89%

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ASTR5420

Please show all work. If you collaborate with other students, write their names at the top of your homework. Please also write the total number of hours you spent on this. If you use any code or plotting routines, print those out and attach them to your hand-written solutions.

1. **(10%) Red Alert!! Romulans from the future have deposited "red matter" into the core of the planet Vulcan, quickly converting it into a black hole. Vulcan's crust is disintegrating and collapsing on a dynamical timescale. How long do Scotty and Chekov have to beam up the away team and members of the Vulcan High Council? According to Mass Trek Wiki, planet Vulcan has a radius of 6,792 km and surface gravity g = 1.4 G of Earth's gravity.**

+10/10

$$\tau_{ff} = \frac{\pi}{2}\left(\frac{R^3}{2GM}\right)^{1/2}$$

$$\frac{GM}{R^2} = 1.4g$$

$$M = \frac{1.4gR^2}{G}$$

$$\tau_{ff} = \frac{\pi}{2}\left(\frac{R^3}{2G\left(\frac{1.4gR^2}{G}\right)}\right)^{1/2}$$

$$\tau_{ff} = \frac{\pi}{2}\left(\frac{R}{2.8g}\right)^{1/2}$$

Code:

```
In [9]: G = 6.67e-11
        Rv = 6.792e6
        g = 9.8
        t_ff = (np.pi/2)*np.sqrt(Rv/(2.8*g))
        print('Free fall time: ' +format(t_ff, '.2E') +' seconds')

        Free fall time: 7.81E+02 seconds
```

$$\boxed{t_{ff} = 781 \text{ seconds}}$$

+24/25

2. **(25%) For late-O/early-B stars ($M = 8 - 30M_\odot$), the MS relations are $R = 4.1(M/8M_\odot)^{0.6}R_\odot$ and $L = 2600(M/8M_\odot)^{3.0}L_\odot$ (slightly flatter than relations for AFGK dwarfs).**

(a) **Compute the effective temperature (in K) of a B2V star ($8M_\odot$) and O7V star ($30M_\odot$).**

$$L = 2600\left(\frac{M}{8M_\odot}\right)^{3.0} L_\odot$$
$$L = 4\pi R^2 \sigma T_{eff}^4$$
$$L = 4\pi\left(4.1\left(\frac{M}{8M_\odot}\right)^{0.6} R_\odot\right)^2 \sigma T_{eff}^4$$
$$4\pi\left(4.1R_\odot\right)^2\left(\frac{M}{8M_\odot}\right)^{1.2} \sigma T_{eff}^4 = 2600\left(\frac{M}{8M_\odot}\right)^{3.0} L_\odot$$
$$T_{eff} = \left(\frac{2600L_\odot}{4\pi\left(4.1R_\odot\right)^2\sigma}\left(\frac{M}{8M_\odot}\right)^{1.8}\right)^{1/4}$$
$$T_{eff} = \left(\frac{2600L_\odot}{4\pi\left(4.1R_\odot\right)^2\sigma}\right)^{1/4}\left(\frac{M}{8M_\odot}\right)^{0.45}$$

Code:

```
In [26]: M_s = [8, 30]
         L_o = 3.828e26
         o = 5.67e-8
         R_o = 6.957e8
         T_eff = [((2600 *L_o)/(4 *np.pi *(4.1 *R_o)**2 *o))**0.25 *(m/8)**0.45 for m in M_s]
         print('Temperature 1: ' +format(T_eff[0], '.2E') +' K')
         print('Temperature 2: ' +format(T_eff[1], '.2E') +' K')

         Temperature 1: 2.04E+04 K
         Temperature 2: 3.69E+04 K
```

