92%

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Dr. Moe
ASTR5420
Worked with Alex
I took about 5 hours to complete this

Please show all work. If you collaborate with other students, please write their names at the top of your homework. If you use any code or plotting routines, print those out and attach them to your hand-written solutions.

1. **(10%): Imagine if our Sun was born in a stellar nursery that was not enriched from nucleosynthesis produced by Type Ia supernovae (exploding white dwarfs in binaries). Assume the gas is completely devoid of Si, S, Ca, and Fe. Use Table 1.6 of the textbook to calculate the resulting metallicity of this theoretical star. Express you answer in dex, i.e., log($Z/Z_\odot$). This question should demonstrate that although the most prominent metal absorption lines of solar- type stars include Si, Ca, and Fe and that [Fe/H] is commonly measured to trace stellar metallicities, the bulk metallicity of stellar atmospheres actually comes from a elements like C, O, Ne, and Mg.**

x10

Table 1.6

Element	N_{elem}/N_{tot}
H	9.097×10^{-1}
He	8.890×10^{-2}
O	7.742×10^{-4}
C	3.303×10^{-4}
Ne	1.119×10^{-4}
N	1.021×10^{-4}
Mg	3.458×10^{-5}
Si	3.228×10^{-5}
Fe	3.154×10^{-5}
S	1.475×10^{-5}

$$Z = \sum \frac{n_i}{N} m_i$$

$$Z_* = \frac{7.742\times10^{-4}\,(15.999) + 3.303\times10^{-4}\,(12.011) + 1.119\times10^{-4}\,(20.180) + 1.021\times10^{-4}\,(14.007) + \ldots}{9.097\times10^{-1}\,(1.008) + 8.890\times10^{-2}\,(4.003) + 7.742\times10^{-4}\,(15.999) + 3.303\times10^{-4}\,(12.011) + \ldots}$$

$$Z_* = 0.0161$$ ✓

$$Z_\odot = \frac{7.742\times10^{-4}\,(15.999) + \ldots + 3.228\times10^{-5}\,(28.086) + 3.154\times10^{-5}\,(55.933) + 1.475\times10^{-5}\,(32.066)}{9.097\times10^{-1}\,(1.008) + 8.890\times10^{-2}\,(4.003) + 7.742\times10^{-4}\,(15.999) + 3.303\times10^{-4}\,(12.011) + \ldots}$$

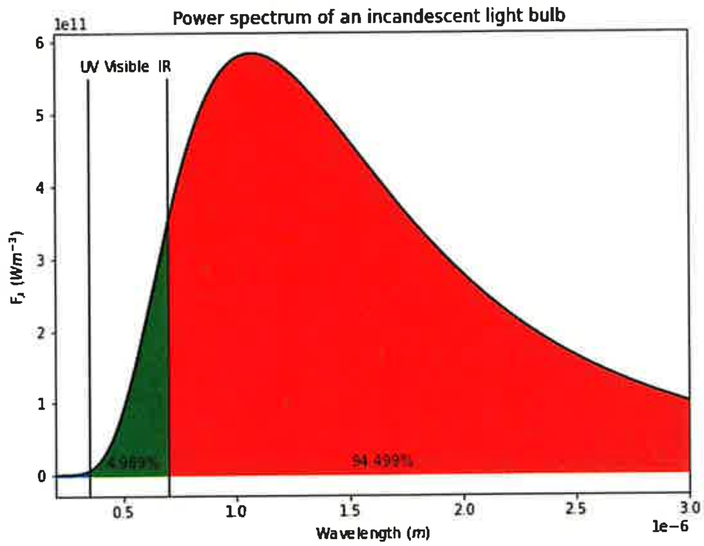
$$Z_\odot = 0.0185$$ ✓

$$\cancel{Z = \log\left(\frac{Z_*}{Z_\odot}\right)}$$ no.

$\log\left(\frac{Z_*}{Z_\odot}\right) =$

$$\boxed{Z_* = -0.0598 \text{ dex}}$$ ✓

Although we ignored the metals that are apparent in spectra and metals that are good indicators of the overall metallicity, Fe, we see that the metallicity of this star with no starting Si, Fe, S and Ca is mostly unchanged. It is slightly metal-poor, as we'd expect, but only very slightly. It is apparent that the bulk of a star's metallicity comes from light α-process elements; C, O, Ne, and Mg.



The calculations for this question were done in python numerically. Above is my code in jupyter notebooks as well as the plot generated from the code.

