92%

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ASTR5420

Please show all work. If you collaborate with other students, write their names at the top of your homework. Please also write the total number of hours you spent on this. If you use any code or plotting routines, print those out and attach them to your hand-written solutions.
This took me about 7 hours to complete.
I collaborated with Alex on this assignment.

1. **(10%): Hertzsprung Gap (HG):**

+10/10

(a) **Calculate the change in binding energy of a $1M_\odot$ star as it expands during the HG from $1.5R_\odot$ at the tip of the MS to $5R_\odot$ at the bottom of the RGB (1^{st} dredge up).**

$$U = -\frac{-3GM^2}{5R}$$

$$\Delta U = U_2 - U_1$$

$$\Delta U = -\frac{3GM^2}{5}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

Code:

```
In [11]: M_o = 2e30
         # NP Mass of the Sun in kg
         R_o = 6.957e8
         # NP Radius of the Sun in m
         G = 6.67e-11
         # NP Gravitational constant
         Rs = np.array([1.5 *R_o, 5 *R_o])
         # NP Defining array of ending and starting radii
         dU = (-3 *G *M_o **2) /(5) *((1 /Rs[1]) -(1 /Rs[0]))
         print('Change in binding energy: ' +format(dU, '.2E') +' Joules')

         Change in binding energy: 1.07E+41 Joules
```

$$\boxed{\Delta U = 1.07 \times 10^{41} \text{ Joules}}$$

(b) **Suppose the star generates an excess $1L_\odot$ that goes into expanding the star during the HG. Compute the corresponding duration of HG evolution. How does this compare to its thermal Kelvin-Helmholtz timescale?**

$$\tau = \frac{\Delta U}{L_\odot}$$

$$\boxed{\tau = 8.93 \times 10^6 \text{ years}}$$

$$\tau_{KH} = \frac{GM_\odot^2}{R_\odot L_\odot}$$

$$\boxed{\tau_{KH} = 3.19 \times 10^7 \text{ years}}$$

Code:

-2% late

first-order approximations, making sure to use M_c and R_c in both relations).

$$R \approx 0.01 R_\odot \left(\frac{M}{0.8 M_\odot}\right)^{-1/3}$$

$$\rho_c = \frac{M_c}{4/3 \pi R_c^3}$$

$$\rho_c = \frac{3M_c}{4\pi R_c^3}$$

$$\rho_c = \frac{3M_c}{4\pi \left(0.01 R_\odot \left(\frac{M_c}{0.8 M_\odot}\right)^{-1/3}\right)^3}$$

$$\rho_c = \frac{3M_c}{4\pi \left(10^{-6}\right) R_\odot^3 \left(\frac{M_c}{0.8 M_\odot}\right)^{-1}}$$

$$\rho_c = \frac{3M_c^2}{4\pi \left(10^{-6}\right) R_\odot^3 \left(0.8 M_\odot\right)}$$

$$T_c = \frac{0.65 G M_c m_H}{k R_c}$$

$$T_c = \frac{0.65 G M_c m_H}{0.01 k R_\odot \left(\frac{M_c}{0.8 M_\odot}\right)^{-1/3}}$$

$$T_c = \frac{0.65 G M_c^{4/3} m_H}{0.01 k R_\odot \left(0.8 M_\odot\right)^{1/3}}$$

Bottom:

$$\boxed{\rho_c = 7.09 \times 10^4 \text{ g cm}^{-3}}$$
$$\boxed{T_c = 1.90 \times 10^8 \text{ K}}$$

Tip:

$$\boxed{\rho_c = 3.59 \times 10^5 \text{ g cm}^{-3}}$$
$$\boxed{T_c = 5.60 \times 10^8 \text{ K}}$$

Code:

```
In [275]: k = 1.38e-23
          # NP Boltzmann constant in mks units
          m_p = 1.67e-27
          # NP Mass of proton in kg
          rho_c = (3 *Mcs **2 *M_o **2) /(4 *np.pi *1e-6 *R_o **3 *(0.8 *M_o))
          # NP Calculating core densities for both stages
          T_c = (0.65 *G *(Mcs *M_o) **(4/3) *m_p) /(0.01 *k *R_o *(0.8 *M_o)\
              **(1/3))
          # NP Calculating core temperatures for both stages
          print('Bottom RGB stellar parameters:'
               +'\nrho_c: ' +format(rho_c[0]/1000, '.2E') +' kg m^-3'
               +'\nT: ' +format(T_c[0], '.2E') +' K')
          # NP Printing result
          print('Tip RGB stellar parameters:'
               +'\nrho_c: ' +format(rho_c[1]/1000, '.2E') +' kg m^-3'
               +'\nT: ' +format(T_c[1], '.2E') +' K')
          # NP Printing result

          Bottom RGB stellar parameters:
          rho_c: 7.09E+04 kg m^-3
          T: 1.90E+08 K
          Tip RGB stellar parameters:
          rho_c: 3.59E+05 kg m^-3
          T: 5.60E+08 K
```

