

Monte Carlo Simulation Method for Calculation of Integral

1 Objectives

- To understand and apply the Monte Carlo method for numerical integration.
- To approximate the value of a definite integral using random sampling.
- To evaluate the accuracy of the Monte Carlo simulation in comparison with the analytical solution.
- To analyze the effect of increasing the number of random samples on the accuracy of the result.

2 Prerequisites

- Basic understanding of definite integrals and probability theory.
- Familiarity with Python programming, especially generating random numbers using libraries such as NumPy.
- Understanding the concept of convergence and the Law of Large Numbers in the context of simulations.

3 Theory

The Monte Carlo method is a statistical technique that uses random sampling to estimate numerical results. It is particularly useful for evaluating definite integrals where analytical solutions are difficult or impossible to obtain.

Consider the integral:

$$I = \int_a^b f(x) dx$$

Using Monte Carlo simulation, this integral can be approximated by:

$$I \approx (b - a) \cdot \frac{1}{N} \sum_{i=1}^N f(x_i)$$

where x_i are uniformly distributed random samples in the interval $[a, b]$, and N is the number of samples.

4 Tasks

1. Simulate Integral Calculation Using Monte Carlo Method

- Define a continuous function $f(x)$ to integrate over a specified interval $[a, b]$.
- Generate a large number of random points x_i in the interval.

- Approximate the integral using the average value of $f(x_i)$.

2. Analyze Convergence and Accuracy

- Repeat the simulation with varying sample sizes (e.g., $N = 100, 1000, 10000$).
- Compare the Monte Carlo results with the exact value of the integral, if known.
- Observe how the accuracy improves with increasing sample size.

3. Visualize the Monte Carlo Process

- Create a graphical representation of random points and the function curve.
- Plot convergence results (error vs number of samples).
- Analyze the efficiency of the method and factors affecting its performance.

5 Expected Outcomes

- Estimation of definite integrals using Monte Carlo simulations with increasing accuracy as the number of samples increases.
- Visualization of the random sampling approach used in the simulation.
- Better understanding of statistical approximation methods and how they compare to traditional analytical techniques.

6 Student Self-Assessment Criteria

- **Accuracy of Estimated Integral:** Compare the simulated result with the exact or numerical solution.
- **Understanding of the Monte Carlo Method:** Evaluate the ability to explain the simulation process and how randomness contributes to the estimation.
- **Analysis of Convergence:** Assess how well the student can analyze and report the relationship between sample size and result accuracy.