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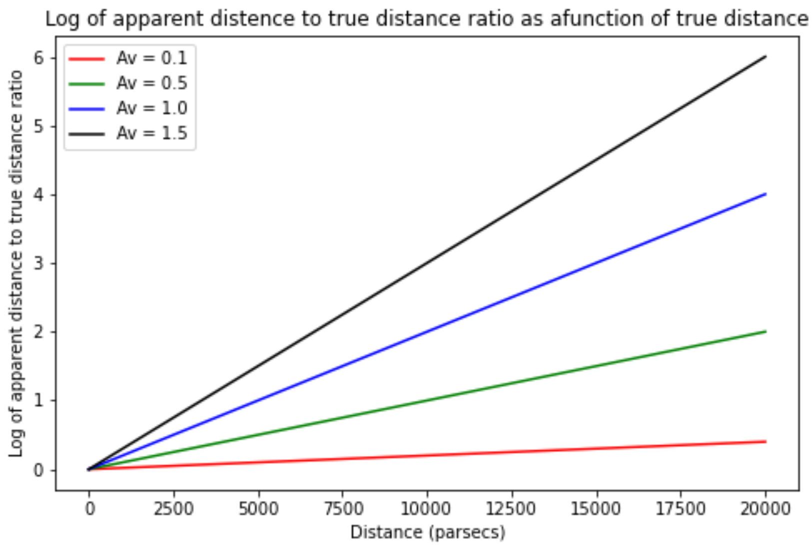
ASTR5465

1. **Simulate the Kapteyn model by assuming that all the stars in the Galaxy have the same absolute magnitude (M v = 0.0) and a space density of 1 star/pc3 . Derive a relationship for the ratio of apparent distance (neglecting extinction) to true distance as a function of true distance assuming a mean extinction of A v /kpc. Next, evaluate your expression for the following values of A v /kpc: 0.1, 0.5, 1.0, 1.5 and graph this ratio vs. distance out to about 20 kpc true distance. Compute the corresponding ratio of apparent density to true density as a function of distance and graph this ratio vs. distance for each of the above values of A v /kpc. Now compute the apparent volume density compared to the true (constant) volume density.**
   **Hint: This can be done by referring to the lecture slide on number counts vs. solid angle if you consider r as the true distance and r + dr as the false distance. Since dr will increase with distance the volume will also appear to increase rapidly with distance and the number density rapidly drop. You can use any software you wish to do this, even Excel.**

$$
\begin{aligned}
m_V - M_V &= 5\log\left[r\right] - 5 + A_V\left(\frac{\text{mag}}{\text{kpc}}\right)\left(r \text{ parsecs}\frac{1 \text{ kpc}}{1000 \text{ parsecs}}\right)\\
m_V - M_V &= 5\log\left[r\right] - 5 + A_V\frac{r}{1000}\\
m_V &= 5\log\left[r\right] - 5 + A_V\frac{r}{1000}\\
5\log\left[r\right] &= m_V + 5 - A_V\frac{r}{1000}\\
\log\left[r\right] &= \frac{m_V}{5} + 1 - \frac{A_V r}{5000}\\
r &= 10^{\frac{m_V}{5}+1-\frac{A_V r}{5000}}\\
m_V - M_V &= 5\log\left[r_{app}\right] - 5\\
m_V &= 5\log\left[r_{app}\right] - 5\\
5\log\left[r_{app}\right] &= m_V + 5\\
\log\left[r_{app}\right] &= \frac{m_V}{5} + 1\\
r_{app} &= 10^{\frac{m_V}{5}+1}\\
\frac{r}{r_{app}} &= \frac{10^{\frac{m_V}{5}+1-\frac{A_V r}{5000}}}{10^{\frac{m_V}{5}+1}}\\
\frac{r}{r_{app}} &= 10^{\frac{m_V}{5}+1-\frac{A_V r}{5000}-\frac{m_V}{5}-1}\\
\frac{r}{r_{app}} &= 10^{-A_V r/5000}
\end{aligned}
$$

$$\boxed{\frac{r_{app}}{r} = 10^{A_V r/5000}}$$




For different values of the extinction, $A_V$, we can see the trend that as the true distance is increased, the apparent distance increases by a power law given by the value of extinction. Thus, without accounting for extinction, objects appear to be much more further away than they actually are as they are observed at greater distances.