(c) **Assume the H-burning shell has solar composition. The bottom of the shell also has similar density and temperature as the core. Estimate the energy production rates ϵ (in erg s^{-1} g^{-1}**

$$\boxed{\dot{M} = 1.23 \times 10^{-11}\ M_\odot\ \mathrm{yr}^{-1}}$$

Tip:

$$\boxed{\dot{M} = 1.17 \times 10^{-7}\ M_\odot\ \mathrm{yr}^{-1}}$$

Code:

```
In [8]: Mdot = 4e-13 *0.5 *(L_s *R_s) /(1)
        # NP Calculating Mdot for both stages
        print('Bottom RGB stellar parameters:'
              +'\nMdot: ' +format(Mdot[0], '.2E') +' Mo yr^-1')
        # NP Printing result
        print('Tip RGB stellar parameters:'
              +'\nMdot: ' +format(Mdot[1], '.2E') +' Mo yr^-1')
        # NP Printing result

        Bottom RGB stellar parameters:
        Mdot: 1.23E-11 Mo yr^-1
        Tip RGB stellar parameters:
        Mdot: 1.17E-07 Mo yr^-1
```

(e) **Estimate the lifetime τ_{TRGB} (in Myr) near the tip of the RGB. To calculate this, first estimate the core mass that corresponds to 50% of the luminosity L_{TRGB} at $M_c = 0.45 M_\odot$. Given your computed growth of the core mass ΔM_c , then estimate the corresponding energy production E using the efficiency for H-fusion you derived in HW #7. Finally compute $\tau_{TRGB} = E/L_{TRGB}$.**

$$L = 200\left(\frac{M_c}{0.3M_\odot}\right)^{7.6} L_\odot$$
$$\frac{L}{L_\odot} = 200\left(\frac{M_c}{0.3M_\odot}\right)^{7.6}$$
$$M_c = 0.3M_\odot\left(\frac{L}{200L_\odot}\right)^{1/7.6}$$

At half L_{TRGB}, find M_c.

$$\boxed{M_{c,1/2} = 0.411 M_\odot}$$

To increase mass this much, find the energy produced. Assume an efficiency of 0.7%, from last homework.

$$E = f\Delta mc^2$$
$$E = f\,(0.45 - 0.411)\,M_\odot c^2$$

$$\boxed{E = 4.94 \times 10^{43}\ \mathrm{J}}$$

Find the duration of this phase.

$$\tau_{TRGB} = \frac{E}{L_{TRGB}}$$

$$\boxed{\tau_{TRGB} = 0.943\ \mathrm{Myrs}}$$

Code:

(b) Derive the PN radius R_{PN}. Assuming the PN shell has thickness $\Delta R_{PN} = 0.1 R_{PN}$, derive an expression for the ion number density $n_{ion} = \rho/\mu m_H$, where $\mu = 1.4$ for solar composition. What is the maximum lifetime of a PN? I.e., at what age τ_{PN} is the PN no longer detectable such that $n_{ion} < 5\ \text{cm}^{-3}$, which is just a few times the average ISM density.

The radius of the planetary nebula will be proportional to the difference in speeds of the wind of the planetary nebula and the wind of the AGB as well as the age of the planerary nebula.

$$\boxed{R_{PN} = (v_{PN} \cancel{- v_{AGB}})\,\tau_{PN}}$$

no, $R_{PN} = v_{PN}\tau_{PN}$

From this relation, and considering the number density, find when planetary nebula is no longer visible ($n_{ion} \approx 5\ \text{cm}^{-3}$).

$$n_{ion} = \frac{\rho}{\mu m_H}$$

$$n_{ion} = \frac{M_{PN}}{V_{PN}}\frac{1}{\mu m_H}$$

$$n_{ion} = \left(\frac{v_{PN} - v_{AGB}}{v_{AGB}}\right)\frac{\dot{M}\tau_{PN}}{4\pi R_{PN}^2 dR}\frac{1}{\mu m_H}$$

$$n_{ion} = \frac{v_{PN} - v_{AGB}}{v_{AGB}}\frac{\dot{M}\tau_{PN}}{4\pi\left((v_{PN} - v_{AGB})\,\tau_{PN}\right)^2 \left(0.1\,(v_{PN} - v_{AGB})\right)}\frac{1}{\mu m_H}$$

