Basic Data Structures: Arrays and Linked Lists

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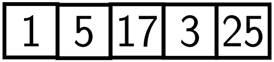
Data Structures Fundamentals
Algorithms and Data Structures

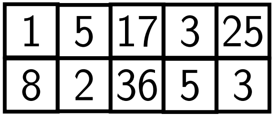
Outline

```
long arr[] = new long[5];
```

```
long arr[5];
```

```
arr = [None] * 5
```





Definition

Array:

Contiguous area of memory

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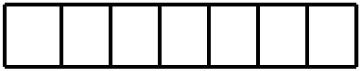
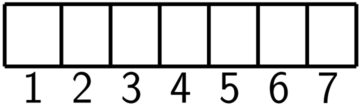
Contiguous area of memory consisting of equal-size elements



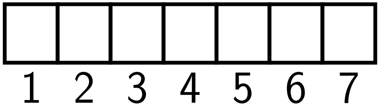
Definition

Array:

Contiguous area of memory consisting of equal-size elements indexed by contiguous integers.

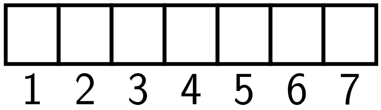


What's Special About Arrays?



What's Special About Arrays?

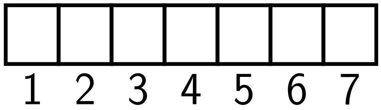
Constant-time access



What's Special About Arrays?

Constant-time access

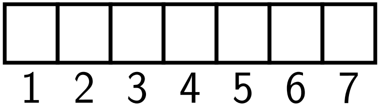
array_addr



What's Special About Arrays?

Constant-time access

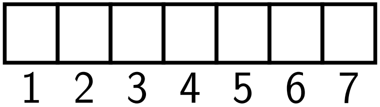
array_addr + elem_size $\times$ ( )



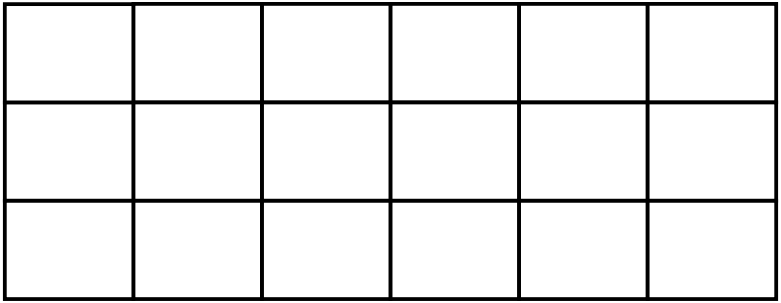
What’s Special About Arrays?

Constant-time access

$\text{array\_addr} + \text{elem\_size} \times (i - \text{first\_index})$



Multi-Dimensional Arrays



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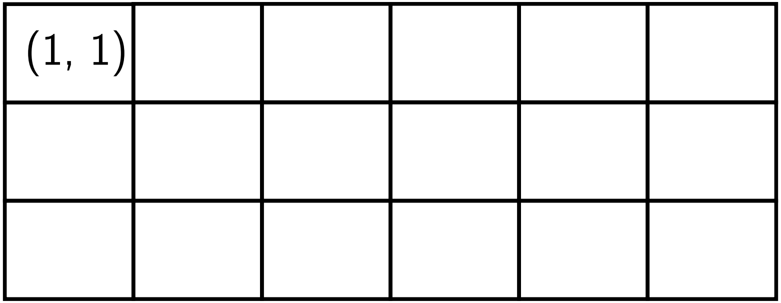

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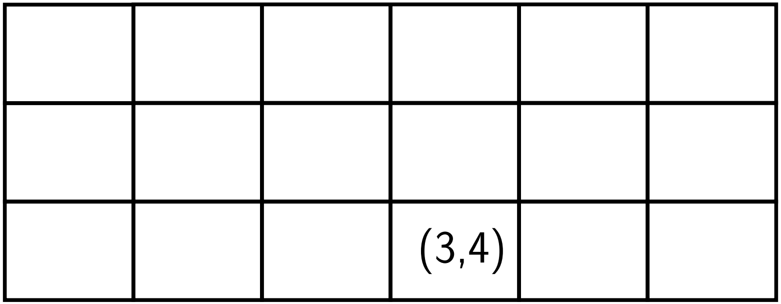

