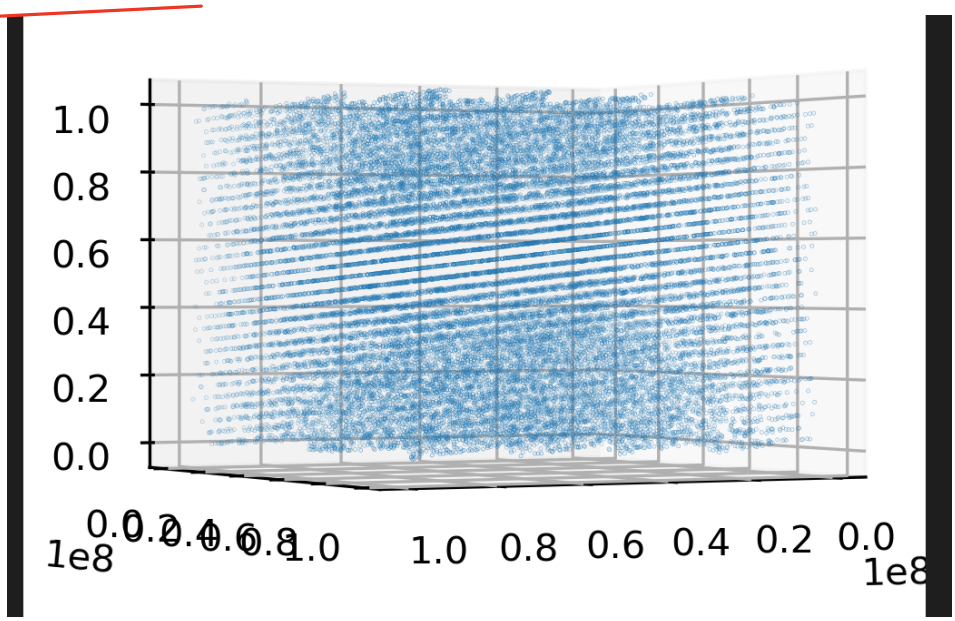
Question #1

a)



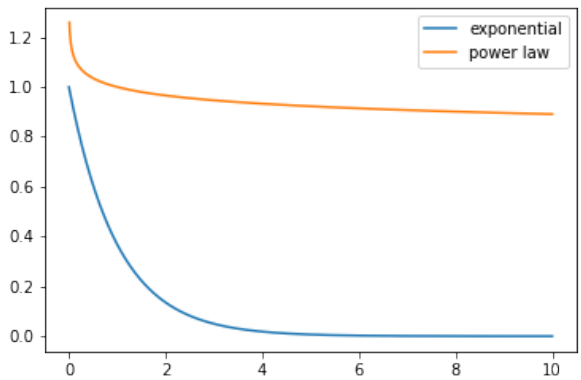
Form this point view we can see the planes generated by the triplets generated with the C RNG. I counted the planes and also get 30.

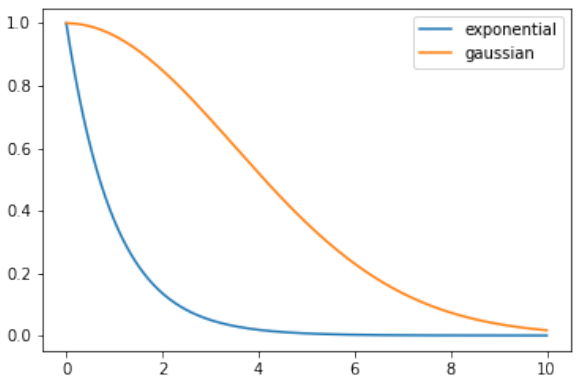
b)

For python's RNG I made a similar plot with randint from 0 to 1E8 with 30000 triplets. I made the plot interactive so that i could change the view easily. After changing the angles for a while I came to the conclusion that they were no such planes.

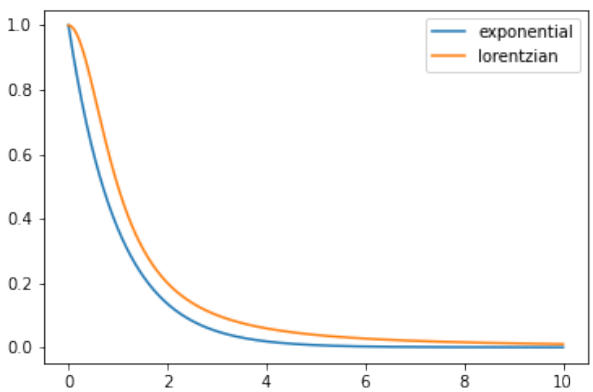
I failed to install libc.dylb

Question 2



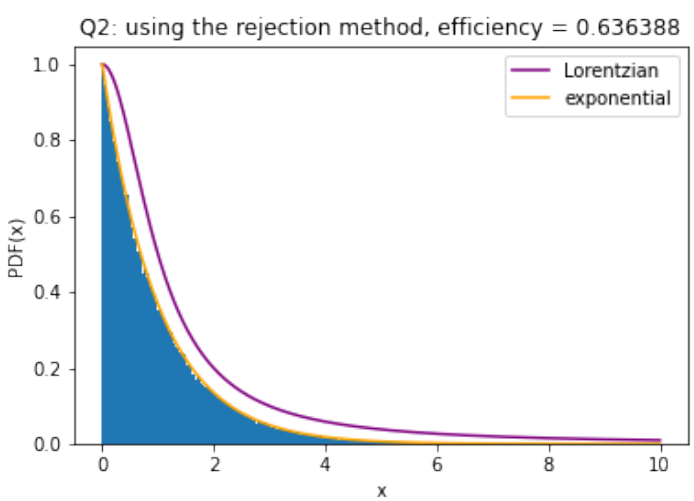


First I plot an exponential against multiple other distro to see which look more appropriate.



Clearly the lorentzian minimize the area between the 2 curves making it the most efficient distro to choose.

I got the following result



The efficiency quoted on the plot is the one I got with the rejection method, but it can be easily predicted using

$$\text{eff} = \frac{\text{area under exponential}}{\text{one under lorentzian}} = \frac{\int_0^\infty dx\, e^{-x}}{\int_0^\infty dx\, \frac{1}{1+x^2}} = \frac{2}{\pi} = 0.6366$$

To optimize it one would need to optimize the ratio of the integrals while making sure their lorentzian is always bigger than the exponential for all x. I tried do it but I would get weird results.

Question # 3

Wikipedia tell me that for a distro h(x), U and V are the following sets

$$U = (0, a) \quad \& \quad V = (b_-, b_+)$$

where $a = \sqrt{\sup h}$, $b_- = \sqrt{\sup\{x^2 h(x) : x \le 0\}}$

& $b_+ = -\sqrt{\sup\{x^2 h(x) : x \le 0\}}$

So for an exponential

$$a = \sqrt{\sup e^{-x}} = 1$$

for b we need to optimize

$$x^2 e^{-x} \Rightarrow 0 = 2x e^{-x} - x^2 e^{-x}$$

$$\Rightarrow x = 2$$

$$\Rightarrow b_+ = \sqrt{2^2 e^{-2}} = \frac{2}{e}$$

$b_- = 0$ because $x \leq 0$

so $u \in (0,1)$ & $v \in (0, 2/e)$

& we want $0 \leq u^2 \leq h(u/v)$

I get