$$n_{ion} = \frac{(v_{PN} - v_{AGB})\,\dot{M}\tau_{PN}}{0.4\pi v_{AGB}\,(v_{PN} - v_{AGB})^3\,\tau_{PN}^3 \mu m_H}$$

$$n_{ion} = \frac{\dot{M}}{0.4\pi v_{AGB}\,(v_{PN} - v_{AGB})^2\,\tau_{PN}^2 \mu m_H}$$

$$\tau_{PN}^2 = \frac{\dot{M}}{0.4\pi v_{AGB}\,(v_{PN} - v_{AGB})^2\, n_{ion}\mu m_H}$$

$$\tau_{PN} = \sqrt{\frac{\dot{M}}{0.4\pi v_{AGB}\,(v_{PN} \cancel{- v_{AGB}})^2\, n_{ion}\mu m_H}}$$

right method!

$$\boxed{\tau_{PN} = 105000 \text{ years}}$$

yet $\tau \approx 50{,}000$ yr if $R_{PN} = v_{PN}\tau_{PN}$

Code:

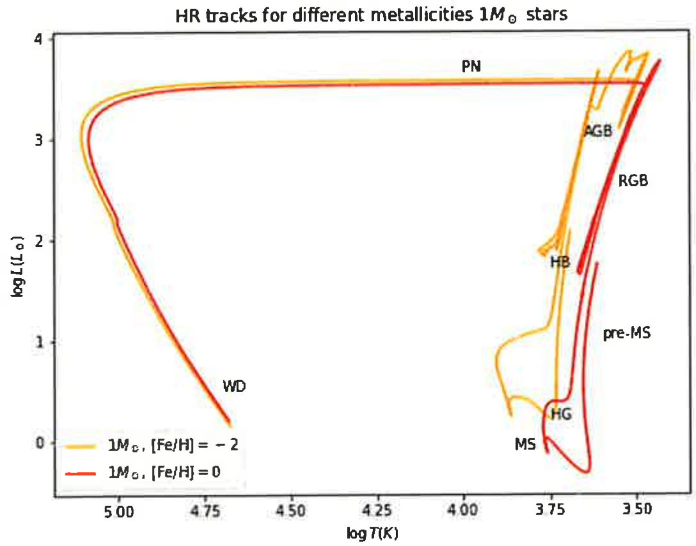
```
In [61]: t_PN = np.sqrt((1e-5 * M_o /(np.pi *10 **7)) /(10000 *(30000 -10000)\
             **2 *5e6 *1.4 *m_p * 0.4 *np.pi))
         # NP Calculating timescale of planetary nebula
         print('PN lifetime: ' +format(t_PN /np.pi /1e7, '.2E') +' years')
         # NP Printing result

         PN lifetime: 1.05E+05 years
```

+33/35

4. **(35%): Download from WyoCourses the MESA evolutionary tracks for $1M_\odot$ and $3M_\odot$ stars at both [Fe/H] = –2 and 0 metallicities. The tables include 77 columns (mostly surface abundances), but all you will need is stellar age (in yr), $\log L/L_\odot$, and $\log T_{eff}/K$.**

   (a) **Plot the four tracks on the same HR diagram. For the solar-mass, solar-metallicity track, label the pre-MS, MS, HG, RGB, HB, AGB, PN, and WD phases of evolution.**



