2019

SEMESTER 1

ENGSCI 760 — Assignment 1

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March 19, 2019

1 Joint Distributions

1.a Pairwise Independence?

Events A, B and C **are** pairwise independent. This is because Event A is independent of B and is also independent of C. Events B and C are also independent of each other. This means the outcomes in any **pair of events** are independent of each other

1.b Mutual indepentence?

Events A, B and C **are not** mutually independent. This is because if we know the outcome of Event B and Event C, then we know with 100% certainty the outcome of Event A. In other words, Event A is **dependent** on the intersection of Events B and C.

2 Markov Chains

2.a One-Step Transition Matrix

$$\mathbf{P} = \begin{array}{c} \begin{matrix} \text{U} & \text{G} & \text{A} & \text{P} & \text{S} \end{matrix} \\ \begin{bmatrix} 0 & 0.4 & 0.3 & 0.2 & 0.1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0.4 & 0.3 & 0.2 & 0.1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{matrix} \text{U} \\ \text{G} \\ \text{A} \\ \text{P} \\ \text{S} \end{matrix}$$

2.b Limiting Distribution

$$\begin{aligned} \text{Proportion of good items} &= \text{Probability that a given item is good} \\ &= \text{Pr(item reaches G in step 1 } \textit{or} \text{ item reaches G in step 2 } \textit{or} \text{ ...)} \\ &= \text{Pr(item good in step 1) + Pr(item good in step 2) + ...} \\ &= 0.4 + (0.2)(0.4) + (0.2)(0.2)(0.4) + (0.2)(0.2)^2(0.4) + ... \\ &= 0.4 + (0.2)(0.4)\sum_{k=0}^{\infty}(0.2)^k \\ &= 0.4 + \frac{0.08}{0.8} \\ &= 0.5 \end{aligned}$$

$$\begin{aligned}
\text{Proportion of average items} &= \Pr(\text{average in step 1}) + \Pr(\text{average in step 2}) + ... \\
&= 0.3 + (0.2)(0.3) + (0.2)(0.2)(0.3) + (0.2)(0.2)^2(0.3) + ... \\
&= 0.3 + (0.2)(0.3)\sum_{k=0}^{\infty}(0.2)^k \\
&= 0.3 + \frac{0.06}{0.8} \\
&= 0.375
\end{aligned}$$

$$\begin{aligned}
\text{Proportion of scrapped items} &= \Pr(\text{scrap in step 1}) + \Pr(\text{scrap in step 2}) + ... \\
&= 0.1 + (0.2)(0.1) + (0.2)(0.2)(0.1) + (0.2)(0.2)^2(0.1) + ... \\
&= 0.1 + (0.2)(0.1)\sum_{k=0}^{\infty}(0.2)^k \\
&= 0.1 + \frac{0.02}{0.8} \\
&= 0.125
\end{aligned}$$

2.c Expected Profit From Unfinished Item

$$\begin{aligned}
\text{Expected profit} &= (50 \times 0.5) + (40 \times 0.375) - (10 \times \sum_{k=0}^{\infty}(0.175)^k) - 20 \\
&= 25 + 15 - \frac{10}{1 - 0.175} - 20 \\
&= \$7.88
\end{aligned}$$

2.d Updated One-Step Transition Matrix

$$\mathbf{P_{new}} = \begin{array}{c} \begin{matrix} \text{U} & \text{G} & \text{A} & \text{S} & P_1 & P_2 & P_3 \end{matrix} \\ \begin{bmatrix} 0 & 0.4 & 0.3 & 0.1 & 0.2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0.4 & 0.3 & 0.1 & 0 & 0.2 & 0 \\ 0 & 0.4 & 0.3 & 0.1 & 0 & 0 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{matrix} \text{U} \\ \text{G} \\ \text{A} \\ \text{S} \\ P_1 \\ P_2 \\ P_3 \end{matrix}$$

3 Hidden Markov Model

3.a createTransitions

3.b createEmissions

3.c HMM

3.d main