92%

Nikhil Patten
2 September 2022
Dr. Moe
ASTR5420
Worked with Alex
I took about 5 hours to complete this

Please show all work. If you collaborate with other students, please write their names at the top of your homework. If you use any code or plotting routines, print those out and attach them to your hand-written solutions.

x10

1. **(10%): Imagine if our Sun was born in a stellar nursery that was not enriched from nucleosynthesis produced by Type Ia supernovae (exploding white dwarfs in binaries). Assume the gas is completely devoid of Si, S, Ca, and Fe. Use Table 1.6 of the textbook to calculate the resulting metallicity of this theoretical star. Express you answer in dex, i.e., log($Z/Z_\odot$). This question should demonstrate that although the most prominent metal absorption lines of solar- type stars include Si, Ca, and Fe and that [Fe/H] is commonly measured to trace stellar metallicities, the bulk metallicity of stellar atmospheres actually comes from a elements like C, O, Ne, and Mg.**

Table 1.6

Element	N_{elem}/N_{tot}
H	9.097×10^{-1}
He	8.890×10^{-2}
O	7.742×10^{-4}
C	3.303×10^{-4}
Ne	1.119×10^{-4}
N	1.021×10^{-4}
Mg	3.458×10^{-5}
Si	3.228×10^{-5}
Fe	3.154×10^{-5}
S	1.475×10^{-5}

$$Z = \sum \frac{n_i}{N} m_i$$

$$Z_* = \frac{7.742\times10^{-4}(15.999) + 3.303\times10^{-4}(12.011) + 1.119\times10^{-4}(20.180) + 1.021\times10^{-4}(14.007) + \ldots}{9.097\times10^{-1}(1.008) + 8.890\times10^{-2}(4.003) + 7.742\times10^{-4}(15.999) + 3.303\times10^{-4}(12.011) + \ldots}$$

$$Z_* = 0.0161$$

$$Z_\odot = \frac{7.742\times10^{-4}(15.999) + \ldots + 3.228\times10^{-5}(28.086) + 3.154\times10^{-5}(55.933) + 1.475\times10^{-5}(32.066)}{9.097\times10^{-1}(1.008) + 8.890\times10^{-2}(4.003) + 7.742\times10^{-4}(15.999) + 3.303\times10^{-4}(12.011) + \ldots}$$

$$Z_\odot = 0.0185$$

$$\cancel{Z = \log\left(\frac{Z_*}{Z_\odot}\right)}$$ no.

$$\log\left(\frac{Z_*}{Z_\odot}\right) =$$

$$\boxed{Z_* = -0.0598 \text{ dex}}$$

Although we ignored the metals that are apparent in spectra and metals that are good indicators of the overall metallicity, Fe, we see that the metallicity of this star with no starting Si, Fe, S and Ca is mostly unchanged. It is slightly metal-poor, as we'd expect, but only very slightly. It is apparent that the bulk of a star's metallicity comes from light α-process elements; C, O, Ne, and Mg.

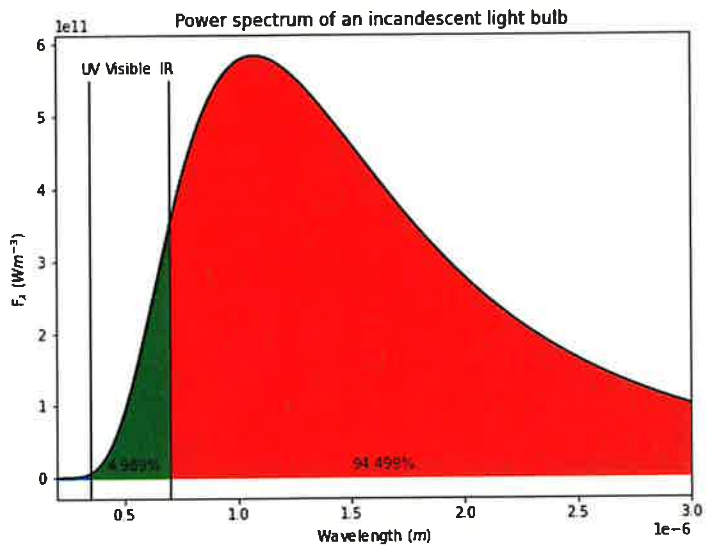
2. (10%): **Consider a 60-Watt incandescent light bulb that emits a blackbody spectrum with a temperature of 2,700 K. What fraction of the power is radiated in the infrared beyond 700 nm? If you replaced the incandescent light bulb with a modern LED that emits only in the visible, how much power (in W) would you need to maintain the same level of visual brightness?**

