92%

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ASTR5420

Please show all work. If you collaborate with other students, write their names at the top of your homework. Please also write the total number of hours you spent on this. If you use any code or plotting routines, print those out and attach them to your hand-written solutions.
This took me about 7 hours to complete.
I collaborated with Alex on this assignment.

1. **(10%): Hertzsprung Gap (HG):**

+10/10

(a) **Calculate the change in binding energy of a $1M_\odot$ star as it expands during the HG from $1.5R_\odot$ at the tip of the MS to $5R_\odot$ at the bottom of the RGB (1^{st} dredge up).**

$$U = -\frac{-3GM^2}{5R}$$
$$\Delta U = U_2 - U_1$$
$$\Delta U = -\frac{3GM^2}{5}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

Code:

```
In [11]: M_o = 2e30
         # NP Mass of the Sun in kg
         R_o = 6.957e8
         # NP Radius of the Sun in m
         G = 6.67e-11
         # NP Gravitational constant
         Rs = np.array([1.5 *R_o, 5 *R_o])
         # NP Defining array of ending and starting radii
         dU = (-3 *G *M_o **2) /(5) *((1 /Rs[1]) -(1 /Rs[0]))
         print('Change in binding energy: ' +format(dU, '.2E') +' Joules')

Change in binding energy: 1.07E+41 Joules
```

$$\boxed{\Delta U = 1.07 \times 10^{41} \text{ Joules}}$$

(b) **Suppose the star generates an excess $1L_\odot$ that goes into expanding the star during the HG. Compute the corresponding duration of HG evolution. How does this compare to its thermal Kelvin-Helmholtz timescale?**

$$\tau = \frac{\Delta U}{L_\odot}$$

$$\boxed{\tau = 8.93 \times 10^6 \text{ years}}$$

$$\tau_{KH} = \frac{GM_\odot^2}{R_\odot L_\odot}$$

$$\boxed{\tau_{KH} = 3.19 \times 10^7 \text{ years}}$$

Code:

−2% late

```
In [18]: L_o = 3.828e26
         # NP Solar luminosity in mks units
         dt = dU /L_o /(np.pi *10 **7)
         # Duration of Hurtzsprung gap
         print('Duration: ' +format(dt, '.2E') +' years')
         # NP Printing result
         tKH = G *M_o **2 /R_o /L_o /(np.pi *10 **7)
         # NP Thermal timescale for the Sun
         print('Kelvin-Helmholtz timescale: ' +format(tKH, '.2E') +' years')
         # NP Printing result

         Duration: 8.93E+06 years
         Kelvin-Helmholtz timescale: 3.19E+07 years
```

The duration of the Hertzsprung gap is (roughly) the same order of magnitude as the thermal Kelvin-Helmholtz timescale!

2. **(40%): Red Giant Branch (RGB):**

+38/40

(a) **Estimate the radii R (in $R_\odot$) and luminosities L (in $L_\odot$) near the bottom ($M_c = 0.20 M_\odot$) and tip ($M_c = 0.45 M_\odot$) of the RGB using the core-mass relations.**

$$L_* \approx 200 L_\odot \left(\frac{M_c}{0.3 M_\odot}\right)^{7.6}$$

$$R_* \approx 30 R_\odot \left(\frac{M_c}{0.3 M_\odot}\right)^{3.7}$$

Bottom:

$$\boxed{L_* = 9.18 L_\odot}$$
$$\boxed{R_* = 6.69 R_\odot}$$

Tip:

$$\boxed{L_* = 4360 L_\odot}$$
$$\boxed{R_* = 134 R_\odot}$$

Code:

```
In [25]: Mcs = np.array([0.2, 0.45])
         # NP Core masses in solar masses
         L_s = 200 *(Mcs /0.3) **7.6
         # NP Luminosity mass relation for RGB
         R_s = 30 *(Mcs /0.3) **3.7
         # NP Radius mass relation for REG
         print('Bottom RGB stellar parameters:'
               +'\nL: ' +format(L_s[0], '.2E') +' L_o'
               +'\nR: ' +format(R_s[0], '.2E') +' R_o')
         # NP Printing result
         print('Tip RGB stellar parameters:'
               +'\nL: ' +format(L_s[1], '.2E') +' L_o'
               +'\nR: ' +format(R_s[1], '.2E') +' R_o')
         # NP Printing result

         Bottom RGB stellar parameters:
         L: 9.18E+00 L_o
         R: 6.69E+00 R_o
         Tip RGB stellar parameters:
         L: 4.36E+03 L_o
         R: 1.34E+02 R_o
```

(b) **Using the mass-radius relations for non-relativistic degenerate gas, estimate the core densities ρ_c (in g cm $^{-3}$) and temperatures T_c (in K) near the bottom and tip of the RGB (Hint: use**

first-order approximations, making sure to use M_c and R_c in both relations).