$$\frac{\rho_{app}}{\rho} = \frac{1/V_{app}}{1/V}$$

$$\frac{\rho_{app}}{\rho} = \frac{V}{V_{app}}$$

$$\frac{\rho_{app}}{\rho} = \frac{4\pi r^2 dr}{4\pi \left(r + dr\right)^2 dr}$$

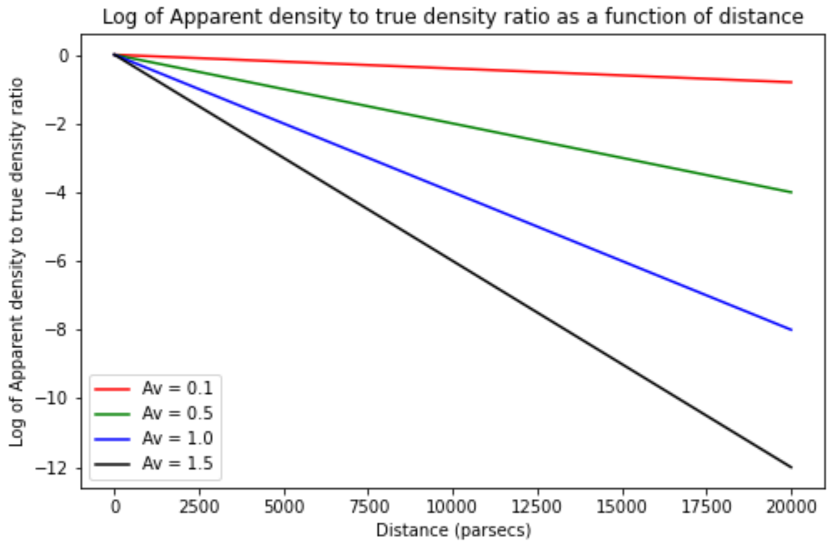
$$\frac{\rho_{app}}{\rho} = \frac{4\pi r^2 dr}{4\pi r_{app}^2 dr}$$

$$\frac{\rho_{app}}{\rho} = \left(\frac{r}{r_{app}}\right)^2$$

$$\frac{\rho_{app}}{\rho} = \left(\frac{1}{10^{A_V r/5000}}\right)^2$$

$$\frac{\rho_{app}}{\rho} = \left(10^{-A_V r/5000}\right)^2$$

$$\boxed{\frac{\rho_{apparent}}{\rho} = 10^{-A_V r/2500}}$$




As shown above, the apparent density appears to drop off as a power law as distance is increased, with the value of extinction determining how severely the density drops off with distance.

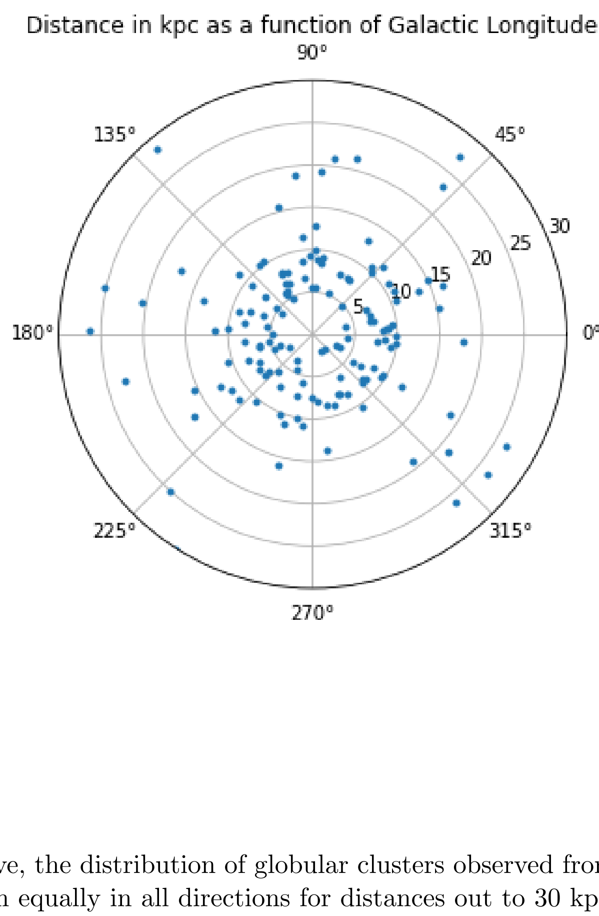
2. **Use the data in Bill Harris' Globular cluster database to recreate Shapley's original figures with modern data. Use IDL, Mathematica, or Python to make your plots. Rescale them and/or limit the axes as necessary to make them look good Specifically, I want to see plots of:**

   (a) a polar plot showing distance vs. Galactic Longitude

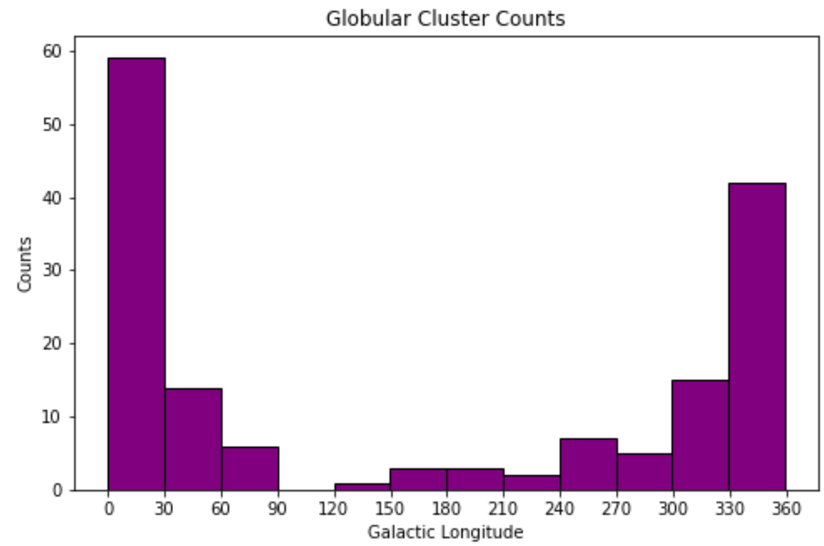
   (b) a histogram showing the number vs. Galactic Longitude

   (c) a plot showing the projected distance (X: along the plane in the direction toward the Galactic Center) vs. Galactic Latitude

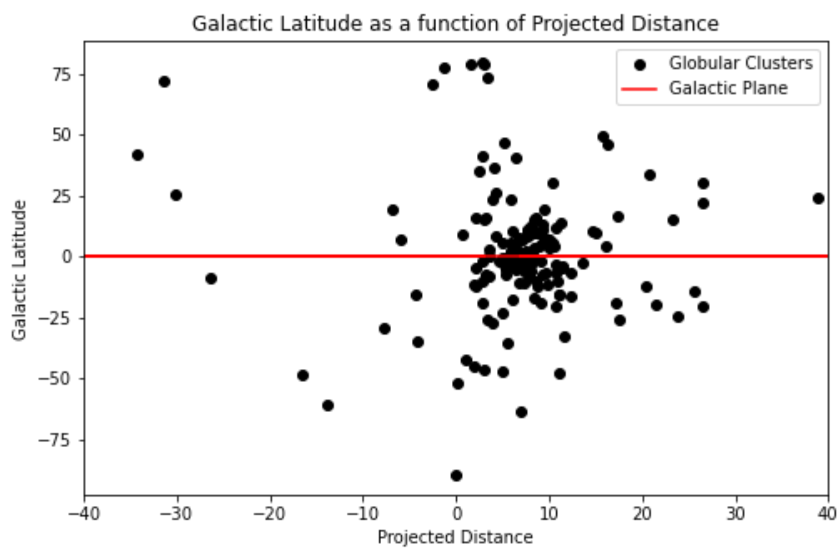
Bill Harris' web site can be found at: http://www.physics.mcmaster.ca/ harris/Databases.html




Illustrated in the plot above, the distribution of globular clusters observed from Earth appears to be spherically distributed around the Sun equally in all directions for distances out to 30 kpc. These observations would have supported the idea that the Sun is very close to the galactic center.




In the above histogram, the globular cluster counts binned for 30 degree increments galactic longitude yields that most globular clusters in this catalog are centered around 0 degrees galactic longitude. This conflicts with the above plot as this histogram suggests that the Sun is near the edge of the galaxy. Being on the outside looking in, we see higher counts of globular clusters than if we are looking away from the galactic center.



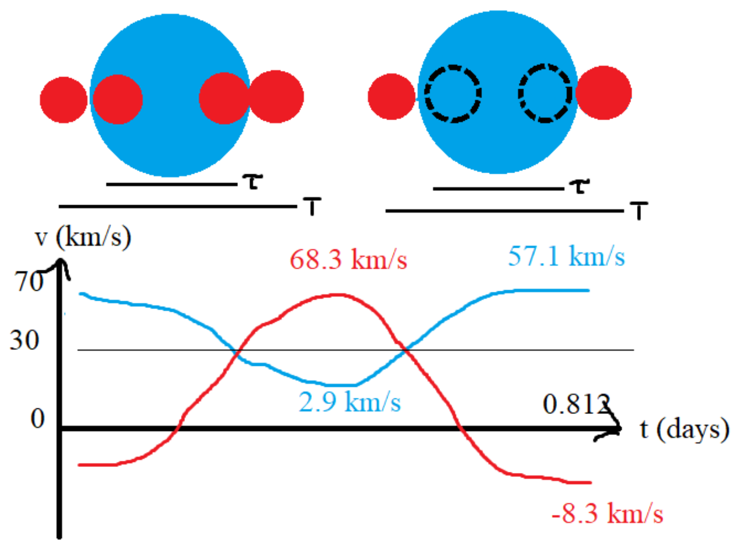

This plot shows how the globular clusters in this catalog are spherically distributed around the galactic plane, but also clump together towards the center. This is again indicative the we are not at the center of the galaxy.

3. **Three stars have apparent visual magnitudes of +2.0, +2.5, and +3.0. What is the apparent magnitude of the combined light of all three stars? Hint: Remember that magnitudes don't add since thery are logarithmic quantities.**

$$m = -2.5 \log \left[ \frac{F}{F_0} \right]$$
$$-\frac{m}{2.5} = \log \left[ \frac{F}{F_0} \right]$$
$$\frac{F}{F_0} = 10^{-m/2.5}$$
$$F = F_0 \left( 10^{-m/2.5} \right)$$
$$F = F_0 \left( 2.512^{-m} \right)$$
$$m_1 = 2$$
$$m_2 = 2.5$$
$$m_3 = 3$$
$$m = -2.5 \log \left[ \frac{F_1 + F_2 + F_3}{F_0} \right]$$
$$m = -2.5 \log \left[ \frac{F_0 \left( 2.512^{-2} \right) + F_0 \left( 2.512^{-2.5} \right) + F_0 \left( 2.512^{-3} \right)}{F_0} \right]$$
$$m = -2.5 \log \left[ \frac{F_0 \left( 2.512^{-2} + 2.512^{-2.5} + 2.512^{-3} \right)}{F_0} \right]$$
$$m = -2.5 \log [0.3216]$$

$$\boxed{m = 1.23}$$

4. **An eclipsing binary has a period of 100 days. Each eclipse lasts 0.812 day and the light remains constant at mid-eclipse for 0.172 day. Spectroscopic data show that the brighter component has a radial velocity which varies between +2.9 and +57.1 km/sec, while the fainter stars varies between -8.3 and +68.3 km/sec. The combined light of the system has m bol = 5.545 normally, m bol = 5.990 during the primary minimum, and m bol = 5.790 during secondary minimum. The parallactic distance is 50 pc. If the radial velocity curves indicate that the orbit is circular and if the plane of the orbit is in the line of sight (i = 90o ), determine as many characteristics of the two stars as possible.**




$P = 100$ days
$T = 0.812$ days
$\tau = 0.172$ days
$m = 5.545$
$m_1 = 5.990$
$m_2 = 5.790$
$d = 50$ pc

$$M_A v_{Amax} + M_B v_{Bmin} = M_A v_{Amin} + M_B v_{Bmax}$$
$$M_A v_{Amax} - M_A v_{Amin} = M_B v_{Bmax} - M_B v_{Bmin}$$
$$M_A \Delta v_A = M_B \Delta v_B$$
$$\frac{M_A}{M_B} = \frac{\Delta v_B}{\Delta v_A}$$

$$\boxed{\frac{M_A}{M_B} = 1.413}$$

Using conservation of momentum, the mass ratio of the two stars was found to be related to their changes in velocity as they orbit each other.

$$v_{cm} = \frac{v_{Amax} + v_{Amin}}{2}$$
$$v_{cm} = \frac{57.1 + 2.9}{2}$$

$$\boxed{v_{cm} = 30 \text{ km s}^{-1}}$$

Using the radial velocity data, it was found that the stars orbit each other with a speed of 30 km $s^{-1}$.

$$v_{cm} = \frac{2\pi a}{P}$$
$$a = \frac{P v_{cm}}{2\pi}$$

$$\boxed{a = 4.13 \times 10^{10} \text{ m}}$$

The semi-major axis can be calculated by virtue of the fact that in 1 period, each star will travel the circumference of its orbit. Using a form of $v = \frac{d}{t}$, distance can be calculated giving the semi-major axis. These stars were found to orbit each other at a distance of $4.13 \times 10^{10}$ m or 0.275 AU.

$$P^2 = \frac{4\pi^2}{G\left(M_A + M_B\right)} a^3$$
$$M_A + M_B = \frac{4\pi^2}{G} \frac{a^3}{P^2}$$
$$1.413 M_B + M_B = 5.57 \times 10^{29}$$
$$M_B = 2.31 \times 10^{29}$$
$$M_A = 3.26 \times 10^{29}$$

$$\boxed{M_A = 0.163 M_\odot,\ M_B = 0.115 M_\odot}$$

Kepler's Third Law yields a mass of $0.115M_\odot$ for the low-mass star, and $0.163M_\odot$ for the high-mass star.

$$2R_A = v_{cm}\left(\tau + \frac{T - \tau}{2}\right)$$
$$R_A = \frac{v_{cm}}{2}\left(\tau + \frac{T - \tau}{2}\right)$$
$$R_A = \frac{30000}{2}\left(0.172 + \frac{0.812 - 0.172}{2}\right) 86400$$
$$R_A = 6.376 \times 10^8$$

$$\boxed{R_A = 0.917 R_\odot}$$

$$2R_B = v_{cm}\left(\frac{T - \tau}{2}\right)$$
$$R_B = \frac{v_{cm}}{2}\left(\frac{T - \tau}{2}\right)$$
$$R_B = \frac{30000}{2}\left(\frac{0.812 - 0.172}{2}\right) 86400$$
$$R_B = 4.15 \times 10^8$$

$$\boxed{R_B = 0.596 R_\odot}$$

Using the eclipse data, both radii can be calculated knowing that the stars each orbit at the same speed about their center of mass and travel their diameters in different times which can be calculated from the given eclipse data.

$$\begin{aligned}
m - m_1 &= -2.5 \log \left[\frac{F_A + F_B}{F_A}\right] \\
5.545 - 5.990 &= -2.5 \log \left[1 + \frac{F_B}{F_A}\right] \\
-0.445 &= -2.5 \log \left[1 + \frac{F_B}{F_A}\right] \\
0.178 &= \log \left[1 + \frac{F_B}{F_A}\right] \\
1.507 &= 1 + \frac{F_B}{F_A}
\end{aligned}$$

$$\boxed{\frac{F_B}{F_A} = 0.507}$$

The flux ratio, and the luminosity ratio, between the two stars is found by finding the difference in magnitudes of when both the stars are in view and the magnitude of when the smaller star is hidden by the larger star.

$$\begin{aligned}
m - M &= 5 \log \left[\frac{d}{10}\right] \\
M &= m - 5 \log \left[\frac{d}{10}\right] \\
M_A &= m_1 - 5 \log \left[\frac{d}{10}\right] \\
M_A &= 5.990 - 5 \log \left[\frac{50}{10}\right] \\
M_A &= 2.495 \\
M_A - M_\odot &= -2.5 \log \left[\frac{L_A}{L_\odot}\right] \\
\log \left[\frac{L_A}{L_\odot}\right] &= -\frac{1}{2.5} \left(M_A - M_\odot\right) \\
\frac{L_A}{L_\odot} &= 10^{0.4(M_\odot - M_A)} \\
M_\odot &= 4.75
\end{aligned}$$

$$\boxed{L_A = 7.98 L_\odot, L_B = 4.05 L_\odot}$$

The absolute magnitude for the large star by itself can be found using the equation of the distance modulus knowing the parallactic distance to these stars. Then by comparing the absolute bolometric magnitude of the large star to the absolute bolometric magnitude of the sun, we can find the Luminosity of the larger star in units of solar luminosities. Using the ratio of luminosities found before, it is then possible to calculate the luminosity of the smaller star. These results are very surprising because so far, we have found that these stars are smaller than the sun, in mass and in size, but are several times more luminous than the sun.

$$\begin{aligned}
L &= \sigma A T^4 \\
T &= \left(\frac{L}{\sigma A}\right)^{1/4}
\end{aligned}$$

$$\boxed{T_A = 10100 \text{ K}, T_B = 10600 \text{ K}}$$

Using the Stefan-Boltzmann equation for a black body, each star's temperature can be estimated. Again, quite surprisingly, we have found that these stars are both nearly twice as hot as the sun.