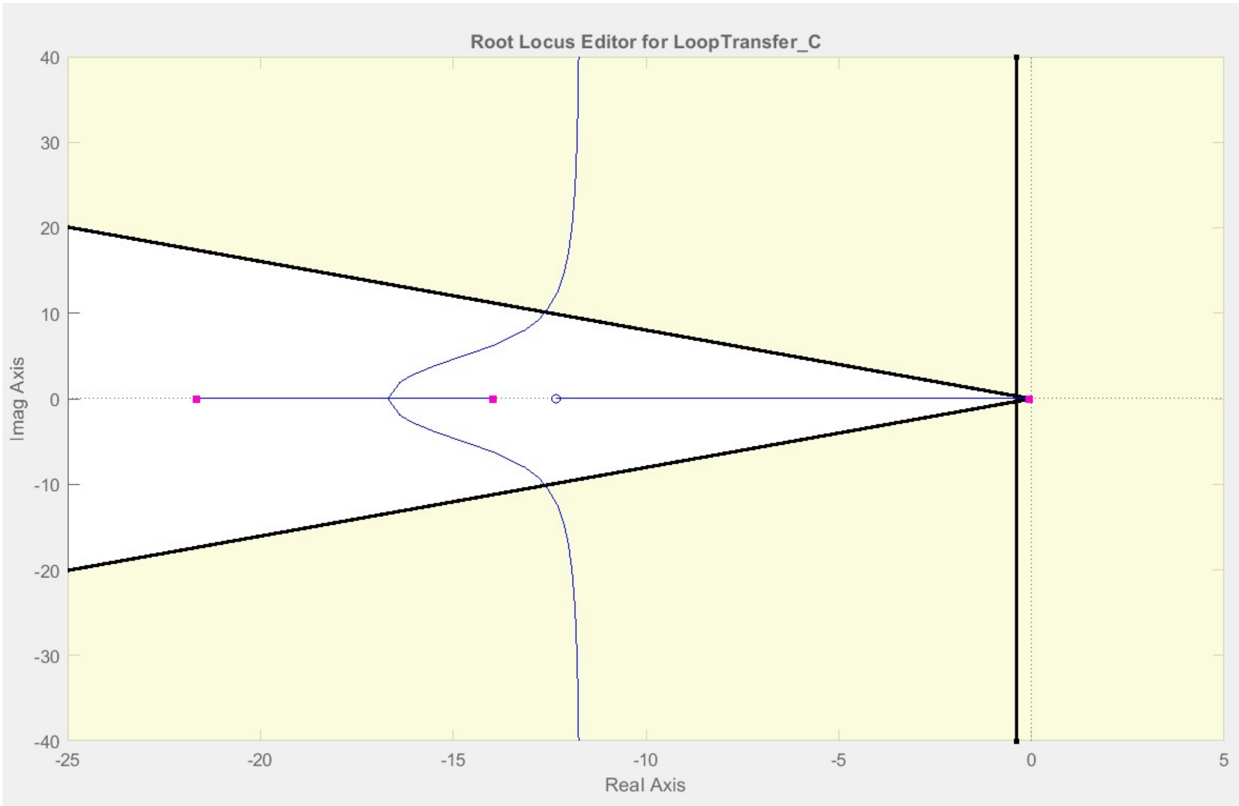
$$\boxed{V_{el}(s) = \frac{\frac{4\Delta}{D} N \ell r K_b \left[(R_f + L_f s)\bar{i}_f + (R_a + L_a s)\bar{i}_a - \frac{4\Delta}{D} N \ell r K_b \bar{V}_{el}\bar{i}_f\right]}{(R_f + L_f s)\left[(R_a + L_a s)(ms+b) + \frac{16\Delta^2}{D^2} N^2 K_b^2 r^2 \ell^2 \bar{i}_f^2\right]} V(s) + \frac{-mg(R_f + L_f s)(R_a + L_a s)}{(R_f + L_f s)\left[(R_a + L_a s)(ms+b) + \frac{16\Delta^2}{D^2} N^2 K_b^2 r^2 \ell^2 \bar{i}_f^2\right]}\alpha(s)} = G_{2,1} V(s) + G_{2,2}\alpha(s)$$

