

Getting started

The Legolas code can be found on [GitHub](#), installation instructions are on the website. Make sure to look at the prerequisites for both Legolas and Pylbo first before running the code.

1 Equilibria

Below is a list of possible setups that can be implemented in the user submodule. We've added a reference figure together with the link to the original work so you can check your implementation and compare with known results.

1.1 Internal kink modes in force-free magnetic fields

This setup is taken from [Goedbloed \(2018\)](#) and corresponds to a cylindrical equilibrium with a force-free magnetic field of constant α and profiles given by

$$\begin{aligned} \rho(r) &= \rho_0 (1 - x^2) & \rho'(r) &= -\frac{2\rho_0}{a}x \\ v_z(r) &= v_{03} (1 - x^2) & v'_z(r) &= -\frac{2v_{03}}{a}x \\ B_\theta(r) &= J_1(\alpha x) & B'_\theta(r) &= \frac{\alpha}{2a} [J_0(\alpha x) - J_2(\alpha x)] \\ B_z(r) &= J_0(\alpha x) & B'_z(r) &= -\frac{\alpha}{a} J_1(\alpha x) \\ T(r) &= \frac{p_0}{\rho(r)} & T'(r) &= \frac{2rp_0}{a^2\rho_0(1 - x^2)^2} \end{aligned}$$

where $x = r/a$ and a denotes the outer wall of the cylinder.

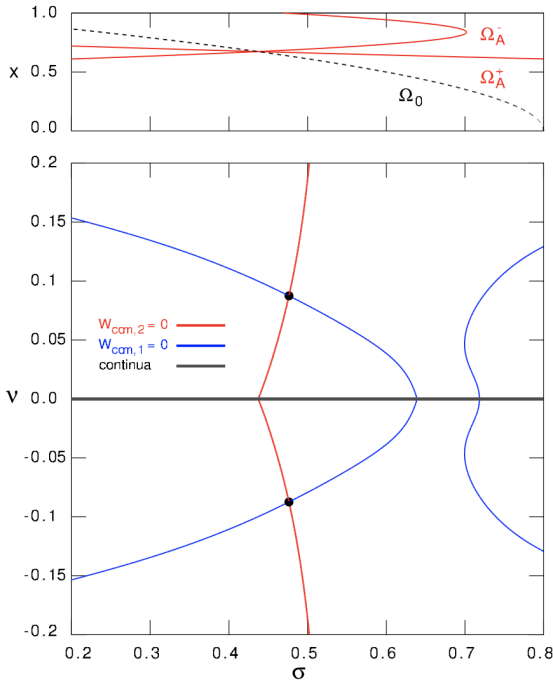


Figure 1: Values $\rho_0 = v_{03} = p_0 = a = m = 1$, $\alpha a = 5$, $k = 0.16\alpha$, incompressible.

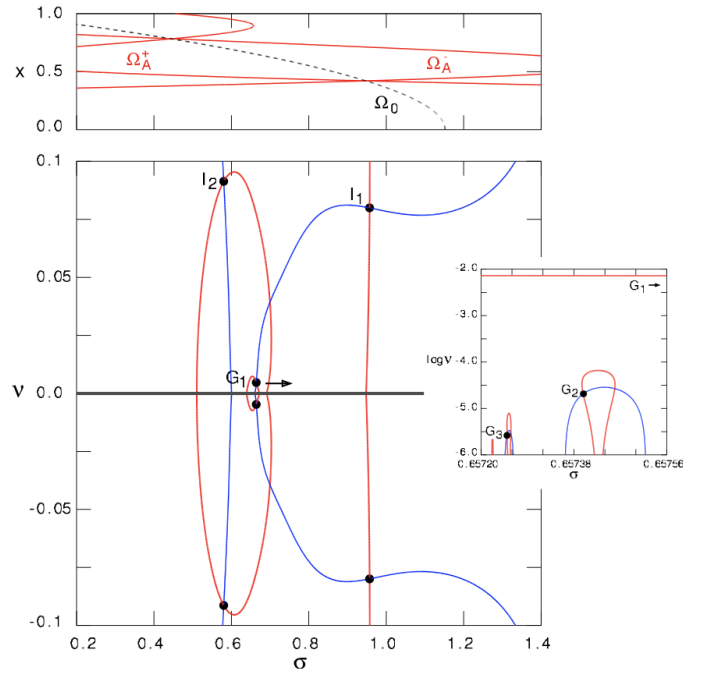


Figure 2: Values $\rho_0 = p_0 = a = m = 1$, $v_{03} = 0.8$, $\alpha a = 8$, $k = 0.16\alpha$, incompressible.

1.2 RTI in rotating theta-pinches

This setup is taken from [Goedbloed \(2018\)](#) and corresponds to Rayleigh-Taylor instabilities in rotating theta-pinches. Introducing the following quantities

$$x = \frac{r}{a} \quad f_x = \alpha^2 (x^2 - r_0^2) \quad f'_x = \frac{2\alpha^2 x}{a}$$

where a denotes the cylinder wall and $r_0 = 0$ in this case. The equilibrium is then given by

$$\begin{aligned} \rho(r) &= \frac{\rho_0}{\cosh^2(f_x)} & \rho'(r) &= \frac{-2\rho_0 f'_x \tanh(f_x)}{\cosh^2(f_x)} \\ v_\theta(r) &= \Omega r & v'_\theta(r) &= \Omega \\ B_z(r) &= B_\infty [\delta + (1 - \delta) \tanh(f_x)] & B'_z(r) &= \frac{B_\infty (1 - \delta) f'_x}{\cosh^2(f_x)} \\ T(r) &= \frac{p_0}{\rho_0} \end{aligned}$$

where

$$p_0 = \frac{1}{2} (1 - \delta)^2, \quad B_\infty = a\sqrt{\rho_0}, \quad \Omega = \alpha\sqrt{2\delta(1 - \delta)}$$

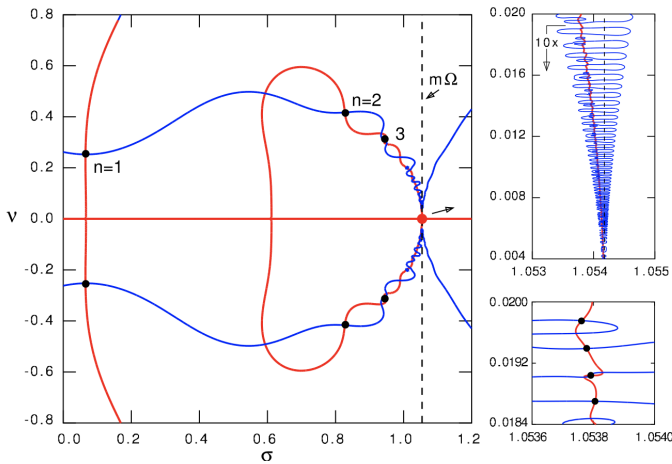


Figure 3: Values $\rho_0 = a = m = 1$, $\alpha = 2$, $\delta = 1/6$, $k = 0$ (hydrodynamics), incompressible.

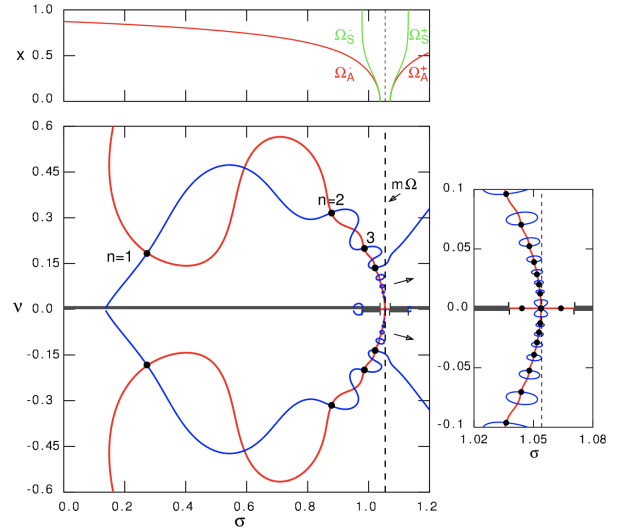


Figure 4: Values $\rho_0 = a = m = 1$, $\alpha = 2$, $\delta = 1/6$, $k = 0.1$ (magnetohydrodynamics), incompressible.

1.3 Magneto-rotational instabilities in accretion disks

This setup is taken from [Goedbloed \(2018\)](#) and models magneto-rotational instabilities in accretion disks. Legolas can model an accretion disk by assuming a cylindrical geometry and letting the grid start from a non-zero value, that is, $r \in [r_0, R]$. In what follows the parameter δ denotes R/r_0 . The equilibrium is given by

$$\begin{aligned} \rho(r) &= r^{-3/2} & \rho'(r) &= -\frac{3}{2}r^{-5/2} \\ p(r) &= p_1 r^{-5/2} & p'(r) &= -\frac{5}{2}p_1 r^{-7/2} \\ v_\theta(r) &= \Omega_1 r^{-1/2} & v'_\theta(r) &= -\frac{1}{2}\Omega_1 r^{-3/2} \\ B_\theta(r) &= B_{\theta 1} r^{-5/4} & B'_\theta(r) &= -\frac{5}{4}B_{\theta 1} r^{-9/4} \\ B_z(r) &= B_{z1} r^{-5/4} & B'_z(r) &= -\frac{5}{4}B_{z1} r^{-9/4} \\ g(r) &= \frac{1}{r^2} \end{aligned}$$

where

$$p_1 = \epsilon^2, \quad B_{z1} = \sqrt{\frac{2p_1}{\beta(1 + \mu_1^2)}}, \quad B_{\theta 1} = \mu_1 B_{z1}, \quad v_{\theta 1} = \sqrt{1 - \frac{5}{2}p_1 - \frac{1}{4}B_{\theta 1}^2 - \frac{5}{4}B_{z1}^2}$$

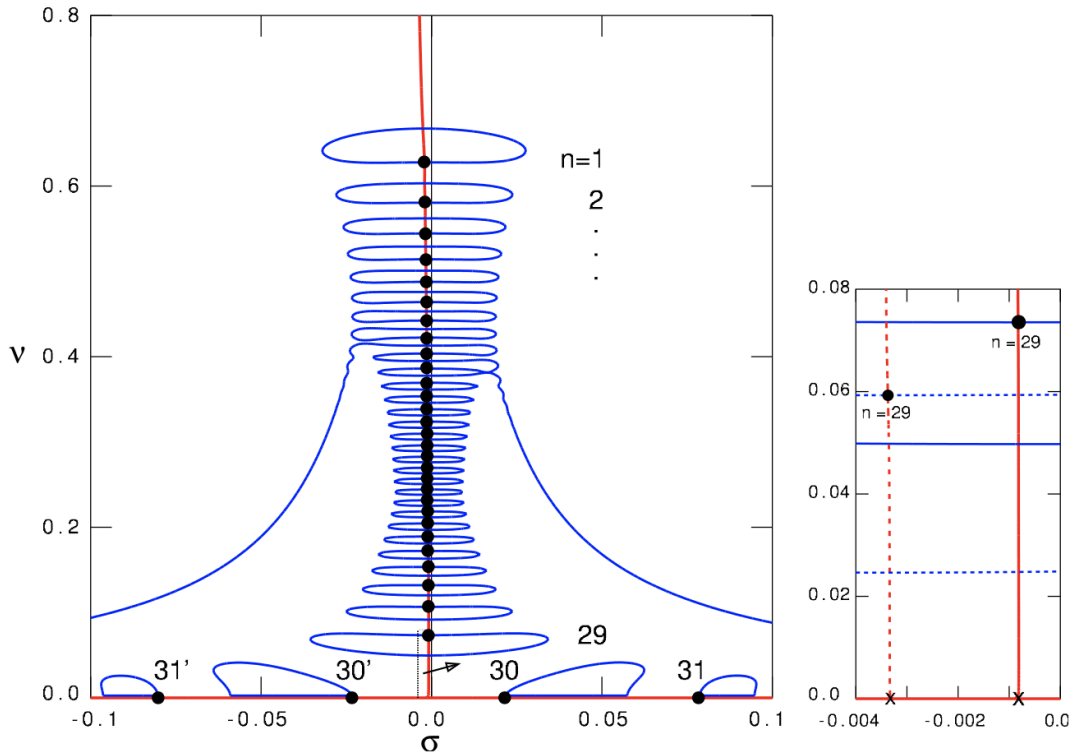


Figure 5: Values $m = 0$, $k = 70$, $\beta = 100$, $\mu_1 = 1$, $\epsilon = 0.1$ and $\delta = 2$.