X10

Python code:

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        from scipy.integrate import quad

In [2]: h = 6.63*10**-34
        c = 3*10**8
        k = 1.38*10**-23
        T = 2700
        l = np.linspace(1*10**-8, 10**-2, 10**7)

In [3]: B = ((2*h*c**2)/(l**5))*((1)/(np.exp((h*c)/(l*k*T)) -1))

In [5]: def blambda(l):
            h = 6.63*10**-34
            c = 3*10**8
            k = 1.38*10**-23
            T = 2700
            return ((2*h*c**2)/(l**5))*((1)/(np.exp((h*c)/(l*k*T)) -1))

In [21]: f = plt.figure()
         f.set_figwidth(8)
         f.set_figheight(6)
         iir = l > 7000*10**-10
         iiuv = l < 3500*10**-10
         iivis = (l >= 3500*10**-10) & (l <= 7000*10**-10)
         plt.plot(l, B, 'k')
         plt.xlim(2000*10**-10, 30000*10**-10)
         plt.xlabel(r"Wavelength ($m$)")
         plt.ylabel('F$_\lambda$ ($Wm^{-3}$)')
         plt.fill_between(l[iir], y1 = B[iir], y2 = 0, color = 'red')
         plt.fill_between(l[iiuv], y1 = B[iiuv], y2 = 0, color = 'blue')
         plt.fill_between(l[iivis], y1 = B[iivis], y2 = 0, color = 'green')
         plt.axvline(x = 7000*10**-10, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.axvline(x = 3500*10**-10, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.text(7000*10**-10 - 3.75*10**-8, 5.6*10**11, 'IR')
         plt.text(3500*10**-10 - 3.75*10**-8, 5.6*10**11, 'UV')
         plt.text(4600*10**-10 - 3.75*10**-8, 5.6*10**11, 'Visible')

         a = 7000*10**-10
         b = np.Inf
         c = 3500*10**-10
         I1 = np.pi*(quad(blambda, a, b)[0])
         I2 = np.pi*(quad(blambda, c, a)[0])
         Ptotal = 5.67*10**-8 *(2700**4)
         print('Ratio of power emittied beyond 7000 Angstroms: ' +str(np.round((I1)/(Ptotal),3)))
         print('Power needed to maintain the same visual brightness: ' +str(np.round(I2*60/Ptotal, 3)) +' W')

         plt.text(4600*10**-10 - 3.75*10**-8, 0.1*10**11, str(np.round(100*I2/Ptotal, 3)) +'%')
         plt.text(1.5*10**-6, 0.1*10**11, str(np.round(100*I1/Ptotal, 3)) +'%')
         plt.savefig('/d/www/nikhil/public_html/ASTR5420/powerspectrum.png')