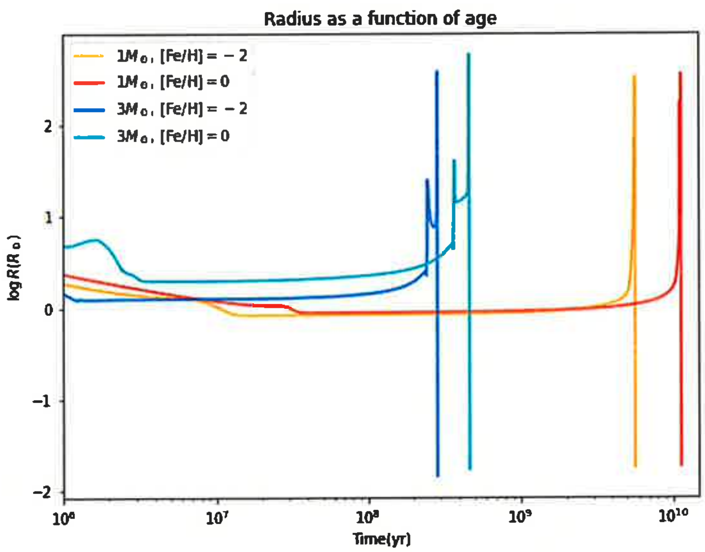
Code:

```
In [66]: plt.figure(figsize = [8, 6])
         # NP Making figure larger
         plt.plot(T1, L1, color = 'orange', label = r'$1M_\odot$,'
             ' [Fe/H]$=-2$')
         plt.plot(T2, L2, 'r', label = r'$1M_\odot$, [Fe/H]$=0$')
         # NP Plotting temperaures and luminosities
         plt.text(3.6, 1, 'pre-MS')
         plt.text(3.85, -0.1, 'MS')
         plt.text(3.75, 0.2, 'HG')
         plt.text(3.55, 2.5, 'RGB')
         plt.text(3.75, 1.7, 'HB')
         plt.text(3.65, 3, 'AGB')
         plt.text(4, 3.65, 'PN')
         plt.text(4.7, 0.5, 'WD')
         # NP Labeling different phases in stellar evolution
         plt.gca().invert_xaxis()
         # NP Flipping x-axis
         plt.legend()
         # NP Making legend
         plt.xlabel(r'$\log T(K)$')
         plt.ylabel(r'$\log L(L_\odot)$')
         plt.title(r'HR tracks for 2 different metalicity $1M_\odot stars')
         # NP Labeling figure
         plt.title(r'HR tracks for different metallicities $1M_\odot$ stars')
         # NP Labeling plot
         plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/HRtracks2.png')
         # NP Saving figure
```

b/c metal-poor stars have less opacity → smaller & hotter

The $1M_\odot$ objects look really similar during the final stages in the star's life. Despite their different metallicities, they very closely resemble each other during the White dwarf and planetary nebula phases. They differ slightly in their pre MS and MS stages. The lower-metallicity star appears slightly up and to the left of the solar-metallicity star, indicating a higher effective temperature and higher luminosity. These two stars most significantly differ during their AGB phases. The lower-metallcity star seems to oscillate in luminosity and temperature during this phase. This difference can be explained by instability in the low-metallicity star due to changes in opacity. The lower-metallicity star is at a higher temperature than the solar-metallicity star. This difference in temperature puts the layers of the low-metallicity star near the temperature in which Hydrogen is ionized. Slight changes in temperature will therefore lead to drastic changes in opacity which casues this star to pulsate.

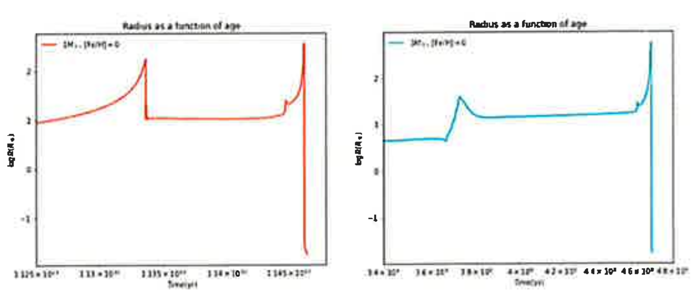
(c) **Describe qualitatively how the $3M_\odot$ solar-metallicity track differs from its $1M_\odot$ counterpart.**



Code:

```
In [233]: plt.figure(figsize = [8, 6])
          # NP Making figure larger
          plt.plot(ages1, R1, color = 'orange', label = r'$1M_\odot$,'
              ' [Fe/H]$=-2$')
          plt.plot(ages2, R2, 'r', label = r'$1M_\odot$, [Fe/H]$=0$')
          plt.plot(ages3, R3, 'b', label = r'$3M_\odot$, [Fe/H]$=-2$')
          plt.plot(ages4, R4, 'c', label = r'$3M_\odot$, [Fe/H]$=0$')
          # NP Plotting radius as a function of age for all stars
          plt.legend()
          # NP Creating legend
          plt.xlabel(r'Time(yr)')
          plt.ylabel(r'$\log R(R_\odot)$')
          plt.title(r'Radius as a function of age')
          # NP Labeling figure
          plt.xscale('log')
          # NP Scaling x-axis to log scale
          plt.xlim(1 *10 **6, 1.5*10**10)
          plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/Radiustime'
              'stars.png')
          # NP Saving figure
```

(e) **Report the maximum radii (in $R_\odot$) during the RGB and AGB for the two solar-metallicity tracks. Explain why $R_{AGB} \approx R_{RGB}$ for $1M_\odot$ stars but $R_{AGB} >> R_{RGB}$ for $3M_\odot$ stars.**



From the plot above, it seems the RGB for the $1M_\odot$ star occurs approximately before 1.135×10^{10} years. For the $3M_\odot$ star, the RGB occurs before 4.2×10^8 years. Find the maximum radii before and after these times for both stars.
Code: