System Simulation

Syllabus

2. System Simulation

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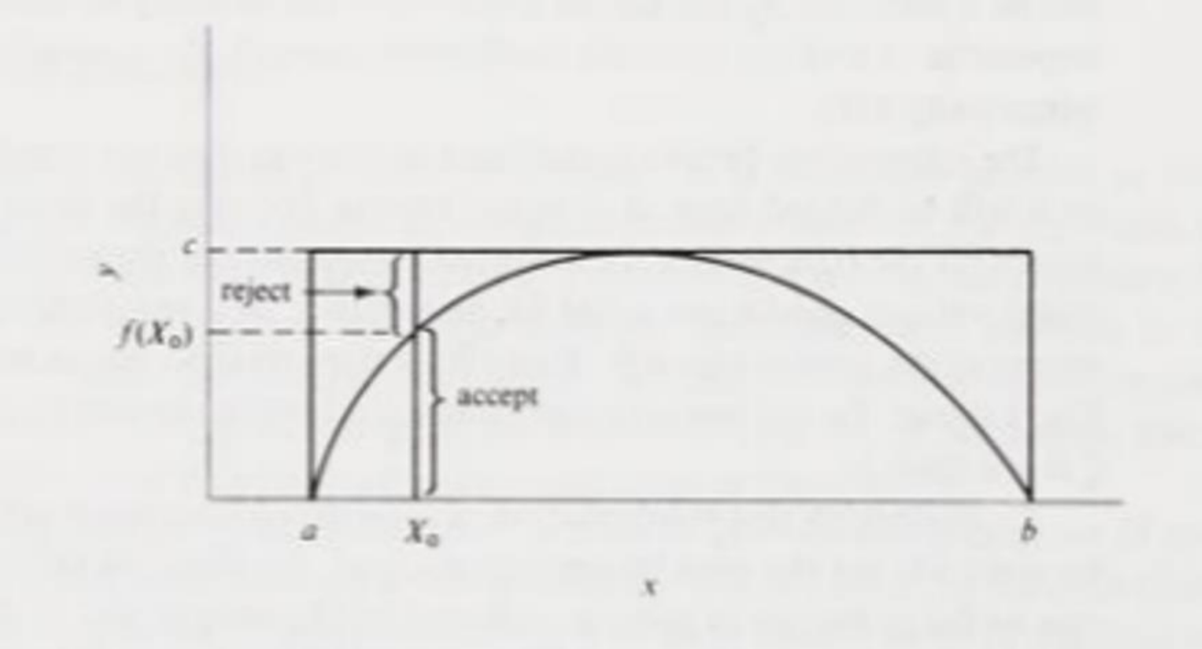
2.1 The Techniques of Simulation- Monte Carlo Method

2.1 The Techniques of Simulation-Monte Carlo Method

- The Monte Carlo Method is the approach of using randomness to describe problems that may have a deterministic solution.
- Monte Carlo Simulation is a type of computational algorithm that uses repeated random sampling to obtain the likelihood of a range of results of occurring.
- Also known as the Monte Carlo Method or a multiple probability simulation, Monte Carlo Simulation is a mathematical technique that is used to estimate the possible outcomes of an uncertain event.
- Monte Carlo Method is based on Law of Large Numbers (LLN): with an increase in the number of measurements the expected value grows to equal the average value.
- The Monte Carlo Method was invented by John von Neumann and Stanislaw Ulam during World War II to improve decision making under uncertain conditions.
- It was named after a well-known casino town, called Monaco, since the element of chance is core to the modeling approach, similar to a game of roulette.

2.1 The Techniques of Simulation-Monte Carlo Method

- Suppose the function f(x) is positive and has lower and upper bound a and b respectively and the function is bounded above by the value c.
- The graph of the function is then contained within a rectangle with sides of length (b-a) and c.
- If we pick points at random within the rectangle and determined whether they lie beneath the curve or not, it is apparent that, providing the distribution of selected points is uniformly spread over the rectangle, the fraction of points falling on or below the curve should be approximately the ratio of the area under the curve to the area of the rectangle.



2.1 The Techniques of Simulation-Monte Carlo Method

- If 'N' points are used and 'n' of them fall under the curve, then approximately

$$\frac{n}{N} = \int_a^b \frac{f(x)\,dx}{c(b-a)} \quad i.e\ I = \frac{n}{N} * c(b-a)$$

- The accuracy improves as the number N increases.
- When it is decided that sufficient points have been taken, the value of integral is estimated by multiplying n with N by the area of rectangle i.e. c(b-a).
- For each point, a value of x is selected at random between a and b say X_0. A second random point selection is made between 0 and c to give Y.
- If $Y \leq f(X_0)$ the point is accepted in the count n, otherwise it is rejected and the next point is picked.

2.2 Problems Depicting Monte Carlo Method

Algorithm to Evaluate the Integration using Monte Carlo Method

1. Find the Range of integration i.e. $b = x_{max}$ and $a = x_{min}$
2. Find Maximum value of the function f(x) in the range of integration i.e. $\mathrm{c} = \max(f(x))$
3. Find Area of Rectangle i.e $\mathrm{A} = \mathrm{c}(\mathrm{b} - \mathrm{a})$
4. Generate a random sequence of points X_0 = (x, y) and check to see if the points are under the curve defined by f(x) or not.