         Ratio of power emittied beyond 7000 Angstroms: 0.945
         Power needed to maintain the same visual brightness: 2.993 W
```

Plot of incandescent light bulb:



The calculations for this question were done in python numerically. Above is my code in jupyter notebooks as well as the plot generated from the code.

For an incandescent bulb at 2700K emitting as a blackbody, approximately 94.5% of the radiated power is emitted in wavelengths beyond 7000Å while only $\sim$ 5% of the power is in the visible range. An LED bulb would therefore use approximately 5% of the power of an incandescent bulb, since the incandescent bulb wastes so much energy emitting in the IR. To match the power output in visible wavelengths of a 60W incandescent bulb, an LED bulb would use approximately 3W.

3. **(10%): The Lyman (n = 1), Balmer (n = 2), Paschen (n = 3), and Brackett (n = 4) series of hydrogen occur in the UV, visible, near-infrared, and mid-infrared, respectively. "Excited" about the images from the James Webb Space Telescope, you thought you should learn more about mid-infrared astronomy. Calculate the α, β, and γ transitions of the Brackett series in both eV and μm.**

x10

Bracket: $n_f = 4$.
α, $\Delta n = 1$:

switch, so ΔE is positive
↓ (photons have pos energy)

$$E_n = -\frac{13.6}{n^2}\,\text{eV}$$

$$\Delta E = \left(-\frac{13.6}{(n_f)^2}\right) - \left(-\frac{13.6}{(n_i)^2}\right)\ \text{eV}$$

$$\Delta E = \left(-\frac{13.6}{(4)^2}\right) - \left(-\frac{13.6}{(5)^2}\right)\ \text{eV}$$

$$\boxed{\Delta E = -0.306\ \text{eV}}$$ ✓

$$-\Delta E = \frac{hc}{\lambda}$$

$$\lambda = -\frac{hc}{\Delta E}$$

$$\lambda = -\frac{6.636\times10^{-34}\,\text{Js}\,(3\times10^{8}\ \text{m/s})}{(-0.306\ \text{eV})\,\frac{1.6\times10^{-19}\ \text{J}}{1\ \text{eV}}} \times \frac{10^{6}\ \mu\text{m}}{1\ \text{m}}$$

$$\boxed{\lambda = 4.07\ \mu\text{m}}$$

β, $\Delta n = 2$:

$$E_n = -\frac{13.6}{n^2}\ \text{eV}$$
$$\Delta E = \left(-\frac{13.6}{(n_f)^2}\right) - \left(-\frac{13.6}{(n_i)^2}\right)\ \text{eV}$$
$$\Delta E = \left(-\frac{13.6}{(4)^2}\right) - \left(-\frac{13.6}{(6)^2}\right)\ \text{eV}$$

$$\boxed{\Delta E = -0.472\ \text{eV}}$$

$$-\Delta E = \frac{hc}{\lambda}$$
$$\lambda = -\frac{hc}{\Delta E}$$
$$\lambda = -\frac{6.636\times10^{-34}\text{Js}\,(3\times10^{8}\ \text{m/s})}{(-0.472\ \text{eV})\,\frac{1.6\times10^{-19}\ \text{J}}{1\ \text{eV}}} \times \frac{10^{6}\ \mu\text{m}}{1\ \text{m}}$$

$$\boxed{\lambda = 2.63\ \mu\text{m}}$$

γ, $\Delta n = 3$:

$$E_n = -\frac{13.6}{n^2}\ \text{eV}$$
$$\Delta E = \left(-\frac{13.6}{(n_f)^2}\right) - \left(-\frac{13.6}{(n_i)^2}\right)\ \text{eV}$$
$$\Delta E = \left(-\frac{13.6}{(4)^2}\right) - \left(-\frac{13.6}{(7)^2}\right)\ \text{eV}$$

$$\boxed{\Delta E = -0.572\ \text{eV}}$$

$$-\Delta E = \frac{hc}{\lambda}$$
$$\lambda = -\frac{hc}{\Delta E}$$
$$\lambda = -\frac{6.636\times10^{-34}\text{Js}\,(3\times10^{8}\ \text{m/s})}{(-0.572\ \text{eV})\,\frac{1.6\times10^{-19}\ \text{J}}{1\ \text{eV}}} \times \frac{10^{6}\ \mu\text{m}}{1\ \text{m}}$$

$$\boxed{\lambda = 2.17\ \mu\text{m}}$$

+2

4. (25%): Consider three main-sequence stars with effective temperatures of 4,000 K (K8V), 10,000 K (A0V), and 40,000 K (O6V).

(a) **Calculate the partition function U_I of neutral hydrogen at these three different temperatures (sum only over the first five levels, i.e. $n_{max} = 5$). Explain why your three answers are not**

significantly different from each other.

$$U_I = \sum_j g_j \exp\left(-\frac{E_j}{kT}\right)$$

$$U_I = \sum_j 2n^2 \exp\left(-\frac{13.6\left(1-\frac{1}{n^2}\right)}{kT}\right)$$

Code:

```
In [6]: T = [4000, 10000, 40000]
        U1 = ([np.sum([2*((i+1)**2)*(np.exp(-(13.6*1.6*10**-19*(1-(i+1)**-2))/(1.38*10**-23*T[ii])))\
                       for i in range(5)]) for ii in range(len(T))])
        print(U1)
        [2.000000000001172, 2.000098716548728, 4.8881488672548095]
```

4000 K:

$$\boxed{U_I = 2.0000}$$ ✓

10000 K:

$$\boxed{U_I = 2.0000}$$ ✓

40000 K:

$$\boxed{U_I = 4.8881}$$ ✓

These numbers are not significantly different from each other because the at these temperatures, the electron in the Hydrogen atom is limited to the ground state, 1s, and the partition function is very close to 2. In the case of the hottest star, the next highest state, 2s, becomes readily available as well as other higher energy states hence why the partition function is greater than 4. ✓

(b) **Calculate the fraction f_I of hydrogen atoms that are neutral in the photospheres of the three stars. Assume a pure hydrogen composition and an electron density of $n_e = 5 \times 10^{15}$ cm^{-3}.**

$$\cancel{f_I} = \frac{n_I}{n_{II}} = \left(2\frac{1}{n_e}\frac{U_{II}}{U_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(\frac{-13.6\text{ eV}}{kT}\right)\right)^{-1}$$

ratio $\quad f_I = \frac{n_I}{n_I + n_{II}} \quad f_I = \frac{\frac{n_I}{n_{II}}}{\frac{n_I}{n_{II}} + 1}$

Code:

