Advanced Physics Lab II
CO-487-A

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Guidelines for Writing a Laboratory Report

Report writing is a generic skill required in many occupations. Engineers, scientists, and also administrators in the financial sector or politics have to provide assessments, evaluations, or analysis and write up their results in form of a report. Good report writing is an essential skill in modern society. The quality of an oral or written report is invariably one of the criteria used in job interviews to evaluate a candidate. Giving a report and being able to structure and express your thoughts, to convince and persuade other people of your ideas and conclusions will be an important factor for advancing your career.

A written report is a mean to disclose, spread or circulate information. At all times the person who reads the report should clearly understand what you are trying to say. Therefore, a report has to be written in a clear and concise style. It must convey the information in a logical step-by-step sequence so that the reader is led inescapably to the same conclusions as the author of the report.

In the reports you have to write in the Physics Lab Course, you will be given specific tasks and objectives towards which you have to work to. In order to meet these objectives, you must develop a logical argument which culminates in a conclusion. The report should therefore follow a carefully considered, logical sequence in order to arrive at that conclusion. Along the way you have to convince the reader (and especially the person who corrects your report) at each stage that you have done the right thing and have made the correct interpretations of your results. You must provide the necessary background, inform the reader what you did and persuade him of your conclusions. If you developed a logical and complete argument, supported by correct scientific facts, then the reader will arrive at the same conclusions. Any errors, missing information, or wrong wording will mislead the reader and will cast doubt on your results and conclusions.

In general, the form, length, content, and emphasis of a report are determined by its purpose, topic and the intended audience. It is clear that there exists no single perfect format of a report. However, the structure of all reports is similar and includes a section that describes the objectives and background (what is known), methods and procedure (what has been done), results and conclusions (what has been measured and what are the implications). This format has evolved over time in practice and in many fields. In the following, we will give you a format which is best suited for writing Physics Lab Reports.

The Physics Laboratory Reports should answer the following general questions:

- What was the goal, what were the objectives of the experiment?

- What is the background, what is the underlying physics of the experiment?

- Which methods or techniques have been used?

- How was the raw data evaluated, what was the result of the experiment, and how does it compare to theory?

- How reliable is the result and what have been possible error sources?

- What conclusion can be drawn?

In general the lab report is a text reporting on the experiment (background, setup, outcome) by addressing all the questions and tasks given in the manual. As said before, a good laboratory report will enable the reader to repeat your measurements and critically assess your evaluation and conclusions. The reader should be able to understand the experiment without consulting other sources except your lab report. This implies that the reader should be able to answer the questions above just by reading your report.

Structure and Content

Here, some guidelines for the content of the different sections of a lab report are given. If you encounter problems by following these guidelines use your common sense and always stick to the general principles of clarity, brevity and a professional scientific style.

The lab report should be independent from any other source of information, *i.e.* all raw and processed data, schemes, and graphs should be given in the report, not referring to your notes or the manual for any experimental data. Related to a more detailed content of the report, all questions posed and tasks given in the manual should be addressed in a report, but it again should not refer to the numbering or the structure of the manual. The development of a logical argument is assisted by an hierarchical order and clear numbering of the following sections which you should use to structure the content of your report:

Abstract

This is usually the last section written, but it should head the report. In the real world an abstract will be the only section read by most readers, so it should briefly and truthfully summarize your report.

The abstract should be a very brief overview of the goals and the main results of the experiment. It shortly states the main objectives of the experiment. It names the main method used, states the main numerical result including error and gives the agreement or disagreement with theory or expectations. Sometimes, especially in case of disagreement, a statement about the main error source is recommended. An abstract does not include any tables, figures, lists, or equations. The abstract must not be longer than a short paragraph (maximum 10 lines).

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Introduction and Theory

Here, the general background and the underlying physics of the experiment should be provided. This should be motivated by a short statement on the objectives and tasks. A historical account of the experiment is not necessary. This section should provide the theoretical background for the experiment and the data evaluation. Cite or derive any equations needed, and discuss all basic formulae necessary for data evaluation and for interpreting your results. Omit derivation of standard formulae but describe the relationship between quantities to be measured and standard formulae. It is essential to define all symbols used. In addition, any assumptions or limitations made by the theory or used for the experiment should be stated (and justified if possible). Give and cite all literature values you might compare your data to. This part should describe the experiment or method in general. Technical or method specific equations, *e.g.* how to read the numerical results from an instrument or what are the correct orders of magnitude of the data belong to the experimental section.

Experimental and Procedure

This section describes the main equipment used for the experiment and details of the procedures you followed. After reading this section, the reader should be able to repeat your experiment and should come to the same results. Hence, you have to list all relevant parameters which may influence your data and results, even if they are not directly used for data analysis. A part list is **not** required.

- Describe shortly the overall measurement configuration and set-up. Always show a sketch or scheme of the set-up and explain what is seen in the text. Highlight important experiment parameters in your schema, *e.g.* length, angle *etc.*. State the chosen parameter ranges and instrumental settings and give the formulae you have to use to get meaningful quantities from the reading of the instruments. Use formulas from theory and give final formula used for the calculated result. This part should have all the information needed to reproduce the set-up independently.

- Write a short but sufficient account of the procedure used to carry out the experiment. Describe the steps you took to obtain the data. Do not forget to describe calibration procedures if they were needed. On the other hand, it is not always necessary to give a step-by-step account of all your activities during the experiment. Avoid unimportant and too specific details (like "a screwdriver was used to turn the red button at the instrument" etc. . .). To be concise, keep in mind that the experimental section is a report on what you actually did and not what one should do like in a manual. It should also not be a word-to-word copy of the lab manual.

- Avoid first person ("I" or "we") and don't write instructions, but what actually has been done!

Results and Data Analysis

This section provides the measured raw data, its mathematical treatment and the numerical results. It is the key part of the lab report. It should be written as a text describing what has been done and obtained. It should not be a mere list of formulas, numbers, keywords or tables. All data and results need to be described and connected by sentences. The raw data which was originally collected should clearly be identified as such. It is important to provide the original and non-manipulated data because later on your experiment may be evaluated in a different way or by someone else. Also, the corrector of your lab report may find that a faulty result was only due to wrong data manipulation and not due to a totally wrong data acquisition, or that you recorded the wrong data but did the right evaluation.

- The raw data has to be directly written in a lab book or a printout has to be attached to the lab book. For the lab report, the scanned raw data from the day of the experiment have to be included in the appendix. The type of data can vary and can include numbers, graphs, images, *etc.* If the raw data can be inserted in a table or figure filling less than half a page, the raw data should be provided in the text. In any case, it should be clearly stated what the original units and orders of magnitude of the raw data have been.

- This section should also describe in a step-like manner and textual form what data analysis and calculations have been performed by citing formulas from theory and procedures. Present the data neatly and clearly in columns and tables including clear headings and units. Sometimes this part may already be combined with the error analysis described in the next section. All equations needed for the data evaluation have to be given here (either by citing from the theory part or by simply restating). In second year reports, you should provide a sample calculation by plugging the actual numerical values and corresponding units in the equations. To do so, it is easier to follow your calculations and to identify unnecessary mistakes.

- Always perform <u>sample calculations</u> with numerical values **and** the corresponding units. This helps to identify errors in formulae and confirm the correct units such as mV, V, kV, *etc.* In addition, be critical to your results (*e.g.* does the order of magnitude make sense?).

- This section concludes with the statement of the final numerical result(s) of your experiment without a detailed discussion (which is part of the next sections!). The

result has to be identified as such and should be stated in a textual context. The end result of the data analysis should be the information which can be used to discuss the outcome of the experiment or project.

- Specify all values with an appropriate number of significant digits and with correct units. Errors should be given with at most two significant digits and the numerical value of a quantity has to have the same precision as the error. You will find more on error analysis in the next section and in the *Error Analysis Booklet*. Guidelines for presenting tables and graphs are also given below.

- Use the most compact layout and lowest number of tables. Avoid rewriting formulas or restating values.

Error analysis

In this section, the numerical validity of the result has to be evaluated including the uncertainties of the measured quantities and their contribution to the uncertainty of the final result. Sometimes, for simple error calculations, this section may be combined with the data analysis.

- You should state or estimate possible error sources of the experiment, especially including the specifications of instruments and reading errors. Sometimes, after listing all error sources, certain errors can be neglected since they are much smaller than others. This may simplify the error calculation, but you should give a justification for it.

- For directly measured quantities, the error is given by the accuracy and/or precision of the instrument as well as statistical error calculation if multiple measurements are performed. This applies *e.g.* for a temperature measured by a thermometer or a length measured by a ruler. The error of a quantity which is derived from measured parameters must be obtained by application of the error propagation theorem, respecting all parameters (see Error Analysis Booklet). This holds *e.g.* if the number of moles of an enclosed amount of gas is calculated from the three directly measured parameters pressure, volume and temperature.

- Always state which type of error you use for calculations. You may refer to a general formula, *e.g.* formula for calculation of the error of a mean value, but you should explicitly give the formulae you obtain by applying error propagation.

- **Never** specify any error with **more than two** significant digits.

Discussion

In this section, you should discuss your observations and results and compare them to your expectations and theory. An unbiased critical judgment and a scientific discussion is at the heart of all science. You should be able to make a reasonable assessment of your experiment. Compared to other sections the discussion part especially demands critical thinking, a clear line of reasoning and creativity. It is even more important that you get a clear picture and a good argument why your experiment did not succeed instead of presenting "perfect" results but with a wrong discussion.

- Some questions you should keep in mind are: What does the data tell me? What is the quality and accuracy of my experiment or procedure? Does my result agree to other results or literature values? If yes or no what does this tell me about the experiment or model I used? Are there any potential systematic error sources in the experimental procedure? Which error sources are significant? What is the dominant error source and how could it be reduced?

- You should not just write down all possible error sources which come to your mind, but only the most likely error sources. Keep in mind that you always have to back-up your claims of error sources at least with numerical estimates.

- Avoid speculations except you have a valid basis of argumentation. Do not over-complicate matter (*e.g.* using quantum mechanics to explain mechanics or optics) and argue on an appropriate level.

- If the expected or literature value is not in the range of uncertainty of the your result or your data does not follow a predicted trend, this has to be stated and possible causes to be named. For example, if you expect that a plot of your data shows an intercept of zero but your experimental result differs from it, name the deviation and its possible causes.

- Any possible systematic errors that cannot be accounted for by error calculation should also be named and estimated. This may especially be important in case your result does not agree with a literature value.

- This is the place to mention observations relevant to the experiment but which have not been recorded numerically or systematically. This may be *e.g.* values changing in time, readings jumping when other equipment is switched on, background noise or light in the room, room temperature or humidity, ... *etc.*.

- Finally, given the main goal of the experiment, you should formulate some suggestions how to improve the experiment or increase the precision of your result in case

you or someone else will repeat it in the future (*e.g.* by modifying the set-up, using another procedure or evaluation method...).

Conclusion

This section of a report is the second most read part. It is not just a restatement of the abstract, instead it looks more into the future. Try to take an overview over the experiment, where you started, what you have reached and where further investigations may want to go. Base all conclusions on your actual results. Explain the implications of your results and put them in a broader context. The conclusions should be also short but a bit longer than the abstract.

The final result has to be restated including its uncertainty. If a physical constant has been measured, the result has to be compared with literature. Main errors and possible improvements of the experiment should be given by summarizing the discussion of the previous paragraph.

References and Appendices

In this section, you should give a list of references as well as all information that decreases the readability of your report if embedded in text in the previous sections.

- You should provide all references you used for preparing the report. This is either done by giving general references for the entire report or by indicating with a number in the text which source you used for a specific statement, equation, literature value or any thought or argument which is not your own. The references are then specified in a numbered list.

  A reference generally contains at least its author(s), title (of book or journal), position in book or journal (*e.g.* chapter or starting page number) and date of publishing. It may also contain the publisher, editor, issue of a journal, and total page numbers. The reader should be able to identify and access the source of information you used.

- In most cases, copies of your original recordings or raw data ("Labnotes") have to be added as an appendix. In case of a long report, the presentation of large tables and graphs or the lengthy derivation of an equation would disturb the readability of the report. These parts should then also be included as appendices.

As stated already, for some experiments it may be favorable to combine some sections in a single paragraph. For example, if intermediate results of different parts of an experiment have to be evaluated, it may be suitable to perform data analysis, error calculation and discussion in one paragraph for each part of the experiment.

Format and Language

Reports are judged not only on technical or scientific content but also on clarity, ease of understanding and on their format and layout. Please adhere to the following guidelines for the format of your physics lab report.

General instructions

- Reports are most effective if they are written in a language and style selected for the background of a specific audience or a principal reader. Try to write your report in such a way that a person familiar with physics but not familiar with this particular experiment would be able to follow what you did and why you did it.

- The report should be written as a coherent and continuous text with full sentences, NOT as a list of words and formulas or as separated statements. Results should be described in a sentence, NOT only presented as numerical values of a variable or as a number in a table. Refer to formulas, tables and figures in the text by their numbers or captions.

- Your guiding principles should be brevity, clarity and concision. Use the minimum number of words to make your point. The quality of a report is in no way related to its length. A typical number of 5 to 10 pages can serve as a guideline. Print on A4 paper and use font size of 10 - 12 with single or 1.5 line spacing.

- Sometimes repetition of information in different sections of the report, perhaps with a different emphasis or detail is necessary.

- The style of the report should be formal and impersonal (avoid "we" or "I"). It should be written in the past tense since you are reporting something you did in the past. Sometimes the present tense may be justified, *e.g.* in the theory section or discussion. Do not give instructions. You are also expected to write legibly, using correct grammar and accurate spelling.

- DO NOT copy from the lab manual, a textbook or the internet. The report should be written in your own words. Normally, the manual provides more information and instructions than you need for the report (especially for "introduction and theory" and "procedure"). Select only the relevant information for your report. In cases when it is inevitable that you have to use additional literature, information from the internet, or parts from the manual, you have to reference paragraphs, figures or formulas accordingly at the end of the lab report in a reference section. *Using the work of another person and presenting it as your own constitutes plagiarism and will be punished according to the code of academic integrity.*

- You can hand-in a hand written report, scanned to pdf, as well as a pdf created by a word-processor. However, the use of a word-processor is strongly encouraged and standard practice today in most organizations but do not waste time with formula editors, just leave a blank line and write the formula on the printout. A handwritten formula is still often more legible than one created with a poor formula editor. In case you do not know how to format a table or graph you might consider to plot it by hand.

- You may underline your layout structure *e.g.* by using boldface formatting for headings and/or italics for figure captions, but take care to format paragraphs, subparagraphs *etc.* consistently. Avoid any overuse of structuring by using too many different formats.

- Include page numbers and use appropriate margins.

- The report should be proof-read in its final layout before submission. As you edit it you may delete unnecessary words, rewrite unclear phrases and clean up grammatical errors. *Both students are equally responsible for the final submitted version of the Labnotes. The report will be written by one of the students.*

Title page

Each lab report should be identified by the title of the laboratory course (including course number), the title of the experiment, the author and coauthor with group number, the date the experiment was performed, and the date of submission.

Quantities and Equations

In general, scientific notation and SI units have to be used in the Physics Labs.

- The decimal fraction of a number is indicated by a decimal point not a comma, *e.g.* "3.1415" and NOT "3,1415".

- Exponents have to be written as superscript, *e.g.* ten to the power of four is given as 10^4 and NOT as 10E4, 10ˆ4 or other notations.

- Numerical values have to be separated by a blank from their units *e.g.* 10 mN not 10mN. For units use upright and not italic fonts.

- Vectors have to be identified as such and should clearly differ from scalars. The best way to identify a vector is by the arrow on top of a letter (you can also add these at the end by hand writing). In cases you are using bold letters to identify vectors, please define your choice of format.

- Equations should be embedded in the text and described by a sentence. All variables or constants have to be defined in the text close to the equation. If more than one equation is used they should be numbered sequentially throughout the text. In general, variables have to be defined only at their first appearance in the text, but it may increase clarity if they are restated when used in sections far apart.

- Equations have to be written with a proper formula editor or by hand. Using *e.g.* an asterisk " * " as a multiplication sign will not be accepted.

- As a general rule NEVER state errors with more than two significant digits. Adjust the significant digits of numerical values to their corresponding error values.

- Also in the running text, variables or constants have to be written using italic fonts (in formula environments, this usually is done automatically. On the other hand, functions are written using upright fonts. Examples are: F_{exp}, c_{vac}, $\sin(\alpha)$, *etc.*

Data tables

Tables of raw data can be included as an appendix. Other tables should be embedded in the text. In any case, you have to refer to the table in the text of the report.

- Captions
  Data tables must be identified by a unique number. This helps to refer to the table from within the text (*e.g.* "Table 3. Electric field and forces"). In addition, a short description of the table contents is mandatory. The table caption should be on top of the table.

- Headers
  Each column row in a table should have a heading that describes the physical quantity that is recorded in the column. The column heading should also show the units of the physical quantity and its appropriate order of magnitude (and the uncertainty in the quantity (if constant)). For example "F / mN" or "force / mN" or for the error "ΔF / mN" (without quotes).

- Data presentation
  Numerical values recorded in a table should be rounded to the appropriate number of significant digits. If the uncertainty is not constant for all values, add an additional column. Do not give unappropriate large number of digits.

Figures and Graphs

In general for layout purposes, every non-numerical or non-textual pieces of information such as graphs, pictures, schemes, *etc.* are called "figures". Any figure should be identified

by a unique number in its caption. The figure caption must include a short description of the details in the figure (*e.g.* "Figure 2. Graph of force versus distance. The force is plotted in dependence of the inverse square of the distance."). The figure caption should help the reader to roughly understand a figure even without reading the related text. Graphs should be prepared on graph paper or with data processing software according to the instructions in the manual. Bad layout due to insufficient (use of) software is not accepted. For clarity and to support the line of argumentation figures are normally inserted into the text. Hand-drawn figures can also be added as appendix. Again, take care to refer to them correctly in the text.

- Caption
  Every figure should be identified by a unique number in its caption, which should also include some short descriptive sentence(s) telling exactly what is plotted at the bottom or after figure.

- Axes and Axes Labels
  Both axes (bottom and left) should be labelled with the physical quantity that is plotted on the axis and the units of the physical quantity in brackets or with a slash, *e.g.* "F [mN]" or "F / mN" (without quotes). For clarity numbers on axis should have a small number of digits.

- Size and Clarity
  All graphs should be produced at a size that is sufficiently large so that the information can be easily read. Choose axis limits so that the region of interest occupies most of the graph area.

- Data Display
  Data points in hand-drawn plots have to be indicated by crosses. Data values should never be connected with a line. Where applicable you should include a fit of a curve that represents the physical relation between the displayed quantities.

- Annotation
  Whenever results of best fit or smoothing procedures to the experimental data are displayed in the graph, the fit/smooth method must be clearly mentioned in its caption. In the case of a linear (straight line) fit, present the values of the slope and the y-intercept as well as their errors within the text (*i.e.* NOT in the graph itself). Be sure to also include the associated units. If possible, describe the physical significance of the slope and intercept.

- Uncertainty Bars
  Whenever the uncertainties of individual data values are known they should be indicated in the graph using error bars.

Final Remarks

The most frequent complain from students about report writing is the excessive amount of time needed for preparing a report. This may be true, but in fact, students who are able to report on their work in a clear, organized and conclusive report receive better grades (not only in physics) than those who can not. Report writing can be indeed time-intensive, but the time is well spent, because report writing is a general skill needed in most professions and it will be most valuable in your future career. The more experience you get the less time you will need in the future to prepare a professional scientific report. The time needed for writing a report can be optimized by proper planning. A good preparation of the experiment and knowing what you will do and what the physics is behind the experiment will facilitate the recording and evaluation of the data. The clarity of writing depends on the clarity of thinking. Word processors and spread sheet software will increase efficient writing and data analysis. Adhering to the given guidelines will also help to save time. Often it is helpful to first make a rough draft of the report and a layout of its structure and content. Thereby, a first selection of important features of the experiment, theory and data should be made. Then after writing the text you should reconsider, edit and polish the report into a final version.

The content of these guidelines has evolved over time and will be used to grade lab reports for undergraduate physics majors at Jacobs University. They also contain information and instructions extracted from standard textbooks, instructions for authors from scientific journals and from other lab report guidelines published on the internet (especially from: www.iit.edu, physics.usask.ca, physics.wku.edu, physics.mq.edu.au). Any ideas and comments for further improvement are welcome.

Two-Electron Spectra

Key Concepts

Diffraction image of a diffraction grating, visible spectral range, single electron atom, Bohr's atomic model, Lyman- Paschen- Brackett- and Pfund series, atomic energy levels, Planck's constant, binding energy, Rydberg's constant, Parahelium, Orthohelium, exchange energy, spin, angular momentum, spin-orbit interaction, singlet series, triplet series, selection rules, forbidden transitions, excitation energy

Summary

The main aim of this experiment is to show the quantization of energy levels in atoms by means of measuring spectral lines.

The spectral lines of hydrogen (H) and mercury (Hg) are examined by means of a diffraction grating. The known spectral lines of Hg are used to determine the grating constant. The wave lengths of the visible lines of the Balmer series of H are measured.

In the second part of the experiment, the spectral lines of two-electron (2e) atoms (here mercury (Hg), cadmium (Cd) and zinc (Zn)) are determined. Important for these spectra is the interaction between the two valence electrons. The prism spectrometer, which is used in this latter part, is to be calibrated with the aid of helium (He).

Experimental Tasks

Get familiar with the goniometer first

- Understand the adjustments of the goniometer for the optical path, avoiding parallax errors.

- Understand the adjustments of the goniometer disc for a reflective grating.

- Use the Na lamp for training in reflection mode, find the Na lines first with your bare eyes.

