97%

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ASTR5420

Please show all work. If you collaborate with other students, write their names at the top of your homework. Please also write the total number of hours you spent on this. If you use any code or plotting routines, print those out and attach them to your hand-written solutions.

+10/10

1. **(10%): For AFGK dwarfs, the mass-luminosity relation scales as $L \propto M^{3.5}$. If this relation continued toward earlier spectral types, then eventually the luminosity would exceed the Eddington limit. Calculate the mass of a star such that its luminosity would equal the Eddington limit assuming $L \propto M^{3.5}$. In reality, in this high-mass regime, massive O and Wolf-Rayet stars actually follow a $L \propto M$ relation limited by the Eddington luminosity.**

$$L_{Edd} = 3.2 \times 10^4 \left(\frac{M}{M_\odot}\right) L_\odot$$

$$L \propto M^{3.5}$$

$$\frac{L}{L_\odot} = 3.2 \times 10^4 \left(\frac{M}{M_\odot}\right) = \left(\frac{M}{M_\odot}\right)^{3.5}$$

$$\left(\frac{M}{M_\odot}\right)^{2.5} = 3.2 \times 10^4$$

$$\boxed{M = 63.4 M_\odot}$$

+20/20

2. **(20%): To first order, the density profile of the Sun can be described as $\rho(r) = \rho_c(1-\frac{r}{R_\odot})^6$.**

(b) **Calculate the mass $M_{r<0.2}$ enclosed within radius $r < 0.2R_\odot$ in units of $M_\odot$.**
Computed numerically in python.

$$M_\odot = \frac{4\pi\rho_c}{R_\odot^6}\int_0^{0.2R_\odot}(R_\odot - r)^6 r^2 dr$$

$$\boxed{M_{r<0.2} = 0.262M_\odot}$$

Code:

```
In [67]: Rrange = np.linspace(0, .2*Ro, 1000)
         M = ((4*np.pi*pc)/(Ro**6))*(Ro-Rrange)**6*Rrange**2
         Min = trapz(M, Rrange)
         print('M_r<0.2 = ' +str(format(Min/Mo, '.2E')) +' Mo')

         M_r<0.2 = 2.62E-01 Mo
```

(c) **As explained in Ex. 2.4 of the textbook, the Sun's central pressure is $P_c = \langle M(r)/r^4\rangle GM/4\pi$, where the crude approximation of $\langle M(r)/r^4\rangle = M_\odot/R_\odot^4$ results in a central pressure that is two orders of magnitude lower than the true value. Instead, the average value $\langle M(r)/r^4\rangle$ is closer to the evaluation of $M(r)/r^4$ toward small radii. Thus assume $\langle M(r)/r^4\rangle = M_{r<0.2}/(0.2R_\odot)^4$, and calculate the central pressure $P_c = GM_\odot M_{r<0.2}/4\pi(0.2R_\odot)^4$ in units of dyn cm^{-2}.**

$$P_c = \frac{GM_\odot M_{r<0.2}}{4\pi\,(0.2R_\odot)^4}$$

$$\boxed{P_c = 1.48 \times 10^{17}\ \text{dyn cm}^{-2}}$$

Code:

```
In [73]: Pc = (G*Mo*Min)/(4*np.pi*(0.2*Ro)**4)
         print('Central Pressure: ' +format(Pc, '.2E') +' Pa')
         print('Central Pressure: ' +format(Pc*1000/100, '.2E') +' dyn/cm^2')

         Central Pressure: 1.48E+16 Pa
         Central Pressure: 1.48E+17 dyn/cm^2
```

+20/20

3. **(20%) Assume that Earth's troposphere is isothermal with temperature $T = 280$ K and composed of 77% N_2 and 23% O_2.**

(a) **Compute the mean molecular weight μ of air.**

$$\mu = \sum_i \frac{n_i}{N} m_i$$

$$\mu = 0.77\,(28.014\ \text{amu}) + 0.23\,(31.999\ \text{amu})$$

$$\boxed{\mu = 28.9\ \text{amu}}$$

Code:

```
In [78]: mu = (0.77*28.014 +0.23*31.999)
         print('mu: ' +format(mu, '.2E') +' amu')

         mu: 2.89E+01 amu
```

```
In [92]: P0 = 101500
         r = 2943
         Pr = np.exp(-r/H)
         print('Atmospheric Pressure: ' +format(Pr, '.2E') +' P_0')

Atmospheric Pressure: 6.97E-01 P_0
```

x15/15

4. **(15%) The giant molecular cloud Orion A is the most active star-forming region in the solar neighborhood. The cloud has mass $M = 10^5 M_\odot$, radius $r = 50$ pc, and mean temperature $T = 60$ K. Compare its mean density ρ to the critical Jeans density ρ_J (both in units of g cm^{-3}) assuming the gas is completely composed of molecular hydrogen H_2. Is $\rho > \rho_J$ as expected?**

$$\rho = \frac{M}{V}$$

$$\rho = \frac{M}{4/3\pi R^3}$$

$$\boxed{\rho = 1.36 \times 10^{-26} \text{ g cm}^{-3}} \times$$

$$\rho_J = \left(\frac{5kT}{\mu m_H G}\right)^3 \left(\frac{3}{4\pi M^2}\right)$$

$$\boxed{\rho_J = 3.47 \times 10^{-20} \text{g cm}^{-3}} \times$$

Code:

```
In [102]: Tc = 60
          Mc = (10**5)*Mo
          rho = (Mc)/((4/3)*np.pi*(50*3.09*10**16)**3)
          print('Cloud Density: ' +format(rho/1000, '.2E') +' g/cm^3')
          rhoJ = (((5*k*Tc)/(2*mh*G))**3)*((3)/(4*np.pi*(Mc**2)))
          print('Jean\'s Density: ' +format(rhoJ/1000, '.2E') +' g/cm^3')

Cloud Density: 1.29E-23 g/cm^3
Jean's Density: 3.83E-26 g/cm^3
```

these are right. ✓

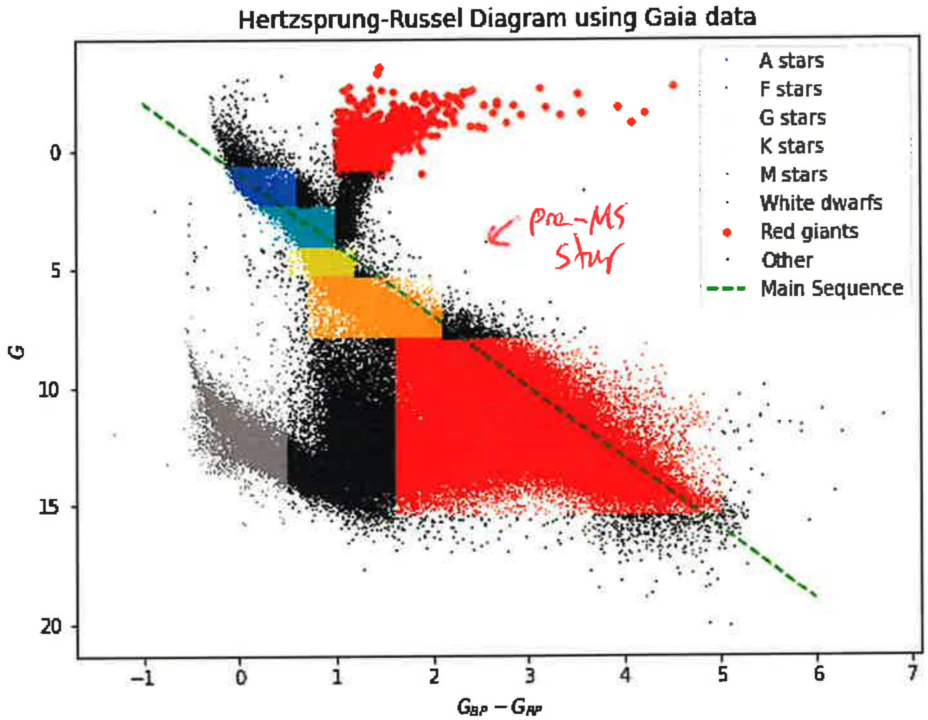
x32/35

As expected, the density of this molecular cloud is greater than the Jean's density. This means that according to the Jean's criterion, this cloud will have active star formation.

5. **(35%) Download from WyoCourses the Gaia table of 942,267 stars brighter than $G < 19$ mag and within $d < 150$ pc (parallaxes > 6.6667 mas). The file is comma separated values (.csv), and the columns are Gaia DR3 ID, RA, Dec, parallax (mas), G, $G_{BP} - G_{RP}$, G_{BP}–G, and $G - G_{RP}$.**

   (a) **Plot a color-magnitude diagram (CMD; MG versus $G_{BP} - G_{RP}$). Label the main-sequence, white dwarfs, and red giant branch. Draw and label the approximate regions bracketing the different spectral types:**

A dwarfs ($0.7 < M_G < 2.4, -0.3 < G_{BP}$–$G_{RP} < 0.6$)
F dwarfs ($2.4 < M_G < 4.2, 0.1 < G_{BP}$–$G_{RP} < 1.0$)
G dwarfs ($4.2 < M_G < 5.4, 0.5 < G_{BP}$–$G_{RP} < 1.2$)
K dwarfs ($5.4 < M_G < 8.0, 0.7 < G_{BP}$–$G_{RP} < 2.1$)
M dwarfs ($8.0 < M_G < 15.5, 1.6 < G_{BP}$–$G_{RP} < 5.0$)



The approximate main sequence is shown as a dotted line in green while each of the points are colored roughly to their spectral type. Also labeled are the approximate groupings of white dwarfs and red giants.

(b) **What fraction of your stars are M-dwarfs? Now consider the subset of stars within $d < 40$ pc? Within this subset, what fraction of stars are M-dwarfs? Explain why this second answer is larger and closer to the true value.**
Code:

```
In [133]: print('Fraction of M dwarfs in the whole set: ' \
                +str(np.round(100*len(GM[iM])/len(GM), 3)) +'%')
          iclose = (distances < 40)
          print('Fraction of M dwarfs closer than 40 pc: ' \
                +str(np.round(100*len(GM[iM & iclose])/len(GM[iclose]), 3)) +'%')

          Fraction of M dwarfs in the whole set: 70.029%
          Fraction of M dwarfs closer than 40 pc: 70.209%
```

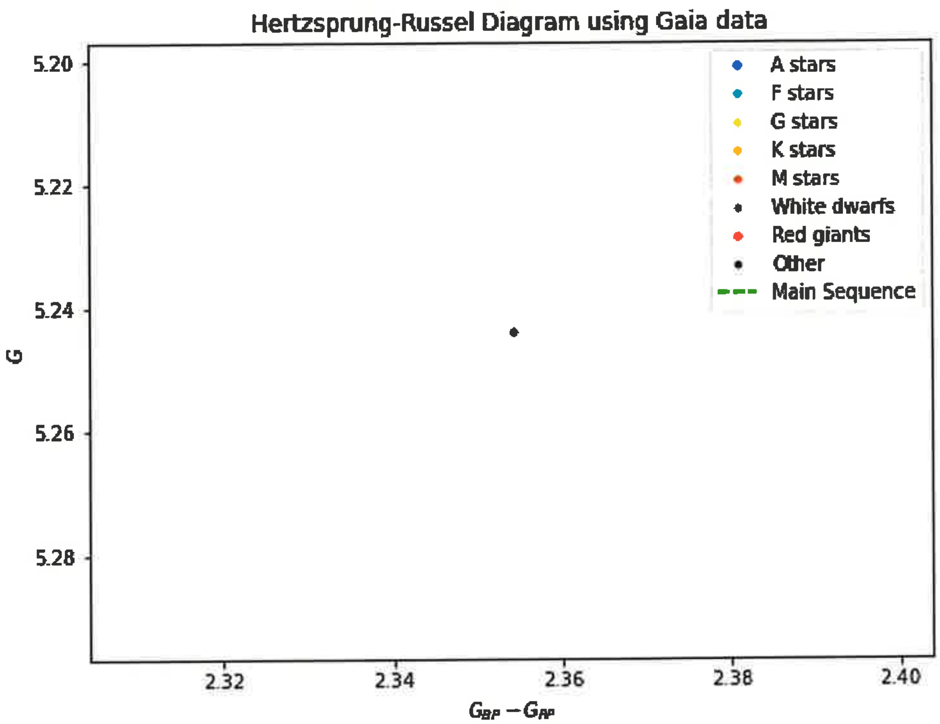
$$\boxed{\frac{M}{N_{total}} = 0.70029, \left(\frac{M}{N_{total}}\right)_{d<40} = 0.70209}$$

The second fraction is slightly larger because at closer distances ($d < 40$ pc), M dwarfs are easier to detect than at larger distances. This is an example of Malmquist bias, which is the effect of exluding intrinsically dimmer objects at larger distances. As a result, the fraction calculated at distances closer than 40 pc is likely closer to the true value of the ratio of M dwarf stars over total stars.

(c) **Identify the hot luminous star near G_{BP}–$G_{RP} = -0.500$ and $MG = 7.079$. Describe its location on the CMD. What are its coordinates? Plug the coordinates into Simbad (https://simbad.u-strasbg.fr/simbad/sim-fcoo). What is its name and type of object? To see the full extent of the object, make sure to select DSS in the image viewer and zoom out to a 1.5° field of view.**

In [146]:
```
plt.figure(figsize = [8, 6])
iA = (GM > 0.7) & (GM < 2.4) & (bpmrp > -0.3) & (bpmrp < 0.6)
iF = (GM > 2.4) & (GM < 4.2) & (bpmrp > 0.1) & (bpmrp < 1)
iG = (GM > 4.2) & (GM < 5.4) & (bpmrp > 0.5) & (bpmrp < 1.2)
iK = (GM > 5.4) & (GM < 8.0) & (bpmrp > 0.7) & (bpmrp < 2.1)
iM = (GM > 8.0) & (GM < 15.5) & (bpmrp > 1.6) & (bpmrp < 5.0)
iW = (GM > 6) & (bpmrp < 0.5)
iRG = (bpmrp > 1) & (GM < 1)
testx = np.linspace(-1, 6.0, 1000)
testline = 3*(testx) +1
plt.plot(bpmrp[iA], GM[iA], '.b', label = 'A stars')
plt.plot(bpmrp[iF], GM[iF], '.c', label = 'F stars')
plt.plot(bpmrp[iG], GM[iG], '.y', label = 'G stars')
plt.plot(bpmrp[iK], GM[iK], '.', color = 'orange', label = 'K stars')
plt.plot(bpmrp[iM], GM[iM], '.r', label = 'M stars')
plt.plot(bpmrp[iW], GM[iW], '.', label = 'White dwarfs', color = 'gray')
plt.plot(bpmrp[iRG], GM[iRG], '.r', label = 'Red giants')
plt.plot(bpmrp[~iA & ~iF & ~iG & ~iK & ~iM & ~iW & ~iRG],\
         GM[~iA & ~iF & ~iG & ~iK & ~iM & ~iW & ~iRG]\
         , '.k', label = 'Other')
plt.plot(testx, testline, '--g', label = 'Main Sequence')
ax = plt.gca()
ax.invert_yaxis()
plt.title('Hertzsprung-Russel Diagram using Gaia data')
plt.xlabel(r'$G_{BP}-G_{RP}$')
plt.ylabel(r'$G$')
plt.legend()
#plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/HertsprungRusselGaia.png')
plt.xlim(-.505, -.495)
plt.ylim(7.079+.05, 7.079-0.05)
xd = bpmrp+.5
yd = GM-7.079
dist = (xd**2 +yd**2)**0.5
print('M_G: ' +str(GM[np.argmin(dist)]) +', G_BP-G_RP: ' +str(bpmrp[np.argmin(dist)]))
print('RA: ' +str(ras[np.argmin(dist)]) +'\nDEC: ' +str(decs[np.argmin(dist)]))
print('Object classified as white dwarf by roughly defined parameters.')
plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/5cplot.png')
```

```
M_G: 7.076387228469315, G_BP-G_RP: -0.49958706
RA: 70.83873938587688
DEC: 46.70156304230519
Object classified as white dwarf by roughly defined parameters.
```


Hertzsprung-Russel Diagram using Gaia data
A stars
F stars
G stars
K stars
M stars
White dwarfs
Red giants
Other
Main Sequence
5.20
5.22
5.24
5.26
5.28
G
2.32
2.34
2.36
2.38
2.40
$G_{BP} - G_{RP}$

Code:

(e) **Finally, plot a color-color diagram (G_{BP}–G versus G–G_{RP}). Describe the MS relation on this diagram.**
Plot: