Binary Search Trees: Splay Trees: Introduction

Daniel Kane

Department of Computer Science and Engineering
University of California, San Diego

Data Structures Fundamentals
Algorithms and Data Structures

Learning Objectives

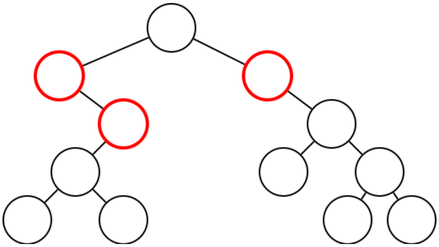
- Understand the motivation behind a splay tree.
- Implement the splay operation.

Non Uniform Inputs

- Search for random elements $O(\log(n))$ best possible.

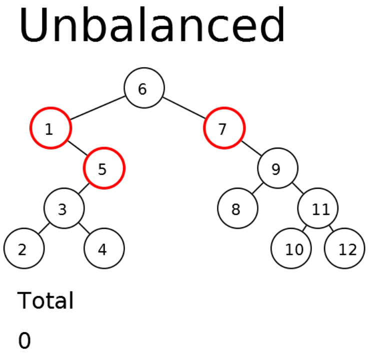
Non Uniform Inputs

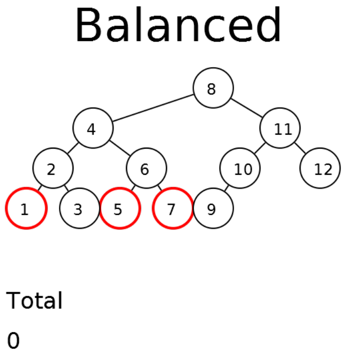
- Search for random elements $O(\log(n))$ best possible.
- If some items more frequent than others, can do better putting frequent queries near root.



Comparison

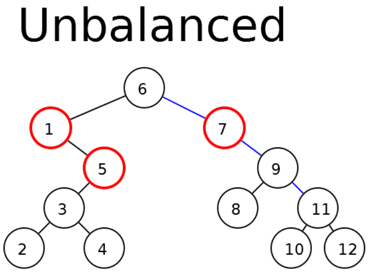
Trees.

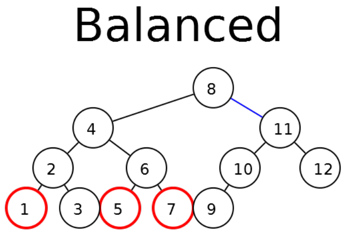




Comparison

`Find 11`





Total

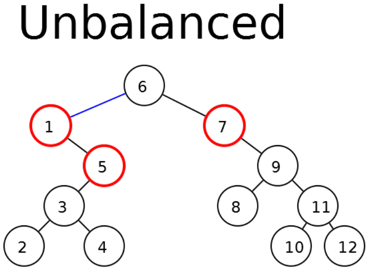
4

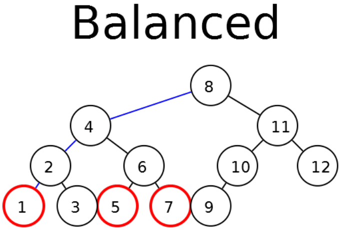
Total

2

Comparison

`Find 1`





Total

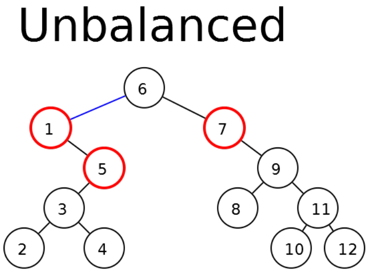
6

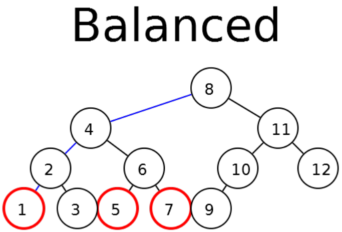
Total

6

Comparison

`Find 1`





Total

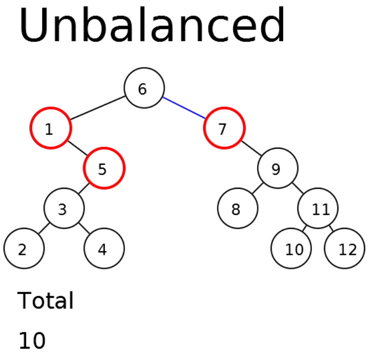
8

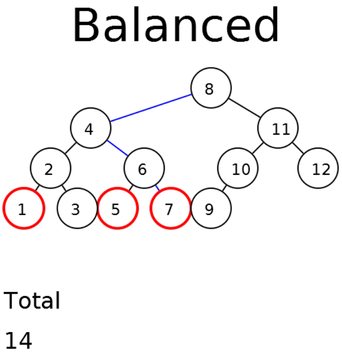
Total

10

Comparison

`Find` 7

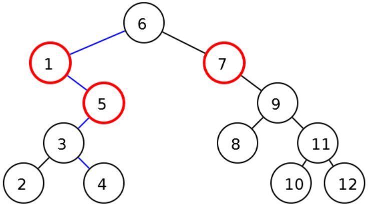




Comparison

`Find 4`

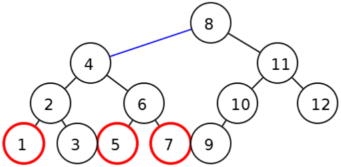
Unbalanced



Total

15

Balanced



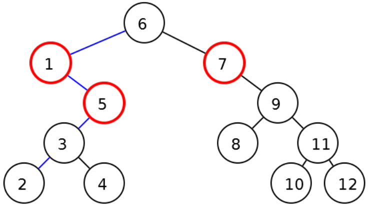
Total

16

Comparison

Find 2

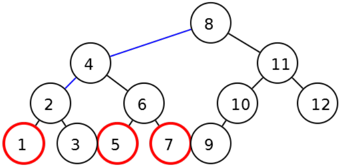
Unbalanced



Total

20

Balanced

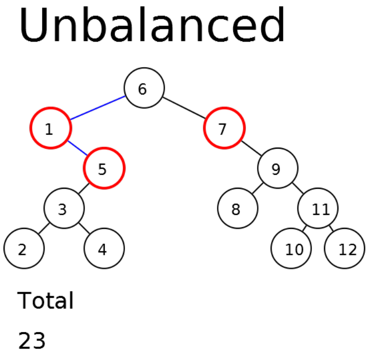


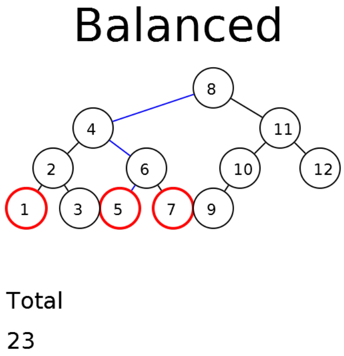
Total

19

Comparison

`Find 5`

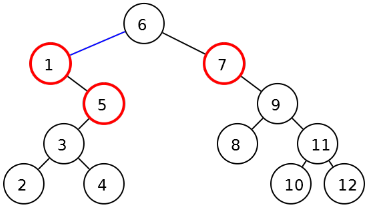




Comparison

`Find 1`

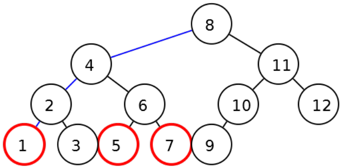
Unbalanced



Total

25

Balanced



Total

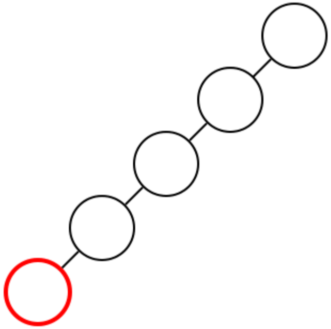
27

Idea

- Want common nodes near root.
- Don't know which those nodes will be.
- Bring the queried node to the root.

Simple Idea

Just rotate to top.



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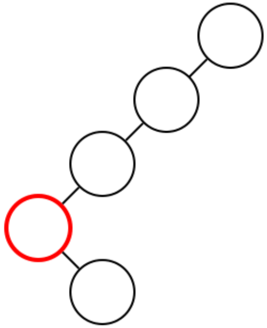

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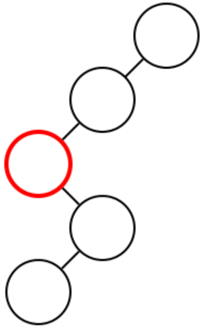
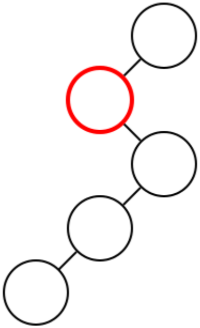


Simple Idea

Just rotate to top.



Simple Idea

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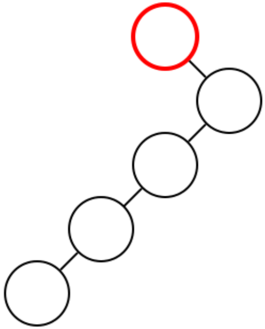
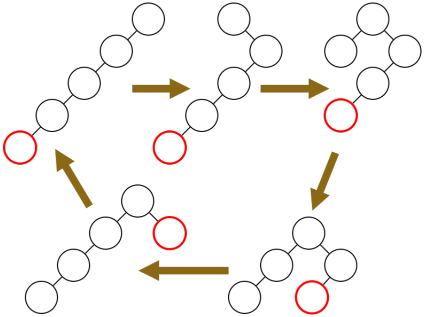


Loop

If you keep doing this you can get stuck in a loop.



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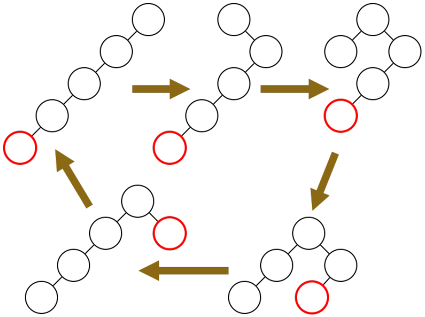
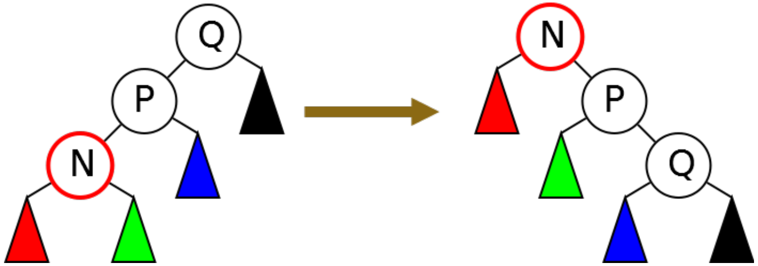


$O(n^2)$ time for $O(n)$ operations. Need something better.

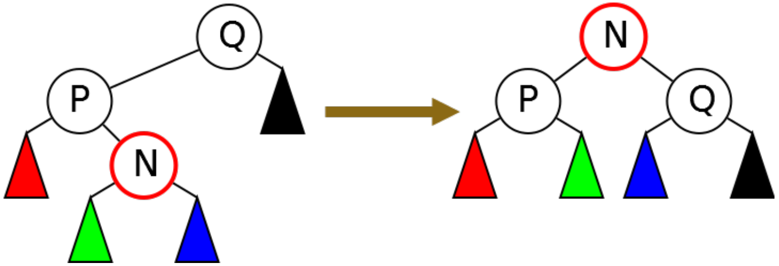
Modification

Zig-Zig



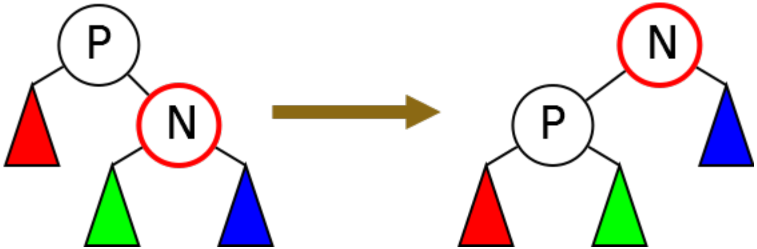
Modification

Zig-Zag



Modification

If just below root:
Zig



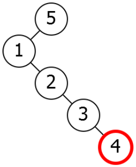
Splay

Splay(N)

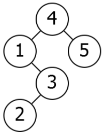
```
Determine proper case
Apply Zig-Zig, Zig-Zag, or Zig as
appropriate
if $N$.Parent $\neq$ null:
  Splay($N$)
```

Problem

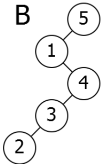
Which of the following is the result of splaying the highlighted node?



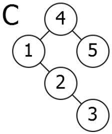
A



B



C



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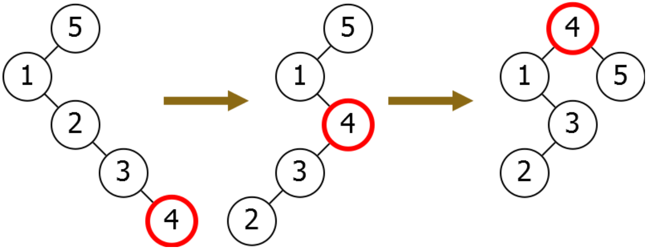

Next Time

How to use the splay operation to rebalance your tree.