Question 1)

I'll do the analysis with the reduced chi-square which is

χ^2/ν where ν = # degree of freedom = number of data point − # of fitted parameters

If the the fit is close to one than it's a good fit if much bigger than it's a bad fit and it it's smaller than our model is overfitting.

For the params dialled by Prof. Siervers I get $\chi^2_\nu = 6.1 \gg 1$ indicating a fairly poor fit.

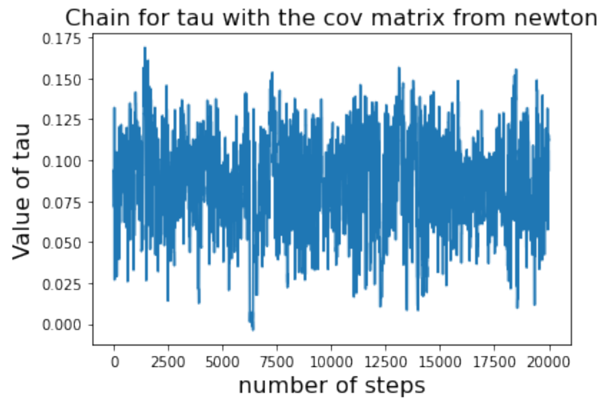
For the set of parameters listed in the question I get $\chi^2 = 3272.2$, $\chi^2_\nu = 1.31$ close to 1 which is could be a an acceptable fit.

Question 2)

```
Parameters from newton
H0 = 68.25672018474549 +/- 1.1178236188569777
omh2 = 0.02240497262888177 +/- 0.0002961743935621618
omch2 = 0.11767675641945229 +/- 0.002296306587742288
tau = 0.0937187371605846 +/- 0.03342057259067247
As = 2.2591952547511157e-09 +/- 1.4373960941937373e-10
ns = 0.9757678152637844 +/- 0.005429224721402755
```

Those are my parameters from newton. The errors where computed by taking the square root of the diagonal entries of the covariance matrix

Question 3)



As we can see from the plot of my chain for tau, the samples oscillate about one value and don't wander off. This suggest that the chain converged.

I evaluated the parameters by averaging over the samples of the chain. To compute the error I computed the std over the chain and divided by sqrt(n-1) where n is the number of samples.

```
Parameters given by the mcmc
H0 = 68.348256525103 +/- 0.007580865436914846
omh2 = 0.022351503254201308 +/- 1.6552286963961998e-06
omch2 = 0.11740554230237912 +/- 1.618558065727534e-05
tau = 0.0853650496499402 +/- 0.0002097276175824919
As = 2.220842282480639e-09 +/- 8.82485162274489e-13
ns = 0.9740768880149552 +/- 4.076048097931655e-05
```

Calculating Ω_Λ

$$\Omega_b + \Omega_c + \Omega_\Lambda = 1 \Rightarrow \Omega_\Lambda = 1 - \frac{1}{h^2}(\Omega_b h^2 + \Omega_c h^2)$$

where $h = \frac{H_0}{100}$

$$\Rightarrow 1 - \frac{100^2}{H_0^2}(\Omega_b h^2 + \Omega_c h^2) = 1 - \frac{100^2}{p_0^2}(p_1 + p_2)$$

p_i = fitted parameters

So for the error

$$\frac{\partial \Omega_\Lambda}{\partial p_0} = \frac{100}{p_0^3}(p_1 + p_2), \quad \frac{\partial \Omega_\Lambda}{\partial p_1} = \frac{-100^2}{p_0^2}, \quad \frac{\partial \Omega_\Lambda}{\partial p_2} = \frac{-100^2}{p_0^2}$$

$$\sigma_\Lambda = \sqrt{\left(\frac{\partial \Omega_\Lambda}{\partial p_0}\sigma_{p_0}\right)^2 + \left(\frac{\Omega_\Lambda}{\partial p_1}\sigma_{p_1}\right)^2 + \left(\frac{\partial \Omega_\Lambda}{\partial p_2}\sigma_{p_2}\right)^2}$$

$$\Rightarrow \boxed{\Omega_\Lambda = 0.70083(7)}$$

Question 4)

```
Parameters given by importance sampling
H0 = 67.89742146356934 +/- 0.018018182362854566
omh2 = 0.022296154798489586 +/- 4.548755138976884e-06
omch2 = 0.11837196183345552 +/- 3.84197610505776e-05
tau = 0.055909991941516475 +/- 0.00014095779724401515
As = 2.096284118154914e-09 +/- 6.016662895811411e-13
ns = 0.971347383243812 +/- 9.091165551842413e-05
```

For the importance sampling I did a weighted average were the weights are computed as follow

$$\text{Weights} = \chi^2_{old} \cdot \text{Gaussian}(x = tau, \mu = 0.054, \sigma = 0.0074)$$

So the samples in the chain that have a value of tau closer to the mean of 0.054 get a bigger weight in the average. I used the fact that that np.average() already has a "weight" kwarg in my advantage. For the error I made a custom weighted_std() function which looks like

mean = average(chain_samples, weights)
variance = average((chain_samples-mean)^2, weights)
std = sqrt(variance)

Now for the new chain I recalculated the covariance matrix using the fact that np.cov() also has a weight kwarg. So I just did np.cov(chain, aweights=weights)

To implement the the constraint in the mcmc I had to make another chi-square function.

