89%

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ASTR5420

Please show all work. If you collaborate with other students, write their names at the top of your homework. Please also write the total number of hours you spent on this. If you use any code or plotting routines, print those out and attach them to your hand-written solutions.

+10/10

1. **(10%) Red Alert!! Romulans from the future have deposited "red matter" into the core of the planet Vulcan, quickly converting it into a black hole. Vulcan's crust is disintegrating and collapsing on a dynamical timescale. How long do Scotty and Chekov have to beam up the away team and members of the Vulcan High Council? According to Mass Trek Wiki, planet Vulcan has a radius of 6,792 km and surface gravity g = 1.4 G of Earth's gravity.**

$$\tau_{ff} = \frac{\pi}{2}\left(\frac{R^3}{2GM}\right)^{1/2}$$

$$\frac{GM}{R^2} = 1.4g$$

$$M = \frac{1.4gR^2}{G}$$

$$\tau_{ff} = \frac{\pi}{2}\left(\frac{R^3}{2G\left(\frac{1.4gR^2}{G}\right)}\right)^{1/2}$$

$$\tau_{ff} = \frac{\pi}{2}\left(\frac{R}{2.8g}\right)^{1/2}$$

Code:

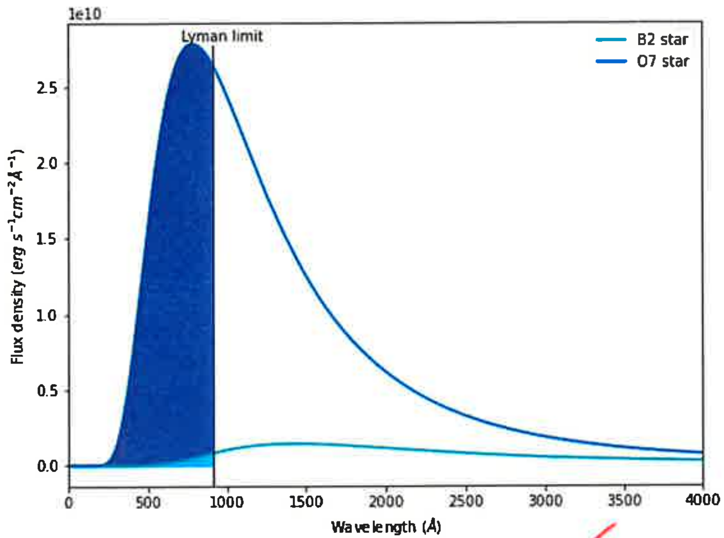
```
In [9]: G = 6.67e-11
        Rv = 6.792e6
        g = 9.8
        t_ff = (np.pi/2)*np.sqrt(Rv/(2.8*g))
        print('Free fall time: ' +format(t_ff, '.2E') +' seconds')

        Free fall time: 7.81E+02 seconds
```

$$\boxed{t_{ff} = 781 \text{ seconds}}$$

+24/25

2. **(25%) For late-O/early-B stars ($M = 8 - 30M_\odot$), the MS relations are $R = 4.1(M/8M_\odot)^{0.6}R_\odot$ and $L = 2600(M/8M_\odot)^{3.0}L_\odot$ (slightly flatter than relations for AFGK dwarfs).**



$\boxed{f_1 = 0.046}$ $\boxed{f_2 = 0.354}$

(c) **Estimate the rate of ionizing photons (in photons s^{-1}) for both stars.**

$$N = \int_0^{912} \frac{B_\lambda}{hc/\lambda} d\lambda$$

Code:

```
In [33]: R_s = [4.1*(m/8)**0.6 *R_o *1e2 for m in M_s]
         N = [np.trapz((B_l[i][ily])/(6.63*10**-34 *3e8 *10**10/wavl[ily]),\
             wavl[ily])*10**-7 *4*np.pi*R_s[i]**2 for i in range(len(B_l))]
         print('Ionizing photon rate: ' +format(N[0], '.2E') +' ph/s')
         print('Ionizing photon rate: ' +format(N[1], '.2E') +' ph/s')

         Ionizing photon rate: 5.79E+45 ph/s
         Ionizing photon rate: 2.03E+48 ph/s
```