$$R \approx 0.01 R_\odot \left(\frac{M}{0.8 M_\odot}\right)^{-1/3}$$

$$\rho_c = \frac{M_c}{4/3 \pi R_c^3}$$

$$\rho_c = \frac{3M_c}{4\pi R_c^3}$$

$$\rho_c = \frac{3M_c}{4\pi\left(0.01 R_\odot \left(\frac{M_c}{0.8M_\odot}\right)^{-1/3}\right)^3}$$

$$\rho_c = \frac{3M_c}{4\pi\left(10^{-6}\right) R_\odot^3 \left(\frac{M_c}{0.8M_\odot}\right)^{-1}}$$

$$\rho_c = \frac{3M_c^2}{4\pi\left(10^{-6}\right) R_\odot^3 \left(0.8 M_\odot\right)}$$

$$T_c = \frac{0.65 G M_c m_H}{k R_c}$$

$$T_c = \frac{0.65 G M_c m_H}{0.01 k R_\odot \left(\frac{M_c}{0.8M_\odot}\right)^{-1/3}}$$

$$T_c = \frac{0.65 G M_c^{4/3} m_H}{0.01 k R_\odot \left(0.8 M_\odot\right)^{1/3}}$$

Bottom:

$$\boxed{\rho_c = 7.09 \times 10^4 \text{ g cm}^{-3}}$$

$$\boxed{T_c = 1.90 \times 10^8 \text{ K}}$$

Tip:

$$\boxed{\rho_c = 3.59 \times 10^5 \text{ g cm}^{-3}}$$

$$\boxed{T_c = 5.60 \times 10^8 \text{ K}}$$

Code:

```
In [275]: k = 1.38e-23
          # NP Boltzmann constant in mks units
          m_p = 1.67e-27
          # NP Mass of proton in kg
          rho_c = (3 *Mcs **2 *M_o **2) /(4 *np.pi *1e-6 *R_o **3 *(0.8 *M_o))
          # NP Calculating core densities for both stages
          T_c = (0.65 *G *(Mcs *M_o) **(4/3) *m_p) /(0.01 *k *R_o *(0.8 *M_o)\
              **(1/3))
          # NP Calculating core temperatures for both stages
          print('Bottom RGB stellar parameters:'
                +'\nrho_c: ' +format(rho_c[0]/1000, '.2E') +' kg m^-3'
                +'\nT: ' +format(T_c[0], '.2E') +' K')
          # NP Printing result
          print('Tip RGB stellar parameters:'
                +'\nrho_c: ' +format(rho_c[1]/1000, '.2E') +' kg m^-3'
                +'\nT: ' +format(T_c[1], '.2E') +' K')
          # NP Printing result

          Bottom RGB stellar parameters:
          rho_c: 7.09E+04 kg m^-3
          T: 1.90E+08 K
          Tip RGB stellar parameters:
          rho_c: 3.59E+05 kg m^-3
          T: 5.60E+08 K
```

(c) **Assume the H-burning shell has solar composition. The bottom of the shell also has similar density and temperature as the core. Estimate the energy production rates ϵ (in erg s^{-1} g^{-1}**

at the bottom and tip of the RGB using Eqn. 6.26 for the CNO cycle (do NOT use $\epsilon \propto \rho T^{20}$, which is valid only near $T_{c,\odot}$). To help visualize how powerful this is, compute the number of Watts = 10^7erg s^{-1} generated from a single cubic centimeter of material at those large densities and temperatures near the core / envelope boundary for both the bottom and tip of the RGB.

$$\epsilon_{CNO} = \frac{4.4\times10^{25}\rho X Z \exp\left(-\frac{15.2}{T_9^{1/3}}\right)}{T_9^{2/3}}$$

$$X \sim 0.7381$$

$$Z \sim 0.0134$$

Bottom:

$$\boxed{\epsilon_c = 3.08\times10^{17}\ \mathrm{erg\ s^{-1}\ g^{-1}}}$$

Tip:

Not one gram! one cc

$$\boxed{\epsilon_c = 2.26\times10^{21}\ \mathrm{erg\ s^{-1}\ g^{-1}}}$$

For one gram of the material near the core, find the corresponding powers in SI units.
Bottom:

need to multiply by ρ

$$\boxed{P = 3.08\times10^{10}\ \mathrm{W}}$$ ~10^{15} W

Tip:

$$\boxed{P = 2.26\times10^{14}\ \mathrm{W}}$$ ~10^{19} W

Code:

```
In [55]: X = 0.7381
         # NP Solar Hydrogen fraction
         Z = 0.0134
         # NP Solar metallicitiy
         eps = (4.4e25 *(rho_c /1000) *X *Z *np.exp(-15.2 /((T_c /1e9) **(1/3))))\
             /((T_c /1e9) **(2/3))
         print('Bottom RGB stellar parameters:'
               +'\neps: ' +format(eps[0], '.2E') +' ergs s^-1 g^-1')
         # NP Printing result
         print('Tip RGB stellar parameters:'
               +'\neps: ' +format(eps[1], '.2E') +' ergs s^-1 g^-1')
         # NP Printing result
         P_cs = eps /(1e7)
         print('Bottom RGB stellar parameters:'
               +'\nP: ' +format(P_cs[0], '.2E') +' W')
         # NP Printing result
         print('Tip RGB stellar parameters:'
               +'\nP: ' +format(P_cs[1], '.2E') +' W')

         Bottom RGB stellar parameters:
         eps: 3.08E+17 ergs s^-1 g^-1
         Tip RGB stellar parameters:
         eps: 2.26E+21 ergs s^-1 g^-1
         Bottom RGB stellar parameters:
         P: 3.08E+10 W
         Tip RGB stellar parameters:
         P: 2.26E+14 W
```

(d) **Estimate the mass-loss rate (in $M_\odot$ yr^{-1}) at the bottom and tip of the RGB.**

$$\dot{M} = 4\times10^{-13}\eta\frac{LR}{M}\ M_\odot\ \mathrm{yr^{-1}}\ (L,\ R,\ M \text{ in solar units})$$

$$\eta \approx 0.5$$

Bottom:

$$\boxed{\dot{M} = 1.23 \times 10^{-11}\ M_\odot\ \mathrm{yr}^{-1}}$$

Tip:

$$\boxed{\dot{M} = 1.17 \times 10^{-7}\ M_\odot\ \mathrm{yr}^{-1}}$$

Code:

```
In [8]: Mdot = 4e-13 *0.5 *(L_s *R_s) /(1)
        # NP Calculating Mdot for both stages
        print('Bottom RGB stellar parameters:'
              +'\nMdot: ' +format(Mdot[0], '.2E') +' Mo yr^-1')
        # NP Printing result
        print('Tip RGB stellar parameters:'
              +'\nMdot: ' +format(Mdot[1], '.2E') +' Mo yr^-1')
        # NP Printing result

        Bottom RGB stellar parameters:
        Mdot: 1.23E-11 Mo yr^-1
        Tip RGB stellar parameters:
        Mdot: 1.17E-07 Mo yr^-1
```

(e) **Estimate the lifetime τ_{TRGB} (in Myr) near the tip of the RGB. To calculate this, first estimate the core mass that corresponds to 50% of the luminosity L_{TRGB} at $M_c = 0.45 M_\odot$. Given your computed growth of the core mass ΔM_c , then estimate the corresponding energy production E using the efficiency for H-fusion you derived in HW #7. Finally compute $\tau_{TRGB} = E/L_{TRGB}$.**

$$L = 200\left(\frac{M_c}{0.3 M_\odot}\right)^{7.6} L_\odot$$

$$\frac{L}{L_\odot} = 200\left(\frac{M_c}{0.3 M_\odot}\right)^{7.6}$$

$$M_c = 0.3 M_\odot \left(\frac{L}{200 L_\odot}\right)^{1/7.6}$$

At half L_{TRGB}, find M_c.

$$\boxed{M_{c,1/2} = 0.411 M_\odot}$$

To increase mass this much, find the energy produced. Assume an efficiency of 0.7%, from last homework.

$$E = f \Delta m c^2$$

$$E = f\,(0.45 - 0.411)\, M_\odot c^2$$

$$\boxed{E = 4.94 \times 10^{43}\ \mathrm{J}}$$

Find the duration of this phase.

$$\tau_{TRGB} = \frac{E}{L_{TRGB}}$$

$$\boxed{\tau_{TRGB} = 0.943\ \mathrm{Myrs}}$$

Code:

```
In [39]: c = 3e8
         # NP Speed of light in m/s
         L_TRGB = L_s[1]
         # NP TRGB Luminosity
         M_halfL = (((L_TRGB /2) /200) **(1 /7.6)) *0.3
         # NP Mass of core when L is half of the tip of red giant branch
         print('Mass at half L_TRGB: ' +format(M_halfL, '.2E') +' Solar masses')
         DE = (0.45 -M_halfL) *c **2 *0.007 *M_o
         # NP Calculating change in energy corresponding to mass growth, assuming H
         # NP fusion efficiency of 0.7% (from homework 7, number 1)
         print('Energy produced during this mass growth: ' +format(DE, '.2E') +' J')
         tau_TRGB = DE / L_TRGB /L_o /np.pi /1e7
         # NP Calculating lifetime in seconds
         print('TRGB lifetime: ' +format(tau_TRGB /1e6, '.2E') +' million years')
         # NP Printing life time in Myrs

Mass at half L_TRGB: 4.11E-01 Solar masses
Energy produced during this mass growth: 4.94E+43 J
TRGB lifetime: 9.43E-01 million years
```

(f) **Essentially all of the RGB mass loss occurs near the tip where L and R are largest. What is the total mass loss (in $M_\odot$) during the RGB?**

$$M_{tot} = \dot{M}_{TRGB}\tau_{TRGB}$$

$$\boxed{M_{tot} = .111M_\odot}$$

Code:

```
In [10]: totalM = Mdot[1] *tau_TRGB
         # NP Calculating total mass loss during RGB phase
         print('Total Mass loss: ' +format(totalM, '.2E') +' Solar masses')
         # NP Printing life time in Myrs

Total Mass loss: 1.11E-01 Solar masses
```

3. **(15%): Transition from Asymptotic Giant Branch (AGB) to Planetary Nebulae (PN):**

+13/15

(a) **At the tip of the AGB, the mass loss becomes a superwind with mass loss rate $\dot{M} = 10^{-5}M_\odot$ yr^{-1} with wind velocity $v_{AGB} = 10$ km s^{-1}. Supposedly single solar-type stars can achieve this, but yours truly believes binary star interactions are necessary to generate a superwind. Once the envelope becomes optically thin and the hot central core becomes exposed, the core generates a faster PN-forming wind of velocity $v_{PN} = 30$ km s^{-1} that sweeps up the prior AGB mass loss. Derive a general expression for the planetary nebula mass M_{PN} in the swept-up shell in terms of age τ_{PN} and the variables defined above. Compute M_{PN} (in $M_\odot$) for $\tau_{PN} = 5{,}000$ yr.**

The mass of the planetary nebula would depend on the difference in speeds between the AGB wind velocity and the planetary nebula wind velocity. If these velocities are the same, we would expect the planetary nebula would have no mass. Additionally, we would expect the planetary nebular mass be inversely proportional to the AGB speed since the higher this speed is, the less mass would be swept up by the planetary nebula. Lastly, we would expect the planetary nebula mass to be directly proportional to the age of the planetary nebula and the mass loss rate for the planetary nebula since these terms give the mass of the "shell" of the planetary nebula.

$$\boxed{M_{PN} = \left(\frac{v_{PN} - v_{AGB}}{v_{AGB}}\right)\dot{M}\tau_{PN}}$$

For $\tau_{PN} = 5000$ years, find M_{PN}.

$$\boxed{M_{PN} = 0.1M_\odot}$$

Code:

```
In [15]: M_PN = 1e-5 * 5000 * (30 -10) /(10)
         # NP Caclulating mass of planetary nebula
         print('M_PN: ' +format(M_PN, '.2E') +' solar masses')
         # NP Printing result

M_PN: 1.00E-01 solar masses
```

(b) **Derive the PN radius R_{PN}. Assuming the PN shell has thickness $\Delta R_{PN} = 0.1R_{PN}$, derive an expression for the ion number density $n_{ion} = \rho/\mu m_H$, where $\mu = 1.4$ for solar composition. What is the maximum lifetime of a PN? I.e., at what age τ_{PN} is the PN no longer detectable such that $n_{ion} < 5$ cm^{-3}, which is just a few times the average ISM density.**

The radius of the planetary nebula will be proportional to the difference in speeds of the wind of the planetary nebula and the wind of the AGB as well as the age of the planerary nebula.

$$\boxed{R_{PN} = (v_{PN} \cancel{- v_{AGB}})\,\tau_{PN}}$$

no, $R_{PN} = v_{PN}\tau_{PN}$

From this relation, and considering the number density, find when planetary nebula is no longer visible ($n_{ion} \approx 5$ cm^{-3}).

$$n_{ion} = \frac{\rho}{\mu m_H}$$
$$n_{ion} = \frac{M_{PN}}{V_{PN}}\frac{1}{\mu m_H}$$
$$n_{ion} = \left(\frac{v_{PN} - v_{AGB}}{v_{AGB}}\right)\frac{\dot{M}\tau_{PN}}{4\pi R_{PN}^2 dR}\frac{1}{\mu m_H}$$
$$n_{ion} = \frac{v_{PN} - v_{AGB}}{v_{AGB}}\frac{\dot{M}\tau_{PN}}{4\pi\left((v_{PN} - v_{AGB})\,\tau_{PN}\right)^2(0.1\,(v_{PN} - v_{AGB}))}\frac{1}{\mu m_H}$$
$$n_{ion} = \frac{(v_{PN} - v_{AGB})\,\dot{M}\tau_{PN}}{0.4\pi v_{AGB}\,(v_{PN} - v_{AGB})^3\,\tau_{PN}^3\mu m_H}$$
$$n_{ion} = \frac{\dot{M}}{0.4\pi v_{AGB}\,(v_{PN} - v_{AGB})^2\,\tau_{PN}^2\mu m_H}$$
$$\tau_{PN}^2 = \frac{\dot{M}}{0.4\pi v_{AGB}\,(v_{PN} - v_{AGB})^2\,n_{ion}\mu m_H}$$
$$\tau_{PN} = \sqrt{\frac{\dot{M}}{0.4\pi v_{AGB}\,(v_{PN} \cancel{- v_{AGB}})^2\,n_{ion}\mu m_H}}$$

right method!