$G_{2,1}$ Poles	$G_{2,1}$ Zeros	$G_{2,1}$ Gain	$G_{2,2}$ Poles	$G_{2,2}$ Zeros	$G_{2,2}$ Gain
-21.67	-12.3262	0.34705	-21.67	-21.67	-9.81
-0.0579			-0.0579	-14	
-13.9714			-13.9714		

* found in Matlab

4. a) See attached RL plot

4. a)



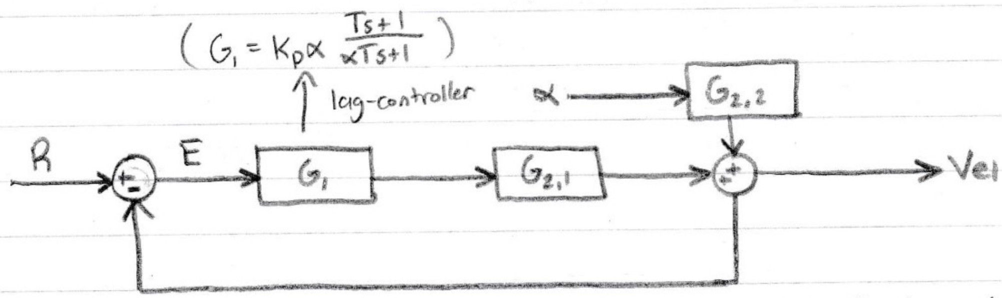
The basic root locus of the transfer function $G_{2,1}$ exists largely in the desired region, so simply increasing the gain on the controller (i.e. a proportional controller) will be sufficient.

4.b) Say $s_{des} = -0.65$

$$|G_{2,2}(s)| = \frac{0.34705(12.3262-0.65)}{(21.67-0.65)(0.65-0.0579)(13.9714-0.65)} = 0.02444$$

$$K = \frac{1}{|G_{2,2}(s)|} = \frac{1}{0.02444} = 40.91 \rightarrow \boxed{K_p = 41}$$

c)



$$\left.\begin{array}{l} E = R - V_{el} \\ V_{el} = EG_1G_{2,1} + \alpha G_{2,2} \end{array}\right\} \begin{array}{l} E = R - EG_1G_{2,1} - \alpha G_{2,2} \\ E(1+G_1G_{2,1}) = R - G_{2,2}\alpha \end{array} \Rightarrow E = \frac{R - G_{2,2}\alpha(s)}{1+G_1G_{2,1}}$$

To start, consider only steady state of velocity set-point change (R)

$$\therefore E(s) = \frac{R(s)}{1+G_1G_{2,1}}$$

* 10 mph = 4.4704 m/s

$$e_{final} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \cancel{s} \frac{1}{1 + K_p\alpha \frac{Ts+1}{\alpha Ts+1} \frac{0.34705(s+12.3262)}{(s+21.67)(s+0.0579)(s+13.9714)}} \cdot \frac{4.4704}{\cancel{s}}$$

$$= \lim_{s\to 0} \frac{4.4704(\alpha Ts+1)(s+21.67)(s+0.0579)(s+13.9714)}{(\alpha Ts+1)(s+21.67)(s+0.0579)(s+13.9714) + K_p\alpha(Ts+1)0.34705(s+12.3262)}$$

$$= \frac{4.4704(1)(21.67)(0.0579)(13.9714)}{(1)(21.67)(0.0579)(13.9714) + (41)\alpha(1)0.34705(12.3262)}$$

* ± ½ mph = ±.2235 m/s

$$= \frac{82.45}{17.527 + 175.4\alpha} < 0.2235 \rightarrow \alpha > 2.003 \rightarrow \alpha = 3$$

Checking to see if $\alpha = 3$ is sufficient for change in road grade

$$E = \frac{-G_{2,2}\alpha(s)}{1+G_1G_{2,1}}$$

* 3° = 0.0523 rad

$$e_{final} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \cancel{s} \frac{9.81 \frac{(s+21.67)(s+14)}{(s+21.67)(s+0.0579)(s+13.9714)}}{1 + K_p\alpha \frac{Ts+1}{\alpha Ts+1} \frac{0.34705(s+12.3262)}{(s+21.67)(s+0.0579)(s+13.9714)}} \cdot \frac{0.0523}{\cancel{s}}$$

$$e_{final} = \frac{0.0523(9.81)\dfrac{(21.67)(14)}{(21.67)(0.0579)(13.9714)}}{1+(41)(3)(1)\dfrac{0.34705(12.3262)}{(21.67)(0.0579)(13.9714)}} = 0.2866 > 0.2235$$

$\hookrightarrow$ increase α

Since $e_{final} > \pm\frac{1}{2}$ mph, increase α to $\alpha = 5$.

