

Simulation and Modeling

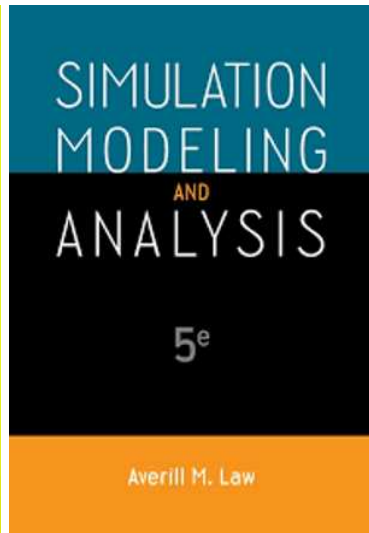
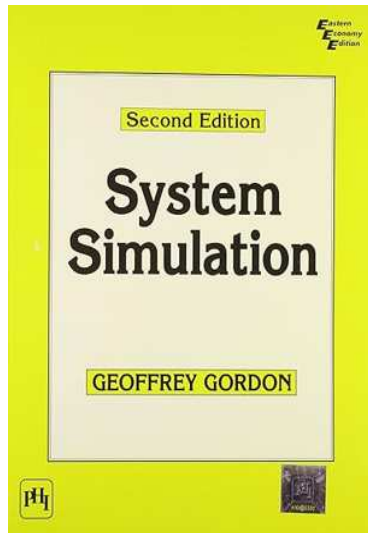
Introduction to Modelling and Simulation

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Syllabus

- 1 System Concept
- 2 System Environment
- 3 Stochastic Activities
- 4 Continuous and Discrete System
- 5 System Modeling
- 6 Types of Models
- 7 Principles of Modeling
- 8 Area of Application of Simulation
- 9 Verification and Validation of Model

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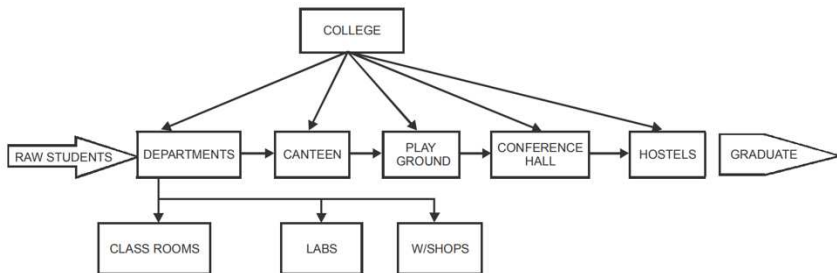
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System Concept (1)

- A system, derived from the Greek word “*systema*”, refers to an organized relationship among functioning units or components, designed to achieve specific objectives.
- Systems are present in daily life, such as transportation, telephone, accounting, production, and computer systems.
- While there are over a hundred definitions of the term, a broad definition is: “A system is any object that performs a function and depends on various entities.”
- Examples include a classroom, college, or university, where each is made up of interconnected components like students, classrooms, and laboratories.
- The systems has three basic implications:
 - A system must be designed to achieve a predetermined objective

System Concept (2)

- Interrelationships and interdependence must exist among the components
- The objectives of the organization as a whole have a higher priority than the objectives of its subsystems.



- As an example of a conceptually simple system (Geoffrey Gordon, 2004), consider an aircraft flying under the control of an autopilot.

System Concept (3)

- A gyroscope in the autopilot detects the difference between the actual heading of aircraft and the desired heading. It sends a signal to move the control surfaces. In response to control surfaces movement, the aircraft steers towards the desired heading.

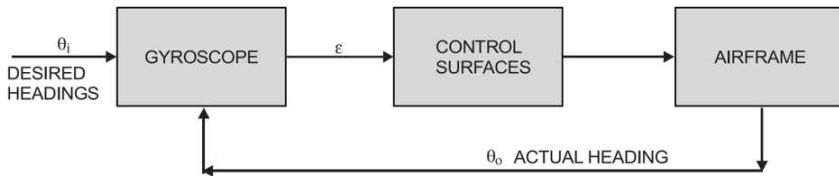


Fig. 0.2: *Model of an autopilot aircraft.*

Components of System (1)

- Three basic components of system are **Entity, Attributes, Activities**.
- A term **entity** will be used to denote an object of interest in a system and the term **attributes** denotes its properties.
- A function to be performed by the entity is called its **activity**.
- For example, if system is a class in a school, then students are entities, books are their attributes and to study is their activity.

1 Entities

- are the fundamental building blocks or components that comprise the system
- can be physical objects, abstract concepts, or even individuals within a social system.
- interact with each other and with the environment to accomplish the system's objectives.

2 Attributes

- are the characteristics or properties of the entities within the system.

Components of System (2)

- define the qualities, behaviors, or states of the entities and influence how they interact with each other and with the environment.
- include **measurable quantities** such as size, weight, temperature, as well as qualitative traits such as color, shape, or function.

