Distributed Lag Model in Time Series Analysis

1 Objectives

The objectives of this lab are to:

- Understand the concept of the Distributed Lag Model (DLM) and its application in time series analysis.
- Learn how to model the dynamic relationship between the dependent and independent variables using lagged values.
- Estimate the parameters of the Distributed Lag Model using ordinary least squares (OLS).
- Interpret the effects of past values of the independent variable on the dependent variable.
- Analyze the significance of lag length in the model.

2 Theory

The Distributed Lag Model(DLM) is used to model the impact of past values of an independent variable (X_t) on the current value of the dependent variable (Y_t).
The general form of the Distributed Lag Model can be written as:

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + \beta_3 X_{t-2} + \cdots + \beta_p X_{t-p} + \epsilon_t$$

Where:

- Y_t is the dependent variable at time t.
- X_t is the independent variable at time t.
- β_0 is the intercept term.
- $\beta_1, \beta_2, \ldots, \beta_p$ are the coefficients that capture the effect of the lagged values of the independent variable on the dependent variable.
- ϵ_t is the error term.
- p represents the number of lags included in the model.

The lag length p determines how many past values of the independent variable are considered in the model. The choice of p depends on the theoretical understanding of the problem and model selection criteria like the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC).

Model Estimation

The model is typically estimated using **Ordinary Least Squares (OLS)**, which minimizes the sum of squared residuals:

$$\text{Minimize} \sum_{t=1}^{T} (Y_t - \hat{Y}_t)^2$$

Where $\hat{Y}_t$ is the predicted value of Y_t based on the estimated coefficients.

3 Tasks

The following tasks should be completed:

1. Estimate the parameters of a Distributed Lag Model using time series data.
2. Analyze the impact of different lag lengths on the model.
3. Interpret the coefficients of the lagged variables.
4. Evaluate the model's fit and significance of the coefficients using statistical tests.
5. Visualize the model results with plots to show the relationship between Y_t and the lagged values of X_t.
6. Test the robustness of the model by varying the number of lags and checking the stability of the coefficients.

4 Expected Outcomes

By the end of this lab, students should be able to:

- Understand the concept and structure of the Distributed Lag Model.
- Estimate the parameters of the model using real data and interpret the results.
- Analyze the relationship between a dependent variable and its lagged independent variables.
- Perform model diagnostics and evaluate the fit of the model.
- Understand the impact of lag length on model estimation and make informed decisions on selecting an appropriate number of lags.

5 Assessment

Students will be assessed on the following:

- **Accuracy of Model Estimation (30%):** Correctly estimate the parameters and interpret the coefficients.

- **Model Interpretation (30%):** Properly interpret the relationship between the dependent and lagged independent variables.
- **Statistical Evaluation (20%):** Evaluate the model fit and significance of the coefficients using appropriate statistical tests.
- **Visualization (10%):** Provide clear and informative plots of the model results.
- **Report (10%):** A well-organized report summarizing the model, its interpretation, and findings.