For an incandescent bulb at 2700K emitting as a blackbody, approximately 94.5% of the radiated power is emitted in wavelengths beyond 7000Å while only $\sim$ 5% of the power is in the visible range. An LED bulb would therefore use approximately 5% of the power of an incandescent bulb, since the incandescent bulb wastes so much energy emitting in the IR. To match the power output in visible wavelengths of a 60W incandescent bulb, an LED bulb would use approximately 3W.

x10

3. **(10%): The Lyman (n = 1), Balmer (n = 2), Paschen (n = 3), and Brackett (n = 4) series of hydrogen occur in the UV, visible, near-infrared, and mid-infrared, respectively. "Excited" about the images from the James Webb Space Telescope, you thought you should learn more about mid-infrared astronomy. Calculate the α, β, and γ transitions of the Brackett series in both eV and μm.**

Bracket: $n_f = 4$.
α, $\Delta n = 1$:

switch, so ΔE is positive ↓ (photons have pos energy)

$$E_n = -\frac{13.6}{n^2}\text{eV}$$

$$\Delta E = \left(-\frac{13.6}{(n_f)^2}\right) - \left(-\frac{13.6}{(n_i)^2}\right) \text{ eV}$$

$$\Delta E = \left(-\frac{13.6}{(4)^2}\right) - \left(-\frac{13.6}{(5)^2}\right) \text{ eV}$$

$$\boxed{\Delta E = -0.306 \text{ eV}}$$

✓

$$-\Delta E = \frac{hc}{\lambda}$$

$$\lambda = -\frac{hc}{\Delta E}$$

$$\lambda = -\frac{6.636 \times 10^{-34}\text{Js}\,(3 \times 10^8 \text{ m/s})}{(-0.306 \text{ eV})\,\frac{1.6\times10^{-19}\text{ J}}{1\text{ eV}}} \times \frac{10^6\ \mu\text{m}}{1\text{ m}}$$

significantly different from each other.

$$U_I = \sum_j g_j \exp\left(-\frac{E_j}{kT}\right)$$

$$U_I = \sum_j 2n^2 \exp\left(-\frac{13.6\left(1-\frac{1}{n^2}\right)}{kT}\right)$$

Code:

```
In [6]: T = [4000, 10000, 40000]
        U1 = ([np.sum([2*((i+1)**2)*(np.exp(-(13.6*1.6*10**-19*(1-(i+1)**-2))/(1.38*10**-23*T[ii])))\
                        for i in range(5)]) for ii in range(len(T))])
        print(U1)

        [2.000000000001172, 2.000098716548728, 4.8881488672548095]
```

4000 K:

$$\boxed{U_I = 2.0000}$$

10000 K:

$$\boxed{U_I = 2.0000}$$

40000 K:

$$\boxed{U_I = 4.8881}$$

These numbers are not significantly different from each other because the at these temperatures, the electron in the Hydrogen atom is limited to the ground state, 1s, and the partition function is very close to 2. In the case of the hottest star, the next highest state, 2s, becomes readily available as well as other higher energy states hence why the partition function is greater than 4.

(b) **Calculate the fraction f_I of hydrogen atoms that are neutral in the photospheres of the three stars. Assume a pure hydrogen composition and an electron density of $n_e = 5 \times 10^{15}$ cm^{-3}.**

$$\cancel{f_I} = \frac{n_I}{n_{II}} = \left(2\frac{1}{n_e}\frac{U_{II}}{U_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(\frac{-13.6\text{ eV}}{kT}\right)\right)^{-1}$$

ratio $\quad f_I = \frac{n_I}{n_I + n_{II}} \quad f_I = \frac{\frac{n_I}{n_{II}}}{\frac{n_I}{n_{II}} + 1}$

Code:

```
In [42]: ne = 5*10**15
         fI = [((100**-3)*(1/ne)*((2*np.pi*9.11*10**-31*1.38*10**-23*T[i])/((6.63*10**-34))**2)**1.5*((2)/(U1[i]))\
               *np.exp((-13.6*1.6*10**-19)/(1.38*10**-23*T[i])))**-1 for i in range(len(U1))]
         print(fI)

         [1081605691866.6566, 14.627556055505707, 3.267160493584352e-05]
```

4000 K:

$$\boxed{\cancel{f_I =} 1.08 \times 10^{12}} = \frac{n_I}{n_{II}}$$

-3

10000 K:

$$\boxed{\cancel{f_I =} 14.6} = \frac{n_I}{n_{II}}$$

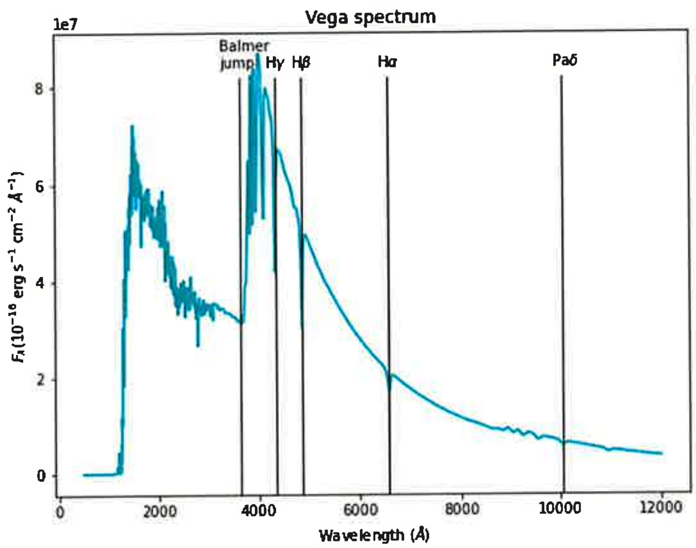
40000 K:

$$\boxed{\cancel{f_I =} 3.27 \times 10^{-5}} = \frac{n_I}{n_{II}}$$

Using the Saha equation, the fraction of neutral hydrogen to ionized hydrogen was found for each star. In accordance with what we expected, the 4000K had the most neutral hydrogen with each hotter star getting a lower f_I.

5. **(45%) Download the spectrum of Vega from WyoCourses. The first column is wavelength λ in Å, the second column is flux density F_λ in units of 10^{-16} ergs^{-1}cm^{-2}Å^{-1}, and the third column is flux density in milli-Janskys**

   (a) **Plot the spectrum and label Hα, Hβ, Hγ, the Balmer jump, and Paδ.**



Code:

```
In [11]: wavl, flux = np.loadtxt('/d/users/nikhil/Downloads/VegaSpectrum.txt', usecols=(0, 1), dtype = float, unpack = True)
         Bmodel = (2*10**-8*6.63*10**-27*(3*10**10)**2)/(wavl*10**-8)**5/\
             (np.exp(6.63*10**-27*3*10**10/(wavl*10**-8*1.38*10**-16*11000))-1)

In [35]: f = plt.figure()
         f.set_figwidth(8)
         f.set_figheight(6)
         plt.plot(wavl, flux, 'c')
         plt.xlabel(r'Wavelength ($\AA$)')
         plt.axvline(x = 6563, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.axvline(x = 4861.35, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.axvline(x = 4340.472, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.axvline(x = 3646, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.axvline(x = 10050, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.text(6563-200, 8.4*10**7, r'H$\alpha$')
         plt.text(4861.35-200, 8.4*10**7, r'H$\beta$')
         plt.text(4340.472-200, 8.4*10**7, r'H$\gamma$')
         plt.text(3646-400, 8.4*10**7, 'Balmer\njump')
         plt.text(10050-200, 8.4*10**7, r'Pa$\delta$')
         plt.ylabel(r'$F_\lambda (10^{-16}$ erg s$^{-1}$ cm$^{-2}$ $\AA^{-1})$')
         plt.title('Vega spectrum')
         plt.savefig('/d/www/nikhil/public_html/ASTR5420/Vegaspectrum.png')
```

   (b) **Calculate the bolometric flux of Vega. Considering Vega is 7.7 pc away, measure the luminosity of Vega (in both erg s^{-1} and L$\odot$).**

$$f = \int_0^\infty f_\lambda d\lambda$$

Code:

```
In [36]: Boloflux = np.trapz(flux, wavl)
         print('Bolometric Flux: ' +str(Boloflux*10**-16) +' erg/scm^2')
         print('Vega Luminosity: ' +str(np.round(Boloflux*10**-16*4*np.pi*(7.7*3.08*10**18)**2,3)) +' erg/s')
         print('Vega Luminosity: ' +str(np.round(Boloflux*10**-16*4*np.pi*(7.7*3.08*10**18)**2/(3.828*10**33),3)) +' L')

Bolometric Flux: 2.8059084627476708e-05 erg/scm^2
Vega Luminosity: 1.9831987785892643e+35 erg/s
Vega Luminosity: 51.808 L
```

$$\boxed{f = 2.806 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2}}$$

Assume $r_{Vega} = 7.7$ pc.

```
In [38]: Tvega = 11000
         R = ((Boloflux*10**-16*4*np.pi*(7.7*3.08*10**18)**2)/(4*np.pi*Tvega**4*5.67*10**-5))**.5
         print('Radius: ' +str(R) +' cm')
         print('Radius: ' +str(R/(6.957*10**10)) +' Rsun')

         Radius: 137880085200.84537 cm
         Radius: 1.9818088210557046 Rsun
```