Check e_{final} again using same process but with $\alpha = 5$

$e_{final} = 0.174 < 0.2235$ $\therefore$ acceptable

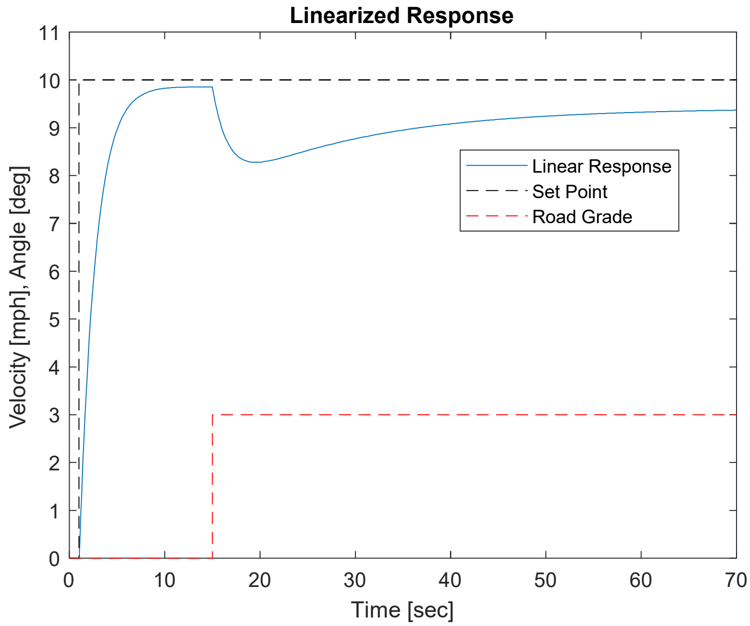
$\rightarrow \boxed{\alpha = 5}$

$\omega_n = 0.65 \rightarrow z_c = \frac{1}{10}\omega_n \rightarrow \boxed{z_c = 0.065} \rightarrow T = \frac{1}{z_c} = 15.38$

$p_c = 1/\alpha T \rightarrow \boxed{p_c = 0.013}$

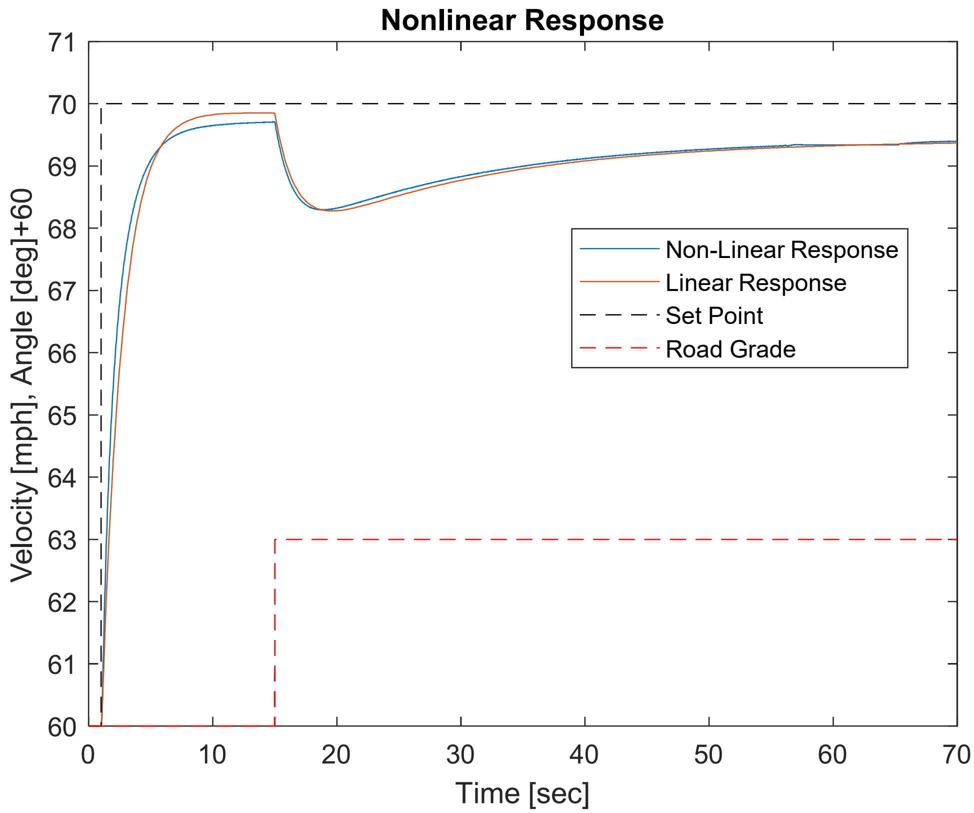
$$\therefore \boxed{G_1 = 41\,\frac{s+0.065}{s+0.013}}$$