I did the following chi-square = (data-get_sprectrum(parms)**2/errs + (pars[3]-0.054)**2/0.0074.

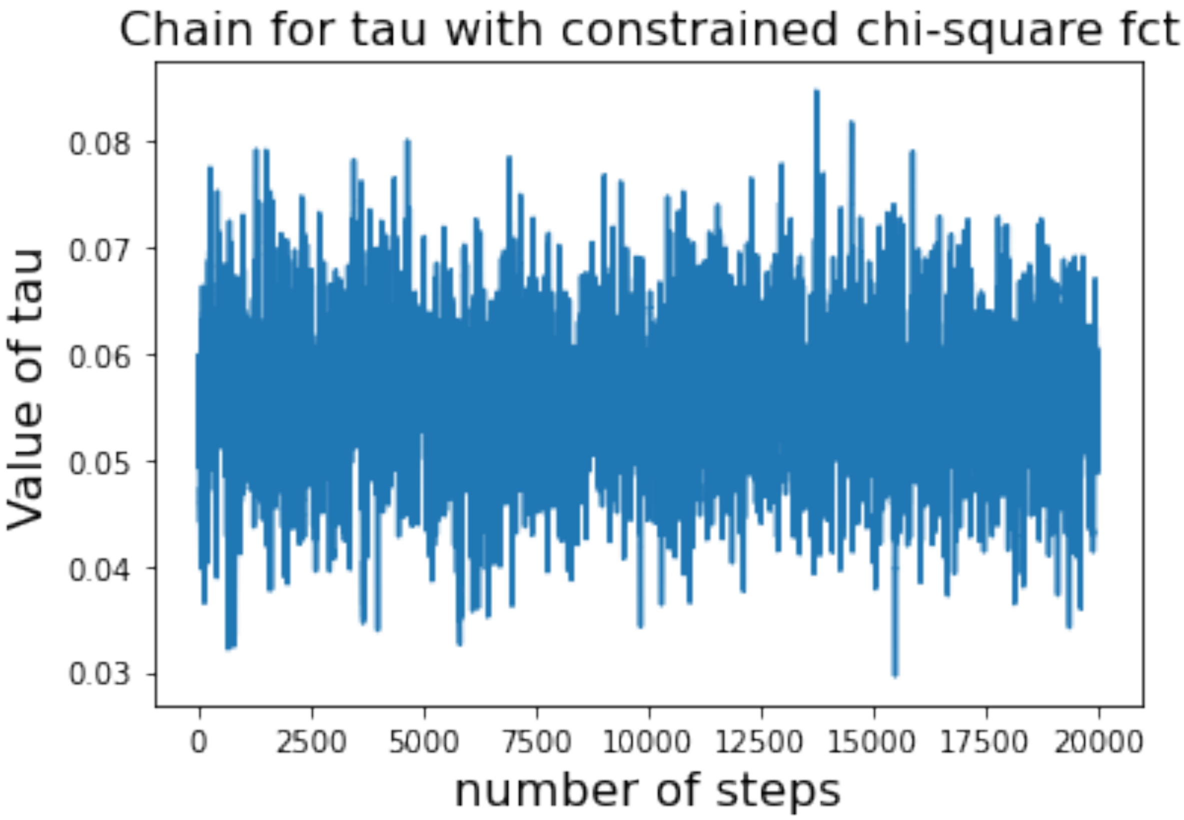
So I essentially just added another chi-square term to our old chi square function accordingly with our new constraint on tau.

I got the following params

```
Parameters given by constrained
H0 = 67.72246602873166 +/- 0.019415139644339476
omh2 = 0.022289497112099634 +/- 4.276946584617312e-06
omch2 = 0.11884210031908733 +/- 4.372976234845353e-05
tau = 0.056046795156113306 +/- 0.00014150690269914092
As = 2.099679828289975e-09 +/- 6.149781193344319e-13
ns = 0.9700726926016747 +/- 0.00010705773915309634
```

Just looking at them with the naked eye we see that they are essentially the same as the one we got with the importance sampling.

Now a plot of the chain



which again doesn't wander off, suggesting that it converged