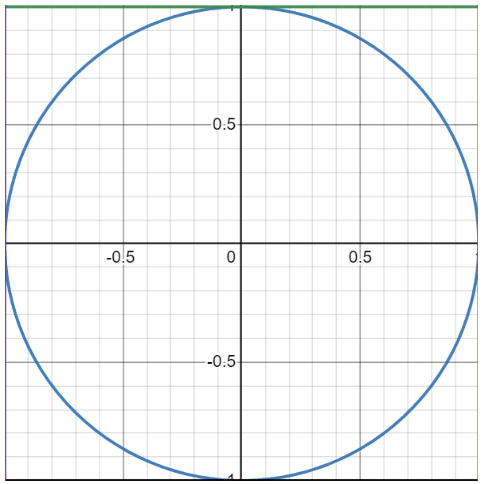
   If $Y \leq f(X_0)$ the point is accepted in the count n, otherwise it is rejected and the next point is generated.
5. Area under the curve, I = $\frac{n}{N} * A_{rectangle}$

2.2 Problems Depicting Monte Carlo Method

Calculate the value of Pie using Monte Carlo Method.

$$\int_{-1}^{1} \sqrt{1 - x^2}\, dx = \pi$$

$$y^2 + x^2 = 1$$



x	y	x^2	y^2	x^2+y^2-1	Accept/Reject
0.626	0.481	0.392	0.232	-0.376	Accept
0.980	0.610	0.961	0.372	0.333	Reject
0.621	0.653	0.386	0.426	-0.188	Accept
0.699	0.700	0.489	0.490	-0.021	Accept
0.816	0.853	0.666	0.728	0.394	Reject
0.532	0.013	0.283	0.000	-0.717	Accept
0.082	0.991	0.007	0.981	-0.012	Accept
0.930	0.787	0.865	0.620	0.485	Reject
0.834	0.596	0.696	0.355	0.051	Reject
0.044	0.285	0.002	0.081	-0.917	Accept
0.597	0.796	0.356	0.634	-0.010	Accept
0.849	0.746	0.720	0.557	0.277	Reject
0.823	0.230	0.677	0.053	-0.270	Accept
0.971	0.232	0.942	0.054	-0.004	Accept
0.242	0.033	0.059	0.001	-0.940	Accept

$$\int_0^{\pi} \sin(x)\, dx = \pi$$

$$\int_0^{2} \sqrt{4 - x^2}\, dx = \pi$$

$$\pi = \int_0^1 \frac{4}{1+x^2} dx$$

b= xmax= 1, a= xmin= -1, c= 2

Area of rectangle = 2*(1+1)= 4

Area of circle = $\frac{n}{N} * A_{rectangle}$

$\pi r^2 = 10/15*4$

$\therefore \pi = 2.66$

2.2 Problems Depicting Monte Carlo Method

Evaluate the following integral using Monte Carlo integration technique. Also solve this integral analytically and find the absolute error and relative error.

$$I= \int_{-2}^{3} x^3 dx$$

Absolute Error = |Iexact-Icalculated|

Relative Error = (Absolute Error/Iexact)

2.3 Comparison of Simulation and Analytical Methods

Drawback of Simulation

- It gives *specific solution rather than general solution*. For example, in the study of automobiles wheel, an analytical solution gives all the condition that can cause oscillation. But each execution of a simulation only tells whether a particular set of condition did or did not cause oscillation. To try to find all such condition required that the simulation must be repeated under many different condition.
- The step by step nature of the simulation technique means that the amount of *computation increases very rapidly* as the amount of detail increases. Coupled with the need to make many runs, the simulation model result in extensive amount of computing.
- Many simulation runs may be *needed to find a maximum* and yet leave undecided the question of whether it is a local or global maximum.

2.3 Comparison of Simulation and Analytical Methods

Drawback of Analytical Technique

- Mathematical technique requires that the *model must be expressed in some particular format*. For example, in the form of linear algebraic equation and continuous linear differential equation.
- There are many simple limitation on a system such as physical stock, finite time delays or non-linear forces which makes a soluble mathematical model insoluble. But simulation removes this limitation.
- The range of problem that can be solve mathematically is limited. Analytical techniques may *struggle to account for uncertainty and variability* in real-world data or parameters. Uncertainty can arise from various sources, such as measurement errors, variability in inputs, or stochastic processes, and adequately incorporating it into analytical models can be challenging.
- Analytical techniques are *most effective for relatively simple systems* with well-defined relationships between variables. However, many real-world problems are highly complex, involving numerous interrelated factors that are difficult to capture analytically.