- Identify the Na lines with the telescope and practice the angle readings (telescope-lock and micrometer screw fine-tuning).

Instrument Calibration and Experimental Identification of spectral lines

- Use the Na lamp for adjustment of the goniometer disc for an incident angle of 60° with respect to the grating normal of the reflective grating.

- Calculate the wavelength from the obtained experimental angle.

- Determine the grating constants (G = groove distance in nm; g = number of grooves per mm) for this grating.

- Calibrate the spectrometer with the He lamp and plot a calibration curve ($\sin(\theta_m)_{exp}$ over λ_{lit}) using the literature values for He lines from below.

- With your calibration at hand determine the wavelength of the most prominent spectral lines for Hg and Zn.

Spectral lines (λ_{lit}) of Helium (subset)

Red	667.82 nm
Yellow	587.56 nm
Green	501.57 nm
Blue-green	492.19 nm
Blue	447.15 nm

Theory

The Bohr model for the hydrogen atom was among the earliest successes of quantum mechanics. It describes the most important properties of the hydrogen atom, especially the so-called Balmer series, quite accurately. The Bohr model assumes that the electrons of an atom rotate around the nucleus on discrete orbits on which the centrifugal force due to the rotation and the Coulomb force exactly balance each other. Postulating quantization according to the wave nature of the electron in quantum mechanics (an integer number of wavelengths must fit on a closed path), the energies of these orbits become discrete values and are indexed by an integer n as follows:

$$E_n = -\frac{1}{8}\frac{e^4 m_e}{\epsilon_0^2 h^2}\frac{1}{n^2} \quad , \quad n = 1, 2, 3.... \quad . \tag{P-2-1}$$

$\epsilon_0 = 8.8542 \cdot 10^{-12}\,\frac{\text{A s}}{\text{V m}}$ is the electric field constant, $e = 1.6021 \cdot 10^{-19}\,\text{C}$ the electric charge of the electron and $m_e = 9.1091 \cdot 10^{-31}\,\text{kg}$ the rest mass of the electron.

The H_2 molecules dissociate to H atoms and / or ions by means of the collision ionization in the spectral tube. Furthermore, the electron of the single H atom will excite to higher energy levels through collision with free electrons. When the excited electron of the H atom takes a transition from an energy level E_m to an energy level E_n with $m > n$, then a photon is emitted which has an energy $E = h\nu$ corresponding to the electron's change in energy. Thus, using (P-2-1), we have:

$$h\nu = \frac{1}{8}\frac{e^4 m_e}{\epsilon_0^2 h^2}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \tag{P-2-2}$$

With $\nu = \frac{c}{\lambda}$, we have:

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \tag{P-2-3}$$

where $R = \frac{1}{8}\frac{e^4 m_e}{\epsilon_0^2 h^3 c} = 1.097 \cdot 10^7 \, \mathrm{m}^{-1}$ is Rydberg's constant, $c = 2.9979 \cdot 10^8 \, \mathrm{m/s}$ is the speed of light and $h = 6.6256 \cdot 10^{-34}$ Js denotes Planck's constant.

For $n < m$ the expression is dominated by the term $1/n^2$. The index n now describes a specific spectral series, while m is changing in this series (see Fig. P-2-1). $n = 1$ is the so-called Lyman series and the spectral range is ultraviolet (UV). The $n = 2$ series is called Balmer series and the spectral range is from UV to red, while $n = 3$ is the Paschen series, $n = 4$ the Bracket series and $n = 5$ the Pfund series. These latter three all have spectral range in infrared. For the Balmer series, the spectral lines with $m = 3$, $m = 4$, $m = 5$, $m = 6$ and $m = 7$ are called the H_α, H_β, H_γ, H_δ and H_ε lines, respectively. As can also be seen from Fig. P-2-1, the energy associated with $m \to \infty$ is the ionization energy (or the binding energy) for an electron in the nth permitted state. The binding energy can be calculated by means of the following equation:

$$E_n = -R \cdot h \cdot c \frac{1}{n^2} \ . \tag{P-2-4}$$
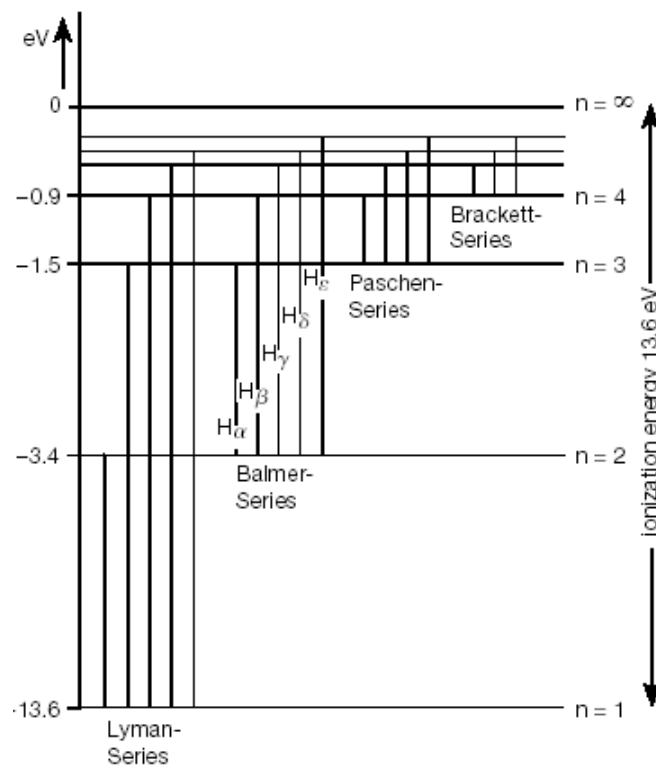
For $n = 1$, we get $E_1 = -13.6 \, \mathrm{eV}$.

Figure P-2-1: Energy levels and spectral series in hydrogen atom [2].

For **atoms which have two valence electrons** like helium (He), mercury (Hg), cadmium (Cd) or zinc (Zn), the total energy of the valence electrons reads:

$$E_{tot} = \frac{\vec{p}_1^2}{2m_e} + \frac{\vec{p}_2^2}{2m_e} - \frac{2e^2}{4\pi\epsilon_0|\vec{r}_1|} - \frac{2e^2}{4\pi\epsilon_0|\vec{r}_2|} + \frac{e^2}{4\pi\epsilon_0|\vec{r}_1 - \vec{r}_2|} \tag{P-2-5}$$

The indices 1 and 2 label the first or second electron, respectively. $\vec{r}_i$ is the position and $\vec{p}_i$ is the momentum of the i-th electron, respectively. The term $E_{int} = \frac{e^2}{4\pi\epsilon_0|\vec{r}_1 - \vec{r}_2|}$ describes the interaction between the two electrons. For $E_{int} = 0$, the energy is simply the sum of two one-electron energies and the discrete energy levels correspond to the sum of energies of single electrons in an atom (see (P-2-1)):

$$E_{n,m}^0 = -\frac{1}{8}\frac{e^4 m_e}{\epsilon_0^2 h^2}\left(\frac{1}{n^2} + \frac{1}{m^2}\right) \quad , \quad n, m = 1, 2, 3... \tag{P-2-6}$$

The eigenfunction $\psi(\vec{r}_1, \vec{r}_2)$ of the Hamiltonian without interaction then simply is a product of the eigenfunctions of the two single-electrons:

$$\psi(\vec{r}_1, \vec{r}_2) = \psi_{n_1,l_1,m_{l_1}}(\vec{r}_1)\psi_{n_2,l_2,m_{l_2}}(\vec{r}_2) \ . \tag{P-2-7}$$

$l_i, i = 1, 2$ is the angular momentum of the electron and $m_{l_i} = -l_i, -l_i+1, ..., l_i-1, l_i$ is the z-component of l_i. If we now consider the influence of the electron-electron interaction, the first order correction $E^{(1)}$ to the energy is:

$$E^{(1)} = \frac{e^2}{4\pi\epsilon_0}\int |\psi_{n_1,l_1,m_{l_1}}(\vec{r}_1)|^2\left(\frac{1}{|\vec{r}_1 - \vec{r}_2|}\right)|\psi_{n_2,l_2,m_{l_2}}(\vec{r}_2)|^2 d\vec{r}_1 d\vec{r}_2 \ . \tag{P-2-8}$$

The term on the right hand side of this equation is the so-called "Coulomb integral". Numerically it evaluates to $E^{(1)} \approx 5.45 \cdot 10^{-18}\,\text{J} \equiv 34\,\text{eV}$.

If we now consider the excited states of atoms which are **two electron systems**, we note -in the case of vanishing interactions between the electrons- that when two electrons occupy different orbitals, the wave functions are either $\psi_{n_1,l_1,m_{l_1}}(\vec{r}_1)\psi_{n_2,l_2,m_{l_2}}(\vec{r}_2)$ or $\psi_{n_2,l_2,m_{l_2}}(\vec{r}_1)\psi_{n_1,l_1,m_{l_1}}(\vec{r}_2)$. These two states then have the same energy, i.e. they are degenerated and the total energy E^0 is the sum of the two one-electron energies $E_{(1)}^0$, $E_{(2)}^0$: $E^0 = E_{(1)}^0 + E_{(2)}^0$. If we now take into account the interaction energy, we find

$$E_\pm = E_{(1)}^0 + E_{(2)}^0 + E^{(1)} \pm E^{(ex)} \tag{P-2-9}$$

where $E^{(ex)}$ is the exchange integral with

$$E^{(ex)} = \frac{e^2}{4\pi\epsilon_0}\int \psi_{n_1,l_1,m_{l_1}}(\vec{r}_1)\psi_{n_2,l_2,m_{l_2}}(\vec{r}_2)\left(\frac{1}{|\vec{r}_1 - \vec{r}_2|}\right)\psi_{n_2,l_2,m_{l_2}}(\vec{r}_1)\psi_{n_1,l_1,m_{l_1}}(\vec{r}_2)d\vec{r}_1 d\vec{r}_2 \ . \tag{P-2-10}$$

Clearly, the degeneracy is lifted by the electron repulsion with $E_+ - E_- = 2E^{(ex)}$. Note that the state ψ_+ is symmetric under exchange of particles, while ψ_- is antisymmetric. Since each electron has spin $s = 1/2$, we expect the total spin S to be $S = 0$ ("para helium") or $S = 1$ ("ortho helium"). Thus the total momentum is $J = S + L$, where L is the total angular momentum. $S = 0$ corresponds to a singlet with $J = L$, while $S = 1$ corresponds to a triplet with $J = L + 1, L, L - 1$. E.g. for a $1s^1 np^1$ state (which is a possible excited state of helium) we would expect $^1P_1$, $^3P_2$, $^3P_1$ and $^3P_0$ states. The notation is such that the big letter denotes the total angular momentum L (with S corresponding to $L = 0$, P to $L = 1$ etc.), the upper index shows whether it is a singlet or triplet state and the lower index is the total momentum $J = S + L$. Thus, for singlets, we would always have terms of the form $^1S_0$, $^1P_1$, $^1D_2$ etc.

It now turns out for helium, that all singlet states are symmetric, while all triplet states are antisymmetric. Considering the selection rules govering transitions between states of the form ψ_+, ψ_-, we find that only transitions between symmetric (resp. antisymmetric) states are allowed. Transitions of the form symmetric $\leftrightarrow$ antisymmetric are forbidden. This is due to the vanishing of the transition dipole moment for states with different symmetry. The dipole moment is given by:

$$\mu_\pm = -e \int \psi_+(\vec{r}_1, \vec{r}_2)(\vec{r}_1 + \vec{r}_2)\psi_-(\vec{r}_1, \vec{r}_2)d\vec{r}_1 d\vec{r}_2 . \tag{P-2-11}$$

Since electrons are identical particles, i.e. $\vec{r}_1$ and $\vec{r}_2$ can be exchanged without changing the physics, we find that $\mu_\pm = -\mu_\pm \rightarrow \mu_\pm = 0$. Thus, for helium, we have the selection rule

$$\Delta S = 0 . \tag{P-2-12}$$

Independent of the spin-angular momentum interaction, we have

$$\Delta J = 0, \pm 1 \tag{P-2-13}$$

and if the spin-angular momentum interaction is small (like in helium), we have in addition

$$\Delta L = 0, \pm 1. \tag{P-2-14}$$

Detailed calculations lead to the He spectrum shown in Fig. P-2-2 with detailed values given in Table P-2-1.

Table P-2-1: Spectrum of helium I

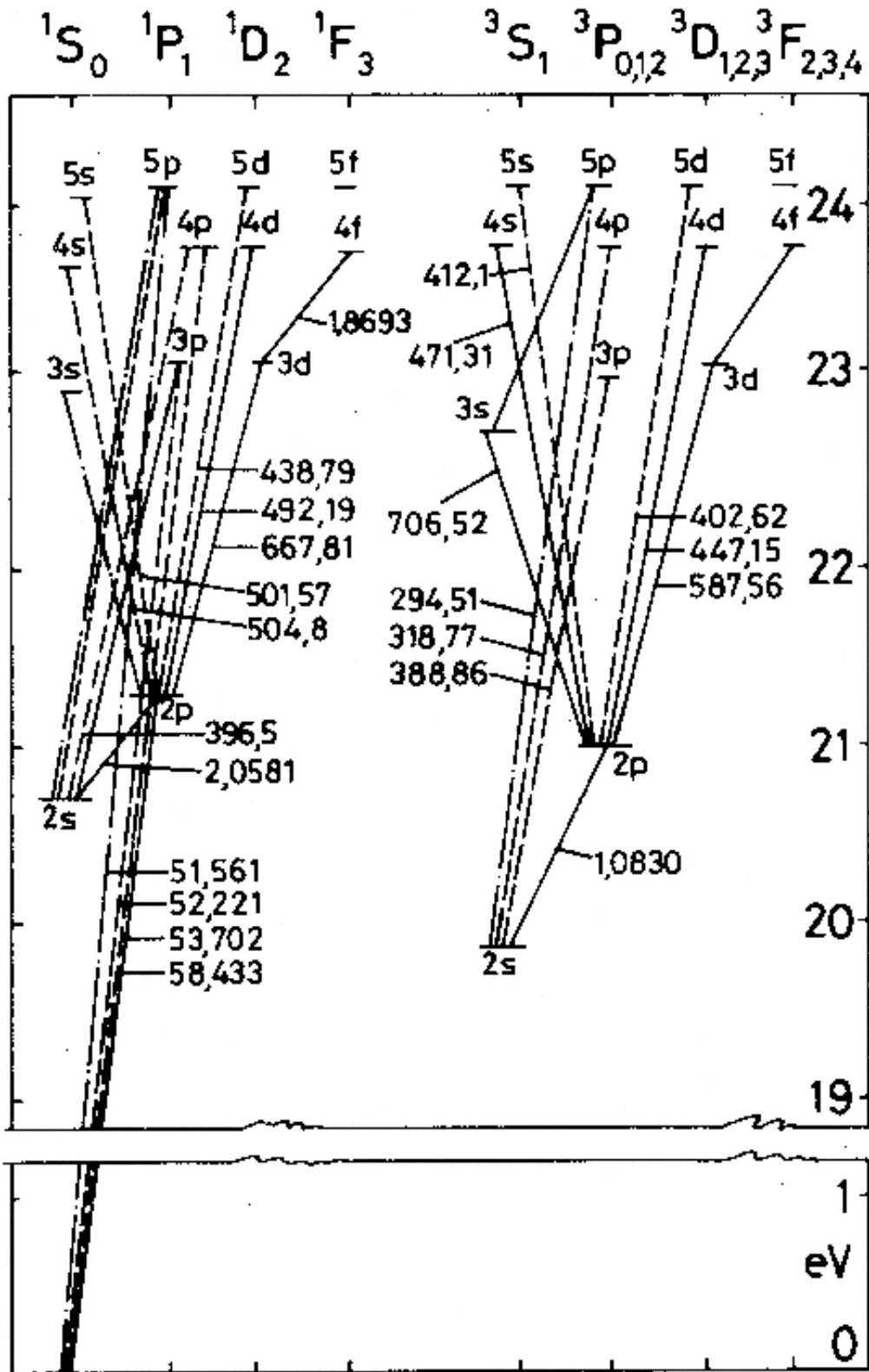| Color	λ/nm	Transition	Relative intensity
red	706.5	$3\ ^3S \rightarrow 2\ ^1P$	5
red	667.8	$3\ ^1D \rightarrow 2\ ^1P$	6
red	656.0	He II	4-6
yellow	587.6	$3\ ^3D \rightarrow 2\ ^3P$	10
green	504.8	$4\ ^1S \rightarrow 2\ ^1P$	2
green	501.6	$5\ ^1S \rightarrow 2\ ^1P$	7
green	492.2	$4\ ^1D \rightarrow 2\ ^1P$	4
blue	471.3	$4\ ^3S \rightarrow 2\ ^3P$	3
blue	447.1	$4\ ^3D \rightarrow 2\ ^3P$	6
blue	438.8	$5\ ^1D \rightarrow 2\ ^1P$	3
violet	414.4	$6\ ^1D \rightarrow 2\ ^1P$	2
violet	412.1	$5\ ^3S \rightarrow 2\ ^3P$	3
violet	402.6	$5\ ^3D \rightarrow 2\ ^3P$	5
violet	396.5	$4\ ^1P \rightarrow 2\ ^1S$	4
violet	388.9	$3\ ^3P \rightarrow 2\ ^3S$	10

Figure P-2-2: Energy levels and spectral series in helium atom [2]

He, Hg, Cd and Zn are also two-electron systems. However, in comparison to helium, the spin-angular momentum interaction is quite pronounced such that only the total momentum J is a conserved quantity. This is why the selection rule $\Delta S = 0$ does not hold any longer here. This can be clearly seen from Figs P-2-3 and P-2-4.

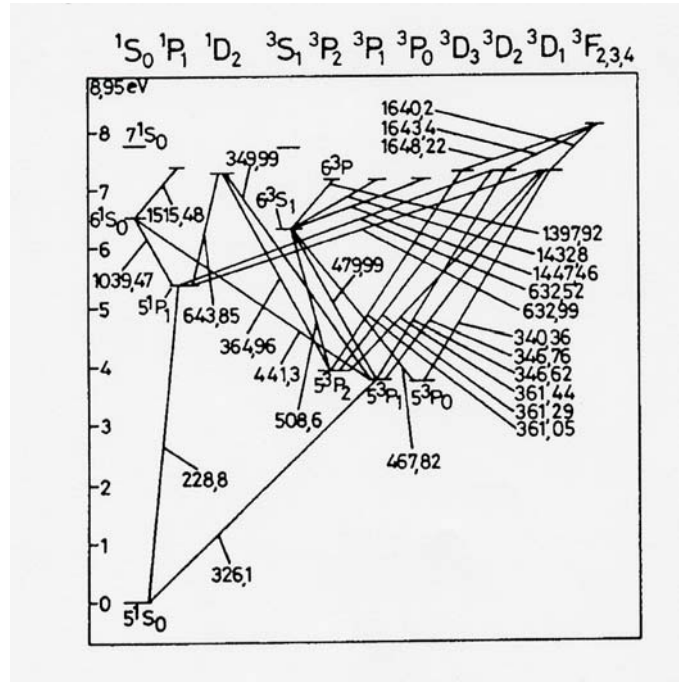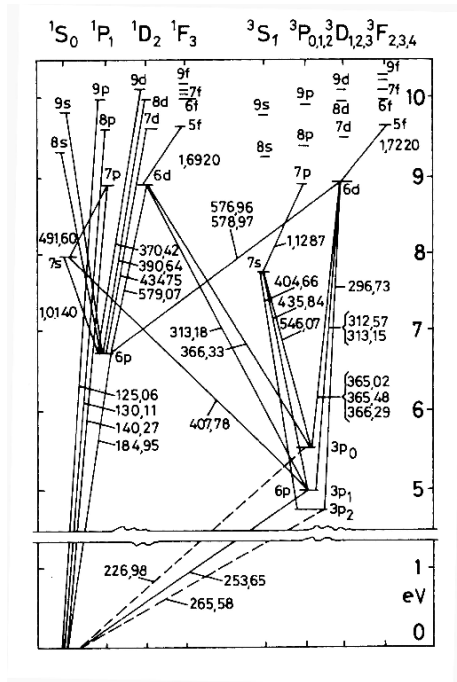Figure P-2-3: Energy levels and spectral series in mercury (Hg) atom [2]

Figure P-2-4: Energy levels and spectral series in cadmium atom [2]

Note:

The treatment of more complex atoms requires quantum mechanics. In this case, the energies of the states are determined by the eigenvalues of the hamiltonian of the atom. For atoms similar to hydrogen, calculations yield the same results as Bohr's atomic model.

Diffraction Grating (Reflection Grating)

Besides transmission gratings where either the phase or transmissivity of a transparent substrate is modulated also a periodic variation of the reflectivity can be used to obtain diffraction patterns. Most monochromators used in the NIR/VIS/UV spectral range are based on the light dispersion facilitated by reflection gratings. In the following, a short overview of the basics of reflection gratings is given.

Figure P-2-5 shows a small part of the grating surface. Only the surfaces containing points A and B are reflecting light while the grooves do not contribute. The distance between the reflecting surfaces is d. Let us consider a plain wave entering the grating from left under an angle α relative to the surface normal. The observer on the right side sees the light "reflected" (why did I use quotation marks here?) by the grating under an angle θ_m again measured relative to the grating normal. Comparing the path length of the two light rays one realizes that they are different as soon as $\alpha \neq \theta_m$ (for $\alpha = \theta_m$, it would make sense to leave away the quotation marks just mentioned above – why?). The path lengths difference Δr can be expressed by the following equation:

$$\Delta r = \overline{AD} - \overline{BC} = d \sin \theta_m - d \sin \alpha \,. \tag{P-2-15}$$
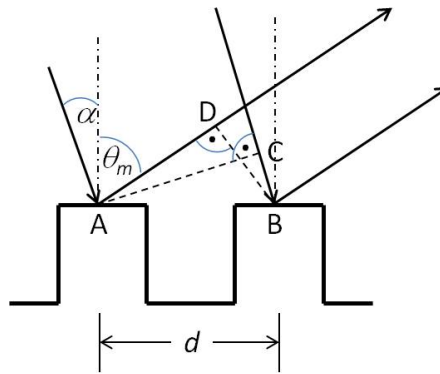
Figure P-2-5: Schematic drawing of a grating. Two incident and outgoing rays are shown. The path lengths difference determines the interference behavior.

Due to this path lengths difference interferences will occur, which are determined by the phase difference $\Delta \phi = \Delta r \, k$, where k is the angular wave number, *i.e.* the magnitude of the wave vector. Its relation to the wavelength ($k = 2\pi/\lambda$) explains the dispersive behavior of the grating.

Constructive interference can be found if the following condition is fulfilled:

$$d(\sin \theta_m - \sin \alpha) = m\lambda \,, \tag{P-2-16}$$

where m is the order of the interference maximum. The dependence on the wavelength vanishes for zero-order diffraction ($\alpha = \theta_m = \theta_0$).

Now, also an answer to the questions above can be given. The condition $\alpha = \theta_0$ is nothing else than the law of reflection. Only here, it makes sense to talk about the reflection of the light. The light observed under angles θ_m for $m \neq 0$ arises from light diffraction. For the grating given in Fig. P-2-5 the intensities of the higher order diffraction maxima will decrease with rising (absolute) order, since the deviation from the reflection law increases. While for transmission gratings this behavior cannot be influenced, the reflection grating offers a possibility to enhance the intensity also of higher diffraction orders. The grating shown in Fig. P-2-6 (taken from Wikipedia) is called "blazed grating". Here, the grating normal and the normal of the reflecting surfaces are not equal. The surface normal is tilted by an angle θ_B, the so-called blaze angle. Again, we assume that the incident light falls onto the grating under an angle α relative to the grating normal. Let us assume that a certain diffraction order ($m = 1, 2, \ldots$) can be occurs under the angle β, again measured relative to the grating normal. It becomes obvious that by adjusting the blaze angle the law of reflection can be fulfilled for the reflecting surfaces. Such, certain orders (for a certain wavelength range – why?) can be favored, *i.e.* their intensities can be enhanced. A special arrangement is indicated in the drawing. In the so-called "Littrow configuration" the angle of incidence α is equal to the angle of diffraction β, which is achieve by choosing $\alpha = \theta_B = \beta$. This arrangement is e.g. used to substitute end mirrors in tunable lasers. By changing the tilt of the grating different wavelengths will be aligned along the resonator axis of the laser.
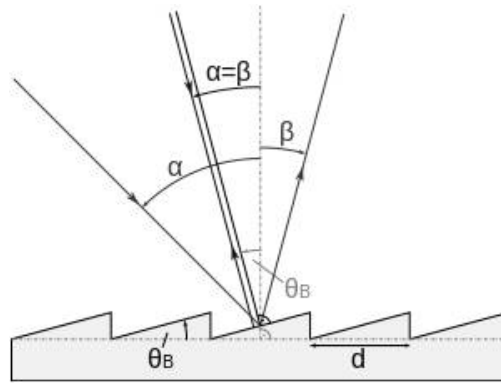
Figure P-2-6: Schematic drawing of a blazed grating. The blaze angle θ_B can be chosen such that the law of reflection is approximately fulfilled for a certain order of diffraction within a certain wavelength range. In the so-called Littrow configuration with $\alpha = \theta_B = \beta$ a retro-reflective grating design is achieved, which can be used as wavelength-selective mirror.

Using a reflection grating for light dispersion allows for the separation of different wave-

length. In order to characterize the efficiency of the grating, a spectral resolution is introduced. The "angle dispersion" describes the dependence of the angle θ_m on the wavelength λ for a fixed angle of incidence α. The bigger this value $d\theta_m/d\lambda$, the higher the spectral resolution, which can be achieved with the grating. The angle dispersion can be derived from equ. P-2-16 by differentiation with respect to λ:

$$\frac{d}{d\lambda}\left[d(\sin\theta_m - \sin\alpha)\right] = \frac{d}{d\lambda}(m\lambda)\,. \tag{P-2-17}$$

From this, the angle resolution can be easily derived:

$$\frac{d\theta_m}{d\lambda} = \frac{m}{d\cos\theta_m}\,. \tag{P-2-18}$$

Obviously, the resolution is increased when higher orders m are used (what difficulties limit this improvement of $d\theta_m/d\lambda$?). Also a decrease of d will improve the resolution. However, there is another important factor, which will be explained in more detail in the optics lectures. Calculating the intensity distribution (maxima, minima) resulting from the diffraction from the grating, one immediately finds that of course each principal maximum (m^{th} order diffraction) has a certain width for each wavelength value. It can be shown that these maxima become narrower with increasing number N of contributing reflecting surface elements. Since even for a very good angle dispersion, two wavelengths cannot be resolved if the corresponding maxima are not narrow enough, the effective number N has to be kept as high as possible. In consequence, in a monochromator lenses (or curved mirrors) are always used such that gratings are fully illuminated. The "chromatic resolving power" of a grating is given by

$$\frac{\lambda}{\Delta\lambda} = mN\,, \tag{P-2-19}$$

where $\Delta\lambda$ is the smallest resolvable wavelength difference based on the Rayleigh criterion (what does this mean?).

Experimental method

If light of wavelength λ impinges on a grating with constant d, it is diffracted. Intensity peaks occur when the angle of diffraction α fulfills the following condition (see reference 1):

$$d(\sin\theta_m - \sin\alpha) = m\lambda, \quad m = 0, 1, 2, \ldots \tag{P-2-20}$$

Gratings are normally characterized by the density of lines $(1/d)$ instead of the grating constant d.

Identification of spectral lines and corresponding energy-level transition

The angle of diffraction is directly observed using a spectrum goniometer. With help of the vernier scale the angle can be measured with a resolution of half an arc minute. The emission spectra of helium, mercury, cadmium and zinc are analyzed and corresponding transitions determined.

Equipment

Part	Phywe No.	Quantity
Spectrometer/goniometer w. vernier	35635.02	1
Spectral lamp He, pico 9 base	08120.03	1
Spectral lamp Hg 100, pico 9 base	08120.14	1
Spectral lamp Cd, pico 9 base	08120.01	1
Spectral lamp Zn, pico 9 base	08120.11	1
Spectral lamp Na, pico 9 base	08120.07	
Power supply for spectral lamps	13662.93	1
Lamp holder, pico 9, f. spectral lamps	08119.00	2
Tripod base -PASS-	02002.55	1
Diffraction grating in plastic frame and holder		
Reflective grating of unknown gating constant		

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Set-up and Procedure Notes

Carry out the experiment according to the "Experimental Tasks" listed in the beginning of the description of this experiment.

Identification of spectral lines and corresponding energy-level transition

The experimental set-up is similar to the one shown in Fig. P-2-7 (replace the displayed prism with gratings)

The spectrum goniometer and the grating must be set up and adjusted in accordance with the operating instructions available in the lab rooms

The spectral lamps attain their maximum light intensity after a warm-up of approx. 5 minutes, the lamp housing should be set up so as to ensure free circulation of air through the ventilator slit

Before changing the spectral lamps they must be allowed to cool (!)

Use a pocket lamp (mobile phone display) to illuminate scale when recording the angle

Figure P-2-7: Experimental goniometer set-up 2]

Further reading

1. P. W. Atkins and R. S. Friedman, **Molecular Quantum Mechanics**, Oxford University Press, 2001

2. L. v. Alvensleben, **Laboratory Experiments Physics**, Phywe GmbH, 2004, Manuals 5.1.06, 5.1.07, 5.1.08 and 5.1.09

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Particle-Wave Duality

Key Concepts

Bragg reflection, Debye-Scherrer method, lattice planes, graphite structure, material waves, de Broglie equation, Compton effect, Compton wavelength, rest energy, absorption, transmission, conservation of energy and momentum, X-rays, Bragg equation

Summary

The main aim of this experiment is to demonstrate the particle-wave duality. This is done in two steps: 1. by showing the wave character of electrons by electron diffraction and 2. by showing the particle character of electromagnetic wave by the Compton effect. In 1. fast electrons are diffracted from a polycrystalline layer of graphite. With the help of a fluorescent screen, interference rings can be made visible. The interplanar spacing in graphite is determined from the diameter of the rings and the accelerating voltage. In 2. a X-ray beam is directed against a piece of plastic. With the help of a counter tube, the intensity of the X-rays at various angles is measured. The measurements are to be made with an absorber positioned in front of and behind the scatterer, respectively. By this way the Compton wavelength can be determined from the varying intensity attenuation of the X-rays at different wavelengths by using a pre-measured transmission curve.

Experimental Tasks

Electron diffraction

1. Measure the diameter of the two smallest diffraction rings at different anode voltages

2. Determine the electron wavelength taking into account the applied anode voltages

3. Determine the interplanar spacing of graphite from the relationship between the radii of the diffraction rings and the wavelength

Compton scattering
(Requires software MEASURE)

1. **Cu Bremsstrahlung**
   Determine the full emission spectrum of the copper tube. For this, scatter the radiation on the LiF crystal and use the 3 - 55 degree range of deflection angle for the goniometer, Gate time 2 seconds, Step 0.1 degree, Diaphragma 2 mm (program Spectra). Assign the wavelength and the transitions and their energies for the four most dominant signals, compare to literature

2. N_1 **and** N_2
   Determine the transmission T of an aluminum absorber as a function of the wavelength λ of the X-rays by means of Bragg scattering at the LiF crystal using the

Range of 5.5 - 11.5 degree, Gate time 60 seconds, Step 0.3 degree, Diaphragma 2 mm (program Spectra). Plot T vs λ and find a suitable trendline (linear?) to your data points.

Remark:
Using the L-shape Al absorber, put it very close to the crystal!

3. **N_3, N_4 and N_5**

Measure the pulse rates N_3, N_4 and N_5 for the Compton scattering, using a fixed angle of 135° for the Acrylic glass block, Detector angle 90°, Gate time 300 seconds, Diaphragma tube 5 mm (program compton). Only at high (!) pulse rates the deadtime τ of the counter tube needs to be considered!

Determine the transmission coefficients:

$$T_1 = \frac{N_4^*}{N_3^*} \qquad , \qquad T_2 = \frac{N_5^*}{N_3^*}$$

With the help of the T-λ-curve, find the corresponding wavelengths λ_1 and λ_2 for the different counter tube positions. Determine the change of wavelength $\Delta\lambda = \lambda_c = \lambda_2 - \lambda_1$ from this and discuss your findings!

Theory

One of the key concepts of quantum physics is the so-called "particle-wave" duality. This means that -depending on the specific situation- particles (e.g. electrons) can behave like waves and waves (e.g. electromagnetic waves) can behave like particles.

In his theory of matter waves, de Broglie argued that a particle with non-zero rest mass m and momentum p should be attributed a wavelength λ (de Broglie equation):

$$\lambda = \frac{h}{p} \tag{P-3-1}$$

where $h = 6.625 \cdot 10^{-34}$ Js denotes Planck's constant. According to the classical relation $p = mv$ the particle's velocity is given by $v = p/m$. If we accelerate an electron (charge $e = 1.602 \cdot 10^{-19}$ As, rest mass $m_e = 9.109 \cdot 10^{-31}$ kg) by an electric potential difference U_A, the final kinetic energy of the electron corresponds to:

$$\frac{1}{2}m_e v^2 = \frac{p^2}{2m_e} = eU_A \,. \tag{P-3-2}$$

Using (P-3-1), we then have for the wavelength of the electron:

$$\lambda = \frac{h}{\sqrt{2m_e e U_A}} \tag{P-3-3}$$

For the acceleration voltages used we can neglect the error of only 0.5 % which results from the fact, that we have used the rest mass m_e in these equations instead of the relativistic mass. The error is small if the resulting velocity of the electrons is much smaller than the speed of light.

The wave nature of electrons can be observed if a beam of electrons strikes crystals. If the angle between the incoming beam and the lattice plane of the crystal is θ (the so-called Bragg angle), the electron beam is diffracted in dependence on the distance d between the lattice planes. Strong reflection occurs if all the reflected beams originating from each individual lattice plane interfere constructively. The required relation between Bragg angle, wavelength λ of diffracted wave and d is given by the Bragg condition :

$$2d \sin \theta = n \cdot \lambda \,, \; n = 1, 2, \ldots \,. \tag{P-3-4}$$

If a polycrystalline film is used, the orientation of the different crystallites in the film is random. For a given Bragg condition the electron beam is not reflected in a single direction, but is spread out in form of a cone. The angle of the cone with respect to the transmitted beam is twice the Bragg angle.

The Compton effect describes the particle nature of waves. When electromagnetic waves (here X-rays) interact with matter, they can behave as particles, so-called photons. When an incoming photon with momentum $\vec{p}_1$ and frequency ν_1 (see Fig.P-3-1) hits a free target electron (mass m_e) at rest, the photon is scattered under angle α, while the electron is emitted under angle β from the collision point. Due to the conservation of momentum we have:

$$\vec{p}_1 = \vec{p}_2 + \vec{p}_e \tag{P-3-5}$$

with $\vec{p}_e = m_e \vec{v}$, which gives us

$$\vec{p}_e^{\,2} = (\vec{p}_1 - \vec{p}_2)^2 = p_1^2 + p_2^2 - 2p_1 p_2 \cos \alpha \,. \tag{P-3-6}$$

With $p_1 = \frac{h\nu_1}{c}$, $p_2 = \frac{h\nu_2}{c}$, we find

$$p_e^2 = \frac{h^2}{c^2} \left(\nu_1^2 + \nu_2^2 - 2\nu_1 \nu_2 \cos \alpha \right) \,. \tag{P-3-7}$$

The conservation of energy tells us that:

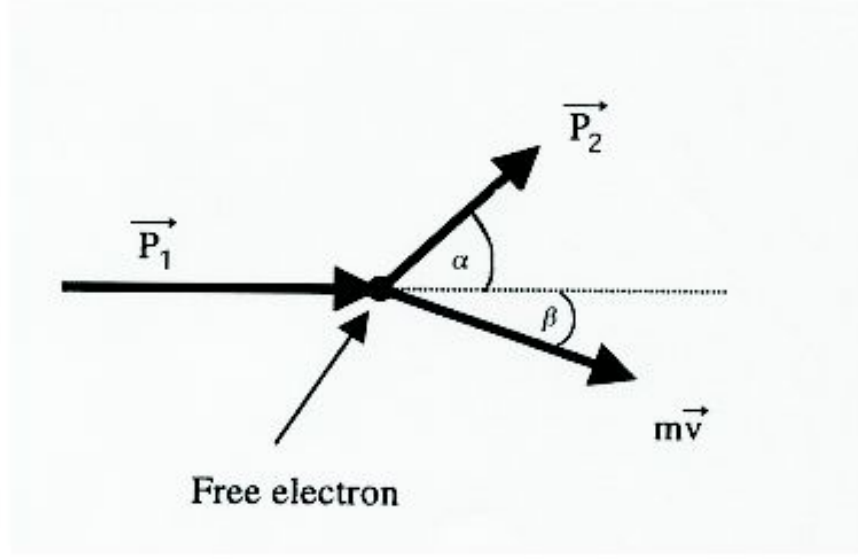$$E_{\nu_1} + E_{e,1} = E_{\nu_2} + E_{e,2} \tag{P-3-8}$$

Figure P-3-1: Momentum relationship in Compton scattering

where $E_{\nu_1} = h\nu_1$ is the energy of the photon before the collision and $E_{\nu_2} = h\nu_2$ is the energy after the collision. $E_{e,1} = m_e c^2$ is the energy of the electron at rest before the collision, $E_{e,2} = \sqrt{m_e^2 c^4 + p_e^2 c^2}$ is the energy of the electron after the collision. Using the formula for the conservation of energy, we thus find :

$$p_e^2 = \frac{(h\nu_1 - h\nu_2 + m_e c^2)^2 - m_e^2 c^4}{c^2} \ . \tag{P-3-9}$$

Equating with (P-3-7), we find:

$$h\nu_1\nu_2(1 - \cos\alpha) = m_e c^2 (\nu_1 - \nu_2) \tag{P-3-10}$$

which by using $\nu = \frac{c}{\lambda}$ becomes

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{h}{m_e c}(1 - \cos\alpha) \ . \tag{P-3-11}$$

where $\lambda_c = \frac{h}{m_e c} = 2.426\,\mathrm{pm}$ is known as the Compton wavelength of the electron and describes the change of wavelength for $\alpha = \frac{\pi}{2}$. It is obvious from this equation, that due to the collision of the photon with the electron, the photon looses energy and thus its wavelength λ increases. For small scattering angles the electron momentum transfer is quite small. Thus the approximation to consider the electrons in a solid as free electrons becomes worse and the experimental values don't agree well with the theoretical prediction, nevertheless, they still show that the difference in wavelength $\Delta\lambda$ decreases with decreasing scattering angle.

Experimental Method

Electron diffraction

In the given experiment, electrons are accelerated in a tube sketched in Fig. P-3-2: After being accelerated, the electron beam strikes a polycristalline graphite film which is
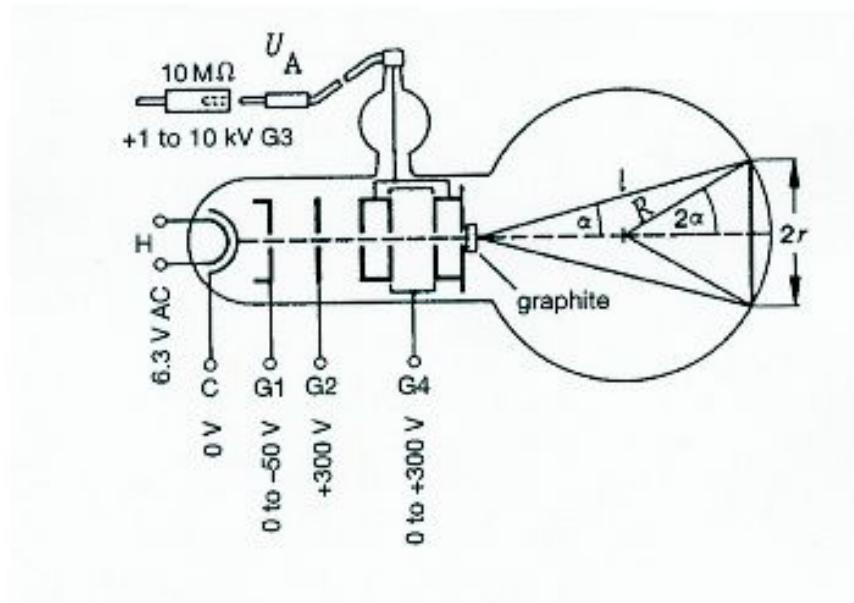

Figure P-3-2: Set-up and power supply to the electron diffraction tube

deposited on a copper grating. Polycristalline graphite consists of layers of carbon atoms which have a certain spacing between them (see Fig.P-3-3). The resulting cone of the electron beam produces interference rings on the fluorescent screen. The Bragg angle θ can then be determined from measuring the radius r of the interference rings as follows: the angle of deviation α (see Fig.P-3-2) is related to θ by $\alpha = 2\theta$. Then, from Fig.P-3-2, we find:

$$\sin(2\alpha) = \sin(4\theta) = \frac{r}{R} \tag{P-3-12}$$

where $R = 65\,\text{mm}$ is the radius of the glass bulb. This equation can be inverted to determine θ in terms of r.

Furthermore, the lattice spacing d can be determined from measured radius r using the mathematical identity $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$. For sufficently small angles α, the approximation $\cos(\alpha) \approx 1$ hold. Accordingly, for small angles of θ we have : $\sin(\alpha) = \sin(2\theta) \approx 2\sin(\theta)$. With these approximations, we obtain (using Bragg condition):

$$r = \frac{2R}{d} \cdot n \cdot \lambda. \tag{P-3-13}$$

The two innermost interference rings ($n = 1$) correspond to reflection at the lattice planes of spacing d_1 and d_2 (see Fig. P-3-3).

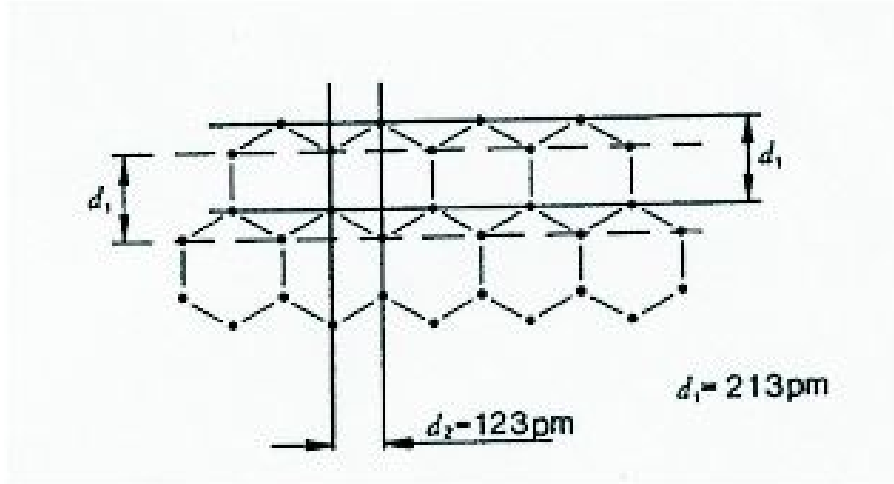
Figure P-3-3: Graphite planes for the first two interference rings

Compton effect

The Compton effect is observed using Cu X-ray tube. The radiation produced with this tube is scattered on a plastic plate and the change of wavelength of the radiation due to the inelastic collision with electrons of the scatterer is determined. The wavelength of the radiation cannot be directly measured in this experiment, only the intensity of radiation is detected with a counter tube. Thus, the measurement combines two wavelength dependent properties, Bragg reflection on a single crystal and the transmission through an aluminium absorber for identification of the wavelength during the Compton scattering. In the first part, the transmission of the X-ray radiation through the aluminium absorber is investigated. For this, an X-ray beam is detected after diffraction on a single LiF crystal in dependence on the deflection angle α_d. According to the Bragg condition (P-3-4), the angle of deflection α_d given by the position of the detecting tube defines the wavelength of the counted radiation. The lattice constant d_{LiF} of the crystal is known as $d_{LiF} = 201.4$ pm. Using a glancing angle $\theta = 7.5° - 11.5°$, i.e. about half of the deflection angle and thus the Bragg angle, the wavelength range covers 55-65 pm.