$$\boxed{\tau_{PN} = 105000 \text{ years}}$$

yet $\tau \approx 50{,}000$ yr if $R_{PN} = v_{PN}\tau_{PN}$

Code:

```
In [61]: t_PN = np.sqrt((1e-5 * M_o /(np.pi *10 **7)) /(10000 *(30000 -10000)\
             **2 *5e6 *1.4 *m_p * 0.4 *np.pi))
         # NP Calculating timescale of planetary nebula
         print('PN lifetime: ' +format(t_PN /np.pi /1e7, '.2E') +' years')
         # NP Printing result

         PN lifetime: 1.05E+05 years
```

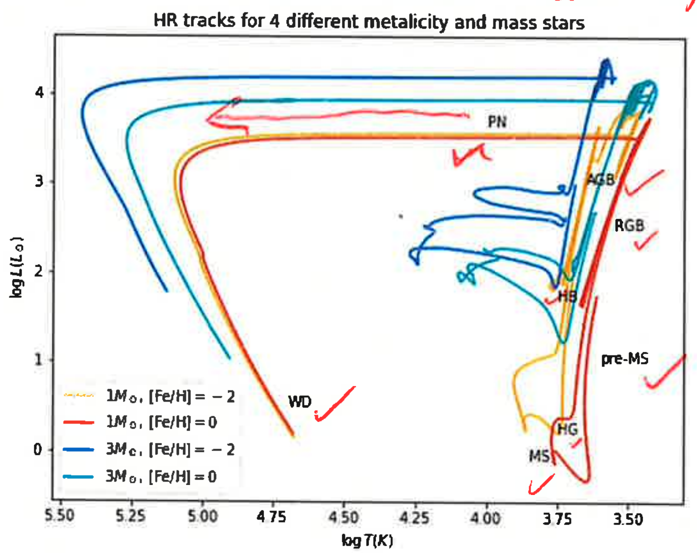
+33/35

4. **(35%): Download from WyoCourses the MESA evolutionary tracks for $1M_\odot$ and $3M_\odot$ stars at both [Fe/H] = −2 and 0 metallicities. The tables include 77 columns (mostly surface abundances), but all you will need is stellar age (in yr), $\log L/L_\odot$, and $\log T_{eff}/K$.**

(a) **Plot the four tracks on the same HR diagram. For the solar-mass, solar-metallicity track, label the pre-MS, MS, HG, RGB, HB, AGB, PN, and WD phases of evolution.**