$\boxed{R = 1.38 \times 10^{11} \text{ cm}}$ $\boxed{R = 1.98 R_\odot}$

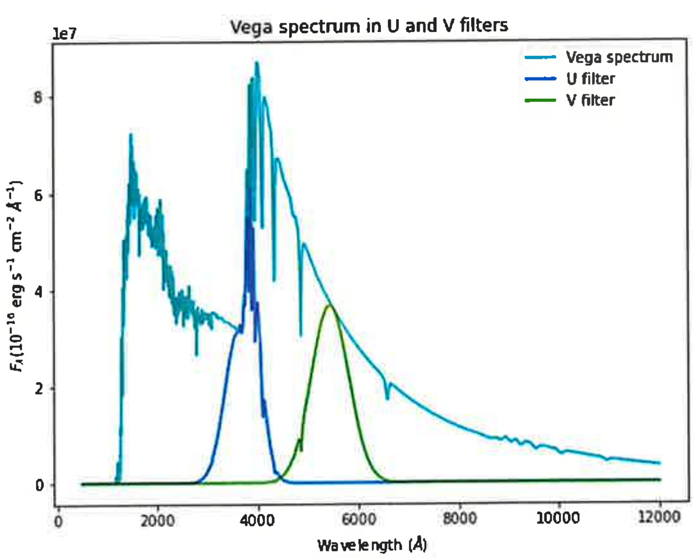
(e) **Calculate the flux of Vega in the U and V filters (in erg s-1 cm-2). Assume both passbands can be described by Gaussians with full throughput at their centers, where U has a central wavelength of 3650 Å and dispersion of 280 Å while V has a central wavelength of 5510 Å and dispersion of 370 Å.**

To measure the flux in U and V filters, I created a Gaussian function centered on the U and V wavelengths with dispersions corresponding to each filter. I then multiplied the Vega spectrum by the U and the V Gaussians to represent how much flux is captured in each filter. Integrating over all wavelengths for each filter should give the flux measured in each filter.

Code:

```
In [39]: f = plt.figure()
         f.set_figwidth(8)
         f.set_figheight(6)
         U = np.exp(-(wavl-3650)**2/(2*280**2))
         V = np.exp(-(wavl-5510)**2/(2*370**2))
         plt.plot(wavl, flux, 'c', label = 'Vega spectrum')
         plt.plot(wavl, U*flux, 'b', label = 'U filter')
         plt.plot(wavl, V*flux, 'g', label = 'V filter')
         plt.xlabel(r'Wavelength ($\AA$)')
         plt.ylabel(r'$F_\lambda (10^{-16})$ erg s$^{-1}$ cm$^{-2}$ $\AA^{-1})$')
         plt.title('Vega spectrum in U and V filters')
         plt.legend()
         Uflux = np.trapz(U*flux, wavl)*10**-16
         Vflux = np.trapz(V*flux, wavl)*10**-16
         print('U flux: ' +str(Uflux) +' erg/scm^2')
         print('V flux' +str(Vflux) +' erg/scm^2')
         fU0 = 417.5*360*10**-11
         fV0 = 363.1*550*10**-11
         mU = -2.5*np.log10(Uflux/fU0)
         mV = -2.5*np.log10(Vflux/fV0)
         print('U mag: ' +str(np.round(mU,3)))
         print('V mag: ' +str(np.round(mV,3)))
         print('U-V: ' +str(np.round(mU-mV,3)))
         plt.savefig('/d/www/nikhil/public_html/ASTR5420/vegauvfilters.png')

         U flux: 3.14591999169175e-06 erg/scm^2
         V flux3.397404892615517e-06 erg/scm^2
         U mag: -0.802
         V mag: -0.577
         U-V: -0.225
```



$\boxed{f_U = 3.15 \times 10^{-6} \text{ erg s}^{-1} \text{ cm}^{-2}}$ $\boxed{f_V = 3.40 \times 10^{-6} \text{ erg s}^{-1} \text{ cm}^{-2}}$

(f) **Measure the U-V color of Vega (in mag). This is roughly the offset between the Vega and ABmag photometric systems. Also, the colors derived from integrating a spectrum are typi-**

(h) **Report the difference Δ(U-V) in mag between the Vega and blackbody spectra. Which one is redder? This effect is called line-blanketing, whereby UV photons are more likely to be absorbed in the photospheres of stars and then re-emitted at longer wavelengths.**

$$\Delta(U-V) = (-0.225) - (-0.835)$$
$$\boxed{\Delta(U-V) = 0.61}$$

Two errors cancelled when taking the difference

Vega is redder than the equivalent blackbody with T_{eff}. This makes sense because as explained in the question, bluer photons from the star are absorbed and re-emitted in redder wavelengths, making the spectrum of the star redder than the predicted black body.