$\boxed{T_1 = 20400\text{ K}}$ $\boxed{T_2 = 36900\text{ K}}$

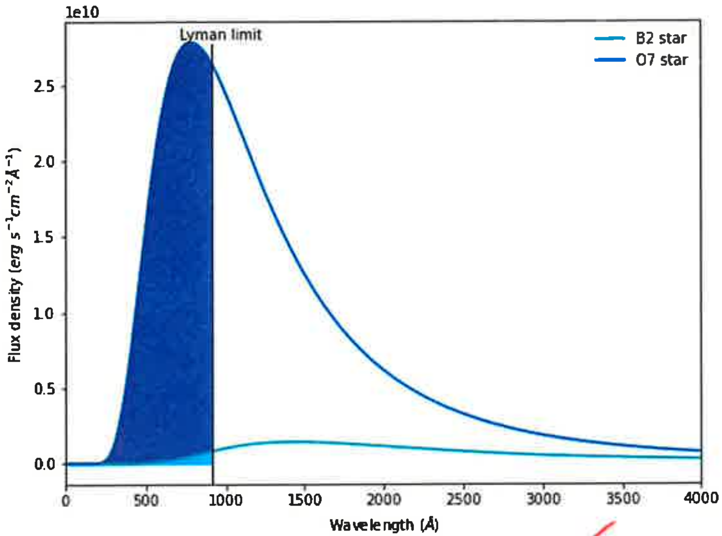
(b) **Assuming blackbody spectra, what fraction of the luminosity is below the Lyman limit (capable of ionizing hydrogen from the ground state) for both stars?** Code:

```
In [89]: wavl = np.linspace(10, 100000, 100000)
         # NP Wavelength range in Angstroms
         h = 6.63e-34
         k = 1.38e-23
         c = 3e8
         B_l = [((2 *h *c**2 *10**43)/((wavl)**5)) *(1/(np.exp(h *c *10**10/\
             ((wavl) *k *t))-1)) for t in T_eff]
         # NP Units of erg s^-1 cm^-2 A^-1 sr^-1

In [92]: ily = wavl < 914
         plt.figure(figsize = [8, 6])
         plt.plot(wavl, B_l[0], 'c', label = 'B2 star')
         plt.plot(wavl, B_l[1], 'b', label = 'O7 star')
         plt.fill_between(wavl[ily], y1 = B_l[1][ily], y2 = 0, color = 'blue')
         plt.fill_between(wavl[ily], y1 = (B_l[0][ily]), y2 = 0, color = 'cyan')
         plt.axvline(x = 914, ymin = 0.00, ymax = 0.95, color = 'k', lw = 1)
         plt.text(914-200, 2.8e10, 'Lyman limit')
         plt.xlim(0, 4000)
         plt.legend()
         plt.xlabel(r'Wavelength ($\AA$)')
         plt.ylabel(r'Flux density ($erg$ $s^{-1}cm^{-2}\AA^{-1}$)')
         plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/OandBstarUV.png')
         print('Fraction beyond Lyman limit: ' +\
             str(np.round(np.trapz(B_l[0][ily], wavl[ily])/np.trapz(B_l[0], wavl), 3)))
         print('Fraction beyond Lyman limit: ' +\
             str(np.round(np.trapz(B_l[1][ily], wavl[ily])/np.trapz(B_l[1], wavl), 3)))

         Fraction beyond Lyman limit: 0.046
         Fraction beyond Lyman limit: 0.354
```

Plot:



$\boxed{f_1 = 0.046}$ $\boxed{f_2 = 0.354}$

(c) **Estimate the rate of ionizing photons (in photons s^{-1}) for both stars.**

$$N = \int_0^{912} \frac{B_\lambda}{hc/\lambda} d\lambda$$

Code:

```
In [33]: R_s = [4.1*(m/8)**0.6 *R_o *1e2 for m in M_s]
         N = [np.trapz((B_l[i][ily])/(6.63*10**-34 *3e8 *10**10/wavl[ily]),\
             wavl[ily])*10**-7 *4*np.pi*R_s[i]**2 for i in range(len(B_l))]
         print('Ionizing photon rate: ' +format(N[0], '.2E') +' ph/s')
         print('Ionizing photon rate: ' +format(N[1], '.2E') +' ph/s')

         Ionizing photon rate: 5.79E+45 ph/s
         Ionizing photon rate: 2.03E+48 ph/s
```

$\boxed{N_1 = 5.79 \times 10^{45} \text{ ph s}^{-1}}$ $\boxed{N_2 = 2.03 \times 10^{48} \text{ ph s}^{-1}}$