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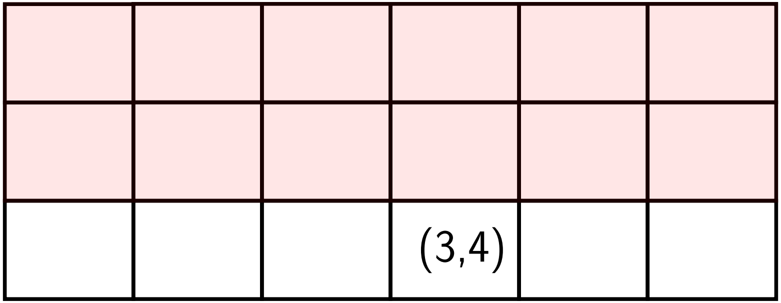
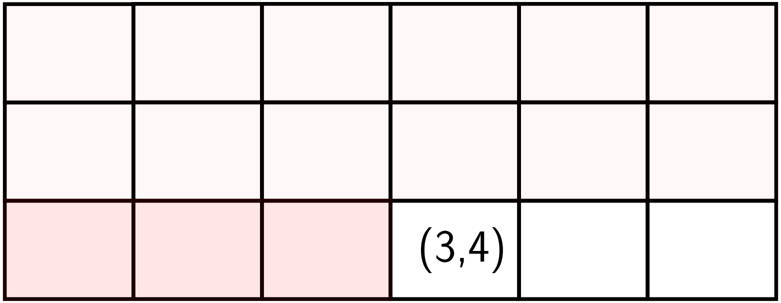


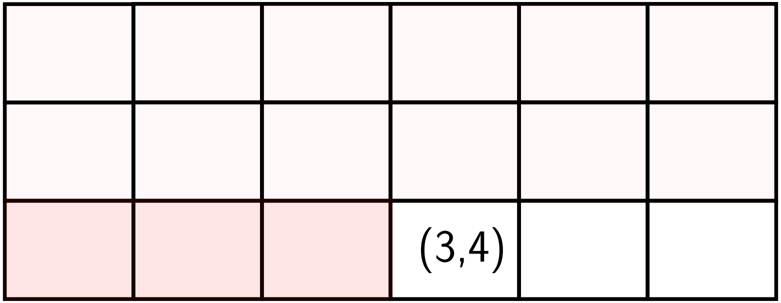
$$(3-1)\times 6$$

Multi-Dimensional Arrays



$$(3-1) \times 6 + (4-1)$$

Multi-Dimensional Arrays



$$\text{elem\_size} \times ((3-1) \times 6 + (4-1))$$

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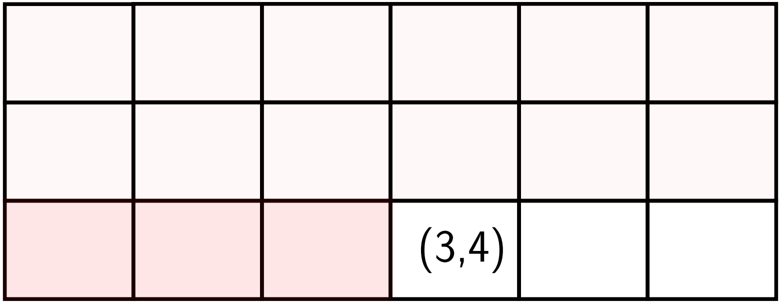
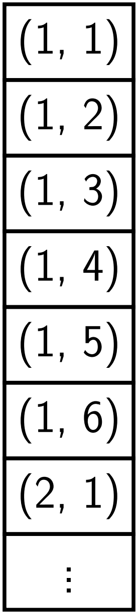
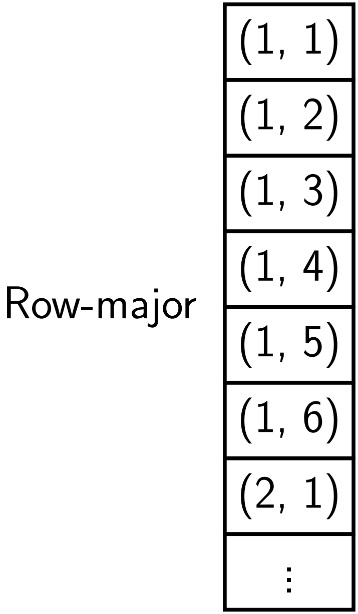


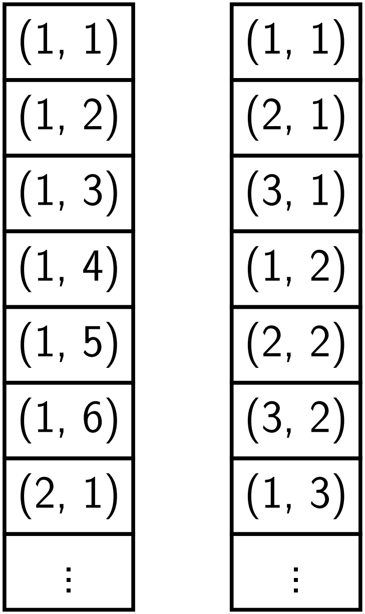
array_addr $+$

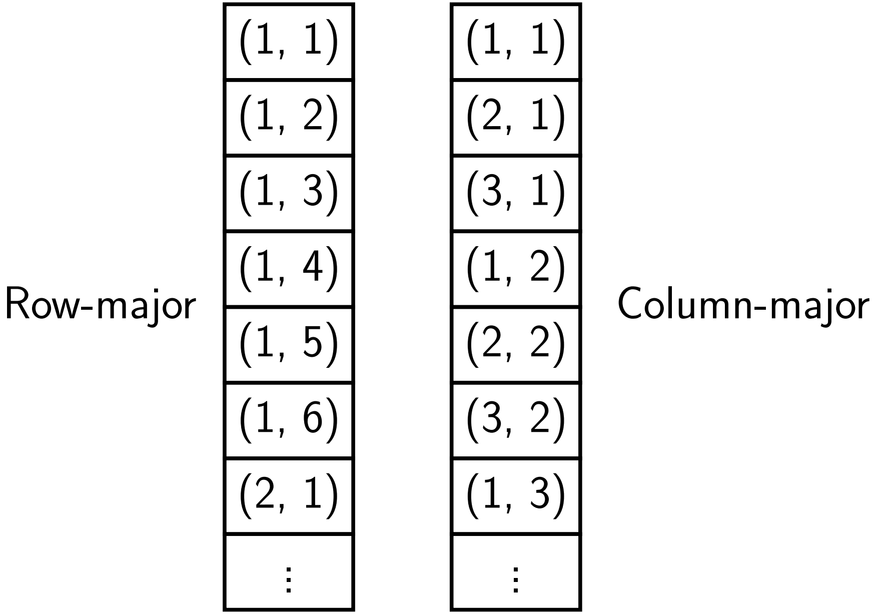
elem_size $\times ((3-1) \times 6 + (4-1))$


(1, 1)
(1, 2)
(1, 3)
(1, 4)
(1, 5)
(1, 6)
(2, 1)
⋮


Row-major
(1, 1)
(1, 2)
(1, 3)
(1, 4)
(1, 5)
(1, 6)
(2, 1)
⋮

Row-major

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(1, 2)
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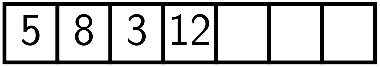
(1, 5)
(1, 6)
(2, 1)
⋮
(1, 1)
(2, 1)
(3, 1)
(1, 2)
(2, 2)
(3, 2)
(1, 3)
⋮
Column-major

Times for Common Operations

	Add	Remove
Beginning		
End		
Middle		

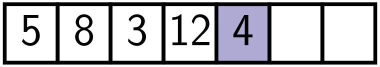
Times for Common Operations

	Add	Remove
Beginning		
End		
Middle		



Times for Common Operations

	Add	Remove
Beginning		
End	$O(1)$	
Middle		



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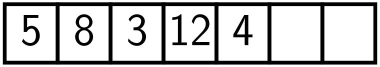

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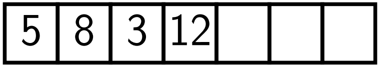
Middle		



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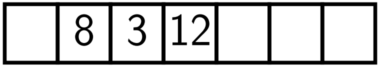

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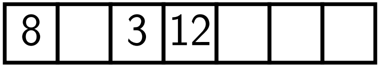

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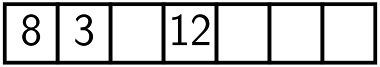
Middle		



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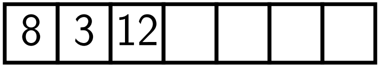

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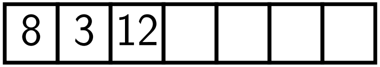

Times for Common Operations

	Add	Remove
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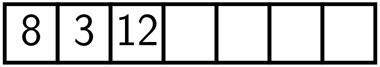

Summary

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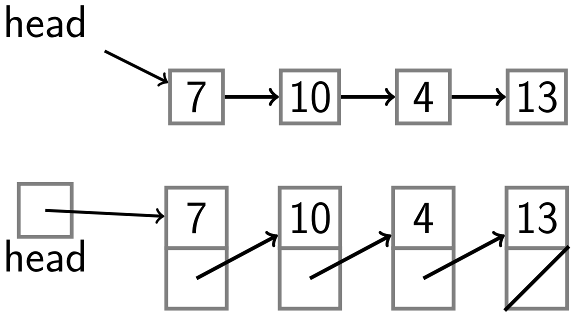
- Array: contiguous area of memory consisting of equal-size elements indexed by contiguous integers.
- Constant-time access to any element.
- Constant time to add/remove at the end.
- Linear time to add/remove at an arbitrary location.

Outline

1. Arrays
2. Linked Lists

Singly-Linked List



Node contains:

- `key`
- `next pointer`

List API

`PushFront(Key)`	add to front

List API

`PushFront(Key)`	add to front
`Key TopFront()`	return front item

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`PushBack(Key)`	add to back also known as `Append`

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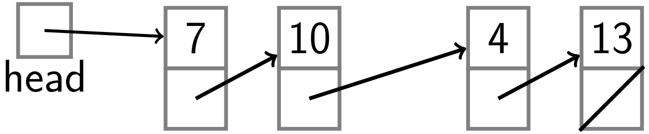
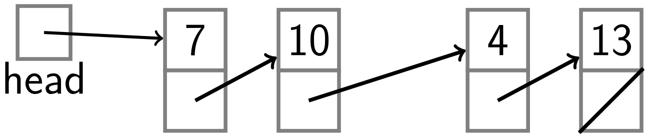
Times for Some Operations



Times for Some Operations

`PushFront`



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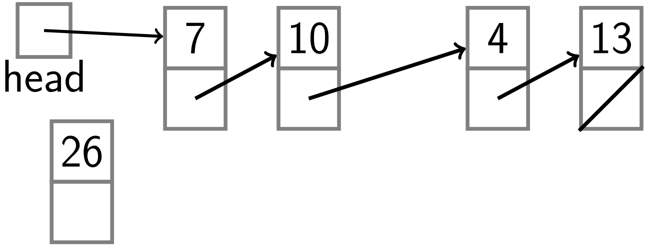

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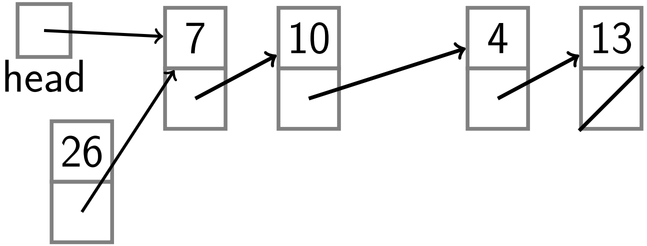
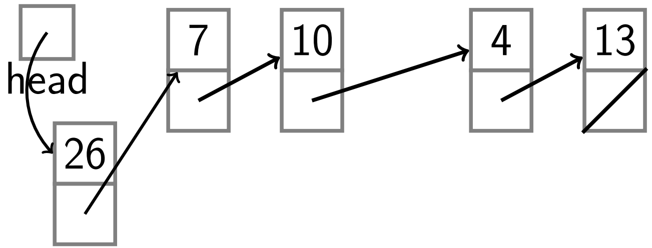


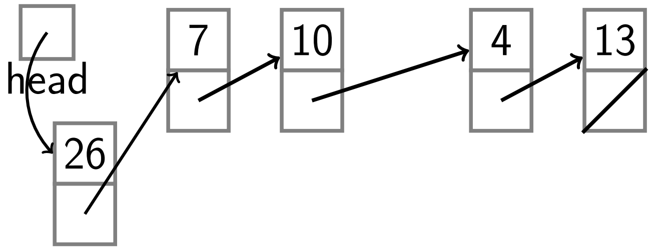
Times for Some Operations

`PushFront` $O(1)$



Times for Some Operations

`PopFront`



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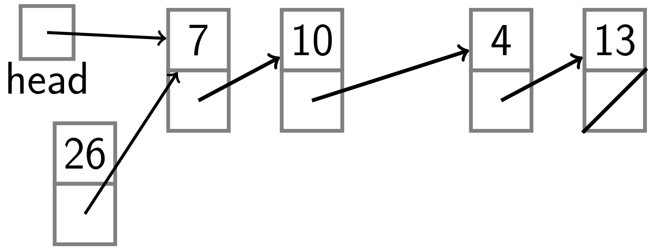

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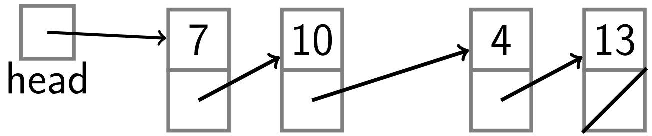
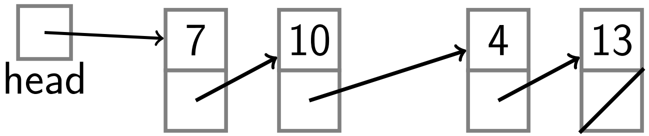


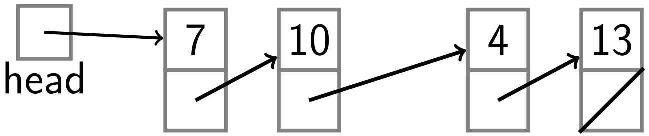
Times for Some Operations

`PushBack`
(no tail)



Times for Some Operations

`PushBack` (no tail) $O(n)$



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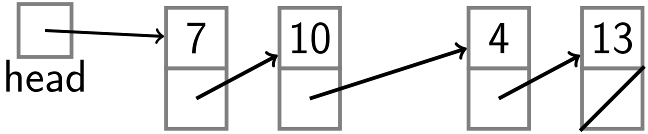

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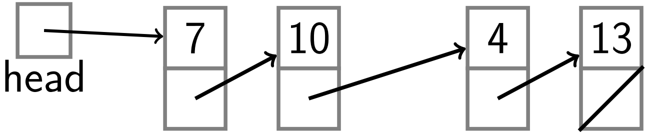

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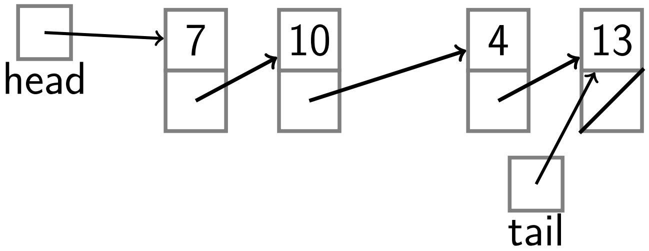

Times for Some Operations

`PushBack`
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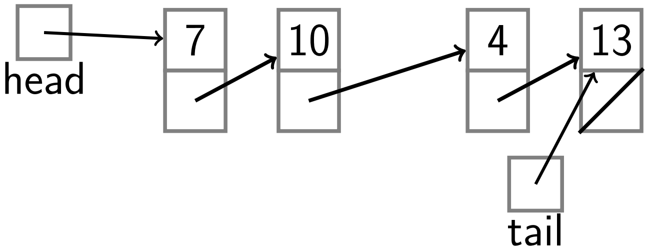

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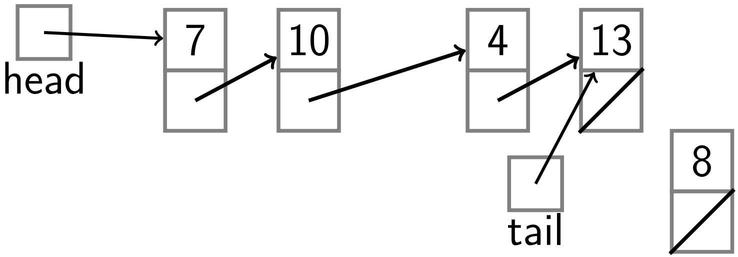

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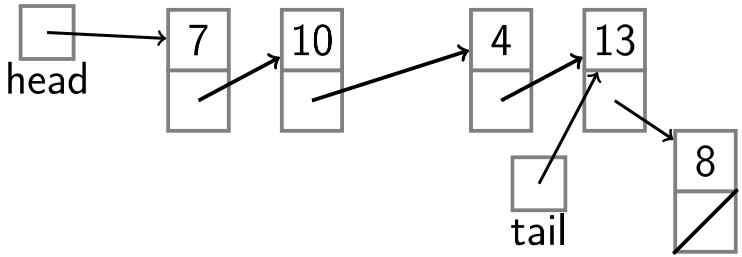

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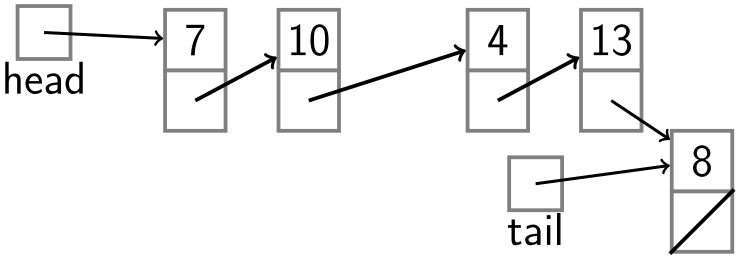

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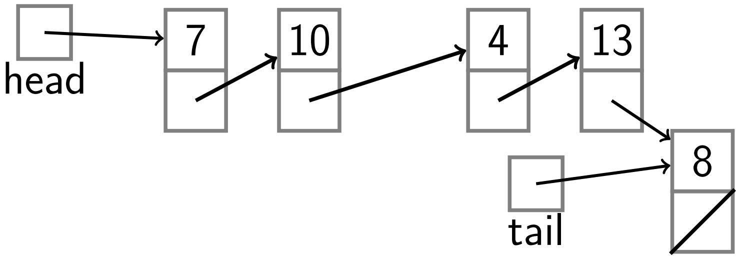

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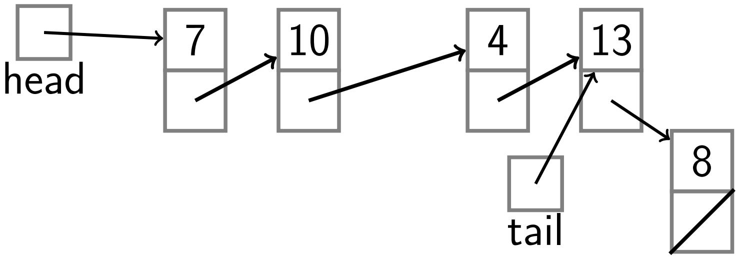

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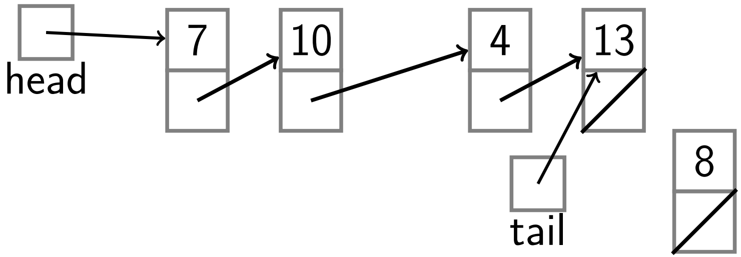

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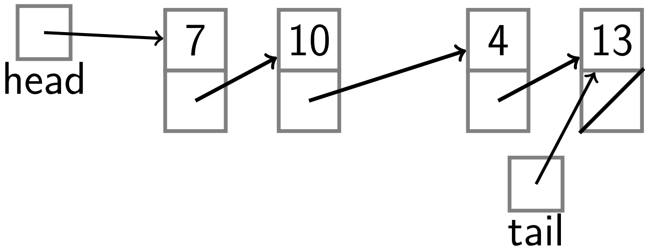

Singly-linked List

PushFront(*key*)

```
$node \leftarrow$ new node
$node.key \leftarrow key$
$node.next \leftarrow head$
$head \leftarrow node$
if $tail =$ nil:
    $tail \leftarrow head$
```

Singly-linked List

PopFront()

```
if $head$ = nil:
  ERROR: empty list
$head \leftarrow head.next$
if $head$ = nil:
  $tail \leftarrow$ nil
```

Singly-linked List

PushBack(*key*)

```
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Singly-linked List

PushBack(key)

```
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$node.key \leftarrow key$
$node.next =$ nil
if $tail =$ nil:
    $head \leftarrow tail \leftarrow node$
```

Singly-linked List

PushBack(*key*)

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$node.key \leftarrow key$
$node.next =$ nil
if $tail =$ nil:
    $head \leftarrow tail \leftarrow node$
else:
    $tail.next \leftarrow node$
    $tail \leftarrow node$
```

Singly-linked List

```
PopBack()
```

Singly-linked List

PopBack()

```
if $head$ = nil: ERROR: empty list
```

Singly-linked List

PopBack()

```
if $head$ = nil: ERROR: empty list
if $head = tail$:
    $head \leftarrow tail \leftarrow$nil
```

Singly-linked List

PopBack()

```
if $head$ = nil: ERROR: empty list
if $head = tail$:
   $head \leftarrow tail \leftarrow$ nil
else:
   $p \leftarrow head$
   while $p.next.next \neq$ nil:
      $p \leftarrow p.next$
```

Singly-linked List

PopBack()

```
if $head$ = nil: ERROR: empty list
if $head = tail$:
   $head \leftarrow tail \leftarrow$nil
else:
   $p \leftarrow head$
   while $p.next.next \neq$ nil:
      $p \leftarrow p.next$
   $p.next \leftarrow$ nil; $tail \leftarrow p$
```

Singly-linked List

```
AddAfter($node, key$)
$node2 \leftarrow$ new node
$node2.key \leftarrow key$
$node2.next = node.next$
$node.next = node2$
if $tail = node$:
    $tail \leftarrow node2$
```

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	
`Find(Key)`	$O(n)$	

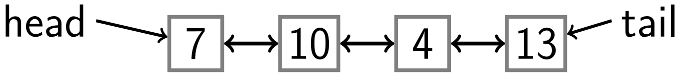
Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	
`Find(Key)`	$O(n)$	
`Erase(Key)`	$O(n)$	

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	
`Find(Key)`	$O(n)$	
`Erase(Key)`	$O(n)$	
`Empty()`	$O(1)$	

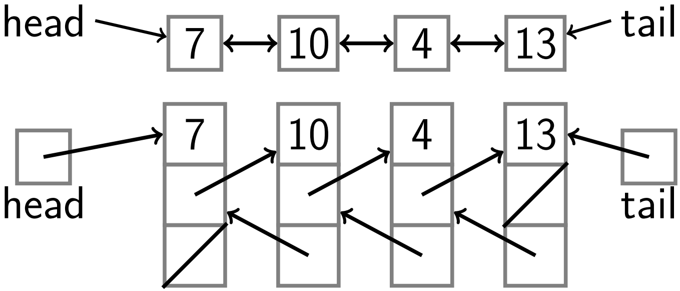
Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	
`Find(Key)`	$O(n)$	
`Erase(Key)`	$O(n)$	
`Empty()`	$O(1)$	
`AddBefore(Node, Key)`	$O(n)$	

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	
`Find(Key)`	$O(n)$	
`Erase(Key)`	$O(n)$	
`Empty()`	$O(1)$	
`AddBefore(Node, Key)`	$O(n)$	
`AddAfter(Node, Key)`	$O(1)$	

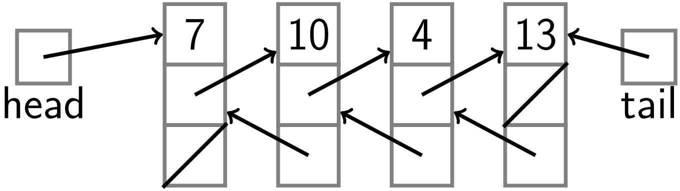
Doubly-Linked List



Doubly-Linked List



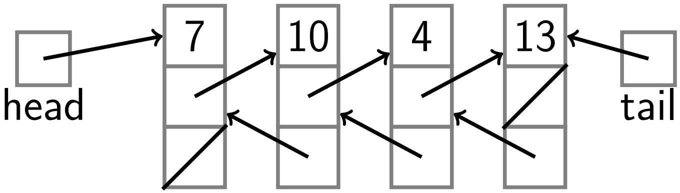
Doubly-Linked List



Node contains:

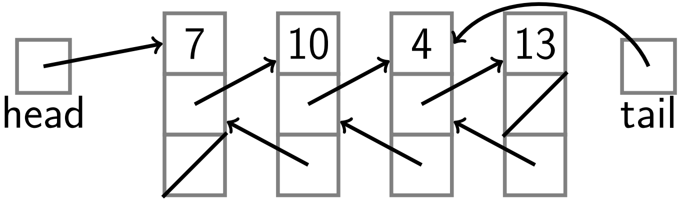
- `key`
- `next` pointer
- `prev` pointer

Doubly-Linked List



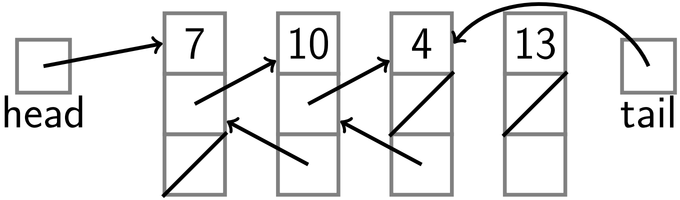
`PopBack`

Doubly-Linked List



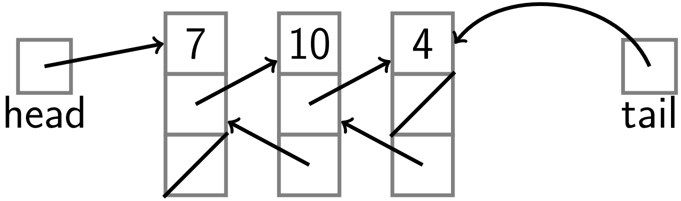
`PopBack`

Doubly-Linked List



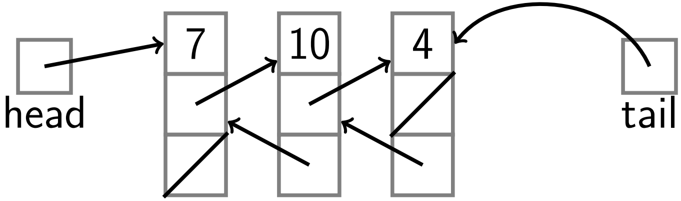
`PopBack`

Doubly-Linked List



`PopBack`

Doubly-Linked List



`PopBack` $O(1)$

Doubly-linked List

PushBack(*key*)

```
$node \leftarrow$ new node
$node.key \leftarrow key$;  $node.next =$ nil
```

Doubly-linked List

PushBack(*key*)

```
$node \leftarrow$ new node
$node.key \leftarrow key$;  $node.next =$ nil
if $tail =$ nil:
    $head \leftarrow tail \leftarrow node$
    $node.prev \leftarrow$ nil
```

Doubly-linked List

PushBack(key)

```
$node \leftarrow$ new node
$node.key \leftarrow key$;  $node.next =$ nil
if $tail =$ nil:
    $head \leftarrow tail \leftarrow node$
    $node.prev \leftarrow$ nil
else:
    $tail.next \leftarrow node$
    $node.prev \leftarrow tail$
    $tail \leftarrow node$
```

Doubly-linked List

`PopBack()`

Doubly-linked List

PopBack()

```
if $head$ = nil: ERROR: empty list
```

Doubly-linked List

PopBack()

```
if $head$ = nil: ERROR: empty list
if $head = tail$:
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```

Doubly-linked List

PopBack()

```
if $head$ = nil: ERROR: empty list
if $head = tail$:
    $head \leftarrow tail \leftarrow$nil
else:
    $tail \leftarrow tail.prev$
    $tail.next \leftarrow$nil
```

Doubly-linked List

```
AddAfter($node$, $key$)

$node2 \leftarrow$ new node
$node2.key \leftarrow key$
$node2.next \leftarrow node.next$
$node2.prev \leftarrow node$
$node.next \leftarrow node2$
if $node2.next \neq$ nil:
    $node2.next.prev \leftarrow node2$
if $tail = node$:
    $tail \leftarrow node2$
```

Doubly-linked List

```
AddBefore($node, key$)
$node2 \leftarrow$ new node
$node2.key \leftarrow key$
$node2.next \leftarrow node$
$node2.prev \leftarrow node.prev$
$node.prev \leftarrow node2$
if $node2.prev \neq$ nil:
    $node2.prev.next \leftarrow node2$
if $head = node$:
    $head \leftarrow node2$
```

Singly-Linked List	no tail	with tai
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	
`Find(Key)`	$O(n)$	
`Erase(Key)`	$O(n)$	
`Empty()`	$O(1)$	
`AddBefore(Node, Key)`	$O(n)$	
`AddAfter(Node, Key)`	$O(1)$	

Doubly-Linked List	no tail	with tai
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	~~$O(n)$~~ $O(1)$	
`Find(Key)`	$O(n)$	
`Erase(Key)`	$O(n)$	
`Empty()`	$O(1)$	
`AddBefore(Node, Key)`	~~$O(n)$~~ $O(1)$	
`AddAfter(Node, Key)`	$O(1)$	

Summary

- Constant time to insert at or remove from the front.

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- With tail and doubly-linked, constant time to insert at or remove from the back.

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- $O(n)$ time to find arbitrary element.
- List elements need not be contiguous.

Summary

- Constant time to insert at or remove from the front.
- With tail and doubly-linked, constant time to insert at or remove from the back.
- $O(n)$ time to find arbitrary element.
- List elements need not be contiguous.
- With doubly-linked list, constant time to insert between nodes or remove a node.