3 Activities

- represent the processes, functions, or behaviors that entities engage in within the system.
- involve the transformation of inputs into outputs, the exchange of information or energy, and the accomplishment of specific tasks or goals.
- are often dynamic and can occur over time, contributing to the overall functioning and behavior of the system.
- activities can be **Endogenous and Exogenous**

1 Endogenous Activities

- internal activities that originate within the system itself.
- driven by the system's own dynamics, interactions, and feedback loops.

Components of System (3)

- influenced by the entities and attributes within the system and the relationships among them.

2 Exogenous Activities

- originate from outside the system and have an impact on the system's behavior.
- external to the system and may include inputs, disturbances, or influences from the environment.
- can affect the system's state, dynamics, and evolution but are not directly controlled by the system itself.

<i>System</i>	<i>Entities</i>	<i>Attributes</i>	<i>Activities</i>
Banking	Customers	Maintaining accounts	Making deposits
Production unit	Machines, workers	Speed, capacity, break-down	Welding, manufacturing
College	Teachers, students	Education	Teaching, games
Petrol pump	Attendants	To supply petrol	Arrival and departure of vehicles

Components of System (4)

Table 1.1 Examples of Systems and Components

<i>System</i>	<i>Entities</i>	<i>Attributes</i>	<i>Activities</i>	<i>Events</i>	<i>State Variables</i>
Banking	Customers	Checking-account balance	Making deposits	Arrival; departure	Number of busy tellers; number of customers waiting
Rapid rail	Riders	Origination; destination	Traveling	Arrival at station; arrival at destination	Number of riders waiting at each station; number of riders in transit
Production	Machines	Speed; capacity; breakdown rate	Welding; stamping	Breakdown	Status of machines (busy, idle, or down)
Communications	Messages	Length; destination	Transmitting	Arrival at destination	Number waiting to be transmitted
Inventory	Warehouse	Capacity	Withdrawing	Demand	Levels of inventory; backlogged demands

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System Environment (1)

• System Environment

- The environment refers to the external factors or conditions that surround and affect a system.
- A system is often affected by changes occurring outside the system. Such changes occurring outside the system are said to occur in the system environment.
- Deciding upon the boundary between the system and its environment is indeed a crucial step in modeling a system, as it helps to define what is included within the system and what is considered external to it.
- This boundary definition influences the scope of the system model and the factors considered in its analysis.

• Exogenous Activities

- The term exogenous is used to describe the activities in the environment that affect the system.
- Exogenous activities influence the system from its external environment.
- **Example:** Strikes in a university system.

System Environment (2)

- **Endogenous Activities**

- The term endogenous is used to describe activities occurring within the system.
- These activities are driven by the internal dynamics and interactions of the system's components.
- **Example:** Sports, cultural functions in a university system.

Based on these activities, a system may be classified as:

- **Open System**

- A system that is affected by exogenous activities from its environment.
- Open systems interact with and are influenced by their environment, exchanging energy, matter, and information across their boundaries.

- **Closed System**

- A system with no exogenous activities, meaning it is not influenced by external factors.
- Closed systems do not exchange matter or energy with their environment; instead, they operate based on internal processes and interactions.

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Stochastic Activities (1)

Deterministic Activities

- The outcome is entirely predictable given the initial conditions and inputs.
- There is no randomness or uncertainty involved in the process.
- The exact outcome of deterministic activities can be precisely determined based on known factors, equations, or rules governing the system.
- **Examples:**
 - Simple mathematical calculations.
 - Classical mechanics problems (e.g., projectile motion).
 - Many engineering design processes where the behavior of the system can be fully understood and predicted with certainty.

Deterministic Activities Characteristics

Stochastic Activities (2)

- Follow clear cause-and-effect relationships, where a specific set of inputs always leads to the same output.
- Represented by deterministic models, which describe the system's behavior using deterministic equations or algorithms.
- Common in systems with well-defined rules, regular patterns, and no inherent randomness or variability.

Stochastic Activities

- The outcome is subject to randomness or uncertainty.
- Even with the same initial conditions and inputs, the outcome may vary each time the activity is performed.
- The random output can often be measured and described in the form of a probability distribution.
- Involve inherent variability in their outcomes due to random factors, chance events, or incomplete information about the system.

Stochastic Activities (3)

- **Examples:**

- Random walks.
- Stock market fluctuations.
- Weather forecasting.
- Radioactive decay.

Stochastic Activities Characteristics:

- Do not have a single, predictable outcome; instead, they are described by probability distributions that represent the range of possible outcomes and their likelihoods.
- Often involve random variables and probabilistic models, where the behavior of the system is simulated or analyzed based on statistical principles.
- Prevalent in systems affected by external influences, inherent randomness, or complex interactions where deterministic modeling may be insufficient to capture the full range of variability.