5.



The controller is able to settle at its steady state vale at the desired 10 seconds and does so without any overshoot, satisfying those two requirements. The set point change falls within the desired error range. The final value of the velocity due to the set point change was 9.851 mph. The road grade change error falls within 0.5 mph of this value (it reaches 9.366 mph), technically satisfying the error requirement. Even though this value of 9.366 mph does not fall within 0.5 mph of 10 mph, it will be considered to be valid since it falls within 0.5 mph of its starting point (i.e. the ending point of the set point change, 9.851 mph).

6.

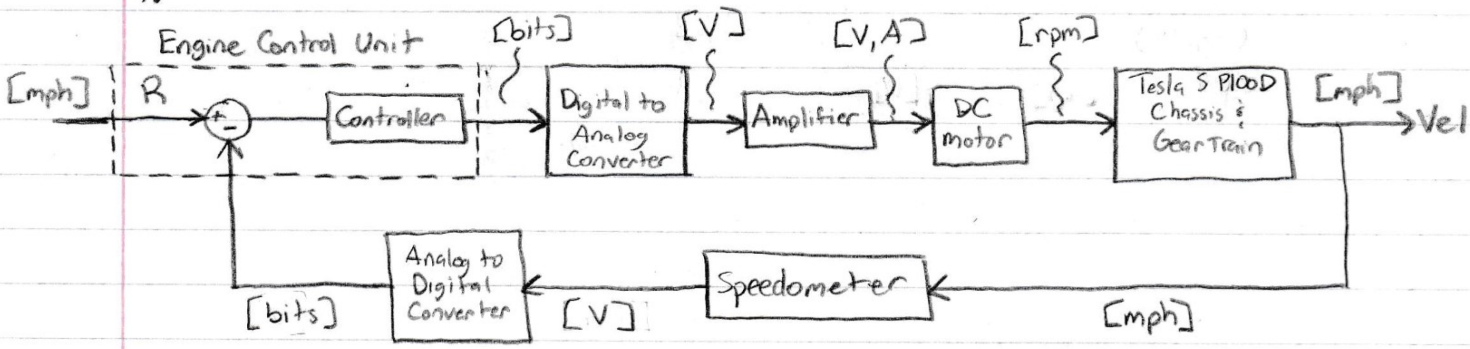


Notes about above plot:

60 was added to the value of the road grade and the linearized response to translate those values up to the 0 to 60 range that the nonlinearized response is displayed on.

The linear response matches the nonlinear response fairly well. The nonlinear response is a little more aggressive than the linear one early on for both the set point and road grade change. For the set point change, the nonlinear response levels off earlier, takes longer to reach its steady state value, and reaches a lower steady state value than the linearized response. The nonlinear and linearized responses are near identical as they reach steady state after a road grade change. The designed controller will likely need to be a little more aggressive as the nonlinear response appears to have a settling time larger than the desired 10 seconds. The nonlinear response still meets the desired overshoot and desired error change.

7.


Engine Control Unit
[mph]
R
+
−
Controller
[bits]
Digital to Analog Converter
[V]
Amplifier
[V, A]
DC motor
[rpm]
Tesla S P100D Chassis & Gear Train
[mph]
Vel
Speedometer
[mph]
[V]
Analog to Digital Converter
[bits]

