

# Getting started

The Legolas code can be found on [GitHub](#), installation instructions are on the website. Make sure to look at the prerequisites for both Legolas and Pylbo first before running the code.

## 1 Equilibria

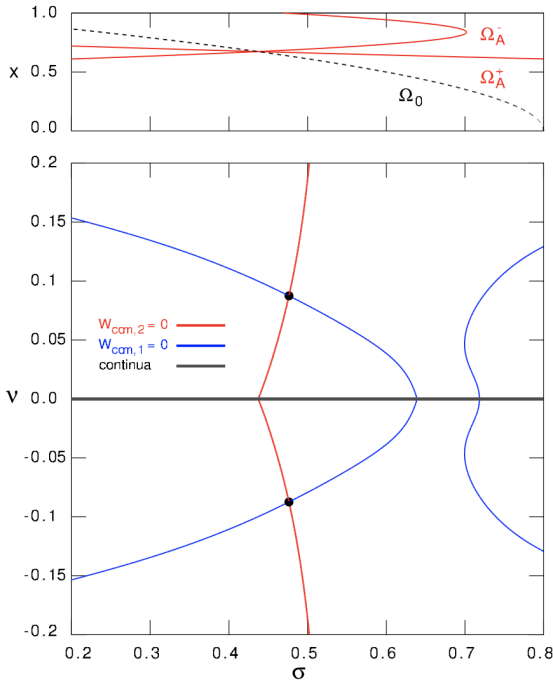
Below is a list of possible setups that can be implemented in the user submodule. We've added a reference figure together with the link to the original work so you can check your implementation and compare with known results.

### 1.1 Internal kink modes in force-free magnetic fields

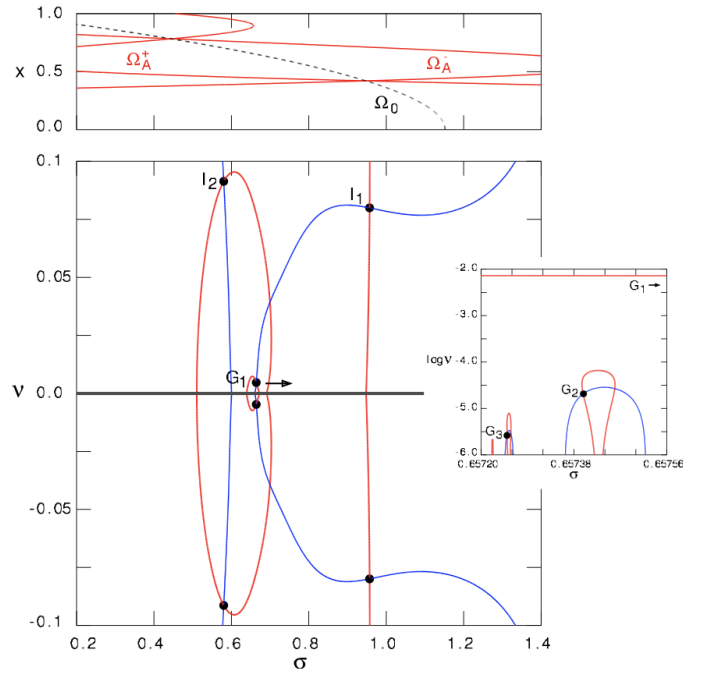
This setup is taken from [Goedbloed \(2018\)](#) and corresponds to a cylindrical equilibrium with a force-free magnetic field of constant  $\alpha$  and profiles given by

$$\begin{aligned}
 \rho(r) &= \rho_0 (1 - x^2) & \rho'(r) &= -\frac{2\rho_0}{a}x \\
 v_z(r) &= v_{03} (1 - x^2) & v'_z(r) &= -\frac{2v_{03}}{a}x \\
 B_\theta(r) &= J_1(\alpha x) & B'_\theta(r) &= \frac{\alpha}{2a} [J_0(\alpha x) - J_2(\alpha x)] \\
 B_z(r) &= J_0(\alpha x) & B'_z(r) &= -\frac{\alpha}{a} J_1(\alpha x) \\
 T(r) &= \frac{p_0}{\rho(r)} & T'(r) &= \frac{2rp_0}{a^2\rho_0(1 - x^2)^2}
 \end{aligned}$$

where  $x = r/a$  and  $a$  denotes the outer wall of the cylinder.