The number of pulses are counted in dependence on the glancing angle without ($N_1(\theta)$) and with ($N_2(\theta)$) the aluminium absorber behind the aperture of the X-ray beam.

If the measured number of pulses is N, then the relative error of the measurement is given by the ratio:

$$\frac{\Delta N}{N} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}} \,. \tag{P-3-14}$$

At high pulse rates N not all incoming photons are recorded due to the dead time $\tau \approx 90\,\mu$s. The true pulse rate N^* must therefore be determined by means of the relation:

$$N^* = \frac{N}{1 - \tau N} \,. \tag{P-3-15}$$

The corrected pulse rates can now be used to calculate the transmission values of aluminium as follows:

$$T(\lambda) = T(\theta) = \frac{N_2^*(\theta)}{N_1^*(\theta)} \tag{P-3-16}$$

$$\text{where} \quad \lambda = 2\,d_{LiF}\,\sin(\theta) \,. \tag{P-3-17}$$

The interpolated plot T over λ is needed for the determination of the difference in wavelengths due to the Compton effect.

The radiation scattered on a plastic plate is then detected without the aluminium absorber and with the absorber before and after the scatterer as given in Fig. P-3-4, a) to c).
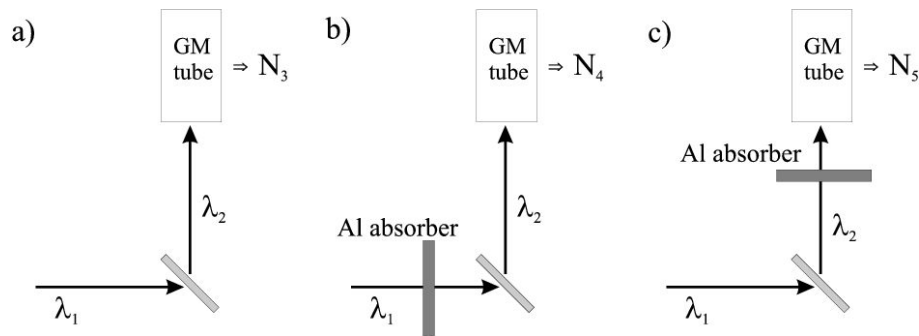

Figure P-3-4: Schematic representation of the 90° Compton scattering arrangement

Now, the rate of the counts in case b) to case a), i.e. N_4^*/N_3^*, gives the transmission which is characteristic for λ_1. Accordingly, $T(\lambda_2)$ is given by the rate of the counts in case c) to a), thus $T(\lambda_2) = N_5^*/N_3^*$.

Equipment

Part	Phywe No.	Quantity
Electron Diffraction Tube and mounting	06721.00	1
Instruction sheet for 06721.00	0672100E.PDF	1
High voltage supply unit, 0-10 kV	13670.93	1
High-value resistor	07160.00	1
Connecting cord, 50 kV, 500nm	07366.00	1
Power supply, 0...600 VDC	13672.93	1
Vernier caliper, plastic	03011.00	1
Connecting cord, 250 mm, red	07360.01	2
Connecting cord, 250 mm, blue	07360.04	2
Connecting cord, 750 mm, red	07362.01	2
Connecting cord, 750 mm, yellow	07362.02	1
Connecting cord, 750 mm, blue	07362.04	1
Connecting cord, 750 mm, black	07362.05	2
X-ray basic unit, 35 kV	09058.99	1
Instruction sheet for 09058.99		1
Goniometer for X-ray unit, 35 kV	09058.10	1
Plug-in module with Cu X-ray tube	09058.50	1
Counter tube, type B	09005.00	1
Lithium fluoride crystal, mounted	09056.05	1
Compton attachment for X-ray unit, 35 kV	09058.04	1
Recommended accessories:		
Software X-ray unit, 35 kV	14407.61	1
Data cable, 2x SUB-D, plug/socket, 9 pole PC	14602.00	1

Set-up and Procedure Notes

Carry out the experiment according to the "Experimental Tasks" listed in the beginning of the description of this experiment. The procedures of both experiments are described in the following. Refer to the instruction sheets of electron diffraction tube (Phywe, 06721.00) and of the X-Ray apparatus (Phywe, 09058.99) for details. The instruction sheets will be available with the experimental parts.

Electron diffraction

Set up the equipment as shown in Figs. P-3-5 and P-3-6. Connect the sockets of the electron diffraction tube to the power supply as shown in Fig. P-3-6. Connect the high voltage to the anode G3 through a $10\,\text{M}\Omega$ protective resistor. Set the voltage G1 and the voltage at grid 4 (G4) and G3 so that sharp and well-defined diffraction rings appear. Read the anode voltage at the display of the HV power supply.
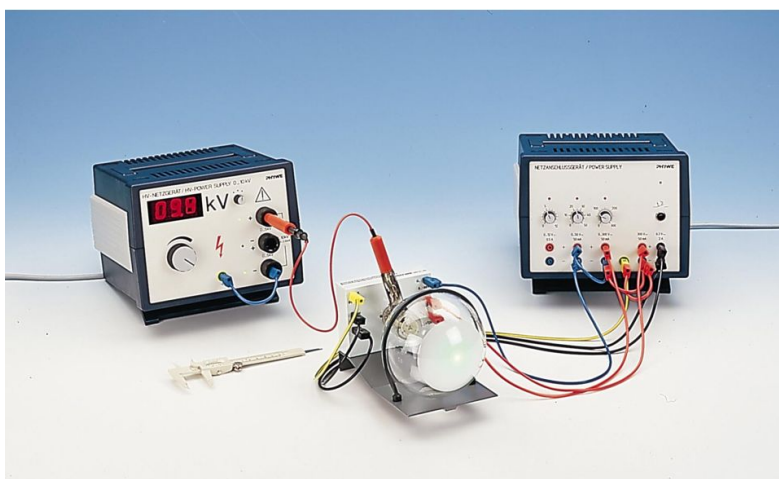
Figure P-3-5: Experimental set-up for electron diffraction

To determine the diameter of the diffraction rings, measure the inner and outer rings with the vernier caliper (in a darkened room) and take the average. Note that there exist a faint further ring close to the the second ring.

Compton scattering

Part 1: Determination of the transmission of aluminium

Fix the diaphragm tube with $2\,\text{mm}$ diameter aperture in the X-ray outlet cylinder of the plug-in module that is to be used. While the X-ray unit is switched off, connect the goniometer to the SUB-D socket in the base plate of the experimenting area. Fit the counter tube in the counter tube holder to the stop and screw it tight. Connect the counter tube cable to the BNC socket in the base plate of the experimenting area. Set

Figure P-3-6: Circuit for electron diffraction experiment

the counter tube holder to the right stop and the goniometer with mounted LiF crystal in the middle position. Lock each of them in these positions. The following settings are recommended: Anod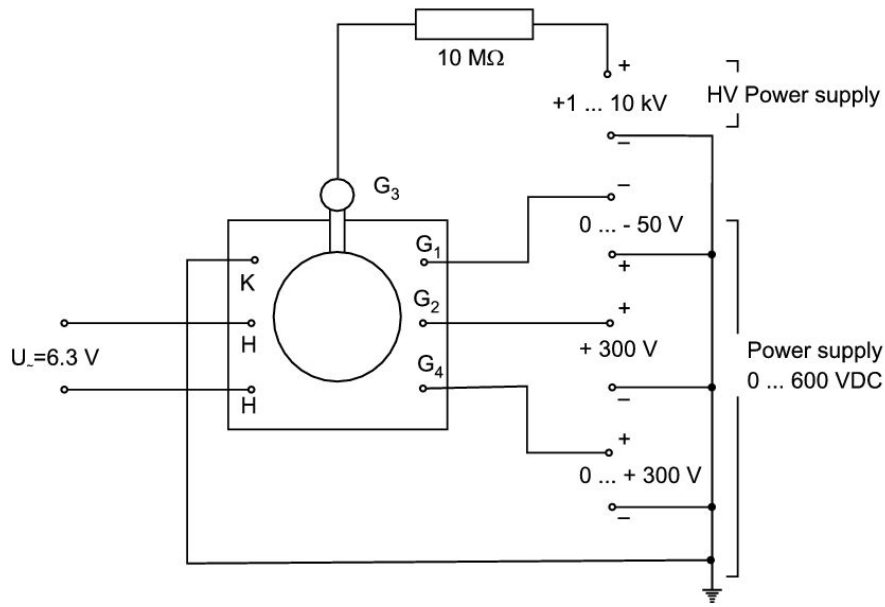e voltage $U_A = 35\,\text{kV}$, Anode current $I_A = 1\,\text{mA}$. Determine the pulse rate $N_1(\theta)$ of the X-rays deflected by the crystal according to the settings outlined in the "Experimental Tasks" section by means of synchronized rotation of the crystal and the counter tube in the angular relationship 2:1. Then, insert the aluminum absorber between the X-ray outlet and the crystal and repeat the above procedure to measure the pulse rate $N_2(\theta)$. Calculate the wavelength λ as a function of the glancing angle θ using the Bragg equation (P-3-4) with lattice constant $d = 201.4\,\text{pm}$ and setting $n = 1$.

Part 2: The determination of Compton scattering
Remove the crystal and replace it with the plastic scatterer. Position this at an angle of 135° (see Fig. P-3-4).
Replace the diaphragm tube with d= 2 mm aperture with the one with d= 5 mm aperture. Turn the counter tube to 90° and measure the following pulse rates: a) using the plastic scatterer but without the aluminum absorber (call the pulse rate N_3), b) using the plastic scatterer and the aluminum absorber in position 1 (call the pulse rate N_4) and c) using the plastic scatterer and the aluminum absorber in position 2 (call the pulse rate N_5). For the measurement of N_5, clamp the absorber on the rods of the counter tube holder. To obtain an appropriate precision, use a measuring time of 300 s for each measurement and take the mean value for the calculation of the transmission coefficient. The "dead time" of the detector and the natural background radiation need to be considered in order to obtain a corrected pulse rate N^*.

Prof. Dr. Arnulf Materny, Faezeh Mohaghegh

NOTE: Never expose the counter tube to unattenuated direct primary radiation for longer times.

Further reading

1. L. v. Alvensleben, **Laboratory Experiments Physics**, Phywe GmbH, 2004, Manuals 5.1.13, 5.4.07 and 5.4.08.

2. M. Alonso and E.J. Finn, **Physics**, Addison-Wesley, 1996

Key Concepts

Bohr's atomic model, quantization of energy levels, electron spin, Bohr's magneton, interference of electromagnetic waves, Fabry-Perot interferometer

Summary

The "Zeeman effect" is the splitting of spectral lines of an atom in a magnetic field. The simplest example is the splitting of one spectral line into three components referred to as the "normal Zeeman effect". In this experiment this latter "normal Zeeman effect" is studied using a cadmium spectral lamp as a specimen. The cadmium lamp is put into different magnetic flux densities and the splitting of the red cadmium line (643.8 nm) is investigated using a Fabry-Perot interferometer. The evaluation of the results should then lead to a fairly precise value for the Bohr's magneton.

Experimental tasks

1. Transverse Zeeman effect

   Perform the measurements for currents of 4, 5, 6, 8 and 9 A.

   (a) Adjust the etalon such that at least 4 rings can be observed.

   (b) Measure the radii $r_{p,a}$ and $r_{p,b}$ of the inner and outer rings established due to the magnetic field between the pole shoes. (p indicates the number of rings, a and b a shift to the left and right, respectively.)

   (c) Compute the values $\Delta_i^{p+1,p}$ for $i = a, b$ and $\delta_{a,b}^p$ for all established rings.

   (d) Comute the mean values Δ and δ.

   (e) Compute $\Delta\tilde{\nu}$.

   (f) Plot $\frac{\Delta\tilde{\nu}}{2}$ versus the magnetic flux density B and add a fit line, find the mean value for $\frac{\Delta\tilde{\nu}}{2B}$ and its standard deviation.

   (g) Determine the Bohr's magneton.

2. Observe qualitatively the longitudinal Zeeman effect and prove that the light is circularly polarized.

Theory

As early as 1862, Faraday investigated whether the spectrum of colored flames changes in a magnetic field-but without success. Only in 1885, the Belgian Fievez was able to demonstrate an effect, but it was forgotten and only rediscovered 11 years later by the Dutchman Zeeman, who studied it together with Lorentz.

This experiment, which was of great importance for the development of the theory of the

atomic shell, can now be carried out with modern equipment in the student's laboratory. The splitting of the Cd-spectral line with $\lambda = 643.8$ nm into three lines – the so-called "Lorentz Triplet" – occurs because the Cd-atom represents a singlet system of total spin $S = 0$. In the presence of a magnetic field, the associated energy levels split into $2L + 1$ components, where L is the total angular momentum. Radiating transitions between the components are possible provided that the selection rules $\mathbf{\Delta M_L = +1}$, $\mathbf{\Delta M_L = 0}$, and $\mathbf{\Delta M_L = -1}$ are fulfilled. Thus a total of nine transitions are permitted in this case. However, the three transitions with the same ΔM_L have the same energy and thus the same wavelength. Consequently, only three lines will appear. The first group with $\Delta M_L = -1$ leads to a σ-line whose light is polarized vertically to the magnetic field. The group with $\Delta M_L = 0$ is the π-line with parallel polarization to the direction of the magnetic field vector. The final group with $\Delta M_L = +1$ again gives a vertically polarized σ-line.

In the absence of an analyzer, all three lines can be seen simultaneously. Each ring, which was observed in the absence of a magnetic field, is split into three rings in the presence of the magnetic field. Inserting the second polarizing filter, which acts as the analyzer, the two σ-lines and the π-line can be observed exclusively if the analyzer is in the vertical position and horizontal position, respectively (transverse Zeeman effect). Turning the electromagnet by 90^0, the light of the spectral lamp parallel to the direction of the field can also be studied since the pole-shoes have been drilled. It can be shown that this light is circular polarized light. Whatever the position of the analyzer may be, each of the rings seen without the magnetic field is now permanently split into two rings in the presence of the magnetic field (longitudinal Zeeman effect). Fig. P-4-1 summarizes these effects.
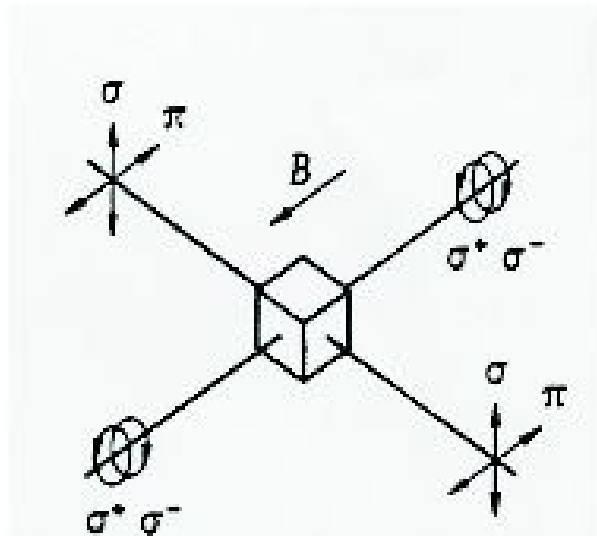

Figure P-4-1: Longitudinal and transverse Zeeman effect

When observing the two σ-lines in the transverse Zeeman effect it is easy to demonstrate that the splitting increases with increasing magnetic field strength. For the quantitative measurement of this splitting, a Fabry-Perot interferometer is used. The **Fabry-Perot etalon** has a resolution of 300000, which means that a wavelength change of approximately 0.002 nm can still be detected. The functioning of this interferometer is explained more detailed in the following section *Experimental Method.*

Preparatory Task:

For a better understanding of the line splitting in a magnetic field, inform yourself about the following:

1. What is the difference between "anomalous" and "normal" Zeeman effect? Which one would you consider to be the "normal" case?

2. Can the anomalous Zeeman effect be described using a classical theory?

3. Describe the normal Zeeman effect using a classical description. Find an expression for the potential energy of the electron in the magnetic field B.

4. What is the Paschen-Back effect?

You should be prepared to answer these questions during the quiz. You may bring some hand-written notes and use them during the quiz.

Experimental Method

The etalon consists of two parallel flat glass plates coated on the inner surface with a partially transmitting metallic layer. Let us consider the two partially transmitting surfaces (1) and (2) in Fig. P-4-2 separated by a distance t. An incoming ray forming an angle Θ with the normal to the plates will be split into the rays AB, CD, EF, *etc.* The path difference between the wave fronts of two adjacent rays (*e.g.* AB and CD) is:

$$\delta = BC + CK \ .$$

Point K is defined so that BK is normal to CD. With

$$CK = BC \cos(2\Theta) \quad \text{and} \quad BC \cos \Theta = t$$

we obtain

$$\delta = BC + CK = BC \left(1 + \cos(2\Theta) \right) = 2BC \cos^2 \Theta = 2t \cos \Theta \ .$$

For a constructive interference to occur, the following has to hold:

$$n\lambda = 2t \cos \Theta \quad , \quad n \text{ integer} \ .$$
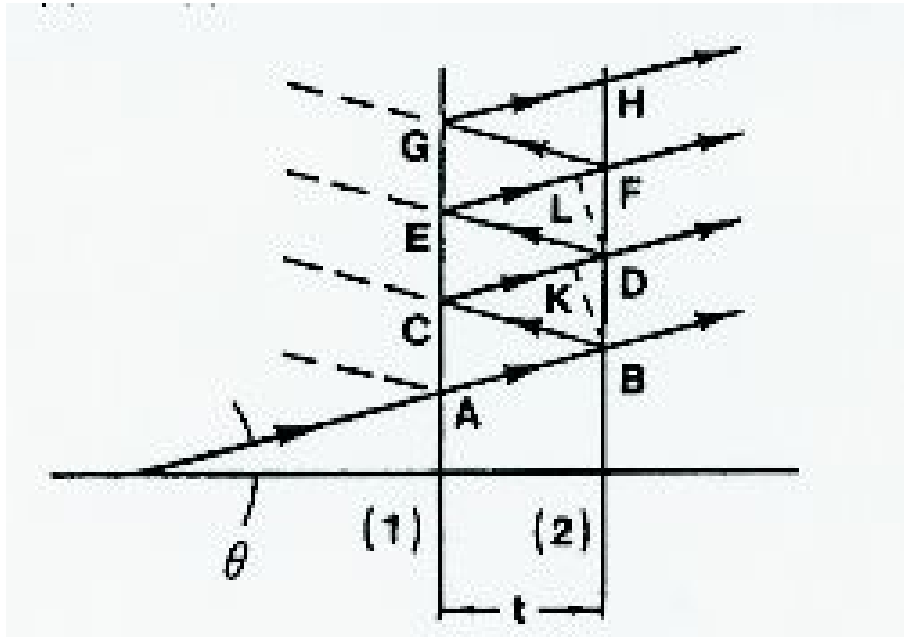
Figure P-4-2: Reflected and transmitted rays at the parallel surfaces (1) and (2) of the etalon. The etalon spacing is t.

If the refractive index of the medium between the plates is $\mu \neq 1$, the equation has to be modified:

$$n\lambda = 2\mu t \cos \Theta \ . \tag{P-4-1}$$

(P-4-1) is the basic interferometer equation. Let the parallel rays B, D, F, *etc.* be focused with the help of a lens with local length f (see Fig. P-4-3).
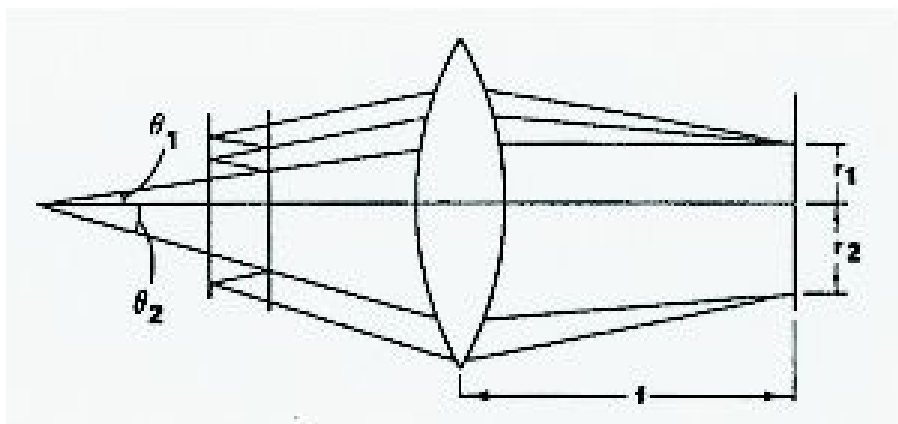
Figure P-4-3: Focusing of the light emerging from a Farby-Pero etalon. Light entering under an angle of Θ is focused onto a ring of radius $r = f\Theta$, where f is the focal length of the lens.