(d) **Assuming a Salpeter IMF (see #4), we expect ≈ 22 times more B3V stars than O7V stars in a zero-age MS population. What produces more ionizing photons: a single O7V star or 22 B2V stars?**

$$N_{tot} = nN_1$$

Code:

```
In [34]: nstars = [22, 1]
         Ntot = [nstars[i] *N[i] for i in range(len(N))]
         print('Total ionizing photon rate: ' +format(Ntot[0], '.2E') +' ph/s')
         print('Total ionizing photon rate: ' +format(Ntot[1], '.2E') +' ph/s')

         Total ionizing photon rate: 1.27E+47 ph/s
         Total ionizing photon rate: 2.03E+48 ph/s
```

$\boxed{N_{tot,1} = 1.27 \times 10^{47} \text{ ph s}^{-1}}$ $\boxed{N_{tot,2} = 2.03 \times 10^{48} \text{ ph s}^{-1}}$

From these results, it seems that one O7V star produces ≈ 20 times more ionizing photons than 22 B2V stars.

(e) Estimate the radius (in pc) of a Strömgren sphere around a single O7V star and a single B3V star assuming surrounding gas has density $n_H = 20\ \mathrm{cm}^{-3}$.

$$R_S = \left(\frac{3N_{tot}}{4\pi\alpha_R n_H^2}\right)^{1/3}$$

$$\alpha_R = 3\times 10^{-13}\ \mathrm{cm}^{-3}\ \mathrm{s}^{-1}$$

$$n_H \sim 10\ \mathrm{cm}^{-3}$$

Code:

```
In [35]: nH = 10
         alpha_R = 3e-13
         R_strom = [((3 *n)/(4 *np.pi *alpha_R *nH**2))**(1/3) for n in Ntot]
         print('Stromgren radius: ' +format(R_strom[0], '.2E') +' cm')
         print('Stromgren radius: ' +format(R_strom[1], '.2E') +' cm')

         Stromgren radius: 1.00E+19 cm
         Stromgren radius: 2.53E+19 cm
```

$\boxed{R_{S,1} = 1.00\times 10^{19}\ \mathrm{cm}}$ $\boxed{R_{S,2} = 2.53\times 10^{19}\ \mathrm{cm}}$

≈ 3pc 8pc

3. (20%) A T Tauri pre-MS star accretes the final $\Delta M/M = 30\%$ of its mass on a thermal (Kelvin-Helmholtz) timescale. In class, we derived the general thermal timescale $\tau_{KH} = GM^2/RL$. For an accreting object, $\tau_{KH} = GM\Delta M/RL$ gives a better approximation.

x 19/20

(a) Estimate the T Tauri disk lifetime (in Myr) assuming $M_* = 1.0M_\odot$, $R_* = 1.5R_\odot$, and $L_* = 1L_\odot$.

$$\tau_{KH} = \frac{GM\Delta M}{RL}$$

$$M = 1M_\odot$$

$$\Delta M = 0.30M$$

$$R = 1.5R_\odot$$

$$L = 1L_\odot$$

Code:

```
In [45]: M_o = 2e30
         DM = 0.3 *M_o
         R_tstar = 1.5 *R_o
         t_kh = (M_o *DM *G)/(R_tstar *L_o)
         print('T Tauri disk lifetime: ' +format(t_kh/(np.pi*10**13), '.2E')\
             +' Myr')

         T Tauri disk lifetime: 6.38E+00 Myr
```

$$\boxed{\tau_{KH} = 6.38\times 10^{0}\ \mathrm{Myr}}$$

(b) What is the disk accretion rate (in $M_\odot\ \mathrm{yr}^{-1}$)?