**Figure 1:** Values  $\rho_0 = v_{03} = p_0 = a = m = 1$ ,  $\alpha a = 5$ ,  $k = 0.16\alpha$ , incompressible.



**Figure 2:** Values  $\rho_0 = p_0 = a = m = 1$ ,  $v_{03} = 0.8$ ,  $\alpha a = 8$ ,  $k = 0.16\alpha$ , incompressible.

## 1.2 RTI in rotating theta-pinches

This setup is taken from [Goedbloed \(2018\)](#) and corresponds to Rayleigh-Taylor instabilities in rotating theta-pinches. Introducing the following quantities

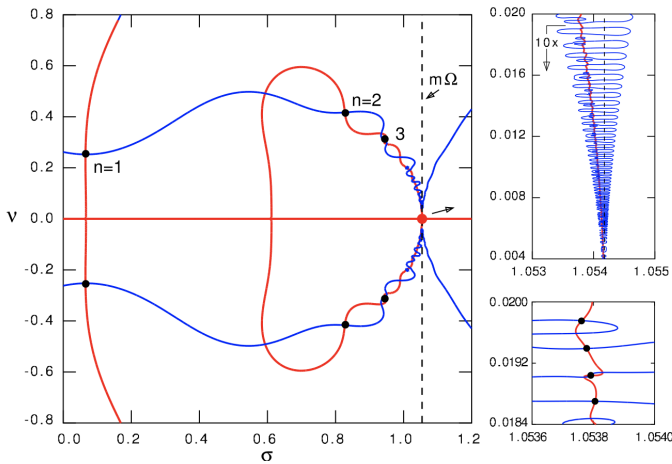
$$x = \frac{r}{a} \quad f_x = \alpha^2 (x^2 - r_0^2) \quad f'_x = \frac{2\alpha^2 x}{a}$$

where  $a$  denotes the cylinder wall and  $r_0 = 0$  in this case. The equilibrium is then given by

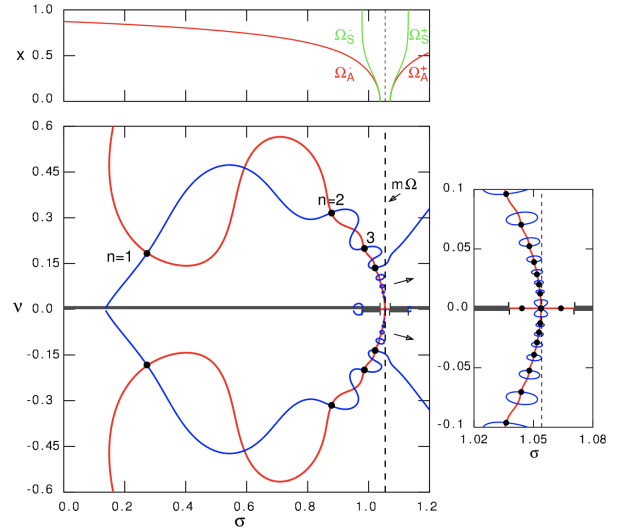
$$\begin{aligned} \rho(r) &= \frac{\rho_0}{\cosh^2(f_x)} & \rho'(r) &= \frac{-2\rho_0 f'_x \tanh(f_x)}{\cosh^2(f_x)} \\ v_\theta(r) &= \Omega r & v'_\theta(r) &= \Omega \\ B_z(r) &= B_\infty [\delta + (1 - \delta) \tanh(f_x)] & B'_z(r) &= \frac{B_\infty (1 - \delta) f'_x}{\cosh^2(f_x)} \\ T(r) &= \frac{p_0}{\rho_0} \end{aligned}$$

where

$$p_0 = \frac{1}{2} (1 - \delta)^2, \quad B_\infty = a\sqrt{\rho_0}, \quad \Omega = \alpha\sqrt{2\delta(1 - \delta)}$$



**Figure 3:** Values  $\rho_0 = a = m = 1$ ,  $\alpha = 2$ ,  $\delta = 1/6$ ,  $k = 0$  (hydrodynamics), incompressible.



**Figure 4:** Values  $\rho_0 = a = m = 1$ ,  $\alpha = 2$ ,  $\delta = 1/6$ ,  $k = 0.1$  (magnetohydrodynamics), incompressible.

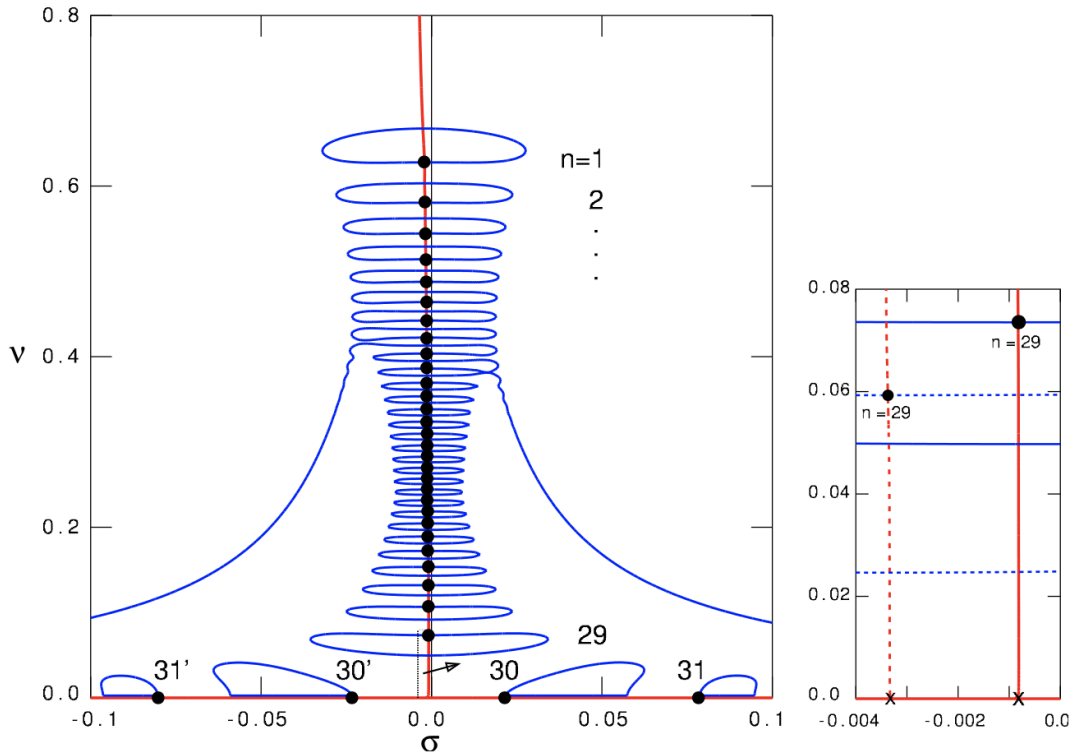
### 1.3 Magneto-rotational instabilities in accretion disks

This setup is taken from [Goedbloed \(2018\)](#) and models magneto-rotational instabilities in accretion disks. Legolas can model an accretion disk by assuming a cylindrical geometry and letting the grid start from a non-zero value, that is,  $r \in [r_0, R]$ . In what follows the parameter  $\delta$  denotes  $R/r_0$ . The equilibrium is given by

$$\begin{aligned} \rho(r) &= r^{-3/2} & \rho'(r) &= -\frac{3}{2}r^{-5/2} \\ p(r) &= p_1 r^{-5/2} & p'(r) &= -\frac{5}{2}p_1 r^{-7/2} \\ v_\theta(r) &= \Omega_1 r^{-1/2} & v'_\theta(r) &= -\frac{1}{2}\Omega_1 r^{-3/2} \\ B_\theta(r) &= B_{\theta 1} r^{-5/4} & B'_\theta(r) &= -\frac{5}{4}B_{\theta 1} r^{-9/4} \\ B_z(r) &= B_{z1} r^{-5/4} & B'_z(r) &= -\frac{5}{4}B_{z1} r^{-9/4} \\ g(r) &= \frac{1}{r^2} \end{aligned}$$

where

$$p_1 = \epsilon^2, \quad B_{z1} = \sqrt{\frac{2p_1}{\beta(1 + \mu_1^2)}}, \quad B_{\theta 1} = \mu_1 B_{z1}, \quad v_{\theta 1} = \sqrt{1 - \frac{5}{2}p_1 - \frac{1}{4}B_{\theta 1}^2 - \frac{5}{4}B_{z1}^2}$$



**Figure 5:** Values  $m = 0$ ,  $k = 70$ ,  $\beta = 100$ ,  $\mu_1 = 1$ ,  $\epsilon = 0.1$  and  $\delta = 2$ .

## References

Goedbloed, J.P.: 2018, The Spectral Web of stationary plasma equilibria. II. Internal modes. *Physics of Plasmas* **25**, 032110. [DOI](#). [ADS](#).