PN needs $T_{eff} > 30{,}000K$

$\log T_{eff} > 4.5$

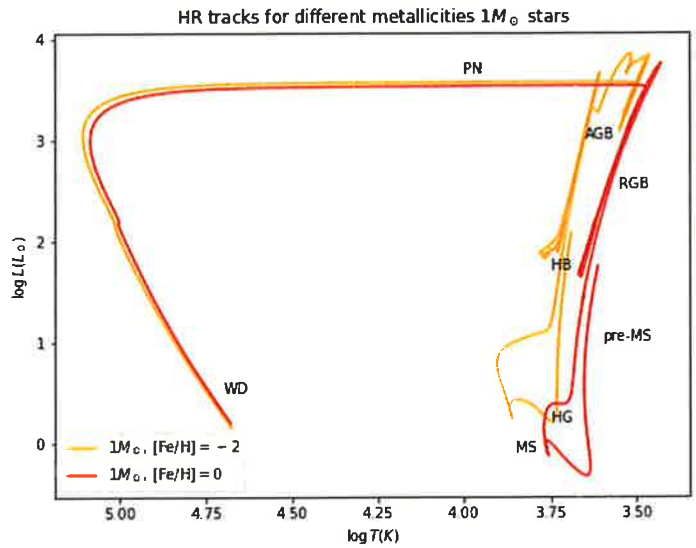


Code:

```
In [15]: L1, T1, R1, ages1 = np.loadtxt('/d/users/nikhil/Downloads/MESA_1p0'
             'Msun_FeHm2p0.txt', usecols = (6, 11, 13, 0), unpack = True)
         L2, T2, R2, ages2 = np.loadtxt('/d/users/nikhil/Downloads/MESA_1p0'
             'Msun_FeH0p0.txt', usecols = (6, 11, 13, 0), unpack = True)
         L3, T3, R3, ages3 = np.loadtxt('/d/users/nikhil/Downloads/MESA_3p0'
             'Msun_FeHm2p0.txt', usecols = (6, 11, 13, 0), unpack = True)
         L4, T4, R4, ages4 = np.loadtxt('/d/users/nikhil/Downloads/MESA_3p0'
             'Msun_FeH0p0.txt', usecols = (6, 11, 13, 0), unpack = True)
         # NP Reading in data for different mass stars with different
         # NP metallicities

In [16]: plt.figure(figsize = [8, 6])
         # NP Making figure larger
         plt.plot(T1, L1, color = 'orange', label = r'$1M_\odot$,'
             ' [Fe/H]$=-2$')
         plt.plot(T2, L2, 'r', label = r'$1M_\odot$, [Fe/H]$=0$')
         plt.plot(T3, L3, 'b', label = r'$3M_\odot$, [Fe/H]$=-2$')
         plt.plot(T4, L4, 'c', label = r'$3M_\odot$, [Fe/H]$=0$')
         # NP Plotting temperaures and luminosities
         plt.text(3.6, 1, 'pre-MS')
         plt.text(3.85, -0.1, 'MS')
         plt.text(3.75, 0.2, 'HG')
         plt.text(3.55, 2.5, 'RGB')
         plt.text(3.75, 1.7, 'HB')
         plt.text(3.65, 3, 'AGB')
         plt.text(4, 3.65, 'PN')
         plt.text(4.7, 0.5, 'WD')
         # NP Labeling different phases in stellar evolution
         plt.gca().invert_xaxis()
         # NP Flipping x-axis
         plt.legend()
         # NP Making legend
         plt.xlabel(r'$\log T(K)$')
         plt.ylabel(r'$\log L(L_\odot)$')
         # NP Labeling axes
         plt.title('HR tracks for 4 different metalicity and mass stars')
         # NP Labeling plot
         plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/HRtracks.png')
         # NP Saving figure
```

(b) **Describe the differences between the metal-poor and solar-metallicity $1M_\odot$ tracks. At what phase of evolution do they differ the most? Explain what causes this difference.**



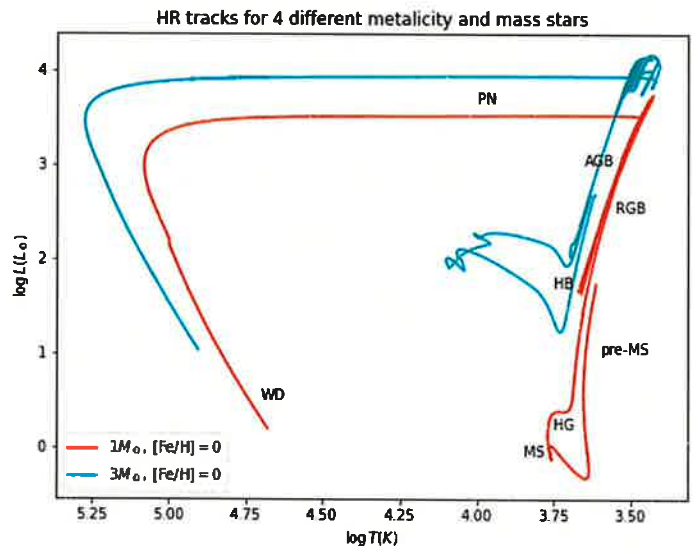
Code:

```
In [66]: plt.figure(figsize = [8, 6])
         # NP Making figure larger
         plt.plot(T1, L1, color = 'orange', label = r'$1M_\odot$,'
             ' [Fe/H]$=-2$')
         plt.plot(T2, L2, 'r', label = r'$1M_\odot$, [Fe/H]$=0$')
         # NP Plotting temperaures and luminosities
         plt.text(3.6, 1, 'pre-MS')
         plt.text(3.85, -0.1, 'MS')
         plt.text(3.75, 0.2, 'HG')
         plt.text(3.55, 2.5, 'RGB')
         plt.text(3.75, 1.7, 'HB')
         plt.text(3.65, 3, 'AGB')
         plt.text(4, 3.65, 'PN')
         plt.text(4.7, 0.5, 'WD')
         # NP Labeling different phases in stellar evolution
         plt.gca().invert_xaxis()
         # NP Flipping x-axis
         plt.legend()
         # NP Making legend
         plt.xlabel(r'$\log T(K)$')
         plt.ylabel(r'$\log L(L_\odot)$')
         plt.title(r'HR tracks for 2 different metolicity $1M_\odot stars')
         # NP Labeling figure
         plt.title(r'HR tracks for different metallicities $1M_\odot$ stars')
         # NP Labeling plot
         plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/HRtracks2.png')
         # NP Saving figure
```

b/c metal-poor stars have less opacity → smaller & hotter

The $1M_\odot$ objects look really similar during the final stages in the star's life. Despite their different metallicities, they very closely resemble each other during the White dwarf and planetary nebula phases. They differ slightly in their pre MS and MS stages. The lower-metallicity star appears slightly up and to the left of the solar-metallicity star, indicating a higher effective temperature and higher luminosity. These two stars most significantly differ during their AGB phases. The lower-metallcity star seems to oscillate in luminosity and temperature during this phase. This difference can be explained by instability in the low-metallicity star due to changes in opacity. The lower-metallicity star is at a higher temperature than the solar-metallicity star. This difference in temperature puts the layers of the low-metallicity star near the temperature in which Hydrogen is ionized. Slight changes in temperature will therefore lead to drastic changes in opacity which casues this star to pulsate.