$\boxed{N_1 = 5.79 \times 10^{45} \text{ ph s}^{-1}}$ $\boxed{N_2 = 2.03 \times 10^{48} \text{ ph s}^{-1}}$

(d) **Assuming a Salpeter IMF (see #4), we expect $\approx$ 22 times more B3V stars than O7V stars in a zero-age MS population. What produces more ionizing photons: a single O7V star or 22 B2V stars?**

$$N_{tot} = nN_1$$

Code:

```
In [34]: nstars = [22, 1]
         Ntot = [nstars[i] *N[i] for i in range(len(N))]
         print('Total ionizing photon rate: ' +format(Ntot[0], '.2E') +' ph/s')
         print('Total ionizing photon rate: ' +format(Ntot[1], '.2E') +' ph/s')

         Total ionizing photon rate: 1.27E+47 ph/s
         Total ionizing photon rate: 2.03E+48 ph/s
```

$\boxed{N_{tot,1} = 1.27 \times 10^{47} \text{ ph s}^{-1}}$ $\boxed{N_{tot,2} = 2.03 \times 10^{48} \text{ ph s}^{-1}}$

From these results, it seems that one O7V star produces $\approx$ 20 times more ionizing photons than 22 B2V stars.

$$\boxed{\frac{dM}{dt} = 4.70 \times 10^{-8}\ M_\odot\ yr^{-1}}$$

(c) **What is the disk luminosity L_{disk} (in $L_\odot$)? What is the ratio L_{disk}/L_*?**

$$L_{disk} = \frac{GM}{2R}\frac{dM}{dt}$$

$$L_{disk} = \frac{G(1M_\odot)}{1.5R_\odot}\frac{dM}{dt}$$

$$L_{disk} = \frac{G(1M_\odot)}{1.5R_\odot}\frac{\Delta M}{\left(\frac{GM\Delta M}{(1.5R_\odot)(1L_\odot)}\right)}$$

$$L_{disk} = \frac{G(1M_\odot)}{1.5R_\odot}\frac{\Delta M\,(1.5R_\odot)\,(1L_\odot)}{GM\Delta M}$$

$$L_{disk} = 1L_\odot$$

Virial thm

$$\boxed{L_{disk} = 1L_\odot}$$ 0.5 $L_\odot$

$$\boxed{\frac{L_{disk}}{L_*} = 1}$$ ≈ 0.5

(d) **What is the temperature of the disk near the star/disk boundary and at $R = 3R_*$? How do these compare to the effective temperature of the star?**

$$L_{disk} = GM\frac{dM}{dt}\frac{1}{R}$$

$$L_{disk} = \sigma\pi R^2 T^4$$

$$\sigma\pi R^2 T^4 = GM\frac{dM}{dt}\frac{1}{R}$$

$$T^4 = \frac{GM}{\pi\sigma}\frac{dM}{dt}\frac{1}{R^3}$$

$$T = \left(\frac{GM}{\pi\sigma}\frac{dM}{dt}\frac{1}{R^3}\right)^{1/4}$$

Code:

```
In [266]: R_tauridiff = [R_tstar, 3*R_tstar]
          T = [((G*M_o)/(np.pi *5.67e-8) *dmdt *1/(r**3))**0.25 for r in R_tauridiff]
          print('Temp. at boundary: ' +format(T[0], '.2E') +' K')
          print('Temp. at 3R_*: ' +format(T[1], '.2E') +' K')
          T_effstar = (L_o/(4 *np.pi *(1.5 *5.67e-8 *(1.5 *R_o)**2)))**0.25
          print('T_eff: ' +format(T_effstar, '.2E') +' K')
          print('Ratio at boundary: ' +format(T[0]/T_effstar, '.2E'))
          print('Ratio at 3R_*: ' +format(T[1]/T_effstar, '.2E'))

          Temp. at boundary: 6.67E+03 K
          Temp. at 3R_*: 2.92E+03 K
          T_eff: 4.26E+03 K
          Ratio at boundary: 1.57E+00
          Ratio at 3R_*: 6.87E-01
```

