

numnums

N.Critser

May 19, 2019

**congruence**

**primality**

**divisability**

**basis-representation**

- **THEOREM:**

Let  $k$  be any integer larger than 1. Then, for each positive integer  $n$ , there exists a representation

$$n = a_0 k^s + a_1 k^{s-1} + \dots + a_s$$

where  $a_0 \neq 0$ , and where each  $a_i$  is nonnegative and less than  $k$ .

This representation of  $n$  is unique.

It is known as the representation of  $n$  to the base  $k$ .

- **PROOF:** Let  $b_k(n)$  be the number of representations of  $n$  to the base  $k$ . We need to show that  $b_k(n) = 1$  and only 1.

Some of the coefficients  $a_i$ , can be equal to zero for a particular representation of  $n$ . But that doesn't effect the representation, so we will exclude terms that are zero.

suppose :  $n = a_0 k^s + a_1 k^{s-1} + \dots + a_{s-t} k^t$ ,

neither  $a_0$  or  $a_{s-t} = 0$ . So subtracting 1 from  $n$  gives,

$$n-1 = a_0 k^s + a_1 k^{s-1} + \dots + a_{s-t} k^t - 1$$

$$n-1 = a_0 k^s + a_1 k^{s-1} + \dots + (a_{s-t}-1) k^t + k^t - 1$$

Since  $\sum_{j=0}^{n-1} x^j = \{\frac{x^n-1}{x-1}\}$

$$n-1 = a_0 k^s + a_1 k^{s-1} + \dots + (a_{s-t}-1) k^t + \sum_{j=0}^{t-1} (k-1) k^j$$

**fundatmental-theorem-of-arithmetic**