(c) **Describe qualitatively how the $3M_\odot$ solar-metallicity track differs from its $1M_\odot$ counterpart.**

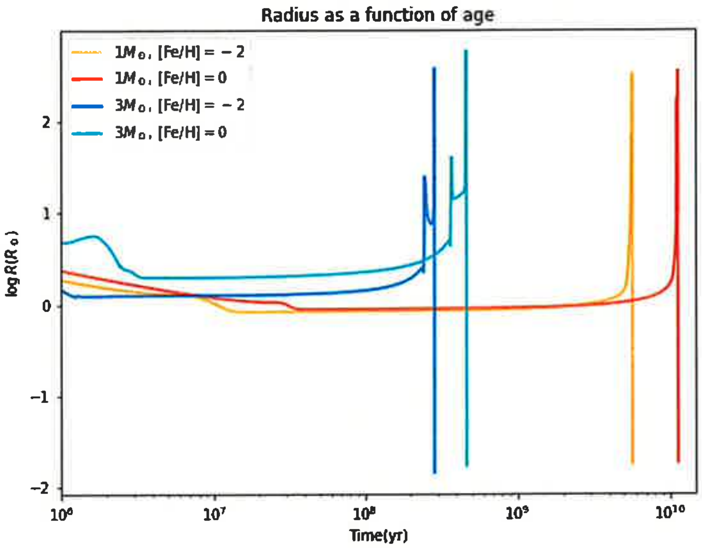


Code:

```
In [67]: plt.figure(figsize = [8, 6])
# NP Making figure larger
plt.plot(T2, L2, 'r', label = r'$1M_\odot$, [Fe/H]$=0$')
plt.plot(T4, L4, 'c', label = r'$3M_\odot$, [Fe/H]$=0$')
# NP Plotting temperaures and luminosities
plt.text(3.6, 1, 'pre-MS')
plt.text(3.85, -0.1, 'MS')
plt.text(3.75, 0.2, 'HG')
plt.text(3.55, 2.5, 'RGB')
plt.text(3.75, 1.7, 'HB')
plt.text(3.65, 3, 'AGB')
plt.text(4, 3.65, 'PN')
plt.text(4.7, 0.5, 'WD')
# NP Labeling different phases in stellar evolution
plt.gca().invert_xaxis()
# NP Flipping x-axis
plt.legend()
# NP Making legend
plt.xlabel(r'$\log T(K)$')
plt.ylabel(r'$\log L(L_\odot)$')
# NP Labeling axes
plt.title('HR tracks for 4 different metalicity and mass stars')
# NP Labeling plot
plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/HRtracks3.png')
# NP Saving figure
```

The $3M_\odot$ star is hotter and more luminous than the $1M_\odot$ star for all phases. In addition, for the pre MS track, the more massive star drops in luminosity, as the $1M_\odot$ star does, but also moves to the left before landing on the MS. This corresponds to the Henyey track, which is more noticeable in massive stars. In addition, during AGB phase, the massive star oscillates in luminosity and temperature, signifying instability from changes in opacity. Finally, the planetary nebula and white dwarf formed by the massive stars is both hotter and more luminous than those formed by the solar mass star.

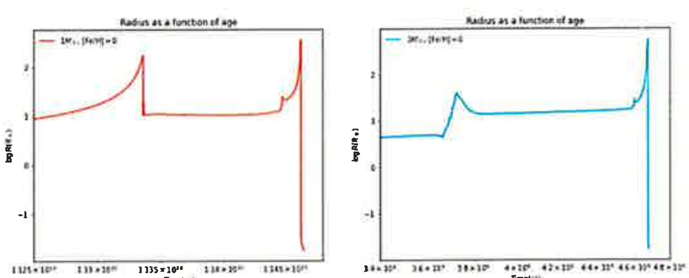
(d) **Separately plot the stellar radii R (in $R_\odot$) as a function of time. Adjust the time scale to feature the different phases of evolution.**



Code:

```
In [233]: plt.figure(figsize = [8, 6])
          # NP Making figure larger
          plt.plot(ages1, R1, color = 'orange', label = r'$1M_\odot$,'
              ' [Fe/H]$=-2$')
          plt.plot(ages2, R2, 'r', label = r'$1M_\odot$, [Fe/H]$=0$')
          plt.plot(ages3, R3, 'b', label = r'$3M_\odot$, [Fe/H]$=-2$')
          plt.plot(ages4, R4, 'c', label = r'$3M_\odot$, [Fe/H]$=0$')
          # NP Plotting radius as a function of age for all stars
          plt.legend()
          # NP Creating legend
          plt.xlabel(r'Time(yr)')
          plt.ylabel(r'$\log R(R_\odot)$')
          plt.title(r'Radius as a function of age')
          # NP Labeling figure
          plt.xscale('log')
          # NP Scaling x-axis to log scale
          plt.xlim(1 *10 **6, 1.5*10**10)
          plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/Radiustime'
              'stars.png')
          # NP Saving figure
```

(e) **Report the maximum radii (in $R_\odot$) during the RGB and AGB for the two solar-metallicity tracks. Explain why $R_{AGB} \approx R_{RGB}$ for $1M_\odot$ stars but $R_{AGB} >> R_{RGB}$ for $3M_\odot$ stars.**



From the plot above, it seems the RGB for the $1M_\odot$ star occurs approximately before 1.135×10^{10} years. For the $3M_\odot$ star, the RGB occurs before 4.2×10^8 years. Find the maximum radii before and after these times for both stars.