```
In [42]: ne = 5*10**15
         fI = [((100**-3)*(1/ne)*((2*np.pi*9.11*10**-31*1.38*10**-23*T[i])/((6.63*10**-34))**2)**1.5*((2)/(U1[i]))\
               *np.exp((-13.6*1.6*10**-19)/(1.38*10**-23*T[i])))**-1 for i in range(len(U1))]
         print(fI)
         [1081605691866.6566, 14.627556055505707, 3.267160493584352e-05]
```

4000 K:

$$\boxed{\cancel{f_I} = 1.08 \times 10^{12}} = \frac{n_I}{n_{II}}$$

-3

10000 K:

$$\boxed{\cancel{f_I} = 14.6} = \frac{n_I}{n_{II}}$$

40000 K:

$$\boxed{\cancel{f_I} = 3.27 \times 10^{-5}} = \frac{n_I}{n_{II}}$$

Using the Saha equation, the fraction of neutral hydrogen to ionized hydrogen was found for each star. In accordance with what we expected, the 4000K had the most neutral hydrogen with each hotter star getting a lower f_I.

(c) **Calculate the fraction n_2/n_I of neutral atoms in the $n = 2$ (Balmer) level for all three stars.**

$$\frac{n_2}{n_I} = \frac{g_2 e^{-E_2/kT}}{U_I}$$

Code:

```
In [43]: Balmer = [(2*(2**2)*np.exp(-(13.6*1.6*10**-19*(1-(2)**-2))/(1.38*10**-23*T[i])))/U1[i] for i in range(len(T))]
         print(Balmer)

         [5.781615712969999e-13, 2.9243829831854417e-05, 0.08510292484271 41]
```

4000 K:

$$\boxed{\frac{n_2}{n_I} = 5.78 \times 10^{-13}}$$

10000 K:

$$\boxed{\frac{n_2}{n_I} = 2.92 \times 10^{-5}}$$

40000 K:

$$\boxed{\frac{n_2}{n_I} = 8.51 \times 10^{-2}}$$

(d) **Combine your answers from (b) and (c) to calculate the overall fraction $n_2/(n_I + n_{II})$ of hydrogen atoms in the $n = 2$ level for the three stars. This question should demonstrate why A0V stars, like Vega, have the strongest Balmer absorption lines.**

$$\frac{n_2}{(n_I + n_{II})} = \frac{\frac{n_2}{n_I}\frac{n_I}{n_{II}}}{\frac{n_I}{n_{II}} + 1}$$

$$\frac{n_2}{(n_I + n_{II})} = \frac{\frac{n_2}{n_I} f_I}{f_I + 1}$$

yes

```
In [44]: Baltotal = [fI[i]*Balmer[i]/(1+fI[i]) for i in range(len(U1))]
         print(Baltotal)