Appendix A
Simulink and Matlab Screenshots

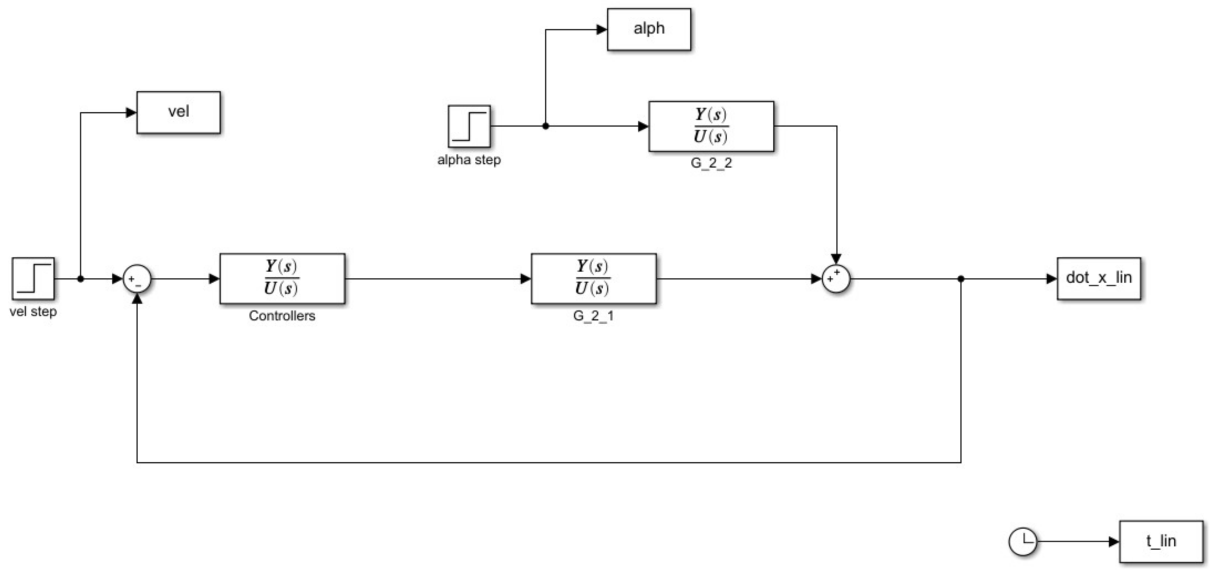


Figure 1: Simulink for linearized system

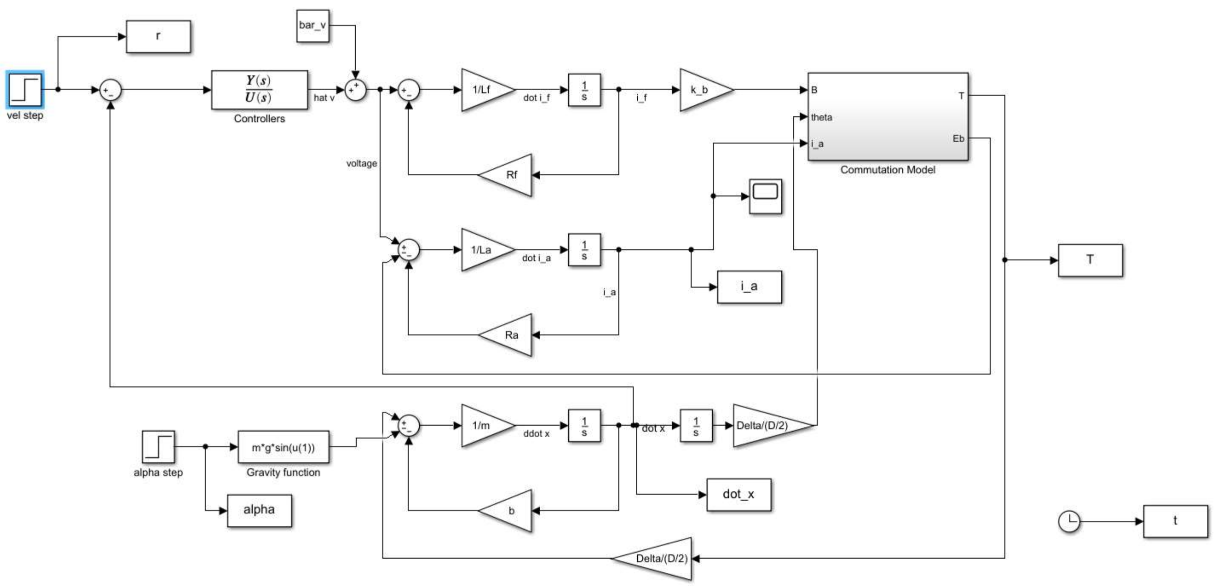


Figure 2: Simulink for nonlinear response

```
%% Code for part 3 solution

clear;
clc;
close all;

%% Parameters used in all simulations
l = 0.5;            % m
r = 1/6;            % m
N = 126;            % # coils
Rf = 6.5;           % ohm
Ra = 0.07;          % ohm
Lf = 0.3;           % H
La = 0.005;         % H
k_b = 1.4e-4;       % H
Delta = 9.73;       % unitless
m = 2110;           % kg
g = 9.81;           % m/sec^2
b = 62;             % N-sec/m
D = 0.48;           % m
t_f = 70;           % sec
bar_vel = 26.8224;  %m/s
bar_alpha = 0;      %deg

%%
%Creates symbols a f v to be used in steady state calculations
syms a f v

%Solves the steady state equations found on paper for bar_i_a, bar_i_f,
%and bar_v
Y = vpasolve(v == Rf*f, v == Ra*a+4*N*k_b*f*Delta*r*l*bar_vel/D, b*bar_vel ==
(4*Delta*N*k_b*l*r/D)*a*f, [a f v]);
bar_v = double(Y.v(2));
bar_i_a = double(Y.a(2));
bar_i_f = double(Y.f(2));

%Creates the transfer functions G_2_1 and G_2_2 found in part 3
%Does so by defining variable 's', inputing the numerators and denominator
%in terms of s, then extracting the coefficients from those expressions.
%These coefficients are converted into doubles, flipped to get in the
%correct order, made into a transfer function, then a zpk model to get the
%poles, zeros and gain for them
syms s
Denom = fliplr(double(coeffs((Rf+Lf*s) * ( (Ra+La*s)*(m*s+b) +
16*Delta^2*N^2*r^2*l^2*k_b^2*bar_i_f^2/(D^2)))));
G_2_1_num = fliplr(double(coeffs((4*Delta*N*l*r*k_b/D) * ( (Rf+Lf*s)*bar_i_f
+ (Ra+La*s)*bar_i_a - 4*Delta*N*l*r*k_b*bar_vel*bar_i_f/D))));