Code:

```
In [274]: print('1M_o  log[RGB radius]: ' +format(np.max(R2[ages2 < 1.135e10]),\
              '.2E'))
          print('1M_o  log[AGB radius]: ' +format(np.max(R2), '.2E'))
          print('3M_o  log[RGB radius]: ' +format(np.max(R4[ages4 < 4.2e8]),\
              '.2E'))
          print('3M_o  log[AGB radius]: ' +format(np.max(R4), '.2E'))
          # NP Printing radii for RGB and AGB for different mass stars

          1M_o  log[RGB radius]: 2.24E+00
          1M_o  log[AGB radius]: 2.55E+00
          3M_o  log[RGB radius]: 1.64E+00
          3M_o  log[AGB radius]: 2.77E+00
```

$$\boxed{\log R_{RGB,1M_\odot} = 2.24}$$
$$\boxed{\log R_{AGB,1M_\odot} = 2.55}$$
$$\boxed{\log R_{RGB,3M_\odot} = 1.64}$$
$$\boxed{\log R_{AGB,3M_\odot} = 2.77}$$

For the $1M_\odot$, the radius at the AGB and RGB are roughly equal, while for the $3M_\odot$ star, the radius of AGB is roughly 1.1 dex larger than the radius of the RGB. This is because For $M > 1.9M_\odot$, the core never becomes fully degenerate. Because the star is more massive, the Helium flash begins earlier and therefore, L and R never become as large as they do for the AGB.

(f) **Finally, report the MS lifetimes for all four tracks. For $1M_\odot$, which lives longer – metal-poor or solar-metallicity? What about for $3M_\odot$?**
Assuming the Hertzsprung gap begins when the radius doubles the main sequence value. — Good choice!
Code:

```
In [232]: radii = np.array([R1, R2, R3, R4])
          # NP Making array of radii
          ages = np.array([ages1, ages2, ages3, ages4])
          # NP Making array of ages
          R_ms = np.array([np.min(radii[i][ages[i] < 1e8]) for i in\
              range(len(radii))])
          # Calculating array of MS radii
          R_hg = R_ms +np.log10(2)
          # Creating array of radii corresponding to Hertzsprung gap
          indices = np.array([np.argmin(np.abs(radii[i][(ages[i] > 1e7) &\
              (ages[i] < ages[i][np.argwhere(radii[i] == max(radii[i]))[0][0]])]\
              - R_hg[i])) for i in range(len(ages))])
          # NP Finding when array of radii are two times the main sequence radii
          # NP Limiting search to after 1e7 years and the time at which the radius
          # NP is a maximum, tip of AGB.
          mslifetimes = np.array([ages[i][(ages[i] > 1e7) &\
              (ages[i] < ages[i][np.argwhere(radii[i] == max(radii[i]))[0][0]])]\
              [indices[i]] for i in range(len(ages))])
          print('MS lifetime for 1M_o Fe/H -2: ' +format(mslifetimes[0], '.2E')\
              +' years')
          print('MS lifetime for 1M_o Fe/H 0: ' +format(mslifetimes[1], '.2E')\
              +' years')
          print('MS lifetime for 3M_o Fe/H -2: ' +format(mslifetimes[2], '.2E')\
              +' years')
          print('MS lifetime for 3M_o Fe/H 0: ' +format(mslifetimes[3], '.2E')\
              +' years')
          # NP Printing results

          MS lifetime for 1M_o Fe/H -2: 5.21E+09 years
          MS lifetime for 1M_o Fe/H 0: 1.04E+10 years
          MS lifetime for 3M_o Fe/H -2: 2.43E+08 years
          MS lifetime for 3M_o Fe/H 0: 3.28E+08 years
```

$$\boxed{\tau_{MS,1M_\odot,[Fe/H]-2} = 5.21 \times 10^9 \text{ years}}$$
$$\boxed{\tau_{MS,1M_\odot,[Fe/H]0} = 1.04 \times 10^{10} \text{ years}}$$
$$\boxed{\tau_{MS,3M_\odot,[Fe/H]-2} = 2.43 \times 10^8 \text{ years}}$$
$$\boxed{\tau_{MS,3M_\odot,[Fe/H]0} = 3.28 \times 10^8 \text{ years}}$$

For $1M_\odot$, the solar metallicity star lives longer. For the $3M_\odot$ star, the same trend continues, as the higher metallicity star lives longer.