         [5.781615712964644e-13, 2.7372530843836927e-05, 2.780358300577962e-06]
```

4000 K:

$$\boxed{\frac{n_2}{n_I + n_{II}} = 5.78 \times 10^{-13}}$$

10000 K:

$$\boxed{\frac{n_2}{n_I + n_{II}} = 2.73 \times 10^{-5}}$$

40000 K:

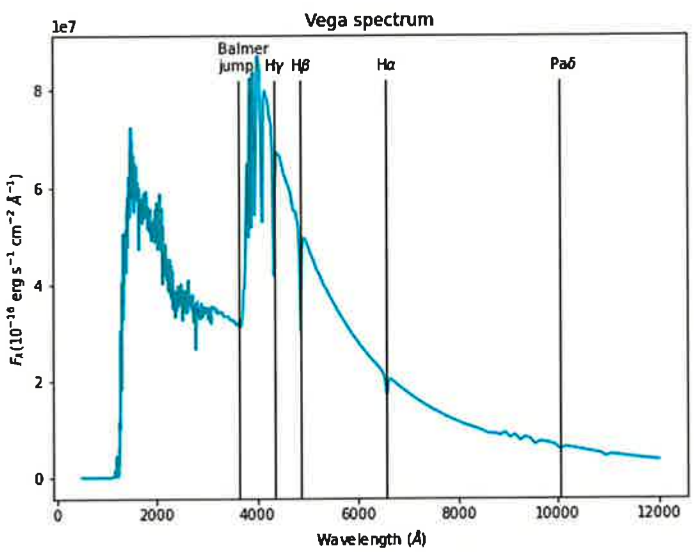
$$\boxed{\frac{n_2}{n_I + n_{II}} = 2.78 \times 10^{-6}}$$

Despite the 40000 K B star having a higher fraction of hydrogen atoms in the $n = 2$ state, we can see from the above calculations that the 10000 K *A*0 star has the highest overall fraction of neutral atoms in the $n = 2$ state. This result makes sense as temperature increases, more hydrogen gets ionized. Although the $n = 2$ state becomes more accessible at higher temperatures, more atoms are getting ionized at the same time. The 10000 K star optimizes these two counteracting pressures resulting in the greatest overall density of hydrogen atoms in the $n = 2$ state.

40/45

5. **(45%) Download the spectrum of Vega from WyoCourses. The first column is wavelength λ in Å, the second column is flux density F_λ in units of 10^{-16} ergs^{-1}cm^{-2}Å^{-1}, and the third column is flux density in milli-Janskys**

(a) **Plot the spectrum and label Hα, Hβ, Hγ, the Balmer jump, and Paδ.**



Code:

```
In [11]: wavl, flux = np.loadtxt('/d/users/nikhil/Downloads/Vegaspectrum.txt', usecols=(0, 1), dtype = float, unpack = True)
         Bmodel = (2*10**-8*6.63*10**-27*(3*10**10)**2)/(wavl*10**-8)**5/\
             (np.exp(6.63*10**-27*3*10**10/(wavl*10**-8*1.38*10**-16*11000))-1)

In [35]: f = plt.figure()
         f.set_figwidth(8)
         f.set_figheight(6)
         plt.plot(wavl, flux, 'c')
         plt.xlabel(r'Wavelength ($\AA$)')
         plt.axvline(x = 6563, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.axvline(x = 4861.35, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.axvline(x = 4340.472, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.axvline(x = 3646, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.axvline(x = 10050, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.text(6563-200, 8.4*10**7, r'H$\alpha$')
         plt.text(4861.35-200, 8.4*10**7, r'H$\beta$')
         plt.text(4340.472-200, 8.4*10**7, r'H$\gamma$')
         plt.text(3646-400, 8.4*10**7, 'Balmer\njump')
         plt.text(10050-200, 8.4*10**7, r'Pa$\delta$')
         plt.ylabel(r'$F_\lambda (10^{-16}$ erg s$^{-1}$ cm$^{-2}$ $\AA^{-1})$')
         plt.title('Vega spectrum')
         plt.savefig('/d/www/nikhil/public_html/ASTR5420/Vegaspectrum.png')
```

(b) **Calculate the bolometric flux of Vega. Considering Vega is 7.7 pc away, measure the luminosity of Vega (in both erg s^{-1} and L$\odot$).**

$$f = \int_0^\infty f_\lambda d\lambda$$

Code:

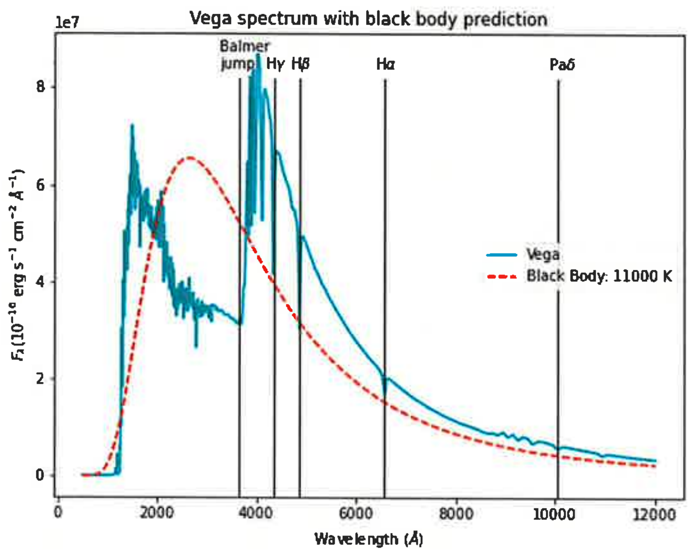
```
In [36]: Boloflux = np.trapz(flux, wavl)
         print('Bolometric Flux:  ' +str(Boloflux*10**-16) +' erg/scm^2')
         print('Vega Luminosity:  ' +str(np.round(Boloflux*10**-16*4*np.pi*(7.7*3.08*10**18)**2,3)) +' erg/s')
         print('Vega Luminosity:  ' +str(np.round(Boloflux*10**-16*4*np.pi*(7.7*3.08*10**18)**2/(3.828*10**33),3)) +' L')