G_2_1 = zpk(tf(G_2_1_num, Denom));
[pole_2_1, zero_2_1] = pzmap(G_2_1);
G_2_2_num = fliplr(double(coeffs(-m*g*(Rf+Lf*s)*(Ra+La*s))));
G_2_2 = zpk(tf(G_2_2_num, Denom));
[pole_2_2, zero_2_2] = pzmap(G_2_2);

%%
%Define parameters used in the simulation, including the initial and final
%velocities for the step having bar_vel subtracted from them
%An increase from 60 mph to 70 mph is given as well as a road grade change
```

```
%of 3 degrees
%Velocities are converted into m/s and angles are converted into rad
v_step_time = 1;
v_initial = 26.8224 - bar_vel;
v_final = 31.2928 - bar_vel;
alpha_step_time = 15;
alpha_initial = 0;
alpha_final = 3*pi/180;
%Getting the transfer functions in a form usable in the simulation
%Namely, in tf form so the .num command works
G1 = tf(zpk([-0.065],[-0.013],[41]));
G_2_1 = tf(G_2_1);
G_2_2 = tf(G_2_2);

%%
%Simulate the linear model and plot the velocity in mph
sim part_3_linear
plot(t_lin, dot_x_lin*2.237);
hold on;
plot(t_lin, vel*2.237, 'k--');
plot(t_lin, alph*180/pi, 'r--');
title('Linearized Response');
legend('Linear Response', 'Set Point', 'Road Grade', 'Location', 'Best');
xlabel('Time [sec]');
ylabel('Velocity [mph], Angle [deg]');
ylim([0 11]);

%Set the initial and final velocities back to their true values for
%non-linear model
v_initial = 26.8224;
v_final = 31.2928;
%Simulate the non-linear model and plot the results in mph
sim part_3_sim_w_car
figure();
plot(t,dot_x*2.237);
hold on;
plot(t_lin, dot_x_lin*2.237+60);
plot(t, r*2.237, 'k--');
plot(t, alpha*180/pi+60, 'r--');
legend('Non-Linear Response', 'Linear Response', 'Set Point', 'Road Grade',
'Location', 'Best');
xlabel('Time [sec]');
ylabel('Velocity [mph], Angle [deg]+60');
title('Nonlinear Response');
ylim([60 71]);
```