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23 September 2022
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ASTR5420

95%

Please show all work. If you collaborate with other students, write their names at the top of your homework. Please also write the total number of hours you spent on this. If you use any code or plotting routines, print those out and attach them to your hand-written solutions.

1. **(20%) Molecular and Dust Extinction and Reddening.** +16/20

   (a) **The intrinsic color of our Sun is $(B-V)_0 = 0.65$ mag, where the B and V central wavelengths are 4450 and 5510 Å, respectively. Compute the reddening $E(B-V)$ and apparent color $(B-V) = (B-V)_0 + E(B-V)$ of the sun near sunset assuming $A_V = 3$ mag of visual extinction and that molecules in our atmosphere attenuate light via Rayleigh scattering. Keep in mind that orange K5V stars have $(B-V)_0 = 1.1$ mag and red M5V stars have $(B-V)_0 = 1.8$ mag. Compute the relative visibility $R_V = A_V/E(B-V)$ of Rayleigh scattering.**

$$A_B = A_V \left(\frac{\lambda_B}{\lambda_V}\right)^{-\alpha}$$
$$\alpha = 1.8$$

where did this come from?

Rayleigh has cross-section $\sigma \propto \lambda^{-4}$

$$\lambda_B = 4450$$
$$\lambda_V = 5510$$
$$A_B = 3\left(\frac{4450}{5510}\right)^{-1.8} \text{ mag}$$
$$A_B = 4.41 \text{ mag}$$
$$E(B-V) = A_B - A_V$$
$$E(B-V) = 4.41 \text{ mag} - 3 \text{ mag}$$

$$\boxed{E(B-V) = 1.41 \text{ mag}}$$

$$(B-V) = (B-V)_0 + E(B-V)$$
$$(B-V)_0 = 0.65 \text{ mag}$$
$$(B-V) = 0.65 \text{ mag} + 1.41 \text{ mag}$$

$$\boxed{(B-V) = 2.06 \text{ mag}}$$

$$R_V = \frac{A_V}{E(B-V)}$$
$$R_V = \frac{3}{1.41}$$

$$\boxed{R_V = 2.13}$$

Code:

```
In [22]: A_V = 3
         # NP Visual extinction in atmosphere
         alpha = 1.8
         # NP Wavelength dependence in atmosphere
         l_B = 4450
         # NP Blue wavelength in Angstroms
         l_V = 5510
         # NP Visual wavelength in Angstroms
         A_B = A_V * (l_B/l_V)**(-1 *alpha)
         # NP B extinction calculation
         print('A_B: ' +format(A_B, '.2E') +' mag')
         # NP Printing result
         EBminusV = A_B - A_V
         # NP Reddening calculation
         print('E(B-V): ' +format(EBminusV, '.2E') +' mag')
         # NP Printing result
         BminusV_0 = 0.65
         # NP Sun's intrinsic color
         BminusV = BminusV_0 +A_B -A_V
         # NP Calculating Sun's observed color
         print('B-V: ' +format(BminusV, '.2E') +' mag')
         # NP Printing result
         R_V = A_V/EBminusV
         # NP Calculating relative visibility, R_V
         print('R_V: ' +format(R_V, '.2E'))
         # NP Printing result
```

```
A_B: 4.41E+00 mag
E(B-V): 1.41E+00 mag
B-V: 2.06E+00 mag
R_V: 2.13E+00
```

(b) **The canonical relative visibility of Milky Way dust / ISM is $R_V = 3.1$. Estimate the wavelength dependence of MW dust extinction. By what other process do dust grains attenuate light? Use your answer to explain why MW dust extinction has a steeper or shallower relative visibility than that from pure Rayleigh scattering.**

$$R_V = \frac{A_V}{E(B-V)}$$

$$R_V = \frac{A_V}{A_B - A_V}$$

$$R_V = \frac{1}{\frac{A_B}{A_V} - 1}$$

$$R_V = \frac{1}{\left(\frac{\lambda_B}{\lambda_V}\right)^{-\alpha} - 1}$$

$$1 = R_V \left(\frac{\lambda_B}{\lambda_V}\right)^{-\alpha} - R_V$$

$$\frac{1 + R_V}{R_V} = \left(\frac{\lambda_B}{\lambda_V}\right)^{-\alpha}$$

$$\log\left(\frac{1 + R_V}{R_V}\right) = -\alpha \log\left(\frac{\lambda_B}{\lambda_V}\right)$$

$$\alpha = \frac{\log\left(\frac{1+R_V}{R_V}\right)}{\log\left(\frac{\lambda_B}{\lambda_V}\right)}$$

$$\alpha_{MW} = \frac{\log\left(\frac{1+3.1}{3.1}\right)}{\log\left(\frac{4450}{5510}\right)}$$

Code:

```
In [28]: R_VMW = 3.1
         # NP Milky Way relative visibility
         alphamw = -1*np.log((1 +R_VMW)/R_VMW)/np.log(l_B/l_V)
         # NP Calculating wavelength dependence for Milky Way relative
         # NP visibility
         print('alpha_MW: ' +format(alphamw, '.2E'))

         alpha_MW: 1.31E+00
```

correct, but explain significance (Mie scattering)

25/25

$$\boxed{\alpha_{MW} = 1.31}$$

2. **(25%) Estimate how long it takes photons produced in the core of the Sun to radiate beyond the photosphere. To compute this, separate the Sun's internal structure into three parts:**

   (a) **The Sun's radiative core within $R < 0.25 R_\odot$ is completely ionized ($T = 10^7$ K) and has a mean density of $\rho = 100\text{g cm}^{-3}$. Calculate the mean free path l_{ph} of a photon, the number $N = (d/l_{ph})^2$ of steps the photons take to travel a linear distance d according to a random walk, and then the timescale $\tau_{rad} = N l_{ph}/c$ it takes the photons to radiate through that distance.**

$$l_{ph} = \frac{1}{\kappa\rho}$$

At large T, ρ:

$$\kappa_T = 0.2\,(1+X)\ \text{cm}^2\ \text{g}^{-1}$$

Calculate mean free path.

$$X \sim 0.5$$
$$\kappa_T = 0.2\,(1+0.5)\ \text{cm}^2\ \text{g}^{-1}$$
$$\kappa_T = 0.3\ \text{cm}^2\ \text{g}^{-1}$$
$$l_{ph} = \frac{1}{100\ \text{g cm}^{-3}\,(0.3\ \text{cm}^2\ \text{g}^{-1})}$$

$$\boxed{l_{ph} = 3.33 \times 10^{-2}\ \text{cm}}$$

Calculate number of steps.

$$N = \left(\frac{d}{l_{ph}}\right)^2$$
$$N = \left(\frac{R_f - R_i}{l_{ph}}\right)^2$$
$$N = \left(\frac{0.25\,(6.957\times10^{10}) - 0}{3.33\times10^{-2}}\right)^2$$

$$\boxed{N = 2.72\times10^{23}}$$

Calculate time scale.

$$\tau_{core} = N\frac{l_{ph}}{c}$$
$$\tau_{core} = (2.72\times10^{23})\,\frac{3.33\times10^{-2}}{3\times10^{10}}$$

Code:

```
In [4]: print('CORE:')
        # NP Printing label for this section
        X = 0.5
        # NP Hydrogen fraction in core
        K_Tc = 0.2* (1 +X)
        # NP Opacity in Sun's core
        print('Opacity: ' +format(K_Tc, '.2E') +' cm^2 g^-1')
        # NP Printing result
        rho_C = 100
        # NP Approximate density in Sun's core
        l_core = 1 /(K_Tc *rho_C)
        # NP Mean free path in Sun's core
        print('l_ph: ' +format(l_core, '.2E') +' cm')
        # NP Printing result
        R_o = 6.957e8
        # NP Defining solar radius in meters
        d_c = 0.25 *(R_o *1e2)
        # NP Calculating distance traveled by photon in cm
        N_core = (d_c/l_core) **2
        # Calculating number of steps taken by photon
        print('Steps taken: ' +format(N_core, '.2E'))
        # NP Displaying result
        c = 3e8
        # NP Defining speed of light in m/s
        t_core = N_core *(l_core /(c *10**2))
        # NP Calculating timescale
        print('Timescale: ' +format(t_core/(np.pi *10**7), '.2E') \
            +' years')
        # NP Printing result

        CORE:
        Opacity: 3.00E-01 cm^2 g^-1
        l_ph: 3.33E-02 cm
        Steps taken: 2.72E+23
        Timescale: 9.63E+03 years
```

$$\boxed{9630 \text{ years}}$$

(b) **The middle radiative layer across $R = 0.25 - 0.70R_\odot$ is mostly ionized ($T = 10^6$ K) and has a mean opacity of $\kappa = 1$ cm^2 g^{-1} and mean density of $\rho = 1$ g cm^{-3}. Calculate l_{ph}, N, and τ_{rad} for this layer as in part (a).**

Calculate mean free path.

$$\kappa = 1 \text{ cm}^2 \text{ g}^{-1}$$

$$l_{ph} = \frac{1}{\kappa\rho}$$

$$l_{ph} = \frac{1}{1 \text{ g cm}^{-3} \left(1 \text{ cm}^2 \text{ g}^{-1}\right)}$$

$$\boxed{l_{ph} = 1 \text{ cm}}$$

Calculate number of steps.

$$N = \left(\frac{d}{l_{ph}}\right)^2$$

$$N = \left(\frac{R_f - R_i}{l_{ph}}\right)^2$$

$$N = \left(\frac{0.7\left(6.957 \times 10^{10}\right) - 0.25\left(6.957 \times 10^{10}\right)}{3.33 \times 10^{-2}}\right)^2$$

$$\boxed{N = 9.80 \times 10^{20}}$$

Calculate time scale.

$$\tau_{mid} = N\frac{l_{ph}}{c}$$
$$\tau_{mid} = \left(9.80\times10^{20}\right)\frac{1}{3\times10^{10}}$$

Code:

```
In [5]: print('Middle:')
        # NP Printing label for this section
        K_Tm = 1
        # NP Opacity in Sun, middle
        rho_m = 1
        # NP Density in Sun, middle
        l_m = 1 /(K_Tm *rho_m)
        # NP Mean free path in Sun, middle
        print('l_ph: ' +format(l_m, '.2E') +' cm')
        # NP Printing result
        d_m = (0.7 -0.25) *(R_o *1e2)
        # NP Calculating distance traveled by photon in cm
        N_m = (d_m/l_m) **2
        # Calculating number of steps taken by photon
        print('Steps taken: ' +format(N_m, '.2E'))
        # NP Displaying result
        t_m = N_m *(l_m /(c *10**2))
        # NP Calculating timescale
        print('Timescale: ' +format(t_m/(np.pi *10**7), '.2E') \
            +' years')
        # NP Printing result

        Middle:
        l_ph: 1.00E+00 cm
        Steps taken: 9.80E+20
        Timescale: 1.04E+03 years
```

$$\boxed{1040\text{ years}}$$

(c) **The outer layer of the Sun across $R = 0.70 - 1.00R_\odot$ is fully convective. In class, we showed that the timescale for eddies to cross a fully convective star is $\tau_{conv} = (M_* R_*^2/L_*)^{1/3}$. For an outer convective envelope, $\tau_{conv} = (M_{env} R_* R_{env}/L_*)^{1/3}$ gives a better approximation, where $M_{env} = 0.02M_\odot$ and $R_{env} = 0.3R_\odot$ is the mass and thickness, respectively, of the Sun's convective envelope. Compute τ_{conv}.**

$$\tau_{conv} = \left(\frac{M_{env}R_*R_{env}}{L_*}\right)^{1/3}$$
$$\tau_{conv} = \left(\frac{(0.02M_\odot)\,R_*L_*\,(0.3R_\odot)}{L_\odot}\right)^{1/3}$$

Code:

```
In [6]: print('Envelope:')
        # NP Printing label for this section
        M_o = 2*10**30
        # NP Defining mass of Sun in kg
        L_o = 3.828*10**26
        # NP Defining bolometric luminosity of Sun
        M_env = 0.02 *M_o
        # NP Defining envelope mass
        R_env = 0.3 *R_o
        # NP Defining envelope radius
        t_env = (M_env *R_o *R_env /L_o) **(1/3)
        print('Timescale: ' +format(t_env/(np.pi *10**7), '.2E') \
            +' years')

        Envelope:
        Timescale: 7.88E-02 years
```

$$\boxed{\tau_{conv} = 7.88 \times 10^{-2} \text{ years}}$$

(d) **Add your answers in a - c to determine the total time (in yr) it takes energy to escape from the center of the Sun.**

$$\tau_{esc} = \tau_{core} + \tau_{mid} + \tau_{env}$$

$$\tau_{esc} = (9630 \text{ years}) + (1040 \text{ years}) + \left(7.88 \times 10^{-2} \text{ years}\right)$$