$$\frac{dM}{dt} = \frac{\Delta M}{\tau_{KH}}$$

$$\frac{dM}{dt} = \frac{0.3M_\odot}{\tau_{KH}}$$

Code:

```
In [51]: dmdt = DM/t_kh
         print('dm/dt = ' +format(dmdt *np.pi *10**7/M_o, '.2E') +' M_o yr^-1')

         dm/dt = 4.70E-08 M_o yr^-1
```

$$\boxed{\frac{dM}{dt} = 4.70 \times 10^{-8}\ M_\odot\ yr^{-1}}$$

(c) **What is the disk luminosity L_{disk} (in $L_\odot$)? What is the ratio L_{disk}/L_*?**

$$L_{disk} = \frac{GM}{R}\frac{dM}{dt}$$
$$L_{disk} = \frac{G\,(1M_\odot)}{1.5R_\odot}\frac{dM}{dt}$$
$$L_{disk} = \frac{G\,(1M_\odot)}{1.5R_\odot}\frac{\Delta M}{\left(\frac{GM\Delta M}{(1.5R_\odot)(1L_\odot)}\right)}$$
$$L_{disk} = \frac{G\,(1M_\odot)}{1.5R_\odot}\frac{\Delta M\,(1.5R_\odot)\,(1L_\odot)}{GM\Delta M}$$
$$L_{disk} = 1L_\odot$$

Virial

x4 mm

$$\boxed{L_{disk} = 1L_\odot}$$ 0.5 $L_\odot$

$$\boxed{\frac{L_{disk}}{L_*} = 1}$$ ≈ 0.5

(d) **What is the temperature of the disk near the star/disk boundary and at $R = 3R_*$? How do these compare to the effective temperature of the star?**

$$L_{disk} = GM\frac{dM}{dt}\frac{1}{R}$$
$$L_{disk} = \sigma\pi R^2T^4$$
$$\sigma\pi R^2T^4 = GM\frac{dM}{dt}\frac{1}{R}$$
$$T^4 = \frac{GM}{\pi\sigma}\frac{dM}{dt}\frac{1}{R^3}$$
$$T = \left(\frac{GM}{\pi\sigma}\frac{dM}{dt}\frac{1}{R^3}\right)^{1/4}$$

Code:

```
In [266]: R_tauridiff = [R_tstar, 3*R_tstar]
          T = [((G*M_o)/(np.pi *5.67e-8) *dmdt *1/(r**3))**0.25 for r in R_tauridiff]
          print('Temp. at boundary: ' +format(T[0], '.2E') +' K')
          print('Temp. at 3R_*: ' +format(T[1], '.2E') +' K')
          T_effstar = (L_o/(4 *np.pi *(1.5 *5.67e-8 *(1.5 *R_o)**2)))**0.25
          print('T_eff: ' +format(T_effstar, '.2E') +' K')
          print('Ratio at boundary: ' +format(T[0]/T_effstar, '.2E'))
          print('Ratio at 3R_*: ' +format(T[1]/T_effstar, '.2E'))

          Temp. at boundary: 6.67E+03 K
          Temp. at 3R_*: 2.92E+03 K
          T_eff: 4.26E+03 K
          Ratio at boundary: 1.57E+00
          Ratio at 3R_*: 6.87E-01
```

$$\boxed{T_{R_*} = 6670\text{ K}}$$
$$\boxed{T_{3R_*} = 2920\text{ K}}$$
$$\boxed{\frac{T_{R_*}}{T_{eff}} = 1.57}$$
$$\boxed{\frac{T_{3R_*}}{T_{eff}} = 0.687}$$

The temperature of the disk at the star/disk boundary and at a radius of 3 times the radius of the star is on the order of magnitude of the effective temperature of the star.

x16/20

4. **(20%): Consider a Kroupa-like IMF $N(M)dM \propto M^{-\alpha}dM$ with a Salpeter slope of $\alpha = 2.35$ across $M = 0.8$–$120M_{\odot}$ and a break toward $\alpha = 1.3$ across $M = 0.08$–$0.8M_{\odot}$.**

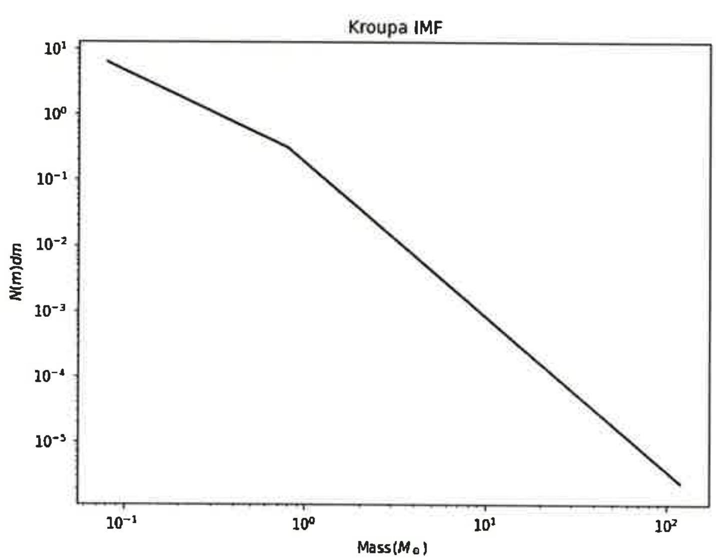
   (a) **What fraction of the stars are M-dwarfs with $M = 0.08$–$0.55M_{\odot}$.**
   Code:

```
In [147]: Ms = np.linspace(0.08, 120, 10000)
          ilow = Ms < 0.8
          ihigh = Ms >= .8
          N_low = (Ms[ilow]/.8)**-1.3
          N_high = (Ms[ihigh]/.8)**-2.35
          N_mdm = np.concatenate((N_low, N_high))
          plt.figure(figsize=[8,6])
          plt.xscale('log')
          plt.yscale('log')
          plt.xlabel(r'Mass($M_\odot$)')
          plt.ylabel(r'$N(m)dm$')
          plt.title('Kroupa IMF')
          normalnmdm = (N_mdm)/(np.trapz(N_mdm, Ms))
          plt.plot(Ms, normalnmdm, 'k')
          plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/Kroupa.png')
```

```
In [148]: mdwarf = (Ms >= 0.08) & (Ms < 0.55)
          print('Fraction of M dwarfs: ' +format(np.trapz(normalnmdm[mdwarf],\
              Ms[mdwarf]), '.2E'))

          Fraction of M dwarfs: 7.19E-01
```

   Plot:



$$\boxed{(N_m)_{m<0.55} = 0.719}$$

   (b) **What fraction of the stars will undergo core-collapse supernovae ($M > 8M_{\odot}$)? Of those, which fraction are early-B stars ($M = 8$–$17M_{\odot}$) versus O stars ($M > 17M_{\odot}$)?**
   Code:

```
In [149]: collapse = (Ms >= 8)
          eB = (Ms > 8) & (Ms < 17)
          iO = (Ms > 17)
          print('Core-collapse fraction: ' +format(np.trapz(normalnmdm[collapse],\
              Ms[collapse]), '.2E'))
          print('Early B fraction: ' +format(np.trapz(normalnmdm[eB], Ms[eB]),\
              '.2E'))
          print('O fraction: ' +format(np.trapz(normalnmdm[iO], Ms[iO]), '.2E'))

          Core-collapse fraction: 7.92E-03
          Early B fraction: 5.19E-03
          O fraction: 2.73E-03
```

$$\boxed{(N_m)_{m>8} = 0.00792}$$ ✓

$$\boxed{(N_m)_{m>8,m<17} = 0.00519}$$ 65% of SN

$$\boxed{(N_m)_{m>17} = 0.00273}$$ 35% of SN

(c) **What fraction of the mass of a stellar population will evolve in 10 Gyr ($M > 1M_\odot$) and subsequently be converted into stellar remnants or returned to the ISM?**

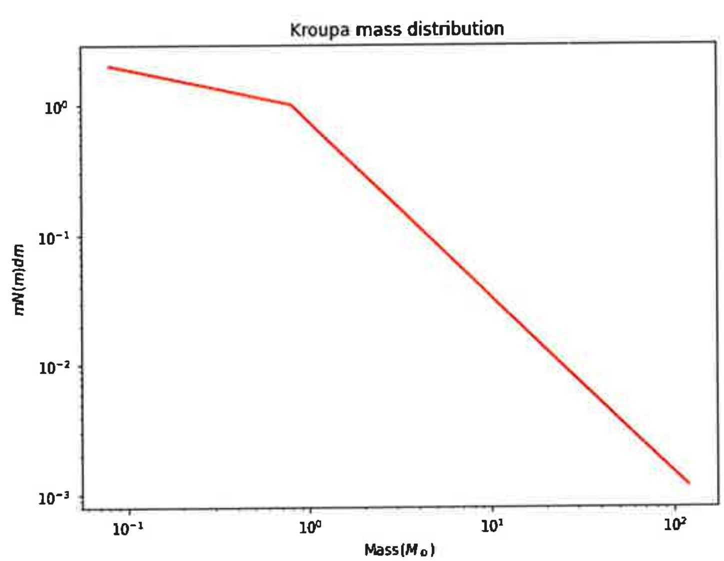
Code:

```
In [159]: plt.figure(figsize=[8, 6])
          MNmdm = (Ms/0.8)*N_mdm
          plt.xscale('log')
          plt.yscale('log')
          MNmdmnorm = MNmdm/np.trapz(MNmdm, Ms)
          plt.plot(Ms, MNmdm, 'r')
          plt.xlabel(r'Mass($M_\odot$)')
          plt.ylabel(r'$mN(m)dm$')
          plt.title('Kroupa mass distribution')
          plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/Kroupamass.png')
```

```
In [161]: tengiga = Ms > 1
          print('Mass fraction of stars greater than solar mass: '\
                +format(np.trapz(MNmdmnorm[tengiga], Ms[tengiga]), '.2E'))

          Mass fraction of stars greater than solar mass: 6.12E-01
```

Plot:



$$\boxed{(MN_m)_{m>1} = 0.612}$$

(d) One early hypothesis for dark matter was faint, low-mass stars, i.e., the Salpeter slope continued all the way down to the brown dwarf / planetary boundary. If this were the case, what would be the ratio of the mass in brown dwarfs and M-dwarfs ($M = 0.013$–$0.55M_\odot$) versus more massive stars ($M > 0.55M_\odot$). By the early 1980s, the census of nearby brown dwarfs and M-dwarfs was sufficiently complete to conclude that the IMF flattens below $M < 0.5M_\odot$ and therefore low-mass stars and brown dwarfs cannot explain dark matter.
Code:

```
In [169]: Mbrown = np.linspace(0.013, 120, 10000)
          bdwarf = (Mbrown > 0.013) & (Mbrown < 0.55)
          print('Brown dwarf mass ratio: ' +format(np.trapz(MNmdmnorm[bdwarf], Mbrown[bdwarf]), '.2E'))

          Brown dwarf mass ratio: 2.47E-01
```

$$\boxed{(MN_m)_{m<0.55,m>0.013} = 0.247}$$

assuming true Kroupa IMF

if Salpeter then M(Mdwarfs + BD) ≈ 3 K-O dwarfs

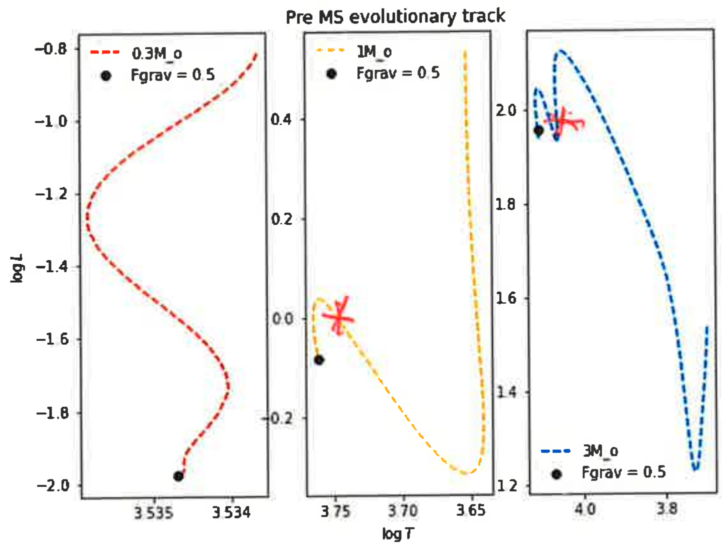
20/25

5. (25%): Download the PISA pre-MS solar-metallicity ($Z = 0.015$) evolutionary tracks (Tognelli et al. 2011) from WyoCourses for $0.3, 1.0,$ and $3.0M_\odot$ stars. The files are in .csv (comma separated values) format, and the columns are log age (yr), log L ($L_\odot$), log Teff (K), log Tcore (K), log ρ_{core} ($g\ cm^{-3}$), and the fractions F_{PP}, F_{CNO}, and F_{grav} of the luminosity that are generated by the proton-proton chain, CNO cycle, and gravitational contraction, respectively. The tracks begin on the pre-MS birthline and end at the zero-age main-sequence (ZAMS) when $F_{grav} = 0$.

(a) Plot luminosity versus effective temperature of the three tracks on a HRD. Indicate for each track where $F_{grav} = 0.5$.
Code:

```
In [324]: ifg = [np.argmin(f-.5) for f in Fgra]
          plt.figure(figsize = [8, 6])
          plt.subplot(1, 3, 1)
          plt.ylabel(r'$\logL$')
          plt.plot(Te[0], lum[0], '--r', label = '0.3M_o')
          plt.plot(Te[0][ifg[0]], lum[0][ifg[0]], 'ok', label = 'Fgrav = 0.5')
          plt.gca().invert_xaxis()
          plt.legend()
          plt.subplot(1, 3, 2)
          plt.title('Pre MS evolutionary track')
          plt.plot(Te[1], lum[1], '--', color = 'orange', label = '1M_o')
          plt.plot(Te[1][ifg[1]], lum[1][ifg[1]], 'ok', label = 'Fgrav = 0.5')
          plt.gca().invert_xaxis()
          plt.xlabel(r'$\logT$')
          plt.legend()
          plt.subplot(1, 3, 3)
          plt.plot(Te[2], lum[2], '--b', label = '3M_o')
          plt.plot(Te[2][ifg[2]], lum[2][ifg[2]], 'ok', label = 'Fgrav = 0.5')
          plt.gca().invert_xaxis()
          plt.legend()
          plt.savefig('/d/www/nikhil/public_html/ASTR5420/images/HRD.png')
```

Plot:



wanted on same HRD

Fgrav=0.5 locations should be slightly earlier

(b) **Describe how the Hyashi and Henyey tracks differ among the three stars. Do $0.3M_{\odot}$ stars ever develop radiative cores or do they remain fully convective?** ⇒ $0.3M_{\odot}$ remain fully convective

The $0.3M_{\odot}$ and the $1M_{\odot}$ stars both show a significant decrease in luminosity while staying at the same temperature while the $3M_{\odot}$ star spends less time on the Hyashi track. As the star's mass increases, it spends more time on the Henyey track.

(c) **At what age does the pre-MS begin for the three stars? Explain why it does not begin at 0 Myr. What is the initial luminosity (in $L_{\odot}$), temperature (K), and radius ($R_{\odot}$) of a $1M_{\odot}$ pre-MS star?**

Code:

```
In [295]: ags = [10**(a[0]) for a in ages]
          print('Pre-main sequence begins at ' +format(ags[0], '.2E') +' years for 0.3M_o star.')
          print('Pre-main sequence begins at ' +format(ags[1], '.2E') +' years for 1M_o star.')
          print('Pre-main sequence begins at ' +format(ags[2], '.2E') +' years for 3M_o star.')

          Pre-main sequence begins at 3.55E+06 years for 0.3M_o star.
          Pre-main sequence begins at 7.07E+05 years for 1M_o star.
          Pre-main sequence begins at 2.67E+05 years for 3M_o star.
```

$$\boxed{t_{0.3M_{\odot}} = 3.55 \times 10^{6} \text{ years}}$$

$$\boxed{t_{1M_{\odot}} = 7.07 \times 10^{5} \text{ years}}$$

$$\boxed{t_{3M_{\odot}} = 2.67 \times 10^{5} \text{ years}}$$

The ages of the pre-MS stars do not begin at 0 years because these stars are obfuscated by accretion disks. The more massive, luminous stars are able to be detected sooner because they are intrinsically brighter, hence the trend as mass increases, the beginning age of the star decreases.

Code:

```
In [318]: print('Inital luminosity: ' +format(10**(lum[1][0]), '.2E') +' L_o')
          print('Initial T: ' +format(10**(Te[1][0]), '.2E') +' K')
          print('Inital radius: ' +format((((10**(lum[1][0]) *L_o)/\
              (4 *np.pi *5.67e-8 *(10**(Te[1][0]))**4))**0.5)/(R_o), '.2E') +' R_o')

          Inital luminosity: 3.40E+00 L_o
          Initial T: 4.49E+03 K
          Inital radius: 3.05E+00 R_o
```

$$\boxed{L_{1M_{\odot},0} = 3.40L_{\odot}}$$

$$\boxed{T_{1M_\odot,0} = 4490\text{ K}}$$

$$\boxed{R_{1M_\odot,0} = 3.05R_\odot}$$

(d) **What are the pre-MS lifetimes of the three stars? Considering the maximum disk age around a $1M_\odot$ T Tauri star is 10 Myr, do you expect to see pre-MS stars (above the ZAMS on the HRD) without disks (no significant near-IR excess)?**
Code:

```
In [296]: ags = [10**(a[len(a)-1])-10**(a[0]) for a in ages]
          print(format(ags[0], '.2E') +' years spent on pre-main sequence for 0.3M_o star.')
          print(format(ags[1], '.2E') +' years spent on pre-main sequence for 1M_o star.')
          print(format(ags[2], '.2E') +' years spent on pre-main sequence for 3M_o star.')

          5.97E+08 years spent on pre-main sequence for 0.3M_o star.
          3.27E+07 years spent on pre-main sequence for 1M_o star.
          3.77E+06 years spent on pre-main sequence for 3M_o star.
```

$$\boxed{t_{0.3M_\odot} = 5.97 \times 10^8\text{ years}}$$

$$\boxed{t_{1M_\odot} = 3.27 \times 10^7\text{ years}}$$

$$\boxed{t_{3M_\odot} = 3.77 \times 10^6\text{ years}}$$

Considering that the maximum disk age of a $1M_\odot$ is greater than the pre-MS lifetime of a $1M_\odot$ star, I would not expect to see ~~any~~ pre-MS stars of this mass without disks and without significant near-IR excess. No $\tau_{disk} = 10$ Myr $< \tau_{pms} \approx 33$ Myr

(e) **What is the luminosity (in $L_\odot$) and radius ($R_\odot$) of a $1M_\odot$ star on the ZAMS? Describe how our Sun has evolved in brightness and size during the past 4.6 Gyr of MS evolution. What fraction of solar radiation did the early Earth receive compared to its current values? This is known as the "faint Sun paradox", whereby the early Earth's equilibrium surface temperature would have been below freezing if it were not for continuous outgassing and bombardment that substantially increased the atmospheric density of greenhouse gasses.**
Code:

```
In [326]: print('ZAMS luminosity: ' +format(10**(lum[1][len(lum[1])-1]), '.2E') +' L_o')
          print('ZAMS T: ' +format(10**(Te[1][len(Te[1])-1]), '.2E') +' K')
          print('ZAMS radius: ' +format((((10**(lum[1][len(lum[1])-1]) *L_o)/\
              (4 *np.pi *5.67e-8 *(10**(Te[1][len(Te[1])-1]))**4))**0.5)/(R_o), '.2E') +' R_o')

          ZAMS luminosity: 8.26E-01 L_o
          ZAMS T: 5.77E+03 K
          ZAMS radius: 9.10E-01 R_o
```

$$\boxed{L_{1M_\odot,ZAMS} = 0.826L_\odot}$$

$$\boxed{T_{1M_\odot,ZAMS} = 5770\text{ K}}$$

$$\boxed{R_{1M_\odot,ZAMS} = 0.910R_\odot}$$

During the past 4.6 Gyr of MS evolution, the Sun has grown more luminous, stayed at nearly the same effective temperature, and increased in size. Assuming the Earth's orbit did not change significantly in the past 4.6 Gyrs, the early Earth would have received ~ 0.826 times the current flux the Earth receives from the Sun (≈ 1440 KW m^{-2}), as the Sun was 0.826 times less luminous.