When Θ fulfills (P-4-1) bright rings with radius

$$r_n = f \tan \Theta_n \approx f \Theta_n \qquad \text{(P-4-2)}$$

appear in the focal plane. *e.g.* for rays nearly parallel to the optical axis, Θ_n can be assumed to be small. With $n_0 = \frac{2\mu t}{\lambda}$ we find from (P-4-1)

$$n = n_0 \cos \Theta_0 = n_0 \left(1 - 2 \sin^2 \left(\frac{\Theta_n}{2} \right) \right)$$

and assuming Θ_n to be small, we have

$$n = n_0 \left(1 - \frac{\Theta_n^2}{2} \right)$$

or

$$\Theta_n = \sqrt{\frac{2(n - n_0)}{n_0}} \qquad \text{(P-4-3)}$$

If Θ_n is to correspond to a bright fringe, n must be an integer. However, n_0, which is related to the interference at the center ($\Theta = 0$ in (P-4-1)) is in general not an integer. If n_1 is the interference order of the first ring, clearly we have that $n_1 < n_0$ since $n_1 = n_0 \cos \Theta_{n_1}$. Thus, we set

$$n_1 = n_0 - \epsilon \ , \quad 0 < \epsilon < 1 \ , \qquad \text{(P-4-4)}$$

where n_1 is the closest integer to n_0. Thus, we have in general for the p-th ring of the pattern (as measured from the center):

$$n_p = (n_0 - \epsilon) - (p - 1) \ . \qquad \text{(P-4-5)}$$

Combining equation (P-4-5) with (P-4-2) and (P-4-3), we obtain for the radii of the rings (putting r_p for r_{n_p}):

$$r_p = \sqrt{\frac{2f^2}{n_0}} \cdot \sqrt{(p - 1) + \epsilon} \ . \qquad \text{(P-4-6)}$$

Note that the difference between the squares of the radii of two adjacent rings is constant:

$$r_{p+1}^2 - r_p^2 = \frac{2f^2}{n_0} \ . \qquad \text{(P-4-7)}$$

If a spectral line is split into two components with only little differing wavelengths λ_a and λ_b, these two components will have the fractional orders ϵ_a and ϵ_b with:

$$\epsilon_a = \frac{2\mu t}{\lambda_a} - n_{1,a} = 2\mu t \tilde{\nu}_a - n_{1,a} \ , \qquad \text{(P-4-8)}$$

$$\epsilon_b = \frac{2\mu t}{\lambda_b} - n_{1,b} = 2\mu t \tilde{\nu}_b - n_{1,b} \ . \qquad \text{(P-4-9)}$$

where the index 1 at n refers to the interference order of the first ring, with the wavenumbers of the components $\tilde{\nu}_a$ and $\tilde{\nu}_b$. If the interference rings do not overlap by a whole order, we have $n_{1_a} = n_{1_b}$ and then the difference in wave numbers between the two components is simply

$$\Delta\tilde{\nu} = \tilde{\nu}_a - \tilde{\nu}_b = \frac{\epsilon_a - \epsilon_b}{2\mu t} \ . \tag{P-4-10}$$

Using (P-4-6) and (P-4-7), we find for the components a, b, respectively

$$\frac{r_{p+1,k}^2}{r_{p+1,k}^2 - r_{p,k}^2} - p = \epsilon_k \quad , \quad k = a, b$$

Substituting ϵ_a, ϵ_b into (P-4-10), we find

$$\Delta\tilde{\nu} = \frac{1}{2\mu t} \left(\frac{r_{p+1,a}^2}{r_{p+1,a}^2 - r_{p,a}^2} - \frac{r_{p+1,b}^2}{r_{p+1,b}^2 - r_{p,b}^2} \right) \ . \tag{P-4-11}$$

From (P-4-7), we find that

$$\Delta_a^{p+1,p} = r_{p+1,a}^2 - r_{p,a}^2 = \frac{2f^2}{n_{0,a}} \approx \frac{2f^2}{n_{0,b}} = r_{p+1,b}^2 - r_{p,b}^2 = \Delta_b^{p+1,p}$$

whatever the value of p may be. Similarly, all values

$$\delta_{a,b}^p = r_{p,a}^2 - r_{p,b}^2$$

must be equal regardless of p. The average δ of these values as well as of the Δ-values (the average value is called Δ in the following) can be taken. With these average values we obtain - setting $\mu = 1$:

$$\Delta\tilde{\nu} = \frac{1}{2t}\frac{\delta}{\Delta} \ . \tag{P-4-12}$$

The mean values Δ and δ are calculated in the following way (Note that not any arbitrary Δ can be used. Only alternating ones are appropriate):

$$\Delta = \frac{1}{4}\sum_{p=1}^{2} \left(\Delta_a^{2p,2p-1} + \Delta_b^{2p,2p-1} \right) \tag{P-4-13}$$

and

$$\delta = \frac{1}{4}\sum_{p=1}^{4} \delta_{a,b}^p \ . \tag{P-4-14}$$

The etalon spacing t is given as $t = 3.00\,\text{mm}$. With the help of (P-4-12), the difference in wave numbers of the two σ-lines can then calculated in dependence on the respective coil currents.

The difference in wave numbers of one of the σ-lines with respect to the central lines is $\Delta\tilde{\nu}/2$. For the radiating electrons this means a change of energy

$$\Delta E = E_{L,M_L} - E_{L-1,M_{L-1}} = hc\frac{\Delta\tilde{\nu}}{2} \ . \tag{P-4-15}$$

On the other hand, the change of energy ΔE is proportional to the magnetic flux density B. The factor of proportionality between ΔE and B is the Bohr magneton μ_B:

$$\Delta E = \mu_B B \ . \tag{P-4-16}$$

Combining (P-4-15) and (P-4-16) leads to the following expression

$$\mu_B = hc\frac{\Delta\tilde{\nu}}{2B} \ . \tag{P-4-17}$$

Then the value of the Bohr magneton can be calculated and compared to the literature value $\mu_{B,Lit.} = 9.27400899(37) \cdot 10^{-24}$ J/T.

Equipment

Part	Phywe No.	Quantity
Fabry-Perot interferometer	09050.02	1
Cadmium lamp for Zeeman effect	09050.01	1
Electromagnet w/o pole shoes	06480.01	1
Pole piece, drilled, conical	06480.03	1
Rot. table for heavy loads	02077.00	1
Power supply for spectral lamps	13662.93	1
Var. transformer, 25 VAC/20 VDC, 12 A	13531	1
Capacitor, electrolyt., 22000 mic-F	06211.00	1
Digital multimeter	07134.00	1
Optical profile-bench, $l = 1000\ mm$	08282.00	1
Base for optical profile-bench, adjustable	08284.00	2
Slide mount, lateral. adjustable, cal.	08082.03	1
Slide mount f. opt. pr.-bench, $h = 30\ mm$	08286.01	5
Slide mount f.opt.pr.-bench, h 80 mm	08286.02	1
Lens holder	08012.00	4
Lens, mounted, f +50 mm	08020.01	2
Lens, mounted, f +300 mm	08023.01	1
Iris diaphragm	08045.00	1
Polarising filter, on stem	08610.00	1
Polarization specimen, mica	08664.00	1
Screen, with aperture and scale	08340.00	1
Plate holder with tension spring	08288.00	1
Swinging arm	08256.00	1
Spirit level	02124.00	1
Connecting cord, 500 mm, red	07361.01	2
Connecting cord, 500 mm, blue	07361.04	2
Sensor Cassy, Leybold	524 010	1
B-Box, Cassy, Leybold	524 038	1
Axial B-Probe, Leybold	516 61	1
Laptop		1

Set-up and Procedure Notes

Carry out the experiment according to the "Experimental Tasks" listed in the beginning of the description of this experiment. The set-up of the experiment is shown in Fig. P-4-4.

Figure P-4-4: Experimental set-up for the Zeeman effect

Set-up of electromagnet

For the experiment, the electromagnet has to be placed on the rotating table for heavy loads. Before placing the magnet on the table it has to be checked that it is horizontally aligned. Use a spirit level to adjust the table appropriately.

The electromagnet is put on the rotating table for heavy loads and mounted with the two pole-shoes with holes in such a way that a gap of 9 mm remains for the Cadmium (Cd)-lamp. The pole-shoes have to be well tightened in such a way that they cannot move later on when the magnetic flux is established. The Cd-lamp is inserted into the gap without touching the pole-shoes. It is connected to the power supply for spectral lamps. The coils of the electromagnet are connected in parallel via an ammeter to the variable power supply (20 V DC, 12 A). A capacitor of 22000 μF is connected in parallel to the power output in order to smooth the DC-voltage.

Set-up of optical bench

The optical bench is placed next to the rotating table with the electromagnet and Cd lamp (see Fig. P-4-5). As the table for the electromagnet, the bench has to be aligned horizontally, too. Use a spirit level to check the alignment.

The elements for the observation of the line splitting are given in the following list. The number in brackets indicates the approximate position on the bench. Adjust

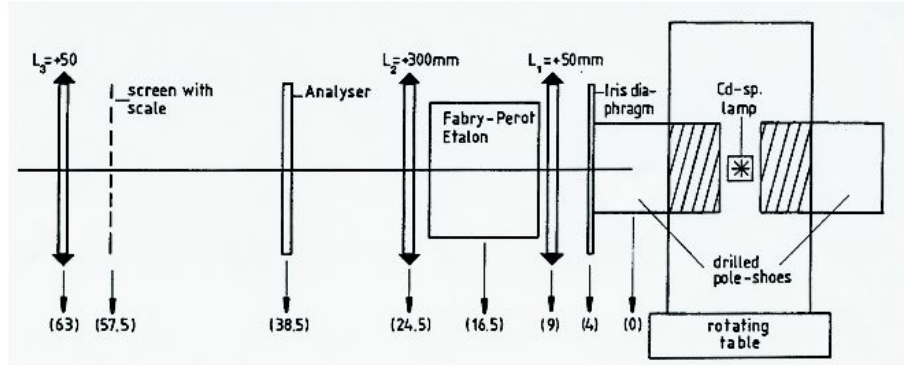the elements according to the given positions and optimize the alignment by slight adjustments of the lenses.

Figure P-4-5: Arrangement of the optical components

(0.0 cm)	Drilled pole-shoes
(4.0 cm)	Iris diaphragm
(9.0 cm)	$L_1 = +50 \ mm$
(16.5 cm)	Fabry-Perot Etalon
(24.5 cm)	$L_2 = +300 \ mm$
(38.5 cm)	Analyzer
(57.5 cm)	Screen with scale
(60-63 cm)	$L_3 = +50 \ mm$

The iris diaphragm is eliminated for initial adjustment and for the observation of the longitudinal Zeeman effect. During the observation of the transverse Zeeman effect, the iris diaphragm is illuminated by the Cd-lamp and thus acts like a light source. The lens L_1 and a lens with $f = 100 \, \text{mm}$ (incorporated in the etalon) create a nearly parallel light beam, which the Fabry-Perot etalon needs for a proper interference pattern. The etalon contains an interchangeable color filter to select the red cadmium line of 643.8 nm for observation. The lens L_2 produces an interference pattern of rings within the plane of the screen. This has a scale mounted on a slide mount, which can laterally be displaced within a precision of 1/100 of a millimeter. The ring system is observed through L_3 and the ring diameters can be measured *e.g.* by systematic displacement of the slash representing the "0" of the scale. The readings should be done in a completely darkened room using a flash-light.

If available, the splitting of rings is observed with a webcam. The picture is then displayed on the screen of a laptop or on the wall using a beamer.

Calibration of magnetic field

For evaluation, the calibration curve of the magnetic flux density versus the coil current has to be traced previously. This is done with the axial hall probe, which is directed through the pole shoes. The magnetic flux is measured for a current of 5 A in dependence on the position of the probe along the axis through the pole shoes. Afterwards, the probe is positioned in the middle of the gap between the pole shoes that should coincide with the maximal flux. The flux is measured as a function of current between $1 - 10$ A.

Initial adjustment

The rotating table with electromagnet, pole-shoes and Cd-lamp (which has already been mounted) is brought to a height of about $16\,\text{cm}$ above the table by using the supporting blocks. Then the electromagnet is adjusted perfectly horizontal. The optical bench, which has all elements but the iris diaphragm mounted is moved closer to the electromagnet in such a way that one of the outlet holes of the pole-shoes coincides with the previous position of the iris diaphragm. L_1 is then adjusted such that the outlet hole is in its focus. All other optical elements (see Fig. P-4-5) are readjusted accordingly.

Measurement of transverse Zeeman effect

The pattern must be centered and sharp, which is eventually achieved by a last, slight movement of the etalon (to the right or the left) and the displacement of L_2 (vertically or horizontally). Provided the ring pattern has been properly established as explained above, the slash of the scale "0" is shifted horizontally along a diameter through the ring pattern until it coincides, *e.g.* with the fourth ring to the left.

A magnetic field corresponding to the respective coil current is established and the splitting of the rings is observed. The current of the coils is chosen to be $8\,\text{A}$ in the beginning (increase in light intensity!) and the interference pattern in axial direction is observed through L_3.

The analyzer is put into the vertical position so that only the two σ-lines appear. The "0" slash is now adjusted to coincide perfectly with the outer ring of the two rings into which *e.g.* the fourth ring has split. The first reading on the socket of the sliding mount is taken. The "0" slash is then moved from left to right through all the rings. The last reading is taken when the "0" slash coincides with the outer ring of the fourth ring to the right. The last reading minus the first reading divided by two then provides the radius $r_{4,b}$.

Measurement of longitudinal Zeeman effect

The electromagnet is turned by 90° to observe the longitudinal Zeeman effect. In the presence of a magnetic field (a coil current of $8\,\text{A}$ is recommended) each of the

rings is split into two rings independent of the position of the analyser.

A $\lambda/4$-plate is generally used to convert linear into elliptically polarized light. In this experiment the $\lambda/4$-plate is used in the opposite way. Namely, by means of the $\lambda/4$-plate, inserted manually between L_2 and the analyser, the light emerging in the longitudinal Zeeman effect is investigated. If the optic axis of the $\lambda/4$-plate is arranged vertically, it is observed that one ring disappears if the analyser forms an angle of $+45°$ with the vertical, while the other ring disappears if the analyser forms an angle of $-45°$ with the vertical. This shows that the light of the longitudinal Zeeman effect is circularly polarized.

Further reading

1. M. Alonso and E. J. Finn, **Physics**, Addison-Wesley, 1996 (very basic)

2. H. Haken and H. C. Wolf, **The Physics of Atoms and Quanta**, Springer, 2004

Faraday and Kerr Effect

Key Concepts

Polarization, elliptically polarized light, optical anisotropy, birefringence, refractive index, electromagnetic field interaction, electron oscillation, Faraday effect, Hall effect, Kerr effect, Verdet's constant, Kerr constant, modulation of light, electro-optical modulator, PLZT element.

Summary

Faraday effect

The angle of maximum extinction of light passing a flint glass cylinder in a magnetic field is investigated as a function of magnetic flux density for different wavelengths. Verdet's constant for flint glass is determined and its change with wavelength displayed.

Kerr Effect

The phase shift of monochromatic, linearly polarized light after traversing a Kerr cell is observed as a function of the applied electric field and the Kerr constant determined.

Experimental tasks

1. **Faraday effect**

   (a) Determination of relationship between magnetic flux density and current

      i. Measure the flux density between the pole pieces along the optical axis with a step width of 5 mm for different coil currents between 0.5 A to 4.0 A.

      ii. Plot the magnetic flux as a function of position for each measurement and determine by interpolation and numerical integration the mean flux density of the gap which is occupied by the flint glass cylinder in the next experiment.

      iii. Plot the mean flux density as a function of coil current.

      iv. Give an approximation of the functional relationship for mean flux density in dependence on coil current.

   (b) Determination of Verdet's constant as a function of wavelengths.

      i. Determine the angle of rotation $(2\Delta\phi)$ in dependence on coil current using the color filters with max. transmission at $\lambda = 440$ nm, 505 nm, 525 nm, 580 nm, and 595 nm.

      ii. Calculate the corresponding magnetic flux densities using the formula determined in part 1(a)iv.

      iii. Determine Verdet's constant from the slope of the graphs for the investigated wavelengths.

      iv. Plot Verdet's constants vs. wavelength.

2. **Kerr effect**

  (a) Determination of Kerr constant of the PLZT element.

      i. Measure the intensity I_0 behind the analyzer if analyzer and polarizer are aligned in the same direction and the voltage applied to the PLZT element is zero.

      ii. Measure the intensity I behind the analyzer with the analyzer perpendicular orientated to the polarizer as a function of applied voltage.

      iii. Determine the phase shift of each measurement.

      iv. Display the normalized intensity vs. applied voltage. Indicate corresponding phase shift for each data point.

      v. Plot the squared voltage vs. resulting phase shift and determine the Kerr constant.

        Hint: Regard that the calculated value for arcsin $\sqrt{\frac{I}{I_0}}$ will be between 0° and 90° although the phase displacement might be larger. Use the knowledge about the procedure when evaluating your result.

Theory

Modern technical applications, laser techniques as well as optical data transfer require fast modulation of the intensity of light. Some of the techniques make use of the fact that some materials become birefringent if an electric or magnetic field is applied. The intensity of electric and magnetic fields can be varied with high frequencies and therefore, allow for a fast change of the optical property of a material.

As described in the manual "Polarization" some materials feature the property of birefringence. That means, a ray of light is divided up into two rays which "see" different refractive indices in the material. Technical applications make use of the fact that the polarization planes of the so-called ordinary and extraordinary rays are perpendicular to each other.

The birefringence depends on the anisotropic structure of material. The effect was first discovered for crystals but is also known for some liquids and applied in Liquid Crystal Displays (LCD). In the following it is mostly referred to crystals, but the physical principles may also be applied to liquids containing orientated molecules.

The anisotropy in birefringent media can in most cases be described by an optical axis. The optical axis defines a certain direction within the material. Light waves having there

electric field vector (polarization direction) parallel to the optical axis see a refractive index n_{eo} (extraordinary) while those being polarized perpendicularly to this symmetry axis will experience an index n_o (ordinary). Therefore, the optical properties of an anisotropic crystal depend on the direction of the ray of light relative to the optical axis of the crystal, which is characterized by the angle α, as well as on its polarization. In the case of unpolarized light or where the polarization vector is neither parallel nor perpendicular to the optical axis, the light ray has to be split up into parallel and perpendicular components (extraordinary and ordinary part, respectively). In contrast to the ordinary ray, the phase velocity of the extraordinary ray varies with the angle α to the symmetry axis in birefringent materials. The different phase velocities lead to a splitting up of the ray if $\alpha \neq 0°$ and $\alpha \neq 90°$. This is illustrated in Fig. P-5-1(a) using Huygens principle.

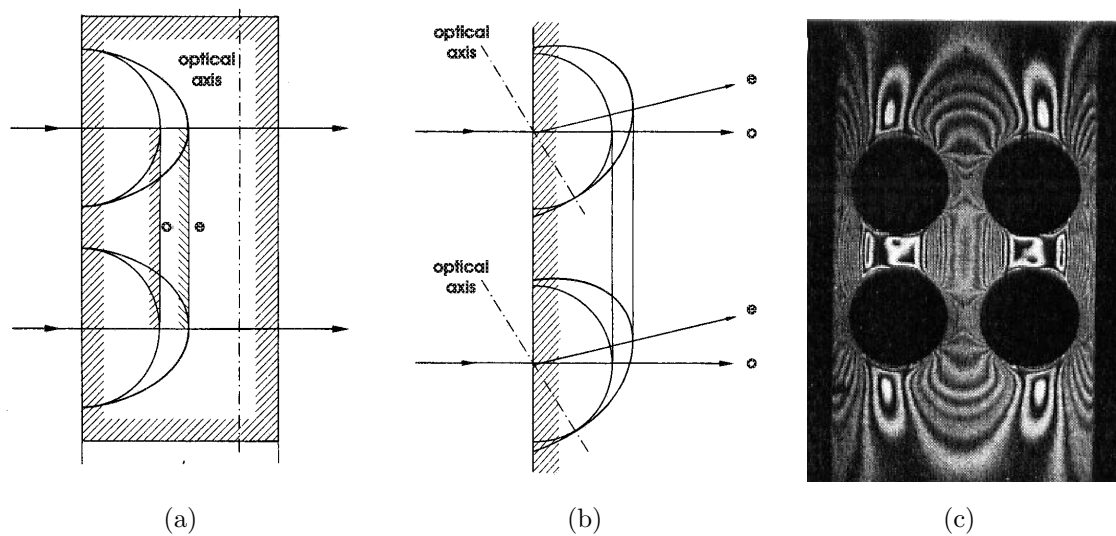(a)   (b)   (c)

Figure P-5-1: a) and b): Sketch for explanation of behavior of ordinary (o) and extraordinary rays (e) in birefringent material; c) example of a technical application using birefringence.

A light ray travelling through the crystal along the symmetry axis ($\alpha = 0°$) has its electric field vector perpendicular to the optical axis in any case. The ray keeps its polarization and direction (it behaves like an ordinary ray in an ordinary material).

If the direction of the ray is perpendicular to the optical axis, also no splitting up of the light beam can be observed. However, ordinary and extraordinary components traverse the crystal with different phase velocities (compare Fig. P-5-1(b)). The superposition of both rays in general leads to elliptically polarized light. In contrast to linearly polarized light, elliptically polarized light usually cannot be extinguished by a polarization filter. Therefore, under these conditions, linearly polarized light cannot be suppressed by the analyzer behind the crystal (exceptions: $\lambda/4$ and $\lambda/2$ plate arrangements, see experiment 10).

For example, glass and plastic can become birefringent if strained. Therefore, the internal strain in dependence on mechanical stress applied to a test object can be observed by the polarization of the transmitted light resulting in intensity variations behind an analyzer. An example is depicted in Fig. P-5-1(c).

Some materials become anisotropic in an electric or magnetic field. These physical phenomena are named Kerr effect and Faraday effect, respectively, after the scientists (J. Kerr, 1875 / M. Faraday 1846) who discovered them. Light can be switched nearly without any time delay using these effects and are therefore used for high-speed photography, measurement of light velocity, as Q-switches for pulsed lasers and for other applications where a high modulation frequency of light is required.

The Kerr effect allows to modulate the light intensity up to a frequency of 200 MHz. Pockels cells (F.C. Pockels, 1893), another example of electro-optical cells, based on the same principle using piezoelectric materials, can be modulated up to 1 GHz. Ferromagnetic materials , e.g. Ga-doted Yttrium-iron garnet (YIG), are used for magneto-optic modulators with frequencies greater than 200 MHz.

Theoretical details of both effects and particularities of the experiments covered by this manual are given in the following paragraphs.

Faraday effect

When a transparent medium is permeated by an external magnetic field, the plane of polarization of a linearly polarized light beam passing through the medium is rotated if the direction of the incident light is parallel to the lines of force of the magnetic field. This is called the "Faraday effect".

For a particular wavelength, the angle of rotation $\Delta\phi$ is proportional to the mean flux-density $\overline{B}$ of the applied magnetic field and the optical path length l through the medium. The proportionality factor V is called Verdet constant:

$$\Delta\phi = V \, l \, \overline{B} \tag{P-5-1}$$

V is a specific constant of the medium and a function of the wavelength λ and the refractive index $n(\lambda)$.

$$V = V' \lambda \frac{dn}{d\lambda} \tag{P-5-2}$$

An atomistic explanation assumes that the oscillating charges in the medium describe a precessional motion in the magnetic field. The frequency is given by the Lamor-frequency:

$$\omega' = \frac{e}{m} B \tag{P-5-3}$$

where e and m are the charge and the mass of the oscillating particle, respectively. Left- and right-circularly polarized light waves have different frequencies with respect to the frequency of the precession. Due to a rotational Doppler effect, the frequency ω of the light is observed by the precessing charge as $\omega + \omega'$ and $\omega - \omega'$ for the differently circularly polarized waves. The refractive index and the phase velocity depend on the interaction with the charges in the medium. Thus the difference of refractive indices for left- and right-circularly polarized light waves depend on the phase difference with respect to the rotating charges. Please note, the Verdet constant increases for large changes of refractive indices with wavelength, i.e. in the vicinity of spectral lines.

In order to demonstrate the Faraday effect experimentally, linearly polarized light is passed through a flint-glass SF_6 cylinder, supported between the drilled pole pieces of an electromagnet. An analyzer arranged beyond the glass cylinder has its polarization plane crossed in relation to that of the polarizer, so that the field of view of the face of the glass cylinder projected on the translucent screen appears dark.

When current flows through the coils of the electromagnet, a magnetic field is produced, permeating the glass cylinder in the direction of irradiation. The rotation now occurring in the plane of oscillation of the light is indicated by resetting the analyzer to maximum extinction of the translucent screen image. After reversing the polarity of the coil current, the experiment is repeated with the opposite magnetic field direction. The difference between the two positions of the analyzer is equal to $2 \cdot \Delta\phi$.

Kerr effect

In the year 1875, Kerr discovered that a glass plate to which a strong electric field was applied became birefringent. It soon became clear that this effect was not based on deformation caused by the electric field as the same phenomenon was observed in liquids and even in gases.

If a medium becomes optically anisotropic when an electric field is applied, the Kerr effect can be studied. The optical axis is built up along the direction of the field vector of the electric field. Therefore, linearly polarized light travels through the media perpendicular to the electric field, i.e. also perpendicular to the optical axis, and a phase difference between the ordinary and extraordinary beam is produced. Therefore, the light which has travelled through the Kerr cell will be elliptically polarized and is no longer extinguished by the analyzer.

If the corresponding refractive indices are designated by n_{eo} and n_o, and l is the distance in the Kerr element covered by the light, then there is a difference in optical paths for the two waves of

$$d\,l = (n_{eo} - n_o) \tag{P-5-4}$$

This corresponds to a phase displacement of

$$\Delta = 2\pi \frac{l}{\lambda}(n_{eo} - n_o) \tag{P-5-5}$$

where λ is the wavelength of the light in vacuum. It can also be shown that the phase displacement is the square of the polarization P. If we assume that the polarization is a linear function of the electric field strength E and the proportionality factor is designated by πK, then the following relation is obtained:

$$\Delta = 2\pi K l E^2 \tag{P-5-6}$$

with K the Kerr constant.
E can be expressed by the applied voltage U and the inter-electrode distance d

$$E = \frac{U}{d} \tag{P-5-7}$$

If analyzer and polarizer are set up perpendicularly to each other and at an angle of 45° to the electric field applied to the Kerr element, the luminous intensity I behind the analyzer is given by equation (P-5-5) and by Malus' law:

$$I = I_0 \cos^2\left(\frac{\pi + \Delta}{2}\right) = I_0 \sin^2 \frac{\Delta}{2} \tag{P-5-8}$$

$$\Rightarrow \qquad \Delta \qquad = 2 \arcsin \sqrt{\frac{I}{I_0}} \tag{P-5-9}$$

I_0 is the luminous intensity behind the analyzer when the polarizer and the analyzer are aligned in the same direction and the electric field on the PLZT element is zero. After substituting (P-5-6) in (P-5-8) and using (P-5-7), equation (P-5-10) is obtained:

$$I = I_0 \sin^2 \frac{\pi K l U^2}{d^2} \tag{P-5-10}$$

Solving the equation for U^2 gives:

$$U^2 = \frac{d^2}{\pi K l} \cdot \arcsin \sqrt{\frac{I}{I_0}} \tag{P-5-11}$$

By plotting U^2 against $\arcsin \sqrt{I/I_0}$ a straight line should be obtained and the Kerr constant can be derived from its slope if the dimensions l and d of the Kerr cell are known.

The relative luminous intensity $\frac{I}{I_0}$ behind the analyzer is measured as a function of the voltage U applied to the Kerr element.

For a phase difference between the two waves of $\lambda/2$ a linearly polarized wave results from the superposition of the two waves after they have passed through the Kerr element.

The applied voltage is therefore called the "half-wave voltage". It is a function of the composition of the Kerr element and of its temperature.

This linearly polarized wave is turned by 90° with respect to the direction of incidence, i.e. parallel to the orientation of the analyzer which is orientated perpendicular to the polarizer. In this case the light intensity behind the analyzer shows a maximum.

The Kerr element used in this experiment has the advantage that the birefringence, represented by the Kerr constant, is more than two orders of magnitude larger than for nitrobenzene and voltages of a few hundred volts are sufficient in order to study the electro-optical effects of the ceramic element. It is transparent for wavelengths from 400 nm to 5.6 μm. Its chemical composition is given by the formula $Pb_{0.9125}$ $La_{0.0875}$ $Zr_{0.65}$ $Ti_{0.35}$. With regard to the transmission of light the PLZT element behaves like a transparent polycrystal. For $\lambda = 633$ nm its transition ratio is more than 60%. With respect to an applied electric field, it behaves like a ferromagnetic substance that is submitted to a magnetic field.

The Kerr effect has usually been demonstrated with nitrobenzene which is highly toxic and requires high voltages of some kV to produce observable effects. The PLZT element which only needs some hundred volts represents an attractive alternative.

Within the PLZT element there are already prepolarized domains which grow or which are reorientated by the applied electric field. In this way, the element becomes optically anisotropic respectively birefringent. Regarding the above mentioned electro-optical modulation, it should be pointed out that the element can deal with modulation frequencies of up to 100 kHz.

Fig. P-5-2 shows a cross-section of the used PLZT element. The active element $\boxed{1}$ of the modulator is a parallelepiped of 8 mm height, 1.5 mm length, and 1.4 mm width.
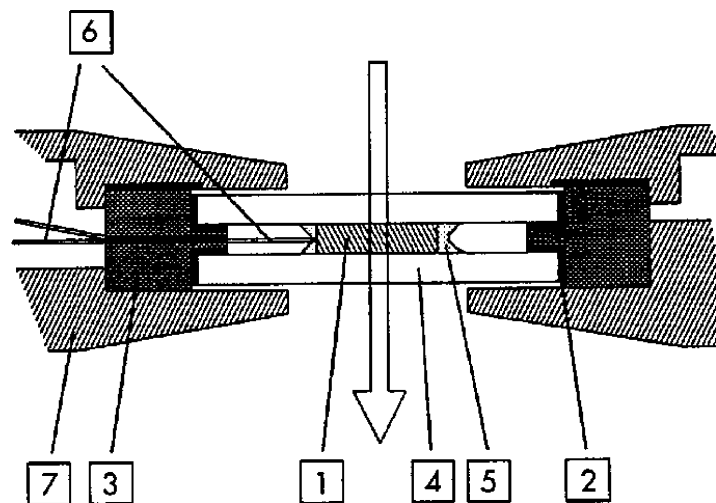

Figure P-5-2: Cross-section of PLZT element.

The width is the distance between the electrodes. The electric field strength is given by the ratio of voltage applied over the distance between the electrodes. The path of the light beam within the element is equal to the length of the element. The active element is encapsulated using silicon hermetics $\boxed{2}$ in addition to an isolating ring $\boxed{3}$ and is glued between two glass plates $\boxed{4}$. As optical glue $\boxed{5}$, Canadian balsam is used. Wires $\boxed{6}$ are fixed on the electrode faces of the element and connected to a BNC socket on frame $\boxed{7}$.

Equipment

Part	Phywe No.	Quantity
Faraday effect		
Glass rod for Faraday effect	06496.00	1
Coil, 600 turns	06514.01	2
Pole pieces, drilled, 1 pair	06495.00	1
Iron core, U-shaped, laminated	06501.00	1
Housing for experiment lamp	08129.01	1
Halogen lamp, 12 V/50 W	08129.06	1
Holder G 6.35 f. 50/100 W halo.lamp	08129.04	1
Double condenser, f 60 mm	08137.00	1
Var.transformer, 25VAC/20VDC, 12A	13531.93	1
Voltmeter 5/15 V DC	07037.00	1
Commutator switch	06034.03	1
Teslameter, digital	13610.93	1
Hall probe, axial	13610.01	1
Lens, mounted, f +150 mm	08022.01	1
Lens holder	08012.00	1
Table top on rod, 18.5×11 cm	08060.00	1
Object holder, 5×5 cm	08041.00	1
Color filter, 440 nm	08411.00	1
Color filter, 505 nm	08413.00	1
Color filter, 525 nm	08414.00	1
Color filter, 580 nm	08415.00	1
Color filter, 595 nm	08416.00	1
Polarizing filter with vernier	08611.00	2
Screen, translucent, 250 × 250 mm	08064.00	1
Optical profile-bench, l 1000 mm	08282.00	1
Base f. opt. profile-bench, adjust.	08284.00	2

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Part	Phywe No.	Quantity
Slide mount f. opt. pr.-bench, h 30 mm	08286.01	2
Slide mount f. opt. pr.-bench, h 80 mm	08286.02	5
Universal clamp	37715.00	1
Connecting cord, 750 mm, red	07362.01	3
Connecting cord, 750 mm, blue	07362.04	3
Kerr effect		
Kerr cell, PLZT-element	08641.00	1
High voltage supply unit, 0-10 kV	13670.93	1
Laser, He-Ne 1.0 mw, 220 V AC	08181.93	1
Polarizing filter, on stem	08610.00	2
Optical profile bench l = 60 cm	08283.00	1
Base f.opt.profile-bench, adjust.	08284.00	2
Slide mount f.opt.pr.-bench, h 30 mm	08286.01	5
Photoelement f. opt. base plt.	08734.00	1
Universal measuring amplifier	13626.93	1
Digital multimeter	07134.00	2
Screened cable, BNC, l 750 mm	07542.11	1
Adapter, BNC-socket/4 mm plug pair	07542.27	1
Connecting cord, 750 mm, red	07362.01	3
Connecting cord, 750 mm, blue	07362.04	2

Set-up and Procedure Notes

Faraday effect

- Determination of flux density as a function of coil current

  The DC output of the power supply is variable between 0 and 20 V DC and is connected via an amperemeter to the coils of the electromagnet which are in series. The electromagnet needed for the experiment is constructed from a laminated U-shaped iron core, two 600-turn coils and the drilled pole pieces, the electromagnet then being arranged in a stable manner on the table on rod.

  In the absence of the flint glass cylinder, the distribution of the magnetic flux-density is determined in the space between the pole pieces. The flux-density is measured in steps of 5 mm using the axial probe of the teslameter which can easily

be moved through one of the holes of the pole pieces if fixed in a universal clamp on a slide mount. The procedure is repeated for different current intensities.

The flux density increases strongly to the center of the gap and decreases to either side. For all coil currents, the ratio of maximum flux-density to mean flux-density (found by numerical integration) should be approximately 1.5 in each case.

The magnetic flux density corresponding to an applied coil current can then be determined from an interpolation of the flux density plotted vs. current.

- Set up and alignment

  The photo of the set up of the equipment and a sketch of the alignment is given in Fig. P-5-3.
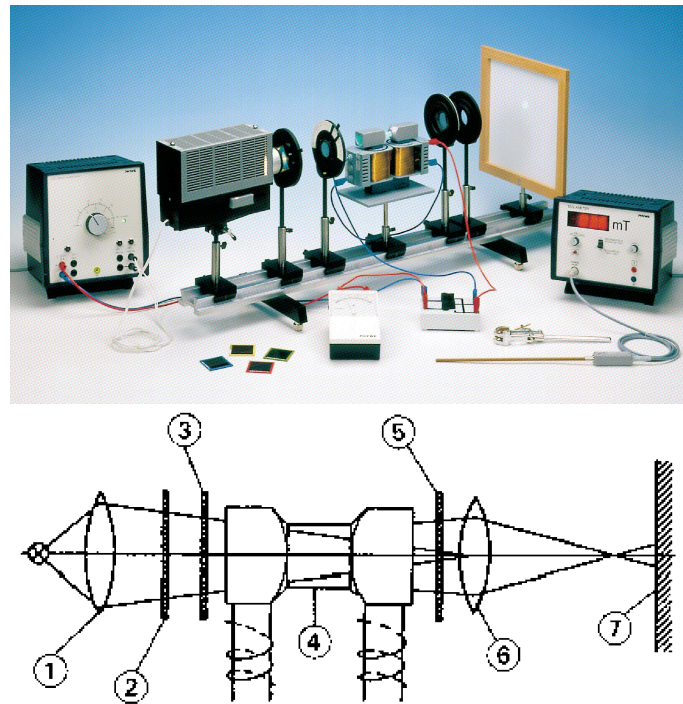

Figure P-5-3: Photo and sketch of experimental setup for observation of faraday effect

After the flux-density distribution without material has been measured, the 30 mm long flint glass cylinder is inserted in the pole piece holes and the table is raised so that the magnet is interpolated in the experimental set-up between the two polarization filters.

The experiment lamp ($\otimes$), fitted with a condensor having a focal length of 6 cm, is fixed on the optical bench. This is followed by the diaphragm holder with colored glass, two polarization filters and a lens holder with a mounted lens of ï¿ = 15 cm. The translucent screen is put in a slide mount at the end of the optical bench. The

ray paths have been traced in Fig. P-5-3. The 50 W experimental lamp is supplied by the 12 V AC constant voltage source. The planes of polarization of the two polarization filters are arranged in parallel. The experiment lamp is switched on and the incandescent lamp moved into the housing until the image of the lamp filament is in the objective lens plane.

The electromagnet is then moved into the path of the image rays and is positioned so that the pole piece holes with the inserted glass cylinder are aligned with the optical axis.

By sliding the objective lens along the optical bench, the face of the glass cylinder is sharply projected onto the translucent screen. Alignment is completed by inserting the colored glass in the diaphragm holder.

The polarizing filter should permanently have a position of $+45°$. In this case the analyzer will have a position of $-45° \pm \Delta\phi$ for perfect extinction with $\Delta\phi$ being a function of the coil current, respectively of the mean flux-density.

The maximum coils current under permanent use is 2 A. However, the current can be increased up to 4 A for a **few minutes** without risk of damage to the coils by overheating.

- Procedure

  If the polarizer and analyzer are crossed, the translucent screen image appears dark. It brightens up when the coil current is switched on and a longitudinal magnetic field is generated between the pole pieces. Adjustment of the analyzer through a certain angle $\Delta\phi$ produces maximum extinction of the light (Position 1).

  If the direction of the magnetic field is reversed by changing the polarity of the coil current, the analyzer must be adjusted in the opposite direction in order to darken the brightened field of view again (Position 2). The difference between position 2 and position 1 of the analyzer is equal to $2 \cdot \Delta\phi$.

  Additional hint: Regarding the judgement about the complete extinction, it may be better to remove the screen and to follow the adjustment of the analyzer by eye-inspection.

Kerr effect

The experiment for the demonstration of the Kerr effect is set up as shown in Fig. P-5-4. The light source is a He/Ne laser with a power of max. 1 mW. The PLZT element is placed between two polarizing filters. As can be seen from this picture, the vertically polarized light from the He/Ne laser impinges on the PLZT element which is set in its holder at $45°$ to the vertical.
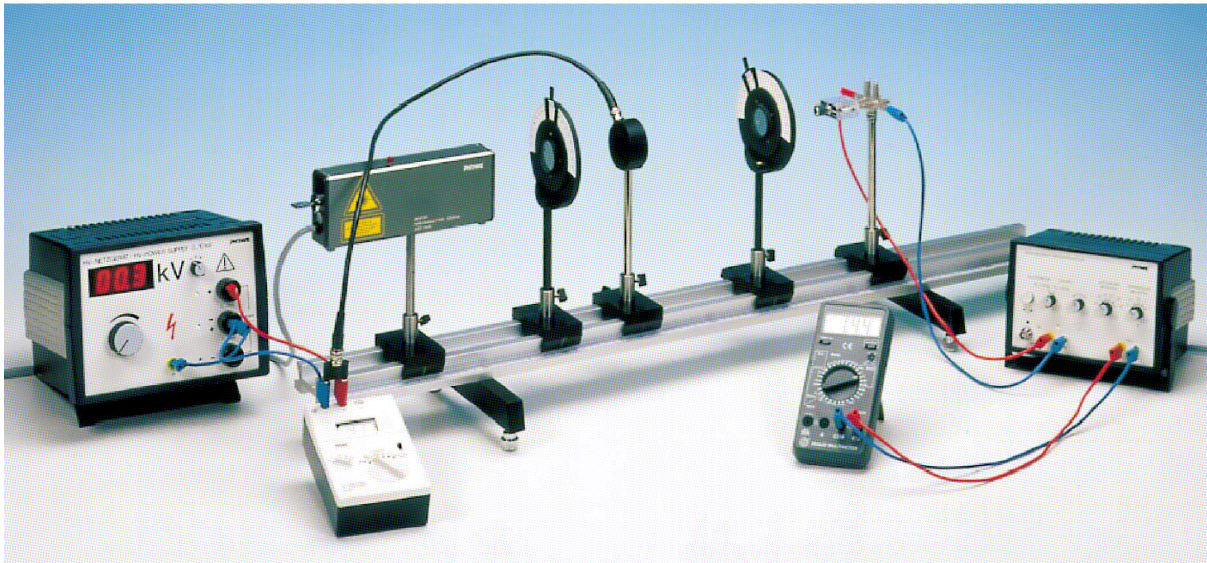
Figure P-5-4: Experimental set-up for demonstrating the Kerr effect.

The PLZT element is connected directly to the HV power supply whose voltage can be altered between 0 and 1200 Volt with a sufficiently high precision by using a digital voltmeter branched in parallel to the HV power supply. A silicon photodiode with amplifier is used as the detector for the luminous intensity behind the analyzer.

The measured values depend on the history of the PLZT-element. Also the number of detectable maxima may vary for different elements. Take sufficient values to elaborate the minima and maxima of the curve.

Safety Warning: Regard the following aspects during measurement in order to get suitable data and to avoid serious damage to you and to the equipment:

- **Do not exceed 1200 V, this will destroy the PLZT element. Take care that the protection earth is connected to the shield of the device.**

- Before performing any measurement of luminous intensity the **He/Ne laser has to be switched on for about one hour** to reach its equilibrium in power emission.

- After changing the voltage applied to the PLZT element, approx. five minutes have to elapse before the crystal structure has readapted and a representative luminous intensity reading can be taken.

- All measurements have to be performed in a darkened room.

- The non-polarized portion of the light (background radiation) must also be taken into account.

Frequently Asked Questions for Self-Monitoring

1. Describe the Faraday effect with induced circular birefringence of transparent dielectric materials.

2. How the brewster angle windows produce the linear polarization in the He-Ne-Laser?

3. Describe the working principle of thin film and birefringent polarizers.

4. Describe the Kerr effect.

5. Find the materials properties of the Kerr (PLZT)-element and discuss it.

6. Draw a sketch of the electrical circuit for light sensing using a photo diode and explain the output signal generation.

Further reading

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  Radiative Transitions, pp. 825–855, esp. the part about laser,
  Wave geometry, pp. 875–907.

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  Probability, pp. 1119–1220,
  Statistics, pp. 1221–1304.

Electron Spin and Nuclear Magnetic Resonance

Key Concepts

magnetic dipole moment, angular momentum, spin, Zeeman effect, resonance condition, paramagnetic materials, nucleon, g-factor, DPPH, Helmholtz coils

Summary

The basic principle of both ESR and NMR is the absorption of high-frequency radiation by materials which are exposed to an external magnetic field. In an external magnetic field spin states split energetically (Zeeman effect) due to the magnetic moment and allow for absorption of high-frequency radiation. For such effect to occur, the material has to possess either total electron (for ESR) or total nuclear (for NMR) magnetic moment that is different from zero value.

> **Preparation:**
> Read the theory part of of the experiment *"Zeeman Effect"* and to try to answer the questions listed there under *"Preparatory Task"*.

Literature

1. H.D. Young and R.A. Freedman, **University Physics**, PEARSON Addison-Wesley, 2008 (chapters 41.2, 41.3, 43.2).

2. M. Alonso and E.J. Finn, **Physics**, Addison-Wesley, 1996 (very basic)

3. H. Haken and H.C. Wolf, **The Physics of Atoms and Quanta**, Springer, 2004

Theory

Electron Spin Resonance (ESR)

Electron spin resonance (ESR) was first discovered by E. K. Zavoisky in 1945. Since then ESR has developed into an important method of investigating molecular and crystal structures, chemical reactions and other problems in physics, chemistry, biology and medicine. ESR is based on the absorption of high-frequency radiation by paramagnetic materials in an external magnetic field which causes an energetic split of the electron spin states.

Electrons in materials possess total angular momentum that comes from two types of rotation: spin (intrinsic form of angular momentum carried by each elementary particle) and orbital motion (rotation of electron around atom nuclei). The spin angular momentum of the electron ($\vec{S}$) is responsible for the spin magnetic moment of the electron:

$$\vec{\mu}_S = -g_e \cdot \mu_B \frac{\vec{S}}{\hbar} \tag{P-6-1}$$

where $\mu_B = \frac{\hbar e}{2m_e}$ is Bohr's magneton, $\hbar = \frac{h}{2\pi}$ and h is Planck's constant, g_e is Landé spin g-factor of the electron, m_e the mass of the electron and e the charge of the electron. The magnitude of the electron spin magnetic moment is given by:

$$|\vec{\mu}_S| = g_e \cdot \mu_B \sqrt{s(s+1)} \tag{P-6-2}$$

where the spin eigenvalue $\left|\vec{S}\right| = \sqrt{s(s+1)} \cdot \hbar$ of the electron with quantum spin number $s = \frac{1}{2}$ was used. In an external static magnetic field $\vec{B}_0$ (which is oriented along z-axis by the definition), the potential energy of the electron spin magnetic moment is given by:

$$E = -\vec{\mu}_S \cdot \vec{B}_0 \tag{P-6-3}$$

The potential energy E is quantized because the z-component of the spin angular momentum (parallel to external magnetic field) and consequently the z-component of the electron spin magnetic moment can only take on discrete values. The z-component of the electron spin angular momentum S_z can only take the values:

$$S_z = m_s \cdot \hbar \quad \text{where formally} \quad m_s = -s, -s+1, \cdots s \tag{P-6-4}$$

m_s is the spin quantum number and only takes half integer values of $m_s = \pm\frac{1}{2}$, since $s = \frac{1}{2}$. The z-component of the spin magnetic moment of the electron is given by (Eqs. (P-6-1) and (P-6-4)):

$$\mu_{S,z} = -g_e \cdot \mu_B \cdot m_s \tag{P-6-5}$$

Each value of the z-component of the electron spin magnetic moment corresponds to a state with a particular potential energy. Therefore, the potential energy splits into the discrete Zeeman levels (Eqs. (P-6-3) and (P-6-5)):

$$E = g_e \cdot \mu_B \cdot m_s \cdot B_0 \tag{P-6-6}$$

Analogous equations are also valid for the orbital angular momentum $(\vec{L})$ and orbital magnetic moment $(\vec{\mu}_L)$ of the electron. The magnitude of the electron orbital angular momentum is given by:

$$\left|\vec{L}\right| = \sqrt{l(l+1)} \cdot \hbar \tag{P-6-7}$$

where l is the orbital quantum number and only takes integer values of $l = 0, 1, \cdots n-1$ with n being the principal quantum number. The principal quantum number is equal to $n = 1 + N$, where N is the number of nodes in the radial wave function of the electrons. The electron orbital magnetic moment and its magnitude are given by:

$$\vec{\mu}_L = -\mu_B \frac{\vec{L}}{\hbar} \tag{P-6-8}$$

$$|\vec{\mu}_L| = \mu_B \sqrt{l(l+1)} \tag{P-6-9}$$

In an external magnetic field, the electron orbital angular momentum can only take discrete values in direction parallel to the field (z-direction):

$$L_z = m_l \cdot \hbar \quad \text{with} \quad m_l = -l, -l+1, \cdots l \tag{P-6-10}$$

where m_l is the magnetic quantum number which can be an integer number or zero. Consequently, the orbital magnetic moment parallel to the static external magnetic field is also quantized. Finally, the potential energy related to the orbital magnetic moment splits into discrete levels:

$$\mu_{L,z} = -\mu_B \cdot m_l \tag{P-6-11}$$

$$E = \mu_B \cdot m_l \cdot B_0 \tag{P-6-12}$$

Electron spin resonance can occur only in paramagnetic materials which are transition metals or rare earth elements (inner electron shells of the atoms are not completely filled) and free radicals (organic molecules that contain individual unpaired electrons). In such materials, spin angular momentum and orbital angular momentum of electrons are coupled in a way that the total angular momentum $\vec{J}$ is different from zero:

$$\vec{J} = \vec{S} + \vec{L} \neq 0 \tag{P-6-13}$$

The magnetic moment associated with the total angular momentum $\vec{J}$ and its magnitude are given by:

$$\vec{\mu}_J = -g_J \cdot \mu_B \frac{\vec{J}}{\hbar} \tag{P-6-14}$$

$$|\vec{\mu}_J| = g_J \cdot \mu_B \sqrt{j(j+1)} \tag{P-6-15}$$

where j is the total angular momentum quantum number with values of $j = l \pm s$, and g_J is Landé g-factor. In an external static magnetic field, the total angular momentum, total magnetic moment and potential energy are quantized. The z-component of the total angular momentum (parallel to the external magnetic field) takes values:

$$J_z = m_j \cdot \hbar \quad \text{with} \quad m_j = -j, -j+1, \cdots j \tag{P-6-16}$$

where m_j is total magnetic moment quantum number. The splitting of energy levels is finally given by:

$$E = g_J \cdot \mu_B \cdot m_j \cdot B_0 \tag{P-6-17}$$

The electron spin resonance is used to measure directly the energy splitting caused by the external magnetic field. The energy difference between two neighboring energy levels is given by:

$$\Delta m_j = \pm 1 \rightarrow \Delta E = g_J \cdot \mu_B \cdot B_0 \tag{P-6-18}$$

To provide the energy ΔE needed to cause a "flip" of the total magnetic moment from one orientation to an other, a high-frequency alternating magnetic field:

$$\vec{B}_1 = \vec{B}_{HF} \cdot \sin(2\pi \cdot \nu \cdot t) \tag{P-6-19}$$

is radiated into the sample perpendicularly to the external static magnetic field $\vec{B}_0$. If the energy $h\nu$ of the alternating field is equal to the energy difference ΔE between two neighboring energy levels (Fig. P-6-1):

$$h\nu = g_J \cdot \mu_B \cdot B_0 \tag{P-6-20}$$

transitions between neighboring energy levels are induced and a resonance absorption of energy from the alternating magnetic field radiated into the sample is observed. In
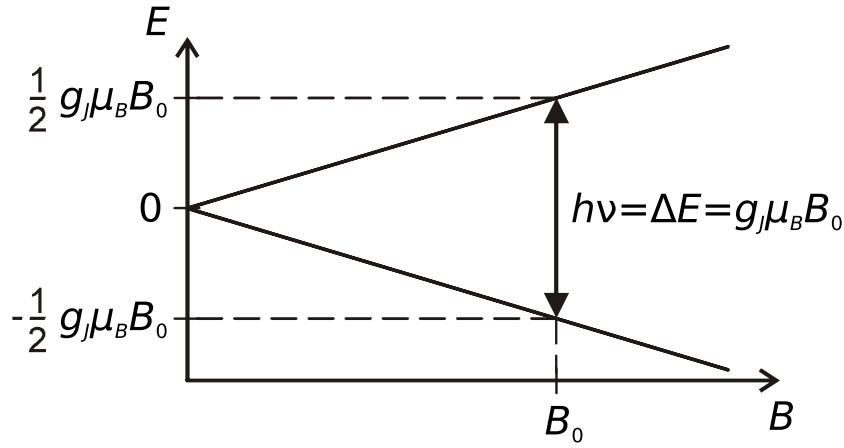

Figure P-6-1: Energy splitting due to the discrete orientation of total magnetic moment in an external magnetic field and resonance condition for electron spin resonance.

numerous compounds, orbital angular momentum is of little significance, and considerations can be limited to the spin of the electrons. For such situation, the Landé factor of the total magnetic moment should be replaced by Landé spin factor of the electron ($g_J = g_e \approx 2.00232$).

Once the resonant absorption of the energy provided by the alternating magnetic field is achieved, the uncertainty of the energy of the transition (δE) can be determined from uncertainty principle:

$$\delta E T \geq \frac{\hbar}{2} \tag{P-6-21}$$

where T is the life-time of the transition state.

By measuring the absorbed energy at a fixed frequency ν as a function of the external static magnetic field B_0, an absorption line with a half-width δB_0 is obtained. From Eq. (P-6-18), the uncertainty of the energy transition is given by:

$$\delta E = g_J \mu_B \delta B_0 \tag{P-6-22}$$

By combining Eqs. (P-6-21) and (P-6-18), the following relation is obtained by using equality in Eq. (P-6-21):

$$T = \frac{\hbar}{2g_J\mu_B\delta B_0} \tag{P-6-23}$$

Equation (P-6-23) shows that the lifetime of the transition level doesn't depend on the frequency ν. In this experiment, Landé factor g_J of the sample and the lifetime of the transition state are evaluated.

Landé factor g_J is determined according to Eq. (P-6-20). In the case of a free atom or ion, the Landé factor lies between $g_J = 1$ if the magnetism is entirely due to orbital angular momentum and $g_J \approx 2.00232$ if only spins contribute to the magnetism. However, the paramagnetic centers studied by means of ESR are inserted into crystal lattices or dissolved in a solvent. In such environment, paramagnetic centers are exposd to strong electric and magnetic fields, which are generated by the surrounding atoms. These fields influence the Zeeman splitting of the electrons, and the value of the g-factor is changed. Landé g-factor often becomes anisotropic, and also a fine-structures can be observed in the ESR spectra. Therefore, the g-factor allows insights into the electron binding and the chemical structure of the sample under consideration.

From the absorption line width, dynamic properties can be determined. The absorption line width is influenced by several processes, which are opposed to an alignment of the magnetic moments. Such processes are spin-spin relaxation and spin-lattice relaxation. The spin-spin relaxation is the interaction between aligned magnetic moments between themselves. The spin-lattice relaxation is the interaction between the magnetic moments and fluctuating electric and magnetic fields, which are caused by lattice oscillations in solids and by thermal motion of the atoms in liquids.

Technical background of ESR

ESR spectrometers developed for practical applications usually work at frequencies of about 10 GHz (microwaves, X-band). The external magnetic fields are in the range of 0.1 to 1 T. In this experiment, the magnetic field B_0 is considerably weaker. It is generated by means of the Helmholtz coils, and can be adjusted to values between 0 and 4 mT by an appropriate current flowing through Helmholtz coils. For easier measurements, a modulated current (50 Hz frequency) is superimposed onto the constant current through Helmholtz coils. The total external magnetic field B_{tot} is therefore composed of a constant field B_0 and a 50 Hz field B_{mod}.

The sample is located in a HF coil which is part of an oscillating circuit. The HF oscillating circuit is excited by a variable frequency oscillator with frequencies between 15 and 130 MHz. If the resonance condition is fulfilled, the sample absorbs energy and the HF oscillating circuit is loaded. As a result, the impedance of the HF oscillating

circuit changes and the voltage at the HF coil decreases. This voltage is converted into the measuring signal by rectification and amplification.

The measuring signal reaches the output of the control unit with a time delay relative to the modulated magnetic field. The time delay can be compensated as a phase shift in the control unit. A two-channel oscilloscope in X-Y-operation displays the measuring signal together with a voltage which is proportional to the magnetic field. The resonance signal can be observed from X-Y figures. The resonance signal is symmetric if the static field B_0 fulfills the resonance condition and if the phase shift φ between the measuring signal and the modulated magnetic field is compensated (see Fig. P-6-2).
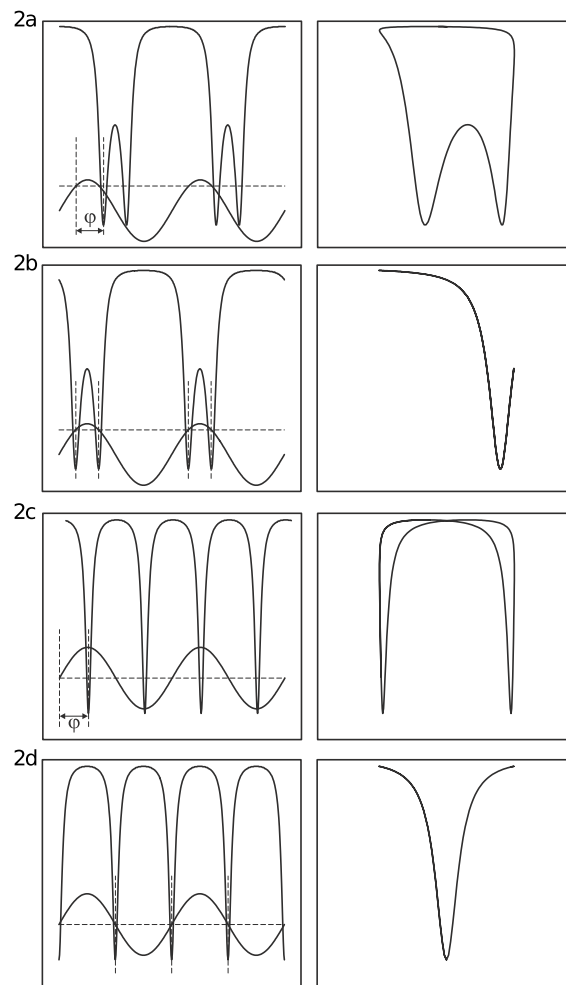

Figure P-6-2: Oscilloscope display of the measuring signal (Y or Channel II) and the modulated magnetic field (X or Channel I). Left: two-channel display with DC coupled channel II, right: X-Y display with AC coupled Channel II. 2a) phase shift φ not compensated, static field B_0 too weak. 2b) phase shift φ compensated, static field B_0 too weak. 2c) phase shift φ not compensated, appropriate statice field B_0. 2d) phase shift φ compensated, appropriate static field B_0.

The sample substance used in this experiment is 1,1-diphenyl-2-picryl-hydrazyl (DPPH). This organic compound is a relatively stable free radical, which has an unpaired valence electron at one atom of the nitrogen bridge (see Fig. P-6-3).
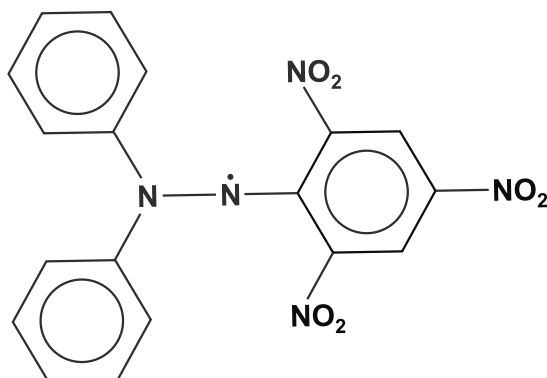
Figure P-6-3: Chemical structure of 1,1-diphenyl-2-picryl-hydrazyl (DPPH).

The orbital motion of the electron is almost canceled by the molecular structure. Therefore the g-factor of the electron is almost equal to that of a free electron. In this polycrystalline form, the substance is very well suited for demonstrating electron spin resonance, because it has an intense ESR line, which, due to exchange narrowing, has a small width.

Nuclear Magnetic Resonance (NMR)

In a very similar way to the electron spin, the total nuclear angular momentum J_N is associated with a magnetic moment, which has the following energy states in an external magnetic field B_0:

$$E = g_{J_N}\mu_N m_{j_N} B_0 \quad \text{with} \quad m_{j_N} = -j_N, -j_N + 1, \cdots j_N \tag{P-6-24}$$

where $\mu_N = 5.051 \cdot 10^{-27} \text{J/T}$ is the nuclear magneton, g_{J_N} is the g-factor of the nucleus, m_{j_N} is the total magnetic moment quantum number of nucleus, and j_N is the total angular momentum quantum number of the nucleus. An orthogonal high-frequency magnetic field with frequency ν applied to the stationary magnetic field excites the transitions between neighboring energy levels when the resonance condition is fulfilled (see Fig. P-6-4):

$$h\nu = E(m_{j_N} + 1) - E(m_{j_N}) \tag{P-6-25}$$

Nuclear magnetic resonance (NMR) is used for the measurement of the magnetic dipole moment of nuclei as well as -vice versa- for the precise measurement of magnetic fields if the dipole moment is known. The internal magnetic fields of crystals can be measured by NMR. The principle is that in a molecule, the magnetic field of the nucleus differs slightly from the outside applied magnetic field. This leads to a small, but precisely measurable shift of the resonance frequency.
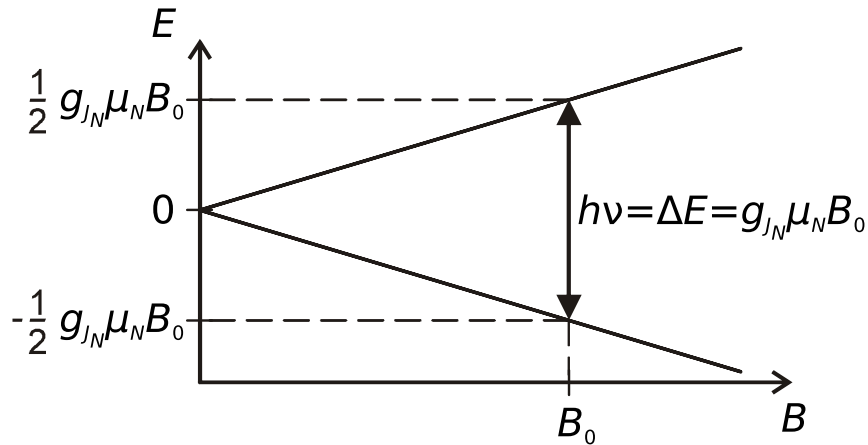

Figure P-6-4: Resonance condition for hydrogen-nuclei.

Equipment

Part	Leybold No.	Quantity
ESR basic unit	51455	1
ESR control unit	51457	1
NMR supply unit	514602	1
NMR kit	514605	1
DC-power supply	521545	1
pair of Helmholtz coils	55506	1
two-channel oscilloscope 303	575211	1
screened cables BNC/4 mm	57524	2
Ammeter $I \leq 3A$	531100	1
Saddle bases	30011	3
Connection cords	50116	5
Sensor Cassy	524 010	1
B-Box	524 038	1
Tangential B-Probe	516 60	1

Set-up and Procedure

ESR experiment

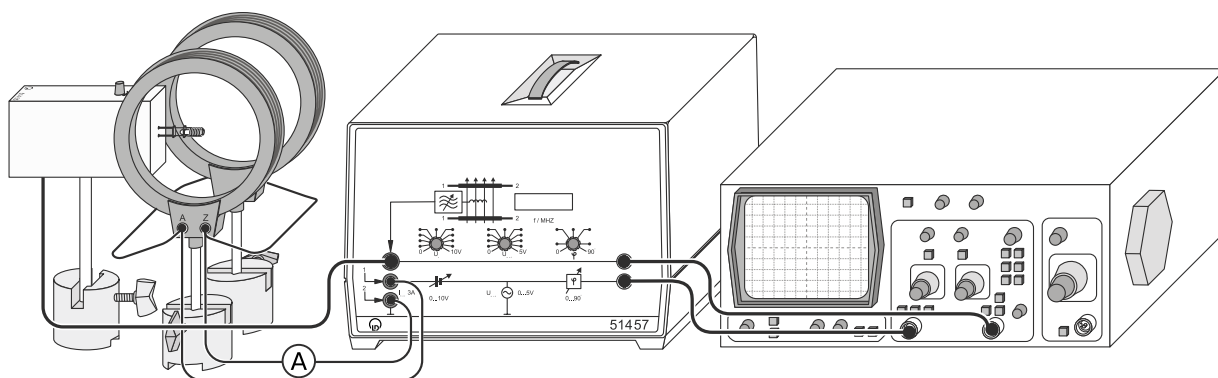The ESR experiment is illustrated in Fig P-6-5 and should be set up according to the following steps:

Figure P-6-5: Experimental set-up for electron spin resonance at DPPH.

1. **Preparation of ESR setup**

   Connect the Helmholtz coils and the ammeter to the ESR control unit according to the cable scheme shown in Figs. P-6-5 and P-6-6. Helmholtz coils are connected

in the parallel, while ammeter measures the total current that flows through both Helholts coils. Connect the ESR basic unit to the ESR control unit via the 6-pole cable. Connect the output **Y** of the ESR control unit to channel II of the two-channel oscilloscope and the output **X** to channel I via BNC cables.

Position the Helmholtz coils parallel to each other at an distance of about 6.8 cm, which is equal to the average radius r. The alignment of the sample between the Helmholtz coils is displayed in detail in Fig. P-6-6.
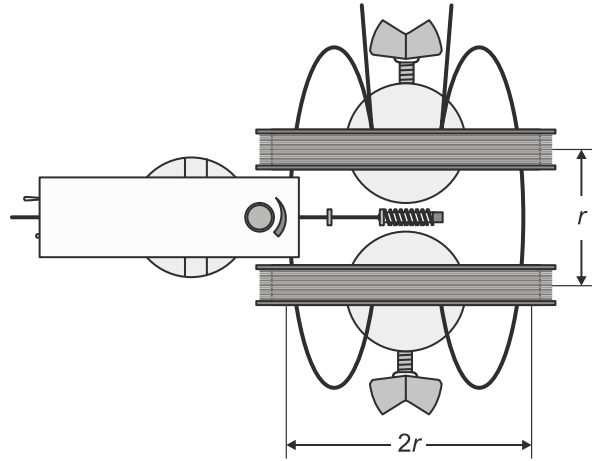

Figure P-6-6: Arrangement of the Helmholtz coils and the ESR basic unit viewed from above.

2. **Determining the resonance magnetic field B_0**

   Connect the plug-in coil 10-30 MHz (coil with largest amount of turns) to the ESR basic unit and insert the DPPH sample so that it is completely in the coil. If possible use gloves while handling the sample. The ESR basic unit should be placed such that the plug-in coil with the DPPH sample is located in the centre of the pair of Helmholtz coils (see Fig. P-6-6). Switch the basic ESR unit on and set the frequency ν to 15 MHz. Set the modulation amplitude U_{mod} to the fourth marking on the scale. Set the phase shift to 0°. Select two-channel operation at the oscilloscope:

   Dual : on, time base : 2 ms/cm, Amplitude I and II : 0.5 V/cm AC

   Slowly enhance the static field at the Helmholtz coils with the voltage U_0 until the resonance signals are equally spaced. Switch the oscilloscope to XY operation and set the phase shift so that the two resonance signals coincide. Vary the direct voltage U_0 until the resonance signal is symmetric. Select a modulation voltage as small as possible. Measure the direct current $2I_0$ through the pair of Helmholtz

coils and note it together with the resonance frequency ν. To measure the current use first the largest range of ammeter (20 A) to avoid burning of the fuse. You can reduce the range of ammeter to 2 A, if the current levels are lower than that. Increase the resonance frequency ν by 5 MHz and adjust the new resonance condition by increasing the direct voltage U_0. Again measure the current $2I_0$ and note it down. Continue increasing the frequency in steps of 5 MHz (use the plug-in coil 30-75 MHz (medium size) for frequencies greater than 30 MHz and the plug-in coil 75-125 MHz (smallest size coil) for frequencies greater than 75 MHz) and repeat the measurements of the direct current $2I_0$.

From the measured values of I_0, the magnetic field B of the Helmholtz coils is calculated as follows:

$$B = \mu_0 \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{n}{r} I \tag{P-6-26}$$

where $\mu_0 = 4\ \pi \cdot 10^{-7}$ H/m, $n = 320$ are the number of turns per coil, $r = 6.8$ cm is the radius of the coil.

The data are evaluated using a plot of ν vs. B_0 to determine the g_J-factor of DPPH from the slope of the resulting curve.

3. **Determination the half-width δB_0**

Select the XY operation at the oscilloscope: Amplitude I and II 0.5 V/cm AC. Adjust the resonance condition for $\nu = 50$ MHz (medium plug-in coil) once more. Extend the resonance signal in the X direction exactly over the total width of the screen (10 cm) by varying the modulation voltage U_{mod}. Switch the ammeter to AC and measure the RMS value of the current $2I_{mod}$, which corresponds to the modulation voltage U_{mod}. Readjust the Y deflection to get signal amplitude that can be easily readable. Make a photo of the signal and by using image processing software read and note the width δU of the resonance signal at half of the maximum of the resonance curve (full width at half maximum (FWHM)). Discuss with lab instructor or teaching assistant about appropriate software tools to do this task. Calculate the uncertainty of the current δI:

$$\delta I = \frac{\delta U}{U_{mod}} I_{mod} \tag{P-6-27}$$

and replace it in Eq. (P-6-26) to get half-width δB_0. Repeat the measurements and calculations for 40 MHz and 60 MHz. Compare the results from all measurements, and discuss the observed behavior. Calculate the errors as well.

The NMR experiment

1. **Preparation of NMR setup**

The experimental set-up of the NMR experiment is shown in Fig. P-6-7. The iron
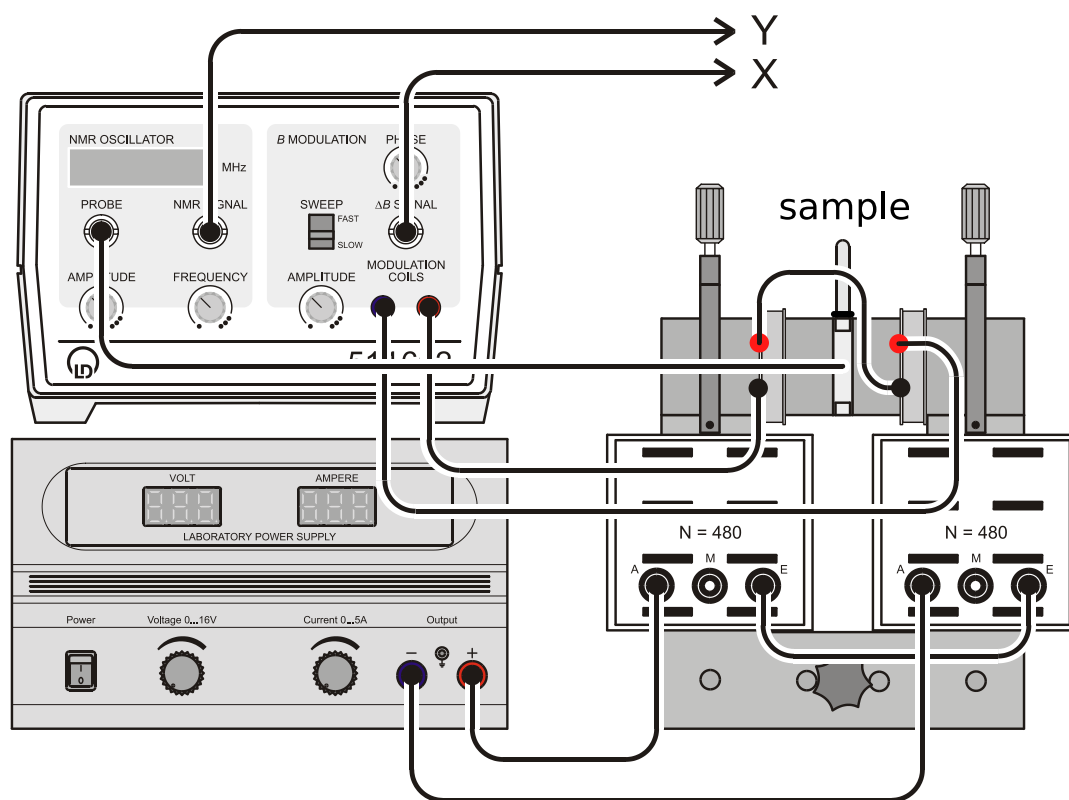
Figure P-6-7: The experimental set-up of the NMR experiment.

core is already placed into the induction coils and the modulation coils are plugged on the ends of the iron core. The samples should be inserted in the sample holder located between the modulation coils. The position of the sample can be adjusted by the rubber ring on it.

The induction coils are connected in series (with opposite direction of windings) to the DC power supply (see Fig. P-6-7). The modulation coils are also connected in series (with the same direction of windings) to the NMR measuring unit.

The ΔB and NMR signal of the operating unit are displayed on channel I and channel II of the oscilloscope, respectively.

2. **Operation of the NMR setup**

   - Switch on DC power supply.

   - For a good signal, the hysteresis curve of the magnet core has to be narrowed. For this, the current through the coils around the metal core is alternately increased and decreased at least 4 times. Afterwards, the coil current is set to approximately 3.1 A.

   - Set the amplitude and frequency of the NMR signal to lowest value (turn the knobs to the right).

- Increase the amplitude of the NMR signal until the red diode above the knob is starts to emit light.

- Increase the frequency of NMR signal until a resonance absorption can be seen. It might be necessary to adjust the amplitude of the NMR signal during the process.

- Optimize the display of the peak by iterative adjustment of amplitude and frequency of the NMR signal as well as the amplitude and phase of B-modulation.

- For determination of the g-factor, the resonance frequency as well as the magnetic field inside the sample holder has to be noted. The magnetic field is measured with a tangential B-probe connected to a Cassy via B-Box.

- Conduct the measurement for 5 different coil currents between 3.1 A and 3.6 A.

Problems and Tasks

1. **The ESR experiment**

    (a) Measure the resonance frequency ν as a function of B_0.

    (b) Determine the g-factor from a plot of ν vs. B_0. Calculate corresponding errors.

    (c) Determine the half-width δB_0 and T lifetime of the transition state for different frequencies. Calculate corresponding errors.

2. **The NMR experiment**

    (a) Optimize the NMR signal for polystyrene ($[C_8H_8]_n$), glycerin ($C_3H_8O_3$) and PTFE ($[C_2F_4]_n$). Measure the resonance frequency of each material as a function of B_0 for at least 5 different values of B_0.

    (b) Determine the g-factor of polystyrene, glycerin and PTFE (teflon) from plots of frequency vs B_0. Determine which nuclei are responsible for the signal in these materials. Hint: calculate the theoretical g-factor from gyromagnetic moment of elements that compose these materials. Calculate corresponding errors.

    (c) **Voluntary Task (5% extra):** Detect the signal of a plant. What causes the signal from a plant?

Nd:YAG Laser

Motivation

The main goal of this experiment is to understand the principles of lasers, in this case a Nd:YAG (Yttrium Aluminium Garnet doped with Neodymium) laser. The Nd:YAG laser of this project experiment follows a modern design. Instead of being pumped by discharge lamps laser diodes are used to excite the active medium. In contrast to the HeNe laser, the Nd:YAG laser is a solid state laser. Solid state lasers are much more efficient than gas lasers and therefore in many cases replace these lasers. Nd:YAG lasers can be used in cw mode (continuous wave; this experiment) or as pulsed lasers. In the MEOS-Manual, which is made available to you for preparation of the experiment, also the so-called Q-switching is described. It will not be part of your experiment, but the information given is of general interest for every physicist.

Key Concepts

Absorption, Spontaneous and Induced Emission, Einstein Coefficients, Population Inversion, Line Broadening, Three-Level and Four-Level System, Meta-Stable States, Rate Equations, Optical Resonator, Resonator Losses, Longitudinal and Transversal Resonator Modes, Stability Criterium, Resonator Types, Gain Profile, Spiking, Laser Diodes, Nd:YAG Term Scheme, Quantum Efficiency and Energy Efficiency, Life Time

Literature

- **Most important and an absolute must for your preparation:**
  Dickmann, *Diodelaser Pumped Nd:YAG Laser*, MEOS Manual, Münster 2003

- Demtröder, *Laser Spectroscopy: Basic Concepts and Instrumentation*, Jacobs University Library – QC454.L3 D46 2003

SAFETY PRECAUTIONS

- **CAUTION: NEVER LOOK DIRECTLY INTO A LASER BEAM**

- You have to be follow all safety regulations taught to you during the laser safety instruction.

- Always stop the laser beam close behind the outcoupling mirror (*e.g.* using the photodiode)

- Alignment only must be performed slightly above the lasing threshold (max. 350 mA).

- For measuring the laser intensity the laser beam must not be focused.

- Always ask the teaching assistant in case of uncertainties.

- Most accidents happen not with direct laser irradiation, but with secondary reflections. Therefore, use safety goggles whenever possible. Do not wear any reflective material, *e.g.* jewelery, watch, belt buckle, etc.

- Always inform other persons present in the laboratory about possible dangers, *e.g.* during alignment of optical elements like mirrors or lenses.

- **Never touch optical surfaces with your fingers. Never remove dirt from these surfaces *e.g.* by using towels or similar things. If you accidentally touch the surface of any optical part or find dirt on it, do not use it in the experiment, but immediately call the TA who will demonstrate to you how the cleaning is done correctly.**

Targets

General remark: The solution to all the tasks mentioned below can be found in the Manual [Ref. 1]. All part needed are described there. You are free to perform further experiments listed in the manual (if parts are available). If you have own ideas for an interesting experiment, please, first consult the teaching assistant or Prof. Materny.

1. Start-up of diode laser and alignment of collimator

2. Measurement of output power of the laser diode as function of injection current for different temperatures

3. Alignment of focusing lens

4. Investigation of the influence of temperature at constant injection current on the emission wavelength; for this make use of the well-known absorption spectrum of the Nd:YAG crystal rod

5. For a fixed temperature determine the characteristic line of the laser diode (power as function of current)

6. Determine the life time of the upper state of the fluorescing state of the Nd atom

7. Optimally align the Nd:YAG laser

8. Measure the Nd:YAG laser power as function of the pump power; what characteristic values can be extracted from the resulting curve?

9. Try to influence the transverse mode structure of the laser output

10. Run the laser with intra-cavity frequency doubling in order to obtain the second harmonic output at 532 nm

11. If the power tuning range allows for it, demonstrate that frequency doubling is a 2nd order nonlinear optical process

12. Additionally, we provide you with a frequency generator, which can be connected to the diode laser control unit for modulation; if you like, then you may investigate dynamic phenomena, like *e.g.* spiking

Experimental set-up and parts

General overview is shown in figure P-7-1.
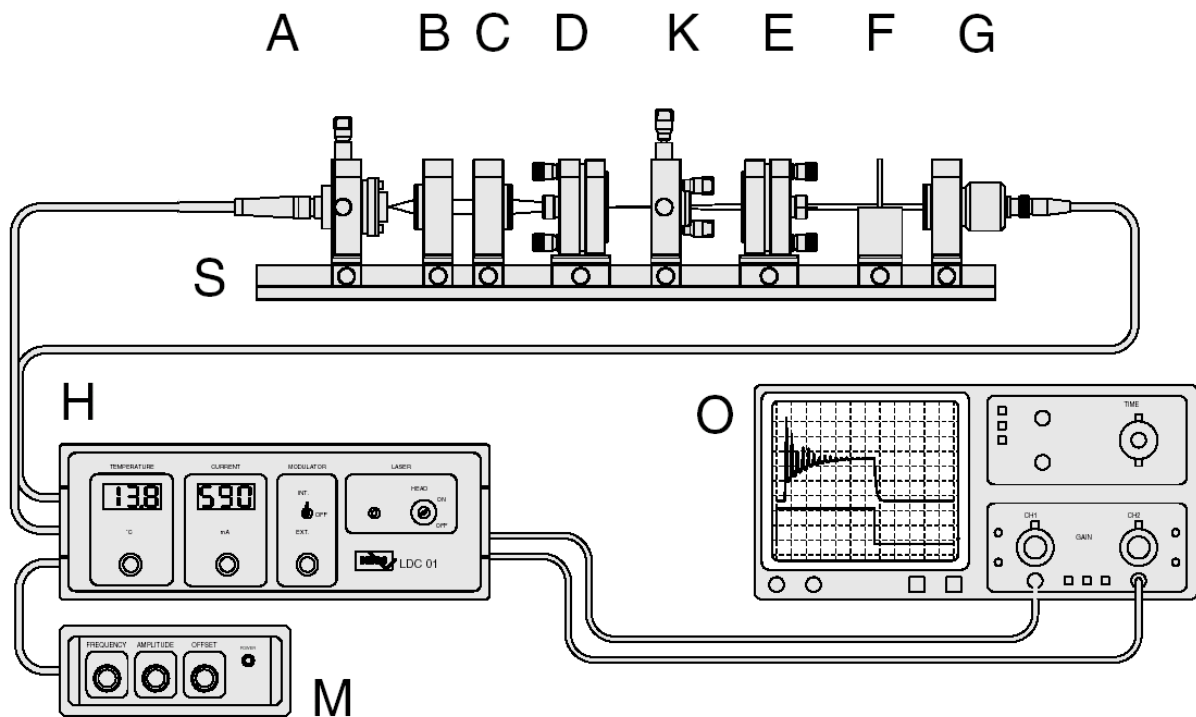The individual modules are:

Figure P-7-1: NdYAG laser setup

A Diodelaser   Precision XY adjustment unit in which a laser diode is mounted. The monitor diode for monitoring the laser output power, a Peltier's cooling element for the control of the diode temperature and a thermistor for the measurement of the temperature are all located inside the laser diode. A warning lamp, which signals the presence of laser radiation, is fitted to the upper side of the module. The diode laser module is connected to the control unit with the connecting lead.

Under no circumstances should this cable be disconnected when the controller LDC01 is powered. This can destroy the expensive laserdiode. For this reason the fixing screws of the connector to the controller are sealed. No guaranty will be granted with a broken seal

B Collimator   The collimator consists of a three-lens system with a short focal length (f=6mm) and a large aperture in order to collimate the strongly divergent laser diode beam.

C Focussing   This unit has the duty to focus the collimated diode laser beam into the YAG rod which is mounted in Module D. The lens has a focal length of 60 mm.

D Mirror+rod   Module D and E form the resonator of the Nd-YAG laser. The adjustable holders have the duty of adjusting the relevant resonator mirror so that the common optical axis is aligned perpendicular to the mirrors. The plane-parallel YAG rod, which is 5 mm long and has a diameter of 5 mm, is located in an exchangeable mount in

Module D. A coating, which is highly reflective at the laser wavelength of 1064 nm, has been vapour deposited onto one end of the rod that also forms the left resonator mirror. The vapour deposited system of layers is designed such that the maximum pump-light radiation can penetrate the highly reflective layer with only 20% losses. The other end of the rod has a vapour deposited, high-quality antireflex layer for 1064 nm in order to keep the internal resonator losses as low as possible. In addition the back side of the YAG rod is coated with a high reflective layer for 532 nm in order to redirect the green light to the output of the resonator. Fine-pitch threaded screws can be set by adjusting knobs which enable the rod holder to be tilted exactly and reproducibly to the optical axis. The laser rod is mounted in a fixture which can be screwed out to allow it to be cleaned.

K f-doubler (optional part) The second harmonic at 532 nm of the fundamental wave at 1064 nm is generated by means of a KTP crystal. The crystal has a size of 3×3×3 mm and is fitted into a special holder to allow the cleaning from time to time. The fitting is mounted into a holder which is inserted into the 5 axes adjustment unit. The module K is additionally supplied with a mounted laser mirror marked as SHG100. This mirror will be used instead of the output coupler(R100-2%) and has a radius of curvature of 100 mm as well as a high reflective coating of >99.98 % to keep as much power as possible of the fundamental wave inside the resonator. Also supplied to the module K is a BG39 filter, which suppresses all infrared radiation including the laser diode radiation and lets the green radiation with a transmission of app. 60 % pass.

F Filter The filter plate holder is delivered with two filters. The colour filter RG1000 suppresses the pumping radiation of 808 nm. The second filter BG39 allows only the radiation of 532 nm to pass.

E Mirror This is the second resonator mirror. Also as with the laser rod, it can be screwed out with its fixture from the holder. The mirror has a diameter of 1/2 inch and a radius of curvature of 100 mm.

G Detector A PIN-photodiode mounted in a housing with click-mechanism and BNC-socket. The inner pin of the BNCsocket is connected to the anode. By means of the attached BNC-cable the detector is connected to the amplifier of module H. A target-screen has been mounted in a click-mount to check on the optical axis during the basic adjustment. If required it can be inserted into the mounting plate of the photodetector.

H Controller The controller unit LDC01 provides reliable and save operation of the expensive laserdiode. It contains the control circuits for power monitoring, temperature con-

trol and current setting. The values for the temperature and the diode current can be read off from two large-format LED displays on the front panel of the unit. The desired values for the temperature and current can be set with precision multi-turn potentiometers. At BNC sockets on the rear of the device analogue output signals of the temperature and current as well the synchronising signal of the internal modulator and the photodiode amplifier of the external photo detector are provided. Via a BNC socket an external modulator can be connected. In addition the controller contains an internal modulator for modulating the laser diode output power for investigation of the dynamic behaviour of the laser as spiking, measuring of lifetimes etc.

M Modulator By means of this modulator, a saw-tooth shaped voltage is generated and applied to the controller unit LDC-01 via its modulation input by a BNC-cable supplied with the module. This module allows the linear modulation of the injection current and of the laserdiodes output power. In this way, a multitude of functional dependencies can be represented on the oscilloscope. The modulating frequency can be adjusted between 10 Hz and 10 kHz. Amplitude and off-set can be adjusted between 0 and 5 Volt. The intensity of the fundamental wave should be as high as possible for an efficient frequency doubling. The module K has to be placed into the resonator. Please make sure that the crystal with its end surfaces is pre-adjusted, approximately vertically and centrally to the resonator axis (eye measurement). According to how well adjusted the doubling crystal is, a green radiation will come out of the exit of the laser. When the adjustment has been carried out to a maximum exiting capacity for the green radiation, the injection current of the laser diode is varied. This is for measurement of the relation $P_{2v} = F(P_v)$.

O Oscillosc. Oscilloscope to visualize modulation and detector signal versus time.

- Aperture (optional) Instead of module K a mode aperture (= adjustable iris) can be inserted to control the number of transverse modes. The adjustable iris can be set to a desired diameter in order to reduce the number of transverse modes. The smallest diameter is about 0.8 mm and is sufficient to force the laser into TEM_{00} mode. The way the iris works can be impressively demonstrated in connection with the second harmonic generation.

A more detailed description is given in Ref. 1 (see escpecially experimental set-up and parts shown on pages 25, 26, 33, and 34).
Additionally, a frequency generator will be required for dynamics experiments (task 12).