Code:

```
In [182]: t_esc = t_core +t_m +t_env
          # NP Calculating total escape time
          print('t_esc: ' +format(t_esc/(np.pi *10**7), '.2E') +' years')
          # NP Printing result

          t_esc: 1.07E+04 years
```

$$\boxed{\tau_{esc} = 10700 \text{ years}}$$

(e) **As a comparison, compute the radiative diffusion timescale τ_{rad} within the convective envelope. Within this layer, the temperature drops to $T = 10^5$ K and bound-bound transitions substantially increase the opacity to $\kappa = 103$ cm^2 g^{-1}. Is $\tau_{conv} << \tau_{rad}$ as expected when convection is the dominant mode of energy transport? (Hint: first use the mass conservation equation).**

$$\tau_{rad} = N\frac{l_{ph}}{c}$$

$$\tau_{rad} = \left(\frac{d}{l_{ph}}\right)^2 \frac{l_{ph}}{c}$$

$$\tau_{rad} = \frac{d^2}{l_{ph}c}$$

$$\tau_{rad} = (R_f - R_i)^2 \frac{\kappa\rho}{c}$$

Use mass conservation to calculate density.

$$dM = 4\pi r^2 \rho dr$$

$$\rho = \frac{dM}{4\pi r^2 dr}$$

Plug in to previous equation for radiative timescale.

$$\tau_{rad} = (R_f - R_i)^2 \frac{\kappa}{c}\left(\frac{dM}{4\pi r^2 dr}\right)$$

$$\tau_{rad} = (R_f - R_i)^2 \frac{dM\kappa}{4\pi r^2 c dr}$$

$$\tau_{rad} = (1R_\odot - 0.7R_\odot)\frac{0.02M_\odot\left(103\text{ cm}^2\text{ g}^{-1}\right)}{4\pi\left(0.85R_\odot\right)^2\left(3\times10^{10}\text{ cm s}^{-1}\right)}$$

Code:

```
In [427]: t_rad = (1-0.7) *R_o *1e2 *0.02 *M_o *1e3 *103 /(4 \
              *np.pi *(0.85 *R_o *1e2)**2 *c *1e2)
          # NP Calculating radiative timescale
          print('Timescale: ' +format(t_rad/(np.pi*10**7), '.2E')\
              +' years')
          # NP Printing result

          Timescale: 2.08E+03 years
```

$$\boxed{\tau_{rad} = 2080\text{ years}}$$

As expected, $\tau_{rad} >> \tau_{conv}$. In fact, τ_{rad} is roughly 10^5 times larger than τ_{conv}.

+20/20

3. **(20%) One application of the linear Eddington approximation is the Eddington-Barbier relation where $F \propto T^4 \propto \tau$. Specifically, $F(\tau) \propto \left[T(\tau)/T_{eff}\right]^4 = 3/4(\tau + 2/3)$, which is valid only below the photosphere ($\tau > 2/3$) where we can assume LTE. For example, at a layer corresponding to an optical depth $\tau = 10$, the flux is 8 times higher and the temperature is $8^{1/4} \approx 1.7$ times hotter than at the photosphere. However, the probability that the photons from that depth escape from the Sun unimpeded is only $e^{-10} \approx 5\times10^{-5}$.**

(a) **What fraction of the escaping stellar flux is emitted from below the photosphere? (Hint: integrate $F(\tau)e^{-\tau}$ and think about your limits of integration.)**

$$\frac{F_{\tau>2/3}}{F_{tot}} = \int_{\tau_1}^{\tau_2} F(\tau)\,e^{-\tau}d\tau$$

$$\frac{F_{\tau>2/3}}{F_{tot}} = \int_{2/3}^{\infty}\frac{3}{4}\left(\tau+\frac{2}{3}\right)e^{-\tau}d\tau$$

$$\frac{F_{\tau>2/3}}{F_{tot}} = \frac{3}{4}\int_{2/3}^{\infty}\tau e^{-\tau}+\frac{2}{3}e^{-\tau}d\tau$$

Solve each integral and evaluate.

$$\int_{2/3}^{\infty} \tau\, e^{-\tau} d\tau$$
$$u = \tau, dv = e^{-\tau} d\tau$$
$$du = d\tau, v = -e^{-\tau}$$
$$\int udv = uv - \int vdu$$
$$\int_{2/3}^{\infty} \tau e^{-\tau} d\tau = -\tau e^{-\tau} + \int e^{-\tau} d\tau$$
$$\int_{2/3}^{\infty} \tau e^{-\tau} d\tau = -\tau e^{-\tau} - e^{-\tau} \Big|_{2/3}^{\infty}$$
$$\int_{2/3}^{\infty} \tau e^{-\tau} d\tau = -0 - 0 + \frac{2}{3} e^{-2/3} + e^{-2/3}$$
$$\int_{2/3}^{\infty} \tau e^{-\tau} d\tau = \frac{5}{3} e^{-2/3}$$
$$\int_{2/3}^{\infty} \frac{2}{3} e^{-\tau} d\tau = \frac{2}{3} \left[-e^{-\tau} \Big|_{2/3}^{\infty} \right]$$
$$\int_{2/3}^{\infty} \frac{2}{3} e^{-\tau} d\tau = \frac{2}{3} \left[0 + e^{-2/3} \right]$$
$$\int_{2/3}^{\infty} \frac{2}{3} e^{-\tau} d\tau = \frac{2}{3} e^{-2/3}$$

Plug back into original equation.

$$\frac{F_{\tau > 2/3}}{F_{tot}} = \frac{3}{4} \left[\frac{5}{3} e^{-2/3} + \frac{2}{3} e^{-2/3} \right]$$
$$\frac{F_{below}}{F_{tot}} = \frac{3}{4} \left[\frac{7}{3} e^{-2/3} \right]$$
$$\frac{F_{\tau > 2/3}}{F_{tot}} = \frac{7}{4} e^{-2/3}$$

Code:

```
In [446]: f1 = (7/4)*np.exp(-2/3)
          # NP Calculating ratio of flux below photosphere
          print('Ratio of flux emitted below photosphere: '\
              +format(f1, '.2E'))
          # NP Printing result

          Ratio of flux emitted below photosphere: 8.98E-01
```

$$\boxed{\frac{F_{\tau > 2/3}}{F_{tot}} = 0.898}$$

(b) **Above the photosphere, the flux decreases more dramatically with respect to optical depth, i.e., $F(\tau) \approx C\tau^3$. Compute the coefficient C assuming continuity at $\tau = 2/3$. Then calculate**

the escaping stellar flux from this optically thin layer above the photosphere.

$$
\begin{aligned}
F(\tau) &= C\tau^3 \\
1 &= C\,(2/3)^3 \\
1 &= C\frac{8}{27} \\
C &= \frac{27}{8} \\
\frac{F_{\tau<2/3}}{F_{tot}} &= \int_{\tau_1}^{\tau_2} F(\tau)\, e^{-\tau}\, d\tau \\
\frac{F_{\tau<2/3}}{F_{tot}} &= \int_{0}^{2/3} \frac{27}{8}\tau^3 e^{-\tau}\, d\tau
\end{aligned}
$$

From integral calculator:

$$\int \tau^3 e^{-\tau} d\tau = \left(-\tau^3 - 3\tau^2 - 6\tau - 6\right) e^{-\tau}$$

Plug back in.

$$
\begin{aligned}
\frac{F_{\tau<2/3}}{F_{tot}} &= \frac{27}{8}\left[\left(-\tau^3 - 3\tau^2 - 6\tau - 6\right) e^{-\tau}\Big|_0^{2/3}\right] \\
\frac{F_{\tau<2/3}}{F_{tot}} &= \frac{27}{8}\left[(6)\, e^0 - \left(\left(\frac{2}{3}\right)^3 + 3\left(\frac{2}{3}\right)^2 + 6\left(\frac{2}{3}\right) + 6\right) e^{-2/3}\right] \\
\frac{F_{\tau<2/3}}{F_{tot}} &= \frac{27}{8}\left[6 - \left(\frac{8}{27} + \frac{4}{3} + 4 + 6\right) e^{-2/3}\right] \\
\frac{F_{\tau<2/3}}{F_{tot}} &= \frac{27}{8}\left[6 - \left(\frac{8}{27} + \frac{36}{27} + \frac{108}{27} + \frac{162}{27}\right) e^{-2/3}\right] \\
\frac{F_{\tau<2/3}}{F_{tot}} &= \frac{27}{8}\left[6 - \frac{314}{27} e^{-2/3}\right]
\end{aligned}
$$

Code:

```
In [447]: f2 = (27/8) *(6 -(314 *np.exp(-2/3)/27))
          # NP Calculating ratio of flux above photosphere
          print('Ratio of flux emitted below photosphere: '\
              +format(f2, '.2E'))
          # NP Printing result

          Ratio of flux emitted below photosphere: 9.84E-02
```

$$\boxed{\frac{F_{\tau<2/3}}{F_{tot}} = 0.0984}$$

(c) **Do your answers from (a) and (b) sum to approximately the total stellar flux?**

$$
\begin{aligned}
F_{total} &= F_{\tau>2/3} + F_{\tau<2/3} \\
F_{total} &= 0.898 F_{tot} + 0.0984 F_{tot}
\end{aligned}
$$

Code:

In [448]:
```
ftot = f1 +f2
# NP Calculating totals
print('Total flux: ' +format(ftot, '.2E'))
# NP Printing Result
```

```
Total flux: 9.97E-01
```

$$\boxed{F_{tot} = .997F_{tot}}$$

My answers for a and b approximately sum to the total stellar flux!

This question should help you understand that stellar spectra are actually combinations of different blackbodies of different temperatures corresponding to different depths (ignoring bound-bound and bound-free transitions). Nonetheless, because the temperature gradient $dT/d\tau$ is relatively small, the convolved spectrum is quite close to a single blackbody with temperature equal to the effective temperature at optical depth $\tau = 2/3$.

+10/10

4. **(10%) We derived the bolometric flux $F = \int B_\nu d\nu \oint \cos\theta d\Omega = \pi \int B_\nu d\nu = \sigma_{SB} T^4$ assuming blackbody radiation emanating from one side of an optically thick surface. Now consider the outgoing radiation pressure P_{rad}, where only the z-component of the outward radiation contributes to P_{rad}, i.e., there as an additional factor of $\cos\theta$. Assume any outgoing photon is reflected back (e.g., by an electron), imparting twice its momentum on the gas. Compute the radiation pressure $P_{rad} = \frac{2}{c} B_\nu d\nu \oint \cos^2\theta d\Omega$ in terms of σ_{SB}, T, and c.**

$$P_{rad} = \frac{2}{c} \int_0^\infty B_\nu d\nu \oint \cos^2\theta d\Omega$$

$$P_{rad} = \frac{2}{c} \int_0^\infty B_\nu d\nu \int_0^{2\pi} \int_0^{\pi/2} \cos^2\theta \sin\theta d\theta d\phi$$

$$P_{rad} = \frac{2}{c} \left(\int_0^\infty B_\nu d\nu \right) \left(\int_0^{2\pi} d\phi \right) \left(\int_0^{\pi/2} \cos^2\theta \sin\theta d\theta \right)$$

Evaluate integral.

$$\int_0^{\pi/2} \cos^2\theta \sin\theta d\theta$$

$$u = \cos\theta, \frac{du}{d\theta} = -\sin\theta$$

$$\int_{\theta=0}^{\theta=\pi/2} u^2 \sin\theta \left(-\frac{du}{\sin\theta} \right)$$

$$-\int_{u=1}^{u=0} u^2 \, du$$

$$\int_0^{\pi/2} \cos^2\theta \sin\theta d\theta = \frac{1}{3} \left[u^3 \Big|_0^1 \right]$$

$$\int_0^{\pi/2} \cos^2\theta \sin\theta d\theta = \frac{1}{3}$$

Plug back in:

$$P_{rad} = \frac{2}{c} \left(\frac{\sigma}{\pi} T^4 \right) (2\pi) \left(\frac{1}{3} \right)$$

$$\boxed{P_{rad} = \frac{4}{3} \frac{\sigma T^4}{c}}$$

x 24/25

5. **(25%) Download the OPAL opacities table for solar abundances from WyoCourses. The 2D text table provides Rosseland mean opacities $\log \kappa_R$ (cm^2 g^{-1}) in rows of $\log T$ (K) and columns of $\log R = \log(\rho/T_6^3)$, where ρ is in units of g cm^{-3} and T_6 is in units of 10^6 K.**

   (a) **For each grid point, determine the corresponding ρ given T and R.**
   Reading in table.

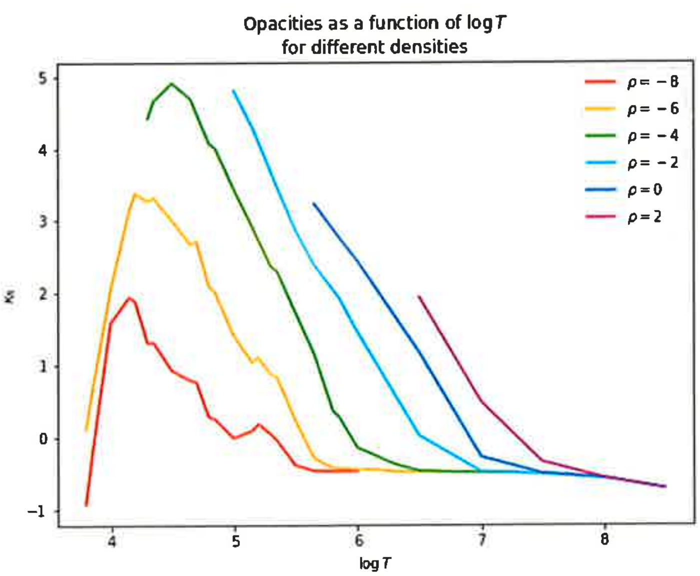
```
In [449]: opa = pd.read_csv('/d/users/nikhil/Downloads/OPAL_SolarComposition.txt')
          # NP Reading in raw text file
          goodopa = opa.to_numpy()
          # NP Converting to numpy array
          betteropa = [np.fromstring(goodopa[i][0], dtype = float, sep = ' ').tolist()\
              for i in range(len(goodopa))]
          # NP Creating a table for opacities and temps., skipping first row
          bestopa = [betteropa[i+1][1:] for i in range(len(betteropa) -1)]
          # NP Creating a list of only opacities
          row_lengths = []
          for row in bestopa:
              row_lengths.append(len(row))
          max_length = max(row_lengths)
          for row in bestopa:
              while len(row) < max_length:
                  row.append(None)
          pristineopa = np.array(bestopa)
          # NP Creating numpy array of opacities to better parse through array
          logTs = np.array([betteropa[i+1][0] for i in range(len(betteropa)-1)])
          # NP Creating array of temperatures
          Tgrid = np.array([[t for r in range(len(logRs))] for t in logTs])
          # NP Creating 2D numpy array of temperatures
          logRs = np.array(np.fromstring(goodopa[0][0][6:], dtype = float, sep = ' '))
          # NP Creating array of R's
          Rgrid = np.array([[r for r in logRs] for t in range(len(logTs))])
          # NP Creating 2D numpy array of temperatures

/tmp/ipykernel_315015/361661287.py:5: DeprecationWarning: string or file could
ed data; this will raise a ValueError in the future.
  betteropa = [np.fromstring(goodopa[i][0], dtype = float, sep = ' ').tolist()
```

   Creating densities

```
In [451]: rhos = np.array([np.array([np.round(r +3 *(t -6) ,3) for r in logRs])\
              for t in logTs])
```

   (b) **Plot $\log \kappa_R$ as a function of $\log T$ for $\log \rho = -8, -6, -4, -2, 0,$ and 2.**



looks good!

   Code:

```
In [453]: ineight = (np.abs(rhos+8) < 0.1)
          insix = (np.abs(rhos+6) < 0.1)
          infour = (np.abs(rhos+4) < 0.1)
          intwo = (np.abs(rhos+2) < 0.1)
          izero = (np.abs(rhos+0) < 0.1)
          itwo = (np.abs(rhos-2) < 0.1)
          # NP Creating indices for each density
          plt.figure(figsize = [8, 6])
          # NP Making figure larger
          plt.plot(Tgrid[ineight], pristineopa[ineight], 'r', label = r'$\rho=-8$')
          plt.plot(Tgrid[insix], pristineopa[insix], color = 'orange', label = r'$\rho=-6$')
          plt.plot(Tgrid[infour], pristineopa[infour], 'g', label = r'$\rho=-4$')
          plt.plot(Tgrid[intwo], pristineopa[intwo], color = 'cyan', label = r'$\rho=-2$')
          plt.plot(Tgrid[izero], pristineopa[izero], color = 'blue', label = r'$\rho=0$')
          plt.plot(Tgrid[itwo], pristineopa[itwo], color = 'purple', label = r'$\rho=2$')
          # NP Plotting opacity as a function of T for each opacity
          plt.xlabel(r'$\logT$')
          plt.ylabel(r'$\kappa_R$')
          # NP Labeling axes
          plt.legend()
          # NP Creating legend
          plt.title(r'Opacities as a function of $\logT$' +'\n'
              'for different densities')
          # NP Creating title
          plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/'
              'opacitiesdensities.png')
          # NP Saving figure
```

(c) **Assuming Kramer's law $\kappa_K = C\rho T^{-7/2}$ for bound-free (photoionization) and free-free (bremsstrahlung) radiation, estimate the coefficient C by evaluating the table at $\log R = -3$ and $T = 10^6$ K.**

$$\begin{aligned}
\kappa_K &= C\rho T^{-7/2} \\
\log\kappa_K &= \log C + \log\rho - 3.5\log T \\
\log C &= \log\kappa_K - \log\rho + 3.5\log T
\end{aligned}$$

At $T = 10^6$ and $\log R = -3$:

$$\boxed{\log C = 24.585}$$

Code:

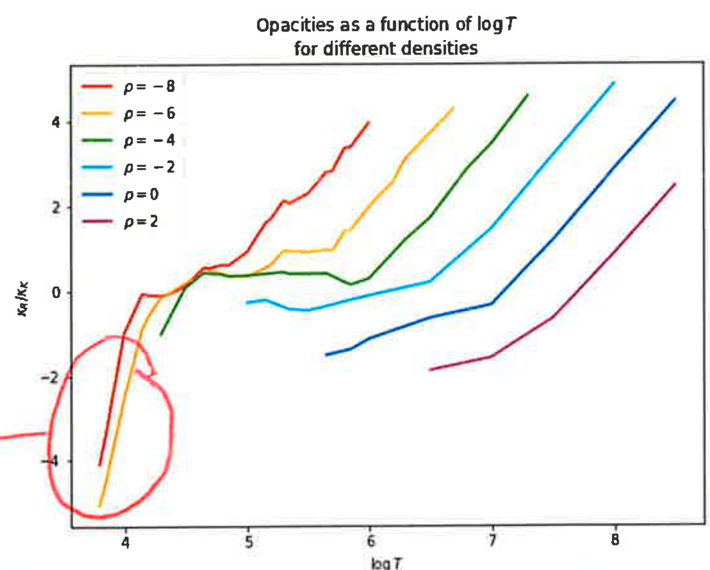
```
In [457]: kramer = pristineopa[np.argwhere(logTs == 6)[0][0]]\
              [np.argwhere(logRs == -3.0)[0][0]]
          # NP Finding Kramer opacity from table
          C = kramer -rhos[np.argwhere(logTs == 6)[0][0]]\
              [np.argwhere(logRs == -3.0)[0][0]] +3.5*\
              (Tgrid[np.argwhere(logTs == 6)[0][0]]\
              [np.argwhere(logRs == -3.0)[0][0]])
          # NP Calculating C
          kgrid = np.array([[C +rhos[t][r] -3.5*Tgrid[t][r] for\
              r in range(len(Tgrid[t]))] for t in range(len(Tgrid))])
          # NP Creating kramer opacities for each point in table
          print('logC = ' +str(C))
          # NP Printing result

          logC = 24.585
```

(d) **Plot the logarithmic ratio $\log(\kappa_R/\kappa_K)$ as a function of $\log T$ for $\log\rho = -8$, -6, -4, -2, 0, and 2. At what temperatures and/or densities are there significant discrepancies between the true Rosseland mean opacities and Kramer's opacity law? Explain why.**
Code:

```
In [458]: plt.figure(figsize = [8, 6])
          # NP Making figure larger
          plt.plot(Tgrid[ineight], pristineopa[ineight]-kgrid[ineight],\
              'r', label = r'$\rho=-8$')
          plt.plot(Tgrid[insix], pristineopa[insix]-kgrid[insix],\
              color = 'orange', label = r'$\rho=-6$')
          plt.plot(Tgrid[infour], pristineopa[infour]-kgrid[infour],\
              'g', label = r'$\rho=-4$')
          plt.plot(Tgrid[intwo], pristineopa[intwo]-kgrid[intwo],\
              color = 'cyan', label = r'$\rho=-2$')
          plt.plot(Tgrid[izero], pristineopa[izero]-kgrid[izero],\
              color = 'blue', label = r'$\rho=0$')
          plt.plot(Tgrid[itwo], pristineopa[itwo]-kgrid[itwo], color\
              = 'purple', label = r'$\rho=2$')
          # NP Plotting difference in log of opacities for each density
          plt.xlabel(r'$\logT$')
          plt.ylabel(r'$\kappa_R/\kappa_K$')
          # NP Labeling axes
          plt.legend()
          # NP Creating legend
          plt.title(r'Opacities as a function of $\logT$' +'\n'
              'for different densities')
          # NP Creating title for plot
          plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/opacity'
              'densitydeviation.png')
          # NP Saving figure
```

Plot:



near photosphere

(small T & ρ) → bound-bound dominate (e.g. Balmer lines strongest at T = 10,000 K)

Generally, extreme deviations from Kramer's opacity law occurs at larger temperatures and densities. This can be explained because Kramer's Law assumes the dominant sources of opacity are bound-free and free-free radiation. At larger temperatures and densities, like in the center of a star, opacity is due to Thompson scattering and not these processes which are assumed to be dominant in Kramer's law. Therefore, at higher temperatures and densities, there will be more of a deviation between opacities calculated by Kramer's law and actual opacities.