         Bolometric Flux: 2.8059084627476708e-05 erg/scm^2
         Vega Luminosity: 1.9831987785892643e+35 erg/s
         Vega Luminosity: 51.808 L
```

$$\boxed{f = 2.806 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2}}$$

Assume $r_{Vega} = 7.7$ pc.

$$\boxed{L = 1.983 \times 10^{35} \text{ erg s}^{-1}} \boxed{L = 51.8L_{\odot}}$$

(c) **Attempt to match a Planck blackbody distribution to the observed Vega spectrum in the Raleigh-Jeans tail, and report the effective temperature. Describe qualitatively how the blackbody differs from the observed spectrum.**



Code:

```
In [37]: f = plt.figure()
         f.set_figwidth(8)
         f.set_figheight(6)
         plt.plot(wavl, flux, 'c', label = 'Vega')
         plt.plot(wavl, Bmodel, '--r', label = 'Black Body: 11000 K')
         plt.xlabel(r'Wavelength ($\AA$)')
         plt.ylabel(r'$F_\lambda (10^{-16}$ erg s$^{-1}$ cm$^{-2}$ $\AA^{-1})$')
         plt.axvline(x = 6563, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.axvline(x = 4861.35, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.axvline(x = 4340.472, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.axvline(x = 3646, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.axvline(x = 10050, ymin = 0.00, ymax = 0.9, color = 'k', lw = 1)
         plt.text(6563-200, 8.4*10**7, r'H$\alpha$')
         plt.text(4861.35-200, 8.4*10**7, r'H$\beta$')
         plt.text(4340.472-200, 8.4*10**7, r'H$\gamma$')
         plt.text(3646-400, 8.4*10**7, 'Balmer\njump')
         plt.text(10050-200, 8.4*10**7, r'Pa$\delta$')
         plt.legend()
         plt.title('Vega spectrum with black body prediction')
         plt.savefig('/d/www/nikhil/public_html/ASTR5420/vegablackbody')
```

$$\boxed{T_{eff} \sim 11000K}$$

At bluer wavelengths, as you approach the Balmer jump, the spectrum shows sharp absorption lines in corresponding to the Balmer series of transitions. Interestingly, the spectrum for Vega also appears to be above the black body spectrum at these same wavelengths. Short of the Balmer jump, the spectrum for Vega is strongly diminished compared to the black body indicating strong absorption of wavelengths before the Balmer jump, which makes sense. There is also strong absorption at what appears to be the Lyman jump, 912Å.

(d) **Estimate the radius of Vega (in both cm and R⊙).**

$$L_* = 4\pi R_*^2 \sigma T^4$$
$$L_* = 4\pi R^2 F$$
$$R = \sqrt{\left(\frac{4\pi R^2 F}{4\pi\sigma T^4}\right)}$$

Code:

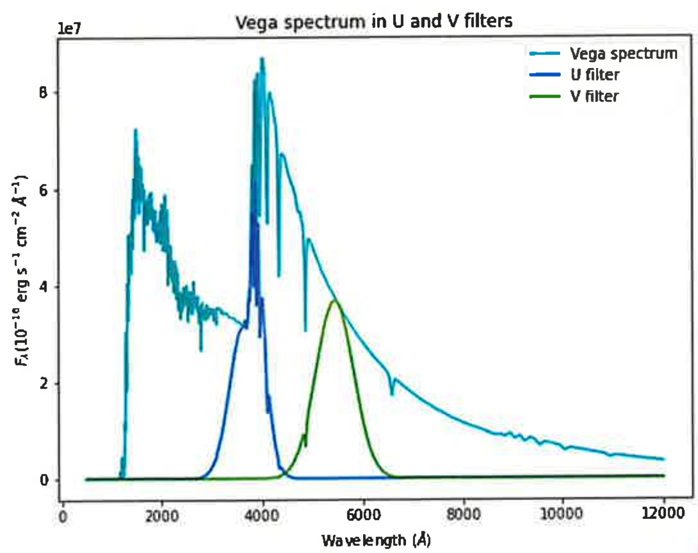
```
In [38]: Tvega = 11000
R = ((Holoflux*10**-16*4*np.pi*(7.7*3.08*10**18)**2)/(4*np.pi*Tvega**4*5.67*10**-5))**.5
print('Radius: ' +str(R) +' cm')
print('Radius: ' +str(R/(6.957*10**10)) +' Rsun')

Radius: 137800005200.84537 cm
Radius: 1.9810000210557046 Rsun
```

$\boxed{R = 1.38 \times 10^{11}\ \text{cm}}$ $\boxed{R = 1.98 R_\odot}$

(e) **Calculate the flux of Vega in the U and V filters (in erg s-1 cm-2). Assume both passbands can be described by Gaussians with full throughput at their centers, where U has a central wavelength of 3650 Å and dispersion of 280 Å while V has a central wavelength of 5510 Å and dispersion of 370 Å.**

To measure the flux in U and V filters, I created a Gaussian function centered on the U and V wavelengths with dispersions corresponding to each filter. I then multiplied the Vega spectrum by the U and the V Gaussians to represent how much flux is captured in each filter. Integrating over all wavelengths for each filter should give the flux measured in each filter.
Code:

```
In [39]: f = plt.figure()
f.set_figwidth(8)
f.set_figheight(6)
U = np.exp(-(wavl-3650)**2/(2*280**2))
V = np.exp(-(wavl-5510)**2/(2*370**2))
plt.plot(wavl, flux, 'c', label = 'Vega spectrum')
plt.plot(wavl, U*flux, 'b', label = 'U filter')
plt.plot(wavl, V*flux, 'g', label = 'V filter')
plt.xlabel(r'Wavelength ($\AA$)')
plt.ylabel(r'$F_\lambda (10^{-16}$ erg s$^{-1}$ cm$^{-2}$ $\AA^{-1})$')
plt.title('Vega spectrum in U and V filters')
plt.legend()
Uflux = np.trapz(U*flux, wavl)*10**-16
Vflux = np.trapz(V*flux, wavl)*10**-16
print('U flux: ' +str(Uflux) +' erg/scm^2')
print('V flux' +str(Vflux) +' erg/scm^2')
fU0 = 417.5*300*10**-11
fV0 = 363.1*550*10**-11
mU = -2.5*np.log10(Uflux/fU0)
mV = -2.5*np.log10(Vflux/fV0)
print('U mag: ' +str(np.round(mU,3)))
print('V mag: ' +str(np.round(mV,3)))
print('U-V: ' +str(np.round(mU-mV,3)))
plt.savefig('/d/www/nikhil/public_html/ASTR5420/vegauvfilters.png')