1.4 RTI in magnetised plasmas

This setup is taken from [Goedbloed, Keppens, and Poedts \(2019\)](#) and corresponds to pure Rayleigh-Taylor instabilities in a Cartesian slab with a unidirectional magnetic field and linear velocity profile. The equilibrium for $0 \leq x \leq 1$ is given by

$$\begin{aligned}
 \rho(x) &= \rho_0(1 - \delta x) & g(x) &= g_0 \\
 p(x) &= p_0 - g_0 \left(x - \frac{1}{2} \delta x^2 \right) & \rho'(x) &= -\rho_0 \delta \\
 v_y(x) &= \xi(x) \sin \theta & p'(x) &= -g_0(1 - \delta x) \\
 v_z(x) &= \xi(x) \cos \theta & v'_y(x) &= \xi'(x) \sin \theta \\
 B_y(x) &= \sin \phi(x) & v'_z(x) &= \xi'(x) \cos \theta \\
 B_z(x) &= \cos \phi(x) & B'_y(x) &= \alpha \cos \phi(x) \\
 & & B'_z(x) &= -\alpha \sin \phi(x)
 \end{aligned} \tag{1}$$

with

$$\begin{aligned}
 \xi(x) &= v_0 + v_1 \left(x - \frac{1}{2} \right) + v_2 \sin \left[\tau \left(x - \frac{1}{2} \right) \right], & \xi'(x) &= v_1 + v_2 \tau \cos \left[\tau \left(x - \frac{1}{2} \right) \right], \\
 \phi(x) &= \phi_0 + \alpha \left(x - \frac{1}{2} \right)
 \end{aligned}$$

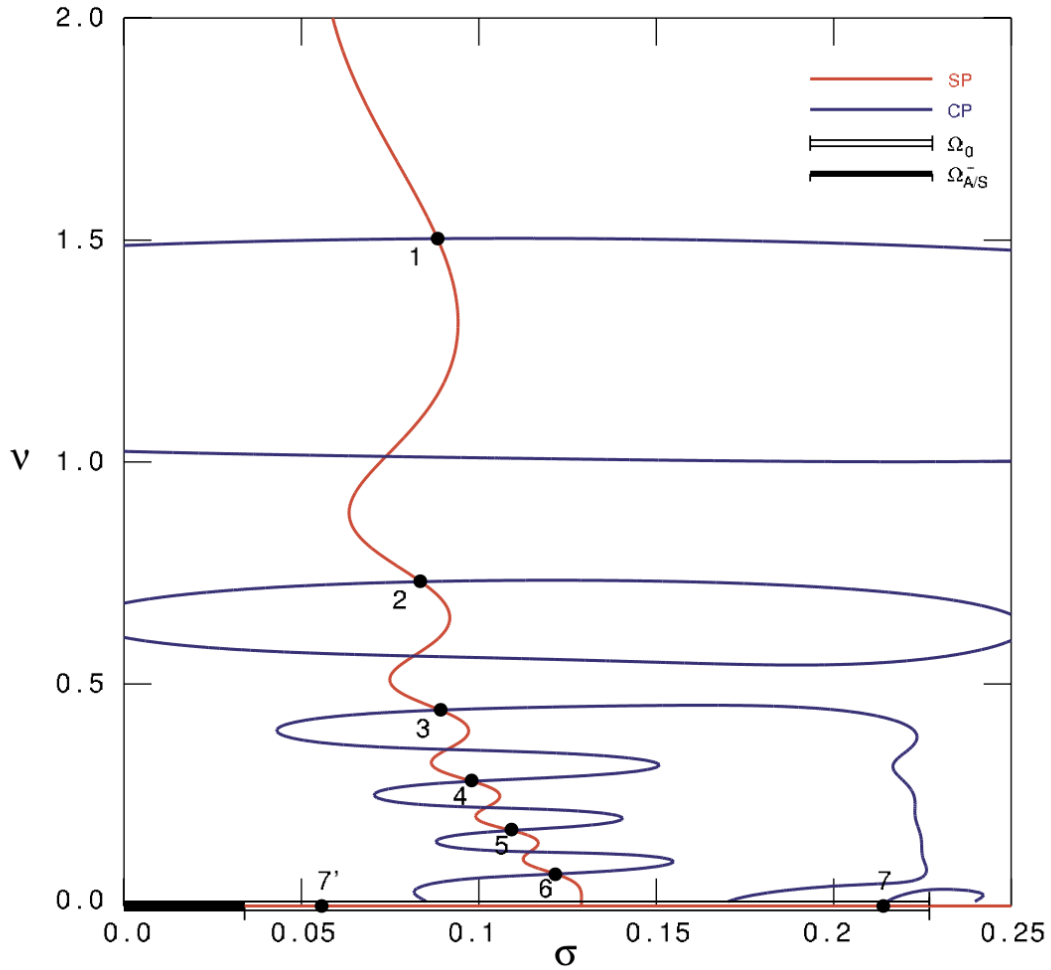


Figure 6: Values $k_y = 0$, $k_z = 1$, $\delta = -5$, $g_0 = 15$, $\rho_0 = 1$, $p_0 = 10^3$, $\phi_0 = -0.35\pi$, $\alpha = 0$, $\theta = 0.35\pi$, $v_0 = 0.2$, $v_1 = 0.6$, $v_2 = 0$, $\tau = 0$.

1.5 KHI in hydrodynamics

This setup uses the same equilibrium prescription as (1) but with different parameters, which corresponds to Kelvin-Helmholtz instabilities of a stationary fluid with a sinusoidal velocity profile.

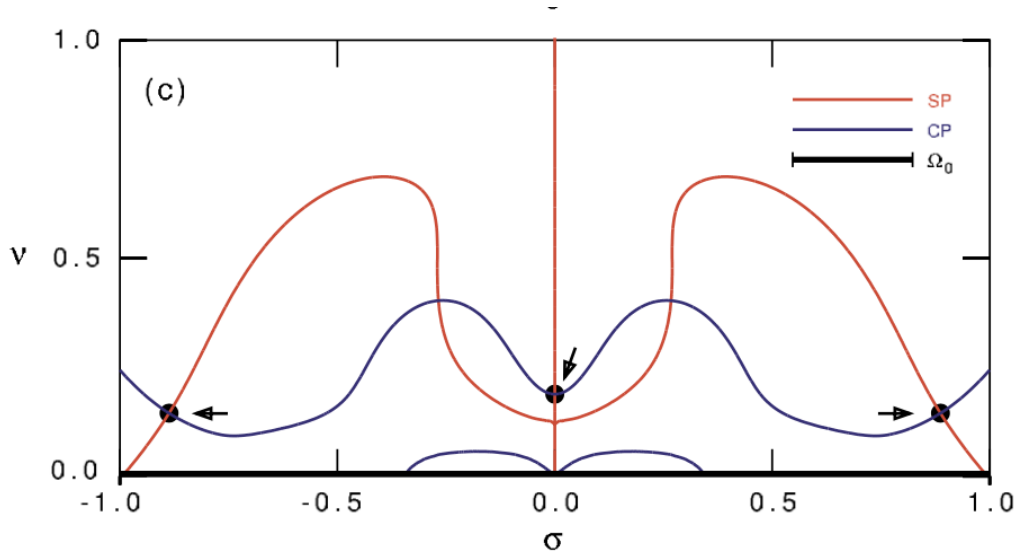


Figure 7: Values $k_y = 0$, $k_z = 1$, $\delta = 0$, $g_0 = 0$, $\rho_0 = 1$, $p_0 = 10^3$, $\phi_0 = 0$, $\alpha = 0$, $\theta = 0$, $v_0 = v_1 = 0$, $v_2 = 1$, $\tau = 11$.

References

- Goedbloed, H., Keppens, R., Poedts, S.: 2019, *Magnetohydrodynamics of Laboratory and Astrophysical Plasmas*, Cambridge University Press, Cambridge, UK. [DOI](#).
- Goedbloed, J.P.: 2018, The Spectral Web of stationary plasma equilibria. II. Internal modes. *Physics of Plasmas* **25**, 032110. [DOI](#). [ADS](#).