$$\boxed{T_{R_*} = 6670\ \text{K}}$$

$$\boxed{T_{3R_*} = 2920\ \text{K}}$$

$$\boxed{\frac{T_{R_*}}{T_{eff}} = 1.57}$$

$$\boxed{\frac{T_{3R_*}}{T_{eff}} = 0.687}$$

The temperature of the disk at the star/disk boundary and at a radius of 3 times the radius of the star is on the order of magnitude of the effective temperature of the star.

```
In [149]: collapse = (Ms >= 8)
          eB = (Ms > 8) & (Ms < 17)
          iO = (Ms > 17)
          print('Core-collapse fraction: ' +format(np.trapz(normalnmdm[collapse],\
              Ms[collapse]), '.2E'))
          print('Early B fraction: ' +format(np.trapz(normalnmdm[eB], Ms[eB]),\
              '.2E'))
          print('O fraction: ' +format(np.trapz(normalnmdm[iO], Ms[iO]), '.2E'))

          Core-collapse fraction: 7.92E-03
          Early B fraction: 5.19E-03
          O fraction: 2.73E-03
```

$$\boxed{(N_m)_{m>8} = 0.00792}$$

$$\boxed{(N_m)_{m>8,m<17} = 0.00519}$$ 65% of SN

$$\boxed{(N_m)_{m>17} = 0.00273}$$ 35% of SN

(c) **What fraction of the mass of a stellar population will evolve in 10 Gyr ($M > 1M_\odot$) and subsequently be converted into stellar remnants or returned to the ISM?**

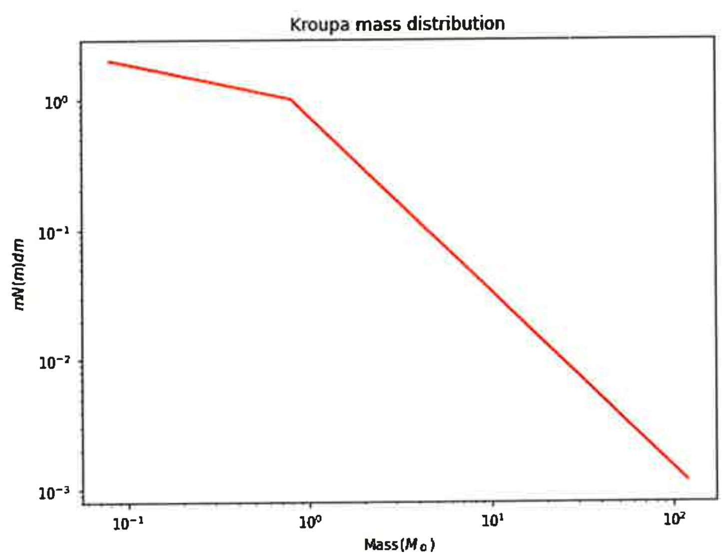
Code:

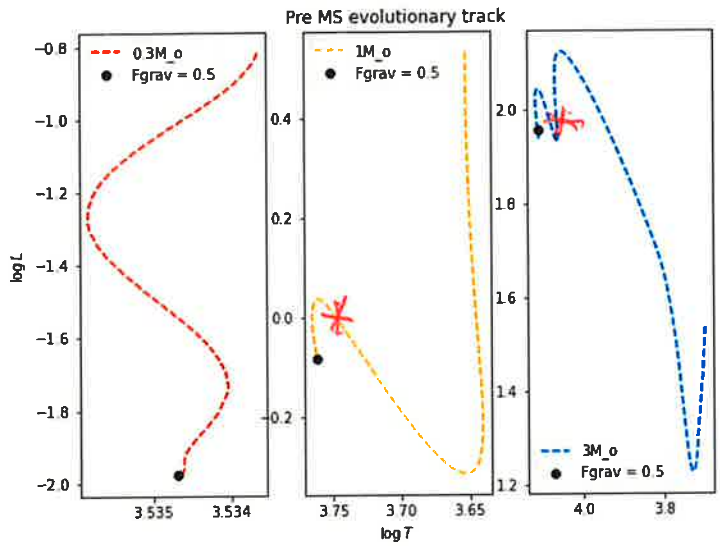
```
In [159]: plt.figure(figsize=[8, 6])
          MNmdm = (Ms/0.8)*N_mdm
          plt.xscale('log')
          plt.yscale('log')
          MNmdmnorm = MNmdm/np.trapz(MNmdm, Ms)
          plt.plot(Ms, MNmdm, 'r')
          plt.xlabel(r'Mass($M_\odot$)')
          plt.ylabel(r'$mN(m)dm$')
          plt.title('Kroupa mass distribution')
          plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/Kroupamass.png')
```

```
In [161]: tengiga = Ms > 1
          print('Mass fraction of stars greater than solar mass: '\
                +format(np.trapz(MNmdmnorm[tengiga], Ms[tengiga]), '.2E'))

          Mass fraction of stars greater than solar mass: 6.12E-01
```

Plot:





wanted on same HRD

Fgrav = 0.5 locations should be slightly earlier

(b) **Describe how the Hyashi and Henyey tracks differ among the three stars. Do $0.3M_\odot$ stars ever develop radiative cores or do they remain fully convective?** ⇒ $0.3M_\odot$ remain fully convective

The $0.3M_\odot$ and the $1M_\odot$ stars both show a significant decrease in luminosity while staying at the same temperature while the $3M_\odot$ star spends less time on the Hyashi track. As the star's mass increases, it spends more time on the Henyey track.

(c) **At what age does the pre-MS begin for the three stars? Explain why it does not begin at 0 Myr. What is the initial luminosity (in $L_\odot$), temperature (K), and radius ($R_\odot$) of a $1M_\odot$ pre-MS star?**

Code:

```
In [295]: ags = [10**(a[0]) for a in ages]
          print('Pre-main sequence begins at ' +format(ags[0], '.2E') +' years for 0.3M_o star.')
          print('Pre-main sequence begins at ' +format(ags[1], '.2E') +' years for 1M_o star.')
          print('Pre-main sequence begins at ' +format(ags[2], '.2E') +' years for 3M_o star.')

          Pre-main sequence begins at 3.55E+06 years for 0.3M_o star.
          Pre-main sequence begins at 7.07E+05 years for 1M_o star.
          Pre-main sequence begins at 2.67E+05 years for 3M_o star.
```

$$\boxed{t_{0.3M_\odot} = 3.55 \times 10^6 \text{ years}}$$

$$\boxed{t_{1M_\odot} = 7.07 \times 10^5 \text{ years}}$$

$$\boxed{t_{3M_\odot} = 2.67 \times 10^5 \text{ years}}$$

The ages of the pre-MS stars do not begin at 0 years because these stars are obfuscated by accretion disks. The more massive, luminous stars are able to be detected sooner because they are intrinsically brighter, hence the trend as mass increases, the beginning age of the star decreases.

Code:

```
In [318]: print('Inital luminosity: ' +format(10**(lum[1][0]), '.2E') +' L_o')
          print('Initial T: ' +format(10**(Te[1][0]), '.2E') +' K')
          print('Inital radius: ' +format((((10**(lum[1][0]) *L_o)/\
              (4 *np.pi *5.67e-8 *(10**(Te[1][0]))**4))**0.5)/(R_o), '.2E') +' R_o')

          Inital luminosity: 3.40E+00 L_o
          Initial T: 4.49E+03 K
          Inital radius: 3.05E+00 R_o
```

$$\boxed{L_{1M_\odot,0} = 3.40 L_\odot}$$