U flux: 3.14591999169175e-06 erg/scm^2
V flux3.397404892615517e-06 erg/scm^2
U mag: -0.802
V mag: -0.577
U-V: -0.225
```



$\boxed{f_U = 3.15 \times 10^{-6}\ \text{erg s}^{-1}\ \text{cm}^{-2}}$ $\boxed{f_V = 3.40 \times 10^{-6}\ \text{erg s}^{-1}\ \text{cm}^{-2}}$

(f) **Measure the U-V color of Vega (in mag). This is roughly the offset between the Vega and ABmag photometric systems. Also, the colors derived from integrating a spectrum are typi-**

cally called pseudo-colors because, in practice, they tend to not be as accurate as photometric colors measured from imaging.

Zero point flux densities:

$$U_{\lambda,0} = 417.5 \times 10^{-11} \text{ erg s}^{-1} \text{ cm}^{-1} \text{ Å}^{-1}$$

$$V_{\lambda,0} = 363.1 \times 10^{-11} \text{ erg s}^{-1} \text{ cm}^{-1} \text{ Å}^{-1}$$

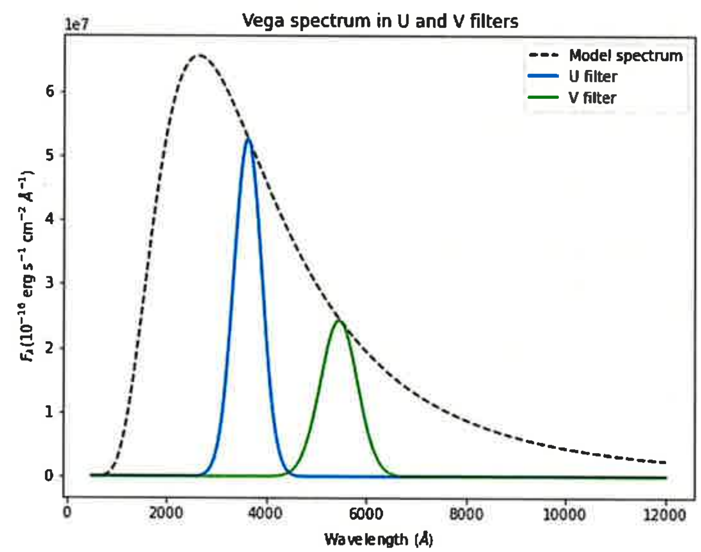
$$m = -2.5 \log \left(\frac{f}{f_{\lambda,0}\lambda} \right)$$

$$\boxed{U = -0.802} \quad \boxed{V = -0.577}$$

$$\boxed{U - V = -0.225}$$

$U-V = 2.5 \log \left(\frac{F_V}{F_U}\right) = 2.5 \log \left(\frac{3.40}{3.15}\right) = 0.08$

(g) **Repeat (e) & (f) for your blackbody spectrum.**



Code:

```
In [41]: f = plt.figure()
         f.set_figwidth(8)
         f.set_figheight(6)
         U = np.exp(-(wavl-3650)**2/(2*200**2))
         V = np.exp(-(wavl-5510)**2/(2*370**2))
         plt.plot(wavl, Bmodel, '--k', label = 'Model spectrum')
         plt.plot(wavl, U*Bmodel, 'b', label = 'U filter')
         plt.plot(wavl, V*Bmodel, 'g', label = 'V filter')
         plt.xlabel(r'Wavelength ($\AA$)')
         plt.ylabel(r'$F_\lambda (10^{-16}$ erg s$^{-1}$ cm$^{-2}$ $\AA^{-1})$')
         plt.title('Vega spectrum in U and V filters')
         plt.legend()
         specUflux = np.trapz(U*Bmodel, wavl)*10**-16
         specVflux = np.trapz(V*Bmodel, wavl)*10**-16
         print('U flux: ' +str(specUflux) +' erg/scm^2')
         print('V flux' +str(specVflux) +' erg/scm^2')
         mUspec = -2.5*np.log10(specUflux/fU0)
         mVspec = -2.5*np.log10(specVflux/fV0)
         print('U mag: ' +str(np.round(mUspec, 3)))
         print('V mag: ' +str(np.round(mVspec, 3)))
         print('U-V: ' +str(np.round(mUspec-mVspec,3)))
         plt.savefig('/d/www/nikhil/public_html/ASTR5420/blackbodyuvfilters.png')

U flux: 3.665109203021643e-06 erg/scm^2
V flux2.2578794052972956e-06 erg/scm^2
U mag: -0.968
V mag: -0.133
U-V: -0.835
```

$U-V = 2.5 \log \left(\frac{2.26}{3.67}\right) = -0.53$

$$\boxed{f_U = 3.67 \times 10^{-6} \text{ erg s}^{-1} \text{ cm}^{-2}} \quad \boxed{f_V = 2.26 \times 10^{-6} \text{ erg s}^{-1} \text{ cm}^{-2}}$$

$$\boxed{U = -0.968} \quad \boxed{V = -0.133}$$

$$\boxed{U - V = -0.835}$$

(h) **Report the difference Δ(U-V) in mag between the Vega and blackbody spectra. Which one is redder? This effect is called line-blanketing, whereby UV photons are more likely to be absorbed in the photospheres of stars and then re-emitted at longer wavelengths.**

$$\Delta(U-V) = (-0.225) - (-0.835)$$
$$\boxed{\Delta(U-V) = 0.61}$$

Two errors cancelled when taking the difference

Vega is redder than the equivalent blackbody with T_{eff}. This makes sense because as explained in the question, bluer photons from the star are absorbed and re-emitted in redder wavelengths, making the spectrum of the star redder than the predicted black body.