Parameterization

Jing JIANG
siliuhe@sina.com

February 24, 2020

Barycentric Mapping

Introduction

According to Tutteąŕs barycentric mapping theorem,

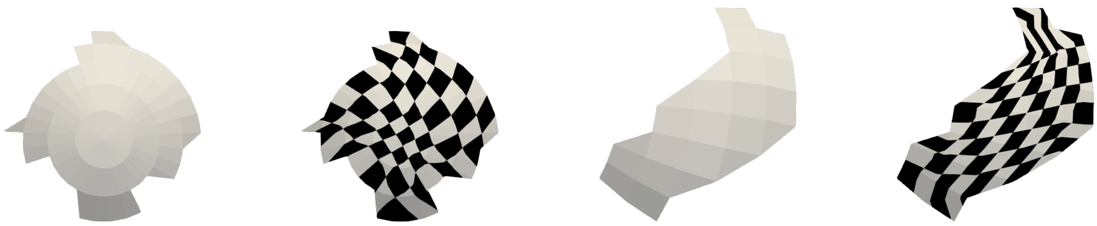
> Given a triangulated surface homeomorphic to a disk, if the (u, v) coordinates at the boundary vertices lie on a convex polygon, and if the coordinates of the internal vertices are a convex combination of their neighbors, then the (u, v) coordinates form a valid parameterization (without self-intersections).

$$\forall i \in 1, 2, ..., n_{in}, \qquad -a_{i,i}\begin{pmatrix} u_i \\ v_i \end{pmatrix} = \sum_{j \neq i} a_{i,j}\begin{pmatrix} u_i \\ v_i \end{pmatrix}$$

Implement

1. judge whether the model is a patch
2. build half-edge structure
3. get the boundary edges and the texture coordinates of the boundary vertices
4. get the texture coordinates of the inner vertices

Result



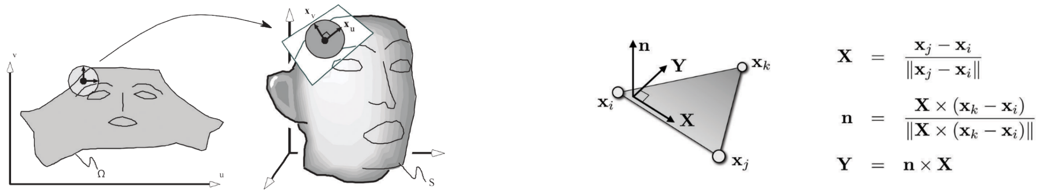
(a) (b) (c) (d)

Figure 1: barycentering mapping, a and c are origin models, b and d are models with texture coordinates

Least Squares Conformal Mapping

Introduction

Conformal mapping: the anisotropy ellipse is a circle for all points of the surface.



(a) (b)

Figure 2: a: mapping $\boldsymbol{X}$ to $\boldsymbol{u}$. b: local X, Y basis in a triangle

Conformal condition: $\boldsymbol{X}_\mu = \boldsymbol{n} \times \boldsymbol{X}_u$, $\nabla\mu = \boldsymbol{n} \times \nabla u$

$$\nabla u = \begin{bmatrix} \partial u/\partial X \\ \partial u/\partial Y \end{bmatrix} = \underbrace{\frac{1}{2A_T}\begin{bmatrix} Y_j - Y_k & Y_k - Y_i & Y_i - Y_j \\ X_k - X_j & X_i - X_k & X_j - X_i \end{bmatrix}}_{=\mathbf{M}_T}\begin{pmatrix} u_i \\ u_j \\ u_k \end{pmatrix}$$

conformal condition,

$$\nabla v = (\nabla u)^\perp = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \nabla u$$

$$\mathbf{M}_T \begin{pmatrix} v_i \\ v_j \\ v_k \end{pmatrix} - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{M}_T \begin{pmatrix} u_i \\ u_j \\ u_k \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

energe to be optimed,

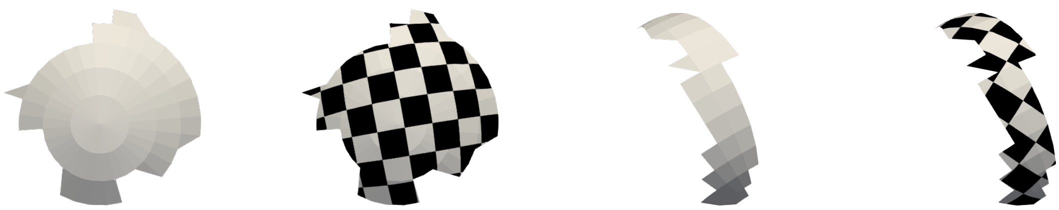
$$E_{\mathrm{LSCM}} = \sum_{T=(i,j,k)} A_T \left\| \mathbf{M}_T \begin{pmatrix} v_i \\ v_j \\ v_k \end{pmatrix} - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{M}_T \begin{pmatrix} u_i \\ u_j \\ u_k \end{pmatrix} \right\|^2$$

$$E_{\mathrm{LSCM}} = \sum_{T=(i,j,k)} \left\| \sqrt{A_T}\mathbf{M}_T \begin{pmatrix} v_i \\ v_j \\ v_k \end{pmatrix} - \sqrt{A_T} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{M}_T \begin{pmatrix} u_i \\ u_j \\ u_k \end{pmatrix} \right\|^2$$

Implement

1. get trianlge area and construct triangle transformation matrix
2. fix two vertices in parameter domain in order to reduce the degrees
3. construct global coefficient matrix and sparse equations
4. solver sparse equation though QR decomposition

Result



(a) (b) (c) (d)

Figure 3: barycentering mapping, a and c are origin models, b and d are models with texture coordinates