Aspect	Simulation	Analytical Methods
Approach	Involves creating virtual models and running experiments within them.	Involves solving mathematical equations or models to obtain solutions.
Complexity	Suitable for complex systems with nonlinear relationships and stochastic elements.	Preferable for simpler systems with well-defined relationships between variables.
Assumptions	Can accommodate a wide range of assumptions and real-world complexities.	Relies on simplifying assumptions, which may not fully capture real-world complexities.
Accuracy	Provides a more accurate representation of real-world systems when the underlying model accurately reflects the system's dynamics.	Can provide precise solutions under ideal conditions, but accuracy may diminish if assumptions are not met.
Computational Resources	Generally, more computationally intensive, especially for complex systems.	Often less computationally demanding, particularly for problems with well-defined analytical solutions.
Flexibility	Offers greater flexibility to model complex, dynamic systems and adapt to changing conditions.	Less flexible, as changes to the model or assumptions may require significant adjustments.
Validation & Verification	Requires rigorous validation and verification to ensure accuracy.	Solutions can be validated mathematically but may lack empirical validation.
Applications	Widely used in engineering, economics, social sciences, and healthcare.	Commonly employed in physics, mathematics, and certain engineering disciplines.

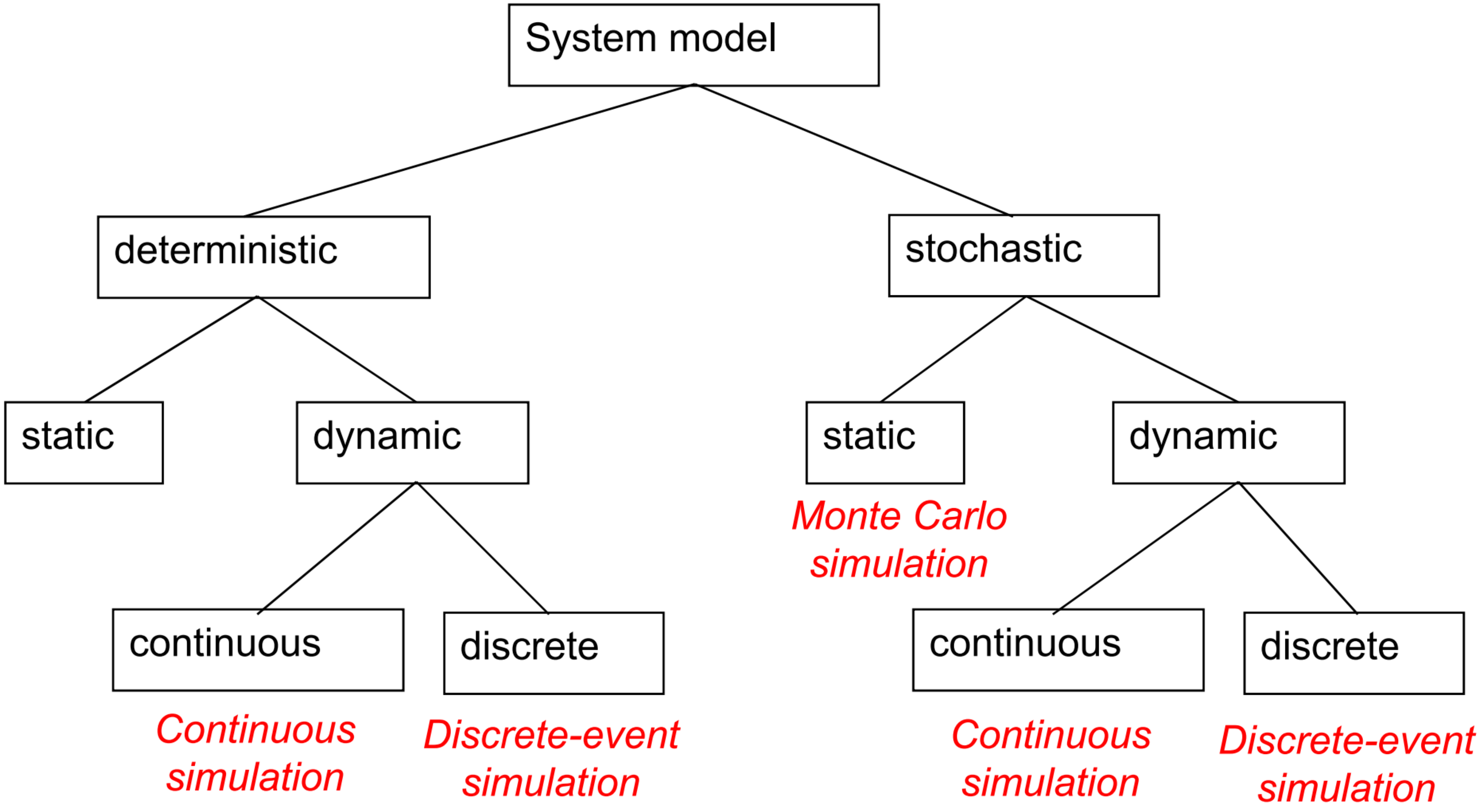
2.4 Experimental Nature of Simulation

- The experimental nature of simulation refers to the process of creating and running simulations to explore and understand complex systems or phenomena.
- Hypothesis Testing: Like traditional experiments, simulations are often used to test hypotheses about how a system behaves under certain conditions.
- Exploration of Scenarios: Simulations enable the exploration of various scenarios that may be difficult, expensive, or even impossible to replicate in the real world. By adjusting parameters and initial conditions, one can simulate different scenarios and observe the resulting outcomes.
- Iterative Process: Simulation is often an iterative process, where researchers refine and improve their models based on the results of previous simulations. This iterative approach allows for a deeper understanding of the system being studied and can lead to more accurate predictions.

2.4 Experimental Nature of Simulation

- The simulation *technique makes no specific attempt to isolate the relationships between any particular variables*: instead, it observes the way in which all variables of the model change with time. Relationships between the variables must be derived from these observations.
- A simulation study of the automobile wheel suspension system that was analyzed in would proceed by following the motion of the wheel under different conditions. The relationship between D, K and M to prevent oscillation, which was previously discovered analytically, would have to be discovered by observing the values that result in the motion being non-oscillatory.
- Simulation is, therefore, essentially an experimental problem-solving technique.
- Many simulation runs have to be made to understand the relationships involved in the system, so the use of simulation in a study must be planned as a series of experiments.

2.5 Types of System Simulation



2.6 Distributed Lag Models

- The market model, discussed in section 1.6 (Static Mathematical Model) was straight forward and too simplistic. When model involves number of parameters and hefty data, one has to opt for computer.
- Models that have the properties of changing only at fixed intervals of time, and of basing current values of the variables on other current values and values that occurred in previous intervals, are called distributed lag models. Griliches Zvi: "Distributed Lags: A Survey,"Econometrica, XXXV, No. 1 (1967), 16–19.
- These are a type of dynamic models, because time factor is involved in them. They are extensively used in econometric studies where the uniform steps correspond to a time interval, such as a month or a year, over which some economic data are collected.
- As a rule, these model consists of linear, algebraic equations. They represent a continuous system, but the one in which data is only available at fixed points in time.

2.6 Distributed Lag Models

- As an example, consider the following simple dynamic mathematical model of the national economy.
- Let, C be consumption, I be investment, T be taxes, G be government expenditure and Y be national income. Then,

$$C = 20 + 0.7(Y - T)$$
$$I = 2 + 0.1Y$$
$$T = 0 + 0.2Y$$
$$Y = C + I + G$$

Equation 1

- This is a static model, but it can be made dynamic by picking a fixed time interval, say one year, and expressing the current values of the variables in terms of values of the previous year.
- Any variable that appears in the form of its current value and one or more previous year's values is called lagged variables. Value of the previous year is denoted by the suffix with-1.
- The static model can be made dynamic by lagging all the variables, as follows;

$$C = 20 + 0.7(Y_{-1} - T_{-1})$$
$$I = 2 + 0.1Y_{-1}$$
$$T = 0.2Y_{-1}$$
$$Y = C_{-1} + I_{-1} + G_{-1}$$

Equation 2

2.6 Distributed Lag Models

- In these equations if values for the previous year (with –1 subscript) is known then values for the current event can be computed. Taking these values as the input, values for the next year can also be computed. In equation (2) we have four equations in five unknown variables.

- It is however not necessary to lag all the variable like it is done in equation (2). Only one of the variable can be lagged and others can be expressed in terms of this variable. We solve equation for Y in equation (1) as,

$$\begin{aligned} Y &= 20 + 0.7(Y - 0.2Y) + I + G \\ &= 20 + 0.56Y + I + G \\ Y &= 45.45 + 2.27(I + G) \end{aligned}$$

- Thus we have, In equations (3) only lagged parameter is Y. Assuming that government expenditure is known for the current year, we first compute I. Knowing I and G, Y and T for the current year is known, and thus C is computed from the last equation. In this problem, lagged model is quite simple and can be computed with hand calculator. But national economic models are generally not that simple and require long computations with number of parameters.

$$\begin{aligned} I &= 2.0 + 0.1Y_{-1} \\ Y &= 45.45 + 2.27(I + G) \\ T &= 0.2Y \\ C &= 20 + 0.7(Y - T) \end{aligned}$$

Equation 3

2.6 Distributed Lag Models

CLASS WORK

Q 1. Find the growth in a national consumption for 5 years using the model given distributed lag model. Assume the initial income Y-1 is 80 and take the government expenditure in the 5 years to be as follows:

Years	G
1	30
2	35
3	40
4	45
5	50

Q2. If demand and supply of a product obey following equations. D = a - bP, S = c +dP and D = S. Here a, b, c, and d are given numbers, convert this model to distributed lagged model.

2.7 Cobweb Models

- In section 1.6, a simple static model of marketing a product had been discussed. In that model two linear equations for demand D and supply S were considered.
- Aim was to compute the probable price and demand of a product in the market subject to a condition that supply and demand should be equal.

$$D = a - bP,$$
$$S = c + dP \qquad \text{Equation 1}$$
$$D = S$$

- But supply of the product in the market depends on the previous year price, and that can be taken as lagged parameter. Thus equations (1) become

$$D = a - bP$$
$$S = c + dP_{-1} \qquad \text{Equation 2}$$
$$D = S$$

2.7 Cobweb Models

- In the equations (2), values of parameters a, b, c, and d can be obtained by fitting a linear curve in the data from the past history of the product. We assign some initial value to the product say P0 and find S from second equation of (2).
- Thus S and D are known and first equation of (2) gives us new value of P. Using this value of P as initial value, we repeat the calculations and again compute P for the next period. If price converges, we say model (2) is stable. Let us take two examples and test whether these models converge or not.

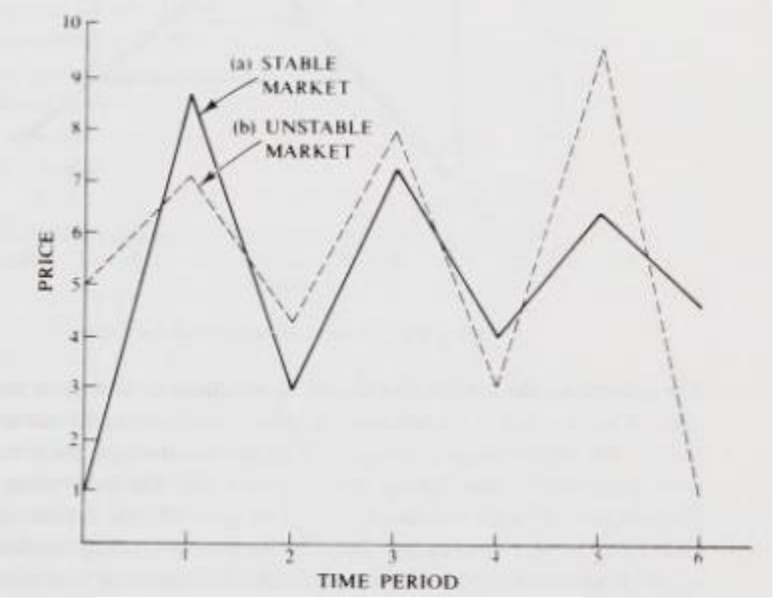
Model 1	*Model 2*
P_0 = 30	P_0 = 5
a = 12	a = 10.0
b = 30	b = 0.9
c = 1.0	c = –2.4
0.9	1.2

Table 1.1: Cobweb model for marketing a product

Model 1		*Model 2*	
i	*P*	*i*	*P*
0	–0.533333	0	7.11111
1	0.382667	1	4.2963
2	0.355187	2	8.04938
3	0.356011	3	3.04527
4	0.355986	4	9.71742
5	0.355987	5	0.821216
6	0.355987	6	12.6828
7	0.355987	7	–3.13265
8	0.355987	8	17.9546
9	0.355987	9	–10.1618
10	0.355987	10	27.3268

2.7 Cobweb Models

- We can see from the table that results in the case of *first model converge even in five steps* where as in *second model they do not converge* et al.
- Data a, b, c, and d for model 2 is such that it does not converge. Thus data of second model is not realistic. These parameters can be calculated from the past history of the product by regression method. This model is called cobweb as it can be graphically expressed as shown in Figure.
- In Fig.2 , we have first drawn supply and demand curves. A line parallel to quantity axis shows that for price equal to one unit, supply is 2 units. If we draw a line parallel to price axis so that it meets demand curve at point marked 1. Thus for the same quantity of supply and demand, price immediately shoots up to more than eight units, due to short supply of product. With this high price, supply shoots up to nine units. Again vertical line equating supply with demand reduces the price to three. We repeat the process and ultimately find that curve converges to optimum value.



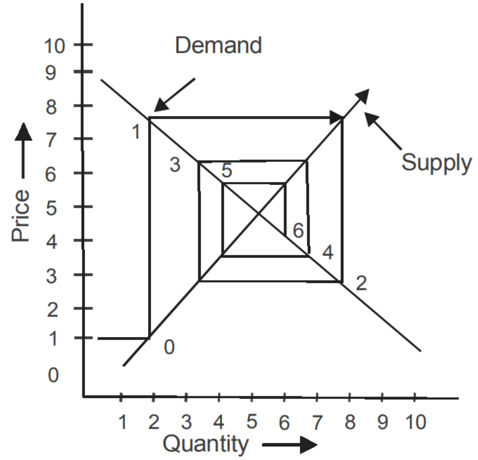


Fig. 1.7: *Cobweb model.*

2.7 Cobweb Models

CLASS WORK

Draw the Cobweb Models for the following market:

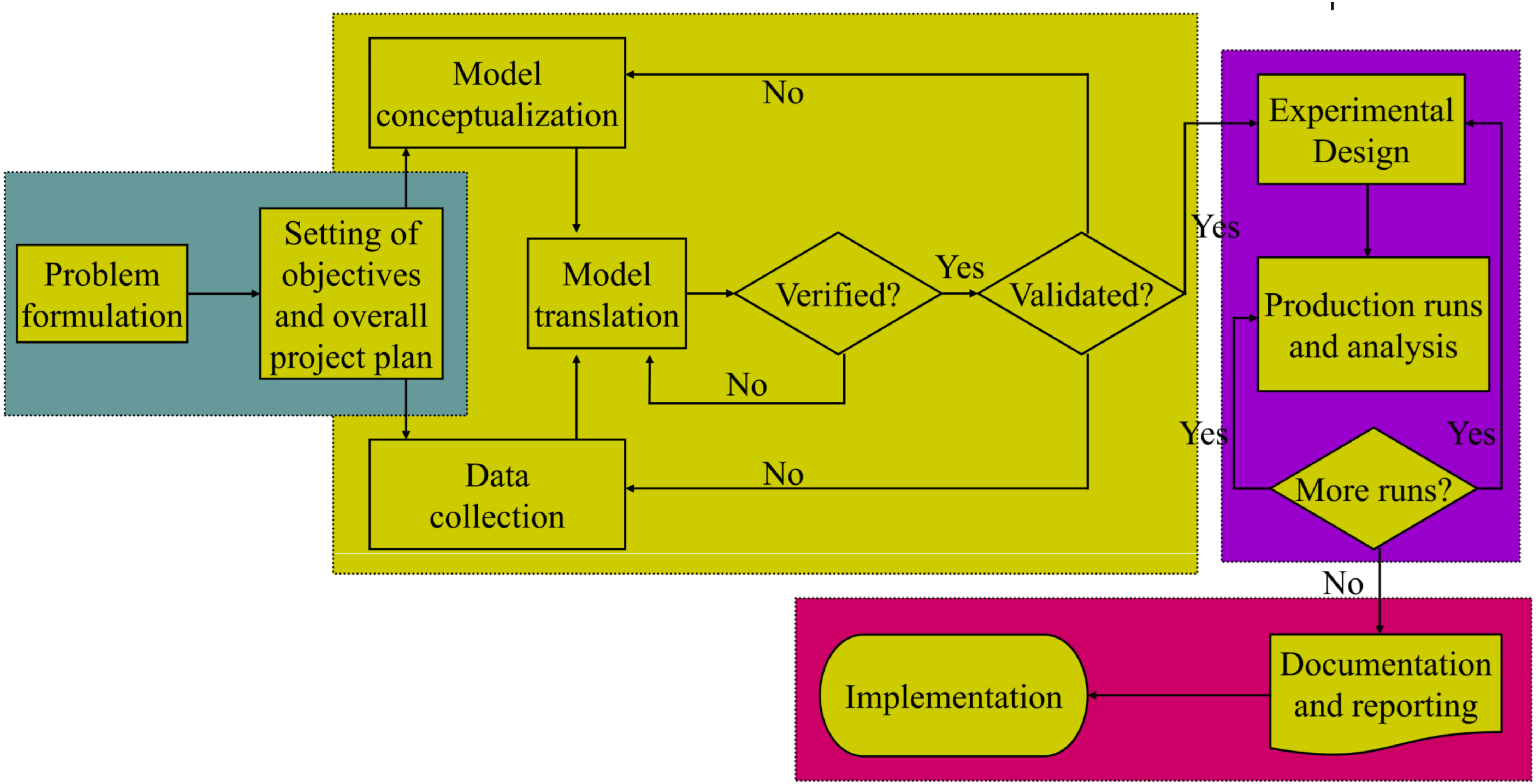
$$D = a + bP,$$
$$S = c - dP$$
$$D = S$$

2.8 Steps of Simulation Study

1. Problem formation
2. Model construction
3. Data Collection
4. Model programming
5. Validation
6. Design of experiment
7. Simulation run and analysis
8. Documentation
9. Implementation

Banks, J., Carson II, J. S., Automation, B., Nelson, B. L., & Nicol, D. M. Discrete-Event System Simulation FOURTH EDITION. P. 12

2.8 Steps of Simulation Study



2.8 Steps of Simulation Study

- **Problem formulation:** Every study *should begin with a statement of the problem*. If the statement is provided by the policymakers, or those that have the problem, the analyst must ensure that the problem being described is clearly understood. If a problem statement is being developed by the analyst, it is important that the policymakers understand and agree with the formulation. Although not shown in Figure, *in some cases problem must be reformulated as the study progresses*. In many instances, policymakers and analysts are aware that there is a problem long before the nature of the problem is known.

- **Setting of objectives and overall project plan:** The objectives indicate *the questions to be answered by simulation*. At this point, a determination should be made concerning whether simulation is the appropriate methodology for the problem as formulated and objectives as stated. Assuming that it is decided that simulation is appropriate, the overall project plan should include a statement of the alternative systems to be considered and of a method for evaluating the effectiveness of these alternatives. It should also include the plans for the study in terms of the number of people involved, the cost of the study, and the number of days required to accomplish each phase of the work, along with the results expected at the end of each stage.

2.8 Steps of Simulation Study

- **Model Conceptualization:** The construction of a model of a system is probably as much art as science. Although it is *not possible to provide a set of instructions that will lead to building successful and appropriate models in every instance*, there are some general guidelines that can be followed. The art of modeling is enhanced by an ability to abstract the essential features of a problem, to select and modify basic assumptions that characterize the system, and then to enrich and elaborate the model until a useful approximation results. Thus, it is best to *start with a simple model* and build toward greater complexity. However, the model complexity need not exceed that required to accomplish the purposes for which the model is intended. Violation of this principle will only add to model-building and computer expenses. It is *not necessary to have a one-to-one mapping between the model and the real system*. Only the essence of the real system is needed.

- **Data Collection:** There is a constant interplay between the construction of the model and the collection of the needed input data. As the complexity of the model changes, the required data elements can also change. Also, since data collection takes such a large portion of the total time required to perform a simulation, it is necessary to begin it as early as possible, usually together with the early stages of model building.

2.8 Steps of Simulation Study

- **Data Collection (contd..):** The objectives of the study dictate, in a large way, the kind of data lobe collected. In the study of a bank, for example, if the desire is to learn about the length of waiting lines as the number of tellers change, the types of data needed would be the distributions of interarrival times (at different times of the day), the service-time distributions for the tellers, and historic distributions on the lengths of waiting lines under varying conditions. This last type of data will be used to validate the simulation model.

- **Model Translation:** Most real-world systems result in models that require a great deal of information storage and computation, to the model must be *entered into a computer-recognizable format*. We use the term "program" even though it is possible to accomplish the desired result in many instances with little or no actual coding. The modeler *must decide whether to program the model in a simulation language or to use special-purpose simulation software*. Simulation languages are powerful and flexible. However, if the problem is amenable to solution with the simulation software, the model development time is greatly reduced. Furthermore, most of the simulation-software packages have added features that enhance their flexibility, although the amount of flexibility varies greatly.

2.8 Steps of Simulation Study

- **Verified?** Verification pertains to the computer program prepared for the simulation model. Is the computer program performing properly? With complex models, it is difficult, if not impossible, to translate a model successfully in its entirety without a good deal of debugging; if the input parameters and logical structure of the model are correctly represented in the computer, verification has been completed. For the most part, common sense is used in completing this step.
- **Validated?** Validation usually is *achieved through the calibration of the model*, an iterative process of comparing the model against actual system behavior and using the discrepancies between the two, and the insights gained, to improve the model. This process is repeated until model accuracy is judged acceptable.
- **Experimental design:** The alternatives that are to be simulated must be determined. Often, the decision concerning which alternatives to simulate will be a function of runs that have been completed and analyzed. For each system design that is simulated, decisions need to be made concerning the length of the initialization period, the length of simulation runs, and the number of replications to be made of each run.

2.8 Steps of Simulation Study

- **Production runs and analysis:** Production runs, and their subsequent analysis, are used to estimate measures of performance for the system designs that are being simulated.
- More Runs? Given the analysis of runs that have been completed, the analyst determines whether additional tuns are needed and what design those additional experiments should follow.
- **Documentation and reporting**. There are two types of documentation: program and progress. Program documentation is necessary for numerous reasons. If the program is going to be used again by the same or different analysts, it could be necessary to understand how the program operates. This will create confidence in the program, so that model users and policymakers can make decisions based on the analysis. Another reason for documenting a program is so that model users can change parameters at will in an effort to team the relationships between input para-meters and output measures of performance or to discover the input parameters that "optimize" some output measure of performance. Project reports give a chronology of work done and decisions made. This can prove to be of great value in keeping the project on course.

2.8 Steps of Simulation Study

- **Implementation**. The success of the implementation phase depends on how well the previous steps have been performed. It is also contingent upon how thoroughly the analyst has involved the ultimate model user during the entire simulation process. If the model user has been thoroughly involved during the model-building process and if the model user understands the nature of the model and its outputs, the likelihood of a vigorous implementation is enhanced. Conversely. if the model and its underlying assumptions have not been properly communicated, implementation will probably suffer, regardless of the simulation models validity.

2.9 Time Advancement Mechanism

Gets Your Hands Dirty: Study the following paper or watch the following video from the provided link and create a 5 page presentation on topic "Time Advancement Mechanism"

- https://www.youtube.com/watch?v=my0Ewj7eN7Q
- Al Rowaei, A. A., Buss, A. H., & Lieberman, S. (2011, December). The effects of time advance mechanism on simple agent behaviors in combat simulations. In *Proceedings of the 2011 Winter Simulation Conference (WSC)* (pp. 2426-2437). IEEE.

2.9 Time Advancement Mechanism (TAM)

- TAM is a fundamental aspect of simulation that *determines how time progresses within the simulated system*. It governs how the simulation updates the state of the system over time. Different time advancement mechanisms are employed depending on the nature of the system being simulated and the requirements of the simulation.
- It's particularly important in two areas (two methods of time advance)
- **Discrete-Event Simulation (DES):** This is commonly used to model systems where events happen at specific points in time, like customer arrivals in a bank or plane landings at an airport. Here, the TAM jumps to the **next event** scheduled in the simulation's timeline.
- **Discrete Time Simulation (DTS):** This simulates time in fixed increments, like running a weather simulation hour by hour. The TAM advances the simulation clock by this predetermined **time step**.

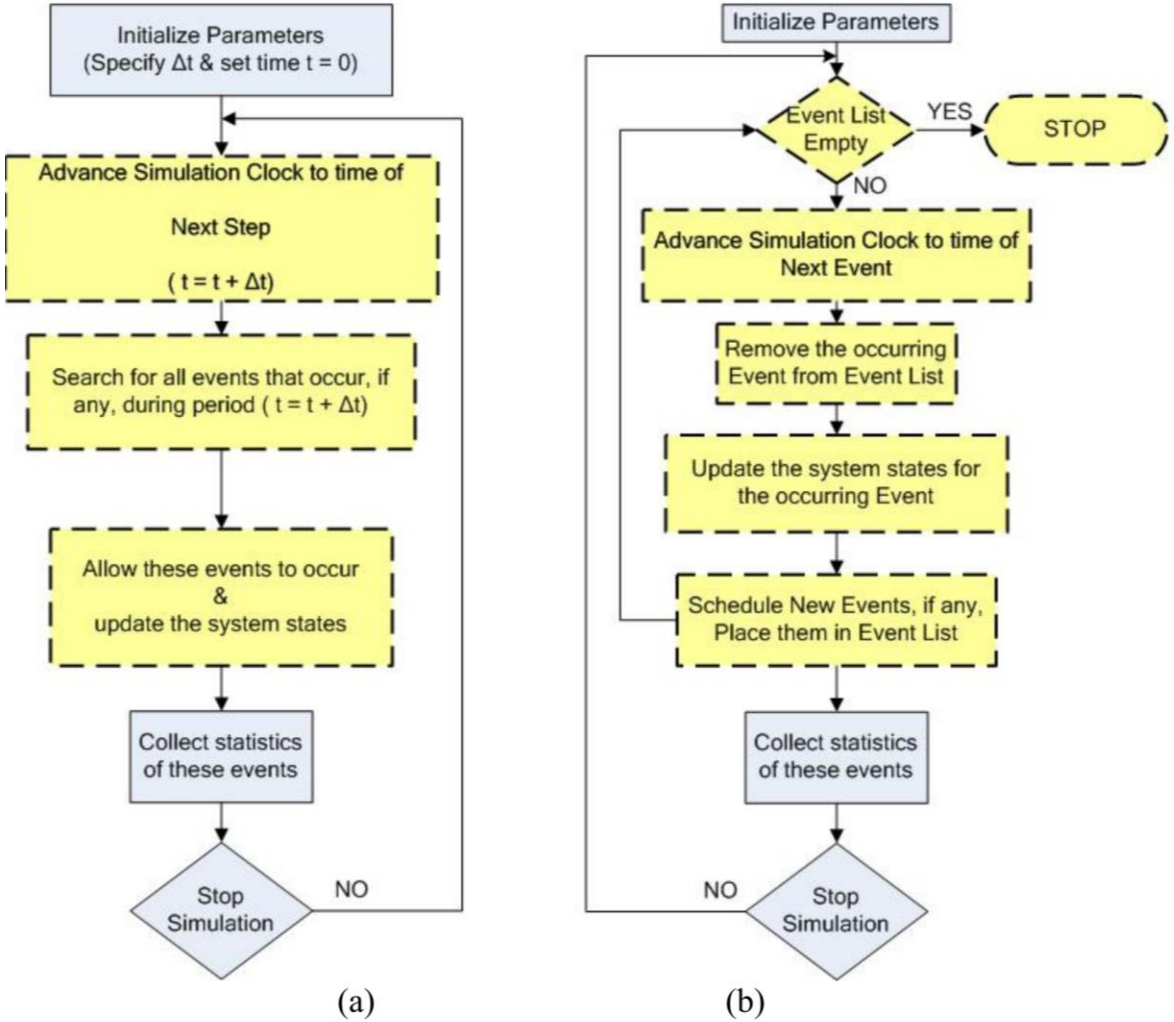


Figure 1: Overview of DTS (a) and DES (b).

2.9 Time Advancement Mechanism

- The choice of TAM depends on the type of simulation and the level of detail needed. Here's a breakdown of the two main approaches: Next-Event Time Advance (NETA) and Fixed Increment Time Advance (FIXA)

Next-Event Time Advance (NETA):

- This method is also called ***Event Oriented Simulation***. In this system, time is incremented when an event occurs. For example in queuing, when a customer arrives, clock is incremented by his arrival time. In such case time period for simulation may be stochastic.
- This is more precise, as it only advances time to the moment the next event occurs. This is ideal for simulations where events are sporadic and require high accuracy.

2.9 Time Advancement Mechanism

Fixed Increment Time Advance (FIXA)

- In fixed time increment model, also called ***Time Oriented Simulation***, events are recorded after a fixed interval of time, which is constant during the simulation period. After the end of each interval, it is noted how many customers have arrived in a queue, and how many have left the server after being served. Attempt in this system is to keep time interval as small as possible, so that minor details of model are monitored.
- This is simpler to implement but can be less accurate, especially if the chosen time step is too large and misses events happening in between. It's efficient for simpler simulations where precise timing isn't crucial.
- Possible in one time interval, only one customer arrives and only one leaves. Fixed time increment simulation is generally preferred for continuous simulation. Numerical methods, where time is taken as independent variable are one such example.

2.10 Queuing Models and its Characteristics

- Let us consider an example where a factory has a large number of semiautomatic machines. Out of these machines, none of the machine fails on 50% of the days, whereas on 30% of the days, one machine fails and on 20% of the days, two can fail. The maintenance staff of the factory, can repair 65% of these machines in one day depending on the type of fault, 30% in two days and 5% remaining in three days. We have to simulate the system for 30 days duration and estimate the average length of queue, average waiting time, and the server loading i.e., the fraction of time for which server is busy. (VP Singh, 173 page)

2.11 Queuing Discipline

2.12 Measures of queues and Single Server Queuing System