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Continuous and Discrete System (1)

Continuous System

- A continuous system is a system where the variables change continuously over time or space.
- The system's state evolves smoothly, without sudden jumps or interruptions.
- For example, the movement of aircraft occurs smoothly.

Modeling Continuous Systems

- Continuous systems are typically modeled using continuous functions or differential equations.
- These equations describe the relationships between the system's variables and their rates of change.
- Time in continuous systems is treated as a continuous variable, meaning it can take on any value within a certain range.

Continuous and Discrete System (2)

- The system's behavior is observed and analyzed over a continuous range of time.

Examples of Continuous Systems

- **Physical Systems:** Fluid dynamics (e.g., fluid flow in pipes).
- **Analog Electronics:** Electronic circuits with continuous signals.
- **Population Dynamics:** Continuous growth models.

Analytical Methods

- Methods such as calculus, Fourier analysis, and Laplace transforms are commonly used to analyze continuous systems.
- These methods enable the solution of differential equations and the study of system behavior over time.

Applications

- Natural sciences (e.g., physics, chemistry, biology).

Continuous and Discrete System (3)

- Engineering (e.g., mechanical, electrical, chemical).
- Other fields where processes evolve smoothly over time or space.

Discrete System

- A discrete system is a system where the variables change at distinct time points or intervals.
- The system's state evolves in discrete steps, with changes occurring only at specific instances in time.

Modeling Discrete Systems:

- Discrete systems are typically modeled using discrete values or events.
- Variables are updated or changed incrementally at discrete time steps or intervals.
- Time in discrete systems is represented as a sequence of discrete time steps.

Continuous and Discrete System (4)

- The system's behavior is observed and analyzed at specific points in time rather than continuously.

Examples of Discrete Systems:

- **Digital Systems:** Digital circuits, computer algorithms.
- **Discrete Event Systems:** Queues, networks.
- **Computer Simulations:** Simulations with discrete time steps.

Analysis Methods

- Difference equations.
- Finite difference methods.
- Discrete-event simulation.

These methods focus on the discrete changes in variables over time.

Applications:

Continuous and Discrete System (5)

- Computer science.
- Digital signal processing.
- Telecommunications.
- Control systems.
- Many other fields where processes are inherently discrete or can be approximated as discrete.

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System Modeling (1)

What is a Model?

- A model is a simplified representation of a system at some particular point in time or space, intended to promote understanding of the real system.
- It is the body of information about the system gathered for the purpose of studying it.
- The purpose of the study determines the nature of the information gathered, so there is no unique model of a system.
- Different models of the system will be produced by different system analysts who are interested in different aspects of the system.

System Modeling

- System modeling is the body of information about a system gathered to study the system.
- The task of deriving a system model is divided into two subtasks:

System Modeling (2)

- Establishing the model structure.
- Supplying the data.

Establishing the Model Structure

- Involves defining the structure of the system model.
- Includes determining the system boundaries and identifying its fundamental components: entities, attributes, and activities.
- Defining the system boundary involves specifying what is included within the system and what lies outside of it. This helps focus the analysis on the relevant components and interactions.

Supplying the Data

- Involves providing the necessary data to populate the system model, including values for attributes and defining relationships between activities.

System Modeling (3)

- **Attribute Values:** Data provides values for the attributes (e.g., numerical values, categorical labels) describing the characteristics of entities within the system.
- **Relationships:** Data also defines relationships between entities and activities in the system. These relationships describe how entities interact to accomplish tasks or achieve objectives.

Example: A Vending Machine

- **Establishing the Model Structure**
 - **System Boundary:** The vending machine, its components, and operations are within the system boundary.
 - **Entities:** Vending machine, products (e.g., snacks, beverages), coins or currency.
 - **Attributes:**
 - Vending machine attributes: capacity, inventory levels, prices of products.
 - Product attributes: type, quantity, price.

System Modeling (4)

- Coin attributes: denomination, quantity.
- **Activities:** Selecting a product, inserting coins, dispensing products, returning change (if applicable).
- **Supplying the Data**
 - **Attribute Values**
 - Vending machine capacity (maximum number of products it can hold).
 - Inventory levels of each product.
 - Prices of products.
 - Denominations and quantities of coins accepted.
 - **Relationships**
 - Users select products and insert coins to initiate the vending process.
 - The vending machine verifies payment and dispenses the selected product.
 - If payment exceeds the product price, the machine returns change.

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Types of Models

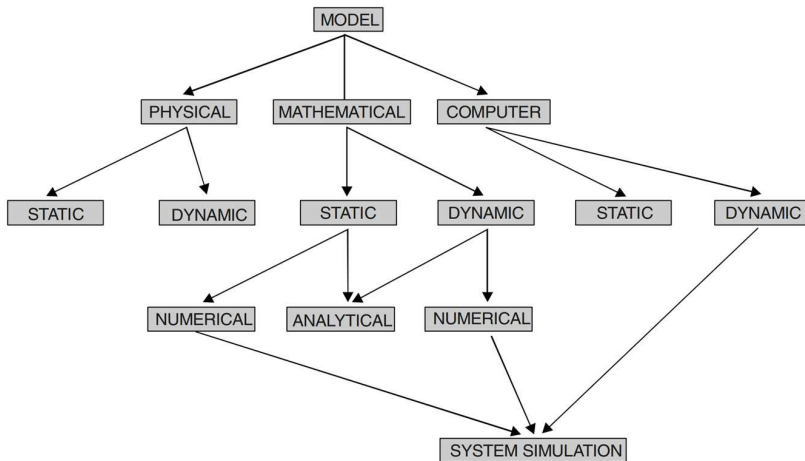


Fig. 1.1: *Different types of models.*

Physical Model (1)

Physical Model

- Physical models are based on an analogy between systems, such as mechanical, electrical, or hydraulic systems.
- The system's attributes are represented by physical measurements such as voltage or the position of a shaft.
- The system's activities are reflected in the physical laws that govern the model.

Example: DC Motor and Vehicle Analogy

- The rate at which the shaft of a direct current (DC) motor turns depends upon the voltage applied to the motor.
- If the applied voltage is used to represent the velocity of a vehicle, the number of revolutions of the shaft is a measure of the distance the vehicle has traveled.

Physical Model (2)

- The higher the voltage (or velocity), the greater the buildup of revolutions (or distance) covered in a given time.

Types of Physical Models

• Static Physical Models

- A static physical model is a scaled-down representation of a system that does not change over time.
- **Example 1:** An architect's scaled-down model of a building before construction, reflecting rooms, outer design, and features.
- **Example 2:** Small water tanks used to simulate the sea for conducting trials; the tank serves as a static model of the ocean.

• Dynamic Physical Models

- A dynamic physical model changes with time or is a function of time.
- Example: In a wind tunnel, small aircraft models (static models) are subjected to varying wind velocities. Pressure profiles are measured with transducers embedded in the model, reflecting dynamic conditions.

Physical Model (3)

- Example: A model of a hanging wheel of a vehicle, where the wheel's motion changes dynamically with time.

Applications

- Physical models are essential in many engineering fields, such as aerodynamics, fluid dynamics, and architecture, where understanding complex systems is difficult through direct observation.
- These models allow for testing in controlled environments, providing insights that are otherwise difficult to obtain.
- Both static and dynamic models are used to simulate real-world systems and predict their behavior under different conditions.

Mathematical Model (1)

Mathematical Model

- A mathematical model uses symbolic notations and mathematical equations to represent a system.
- System attributes are represented by variables, and system activities are represented by mathematical functions that interrelate these variables.

Types of Mathematical Models

• Static Models

- Show the values that system attributes take when the system is in balance.
- Represent the system's equilibrium state.

• Dynamic Models

- Follow the changes over time resulting from system activities.
- Capture how system behavior evolves and changes over time.

Mathematical Model (2)

Techniques for Solving Mathematical Models

- **Analytical Methods:** Use deductive reasoning and mathematical theory to solve the model.
 - **Example:** Linear differential methods to solve differential equations.
- **Numerical Methods:** Involve computational procedures to solve equations, often using mathematical tables and algorithms.
 - **Example:** Using computers to simulate complex equations when exact analytical solutions are difficult or impossible.

Applications of Mathematical Models

- Physical models like scale models are often used to test or simulate systems mathematically.
- Examples include:
 - Shipbuilding: Scale models help in determining precise measurements for hull construction.

Mathematical Model (3)

- Wind tunnels and water tanks: Used in aircraft and ship design to simulate environmental conditions.
- DNA deciphering: Mathematical models help in interpreting complex biological data.

Solving Equations for Complex Systems

- In many cases, equations can only be solved for simple-shaped bodies.
- For complicated shapes, solutions can be approximated by creating a model with the same shape and measuring specific attributes.
- Example: Predicting heat distribution in a body by enclosing it with a space and measuring the charge when the space's surface is electrified in a way that reflects the heat injection process.

Dynamic Physical Model (1)

- Dynamic physical models rely on analogies between different systems to study behaviors.
- Consider a wheel of mass M suspended vertically, with a force $F(t)$ applied. The force varies with time and acts on the wheel.
- The wheel is connected to a spring with stiffness K and a piston with damping factor D . This system can be used to study oscillations in a motor wheel.

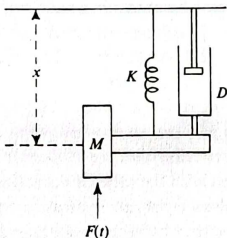
Spring-Mass System

- When the force $F(t)$ is applied, the mass M oscillates under the action of the three forces: spring force, damping force, and external force.
- This system can be used to model the load on beams in a building.
- The system becomes dynamic when the force varies with time, and parameters K and D can be adjusted to control the oscillations.

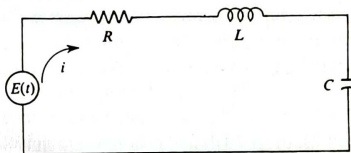
Analogous Electrical System

Dynamic Physical Model (2)

- Consider an electric circuit with inductance L , resistance R , and capacitance C , connected to a time-varying voltage source $E(t)$.
- The system studies the rate of current flow as $E(t)$ varies, with q representing the charge on the capacitance.
- This electrical system is analogous to the hanging wheel system, and both systems can be studied using similar principles.



(a) Mechanical System



(b) Electrical System

Dynamic Physical Model (3)

MECHANICAL SYSTEM	ELECTRICAL SYSTEM
Mass (M)	Inductance (L)
Damping Factor (D)	Resistance (R)
Spring Constant (K)	Capacitance (1/C)
Force F(t)	Applied Voltage E(t)
Displacement (x)	Charge (q)
Velocity (\dot{x})	Current (\dot{q})
Acceleration (\ddot{x})	Voltage (\ddot{q})

Mathematical Model of the Hanging Wheel

- Using Newton's second law of motion, the system for the wheel can be expressed mathematically as:

$$M \frac{d^2 x}{dt^2} + D \frac{dx}{dt} + Kx = F(t) \quad (1)$$

where:

- M is the mass of the wheel,

Dynamic Physical Model (4)

- D is the damping factor,
 - K is the spring constant,
 - x is the displacement of the wheel,
 - $F(t)$ is the applied force as a function of time.
- This equation describes the oscillatory behavior of the wheel under the influence of the applied force, damping, and spring forces.

Electrical System Equations

- The electrical system with inductance, resistance, and capacitance can be described by the following equation:

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = E(t) \quad (2)$$

where:

- L is the inductance,
- R is the resistance,

Dynamic Physical Model (5)

- C is the capacitance,
- q is the charge on the capacitor,
- $E(t)$ is the time-varying voltage.
- This equation models the flow of current in an electrical circuit in response to a time-varying voltage source.

Mechanical and Electrical Analogies

- The mechanical system (spring-mass system) and the electrical system are analogs of each other.
- It is often easier to modify the electrical system for study, making the electrical system a preferred tool for analyzing mechanical systems.

Static Mathematical Model (1)

A static mathematical model represents a system at equilibrium, where the system's variables remain constant, and there is no time-dependent change. These models describe the relationships between variables at a given point without considering time variation.

Characteristics

- No time dependence.
- Describes equilibrium or steady-state conditions.
- Simplifies complex systems by ignoring dynamic changes.

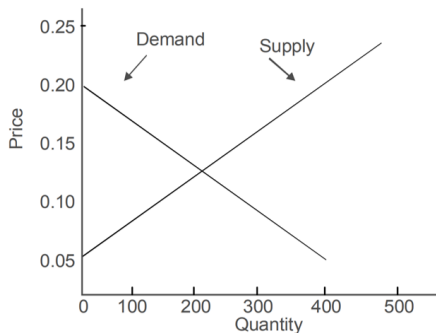
Example: Supply and Demand Model

In marketing a commodity there is a balance between the supply and demand for the commodity. Both factor (demand and supply) depend upon price.

- The law of supply: the quantity of a good supplied rises as the market price rises, and falls as the price falls.

Static Mathematical Model (2)

- The law of demand: the quantity of a good demanded falls as the price rises, and vice versa.



Static Mathematical Model (3)

Demand Function:

$$Q_d = a - bP$$

where Q_d is the quantity demanded and P is the price.

Supply Function:

$$Q_s = c + dP$$

where Q_s is the quantity supplied.

At equilibrium, $Q_d = Q_s$, leading to:

$$P = \frac{a - c}{b + d}$$

- Usually, the demand will be represented by a curve that slopes downwards, and the supply by a curve that slopes upwards, as illustrated in Figure 1.

Static Mathematical Model (4)

- It may not then be possible to express the relationships by equations that can be solved. Some numeric method is then needed to solve the equations.
- Drawing the curves to scale and determining graphically where they intersect is one such method. In practice, it is difficult to get precise values for the coefficients of the model.
- Observations over an extended period of time, however, will establish the slopes (that is, the values of b and d) in the neighborhood of the equilibrium point, and, of course, actual experience will have established equilibrium prices under various conditions.

Static Mathematical Model (5)

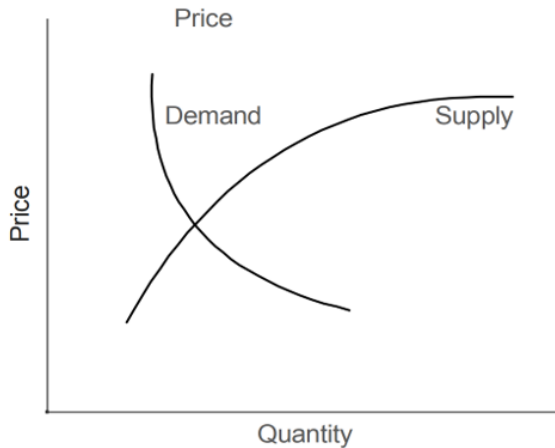


Figure 1: Non-Linear Market Model

Dynamic Mathematical Model (1)

- A dynamic mathematical model describes how the attributes of a system change over time.
- These changes are usually represented using differential equations, which capture the rate of change of system variables.
- The solution to these equations can either be found analytically (using mathematical techniques) or numerically (using computational methods), depending on the complexity of the system being modeled.

Example: Wheel Suspension System

Consider the equation that models the behavior of a **wheel suspension system** in a vehicle, which is a dynamic system. The equation is given by:

$$M \frac{d^2x}{dt^2} + D \frac{dx}{dt} + Kx = F(t) \quad (3)$$

Where:

Dynamic Mathematical Model (2)

- M is the mass of the wheel,
- D is the damping coefficient (resistance to motion),
- K is the stiffness constant (a measure of the system's resistance to deformation),
- $F(t)$ is the external force applied to the system, which is a function of time.

This equation is a second-order linear differential equation that describes how the displacement(x) of the wheel changes over time under the influence of external forces.

Oscillatory Motion: For an oscillatory system, where the system exhibits periodic motion (e.g., vibrations of the suspension system), the system's behavior can be further described in terms of the **damping ratio** ζ and the **frequency of oscillation** ω .

Dynamic Mathematical Model (3)

- The **damping ratio** ζ is a dimensionless measure of damping and is related to the damping coefficient D and mass M by the formula:

$$2\zeta\omega = \frac{D}{M}$$

- The **angular frequency** ω (in radians per second) is related to the stiffness K and mass M by the equation:

$$\omega^2 = \frac{K}{M}$$

Thus, the equation can be rewritten as:

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2x = \omega^2f(x) \quad (4)$$

Where:

Dynamic Mathematical Model (4)

- \ddot{x} is the acceleration (the second derivative of displacement with respect to time),
- \dot{x} is the velocity (the first derivative of displacement with respect to time),
- x is the displacement of the system,
- $f(x)$ is a function that describes how the external force depends on displacement.

Frequency of Oscillation:

- The frequency of oscillation f (in cycles per second) is related to the angular frequency by the formula:

$$\omega = 2\pi f$$

Dynamic Mathematical Model (5)

- This formula helps to understand the frequency of oscillation of the system and can be used to design and control mechanical systems that involve oscillatory motion, such as vehicle suspension systems.

Solutions of Differential Equations

The second-order differential equation is

$$M \frac{d^2x}{dt^2} + D \frac{dx}{dt} + Kx = F(t)$$

First, we solve the homogeneous equation:

$$M \frac{d^2x}{dt^2} + D \frac{dx}{dt} + Kx = 0$$

Dynamic Mathematical Model (6)

Assuming a solution of the form $x(t) = e^{\lambda t}$, we substitute into the homogeneous equation:

$$M\lambda^2 e^{\lambda t} + D\lambda e^{\lambda t} + Ke^{\lambda t} = 0$$

Factor out $e^{\lambda t}$:

$$M\lambda^2 + D\lambda + K = 0$$

The characteristic equation is:

$$M\lambda^2 + D\lambda + K = 0$$

Using the quadratic formula to solve for λ :

$$\lambda = \frac{-D \pm \sqrt{D^2 - 4MK}}{2M}$$

Dynamic Mathematical Model (7)

Case 1: Overdamped When $D^2 > 4MK$, we have two distinct real roots:

$$\lambda_1 = \frac{-D + \sqrt{D^2 - 4MK}}{2M}, \quad \lambda_2 = \frac{-D - \sqrt{D^2 - 4MK}}{2M}$$

The general solution is:

$$x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

Case 2: Critically Damped When $D^2 = 4MK$, we have one repeated root:

$$\lambda = \frac{-D}{2M}$$

The general solution is:

$$x(t) = (C_1 + C_2 t) e^{\lambda t}$$

Dynamic Mathematical Model (8)

Case 3: Underdamped When $D^2 < 4MK$, we have complex conjugate roots:

$$\lambda = \frac{-D \pm i\sqrt{4MK - D^2}}{2M}$$

The general solution is:

$$x(t) = e^{-\frac{D}{2M}t} (C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t))$$

where $\omega_d = \frac{\sqrt{4MK - D^2}}{2M}$ is the damped frequency.

Particular Solution (Forced Response)

Now, consider the full equation with the external force $F(t)$:

$$x(t) = x_h(t) + x_p(t)$$

Case 1: Constant Forcing Function $F(t) = F_0$

Dynamic Mathematical Model (9)

For a constant force, the particular solution is:

$$x_p(t) = A = \frac{F_0}{K}$$

Thus, the full solution is:

$$x(t) = x_h(t) + \frac{F_0}{K}$$

Case 2: Sinusoidal Forcing Function $F(t) = F_0 \sin(\omega t)$

For a sinusoidal forcing function, the particular solution is of the form:

$$x_p(t) = A \sin(\omega t) + B \cos(\omega t)$$

Substitute into the differential equation and solve for A and B .

Complete Solution

Dynamic Mathematical Model (10)

The complete solution is:

$$x(t) = x_h(t) + x_p(t)$$

Where $x_h(t)$ is the homogeneous solution and $x_p(t)$ is the particular solution.

Example: Under Damped Response with Sinusoidal Force

For example, if the system is underdamped with a sinusoidal external force $F(t) = F_0 \sin(\omega t)$, the general solution is:

$$x(t) = e^{-\frac{D}{2M}t} (C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t)) + \frac{F_0}{K} \sin(\omega t)$$

Where ω_d is the damped frequency of oscillation, and $\frac{F_0}{K}$ is the steady-state amplitude.

Dynamic Mathematical Model (11)

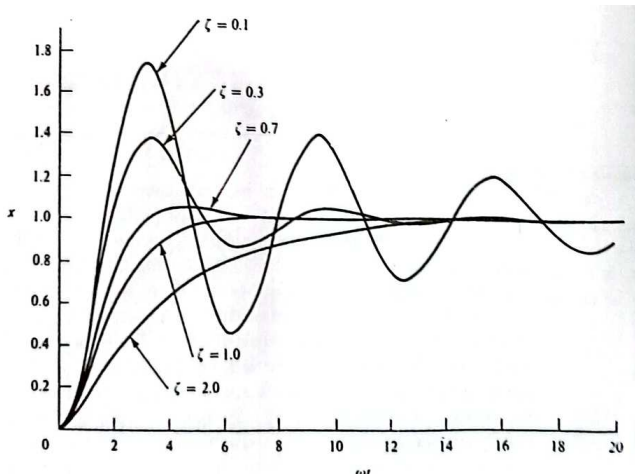


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Principles of Modeling (1)

Block Building

The description of the system should be organized in a series of blocks. The aim is to simplify the specification of the interaction within a system. Block building refers to the process of constructing the model by breaking down the system into manageable blocks or components. Each block represents a distinct aspect or function of the system, making the overall model easier to understand and analyze. By building the model in blocks, it becomes easier to identify relationships between components and to make changes or modifications to specific parts of the system without affecting the entire model.

Relevance

Relevance emphasizes the importance of including only relevant factors and variables in the model. Extraneous details or irrelevant components can clutter the model and make it more complex than necessary. It's essential

Principles of Modeling (2)

to focus on capturing the essential aspects of the system that are directly related to the objectives of the analysis or simulation.

Accuracy

Accuracy involves ensuring that the model accurately represents the real-world system it is intended to simulate or analyze. The model should reflect the actual behavior, relationships, and dynamics of the system as closely as possible. This requires careful selection of input data, appropriate modeling techniques, and validation against empirical observations or data.

Aggregation

Aggregation involves simplifying the model by combining or summarizing similar components or variables. Aggregating similar elements reduces complexity and computational burden while still capturing the essential characteristics of the system. However, it's important to strike a balance between

Principles of Modeling (3)

aggregation and accuracy to ensure that important details are not overlooked.

Validation

Validation is the process of confirming that the model produces reliable and credible results. Validation involves comparing the model's predictions or simulations against real-world data or observations. If the model accurately replicates the behavior of the system and produces results that align with empirical evidence, it is considered validated and can be used with confidence for analysis or decision-making.

Example

A model of a system can be divided into a number of blocks, which in themselves are complete systems. But these blocks should have some relevance to the main system. For example, let us take an example of a school.

Principles of Modeling (4)

Classrooms are blocks of the school. The aim of the school is to impart education to students, and classrooms are required for coaching. Thus, the relevance of classrooms (blocks) with the school is coaching.

Interdependency is one of the important factors of different blocks. Each block should be accurate and tested independently. Then these blocks are to be integrated together. The last step is validation, where the model is tested for its performance.

For validation, the following methods can be used:

- If the model is mathematical, some trivial results can be run for verification.
- If experimental results are available, the model can be checked with these experimental results.

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Simulation (1)

- Simulation is an experimental technique. It is a fast and experimental method of performing experiments under a computer.
- There is no specific unifying theory of computer simulation and no principle guiding the formulation of simulation models.
- Simulation provides an alternative that is cheap, fast, and fills the gap between exact analysis and physical intuition.
- Simulation is the representation of a real-life system by another system, which depicts the important characteristics of the real system and allows experimentation on it. In other words, **simulation is an imitation of reality**.
- Simulation has long been used by researchers, analysts, designers, and other professionals in physical and non-physical experiments and investigations.

Simulation (2)

Naylor et al. defines simulation as follows: *“Simulation is a numerical technique for conducting experiments on a digital computer, which involves certain types of mathematical and logical models over an extended period of real time.”*

Why Simulation is Required?

According to Naylor, some of the reasons why simulation is appropriate are:

- 1 Simulation enables the study of, and experimentation with, the internal interactions of a complex system or a subsystem within a complex system.
- 2 Informational, organizational, and environmental changes can be simulated, and the effect of these alterations on the model's behavior can be observed.

Simulation (3)

- ③ The knowledge gained during the design of a simulation model could be of great value toward suggesting improvements in the system under investigation.
- ④ Changing simulation inputs and observing the resulting outputs can produce valuable insight into which variables are the most important and into how variables interact.
- ⑤ Simulation can be used as a pedagogical device to reinforce analytic solution methodologies.
- ⑥ Simulation can be used to experiment with new designs or policies before implementation, to prepare for what might happen.
- ⑦ Simulation can be used to verify analytic solutions.
- ⑧ Simulating different capabilities for a machine can help determine the requirements for it.

Simulation (4)

- 9 Simulation models designed for training make learning possible without the cost and disruption of on-the-job instruction.
- 10 Animation shows a system in simulated operation so that the plan can be visualized.
- 11 The modern system (factory, wafer fabrication plant, service organization, etc.) is so complex that its internal interactions can only be treated through simulation.

Naylor, TJ, Balintfy, JL, Burdick, DS, and K Chu, *Computer Simulation Techniques*, Wiley, NY, 1966

When to use Simulation ?

Over the years tremendous developments have taken place in computing capabilities and in special purpose simulation languages, and in simulation methodologies. The use of simulation techniques has also become widespread.

Following are some of the purposes for which simulation may be used.

- Simulation is very useful for experiments with the internal interactions of a complex system, or of a subsystem within a complex system.
- Simulation can be employed to experiment with new designs and policies, before implementing
- Simulation can be used to verify the results obtained by analytical methods and reinforce the analytical techniques.
- Simulation is very useful in determining the influence of changes in input variables on the output of the system.
- Simulation helps in suggesting modifications in the system under investigation for its optimal performance.

When Simulation Is Not Appropriate ? (1)

This section is based on an article by Banks and Gibson [1997], who gave ten rules for evaluating when simulation is not appropriate.

1. Common Sense Solution:

Simulation should not be used when the problem can be solved by common sense. An example is given of an automobile tag facility serving customers who arrive randomly at an average rate of 100/hour and are served at a mean rate of 12/hour. To determine the minimum number of servers needed, simulation is not necessary. Just compute:

$$\frac{100}{12} = 8.33$$

which indicates that nine or more servers are needed.

2. Analytical Solution:

When Simulation Is Not Appropriate ? (2)

Simulation should not be used if the problem can be solved analytically. For example, under certain conditions, the average waiting time in the above example can be found from curves developed by Hillier and Lieberman [2015].

3. Direct Experiments:

Simulation should not be used if it is easier to perform direct experiments. An example is a fast-food drive-in restaurant where it was less expensive to stage a person taking orders using a hand-held terminal and voice communication to determine the effect of adding another order station on customer waiting time.

4. Cost Exceeds Savings:

Simulation should not be used if the costs exceed the savings. There are many steps in completing a simulation, and these must be done thoroughly.

When Simulation Is Not Appropriate ? (3)

If a simulation study costs \$20,000 and the savings might be \$10,000, simulation would not be appropriate.

5. and 6. Lack of Resources or Time:

If the resources or time are not available, simulation is not appropriate. If the simulation is estimated to cost \$20,000 and only \$10,000 is available, the suggestion is not to venture into a simulation study. Similarly, if a decision is needed in two weeks and a simulation will take a month, the simulation study is not advised. If no data is available, not even estimates, simulation is not advised.

7. Model Verification and Validation:

If there is not enough time or if the personnel are not available for model verification and validation, simulation is not appropriate.

When Simulation Is Not Appropriate ? (4)

8. Unreasonable Expectations:

If managers have unreasonable expectations, if they ask for too much too soon, or if the power of simulation is overestimated, simulation might not be appropriate.

9. Complex or Undefined Behavior:

If the system behavior is too complex or can't be defined, simulation is not appropriate. Human behavior is sometimes extremely complex to model.

Banks, J., & Gibson, R. (1997). 10 rules for determining when simulation is not appropriate. IIE Solutions, 29(9).

Hillier, F. S., & Lieberman, G. J. (2015). Introduction to operations research. McGraw-Hill.

Area of Application of Simulation (1)

Types of Simulation Models (1)

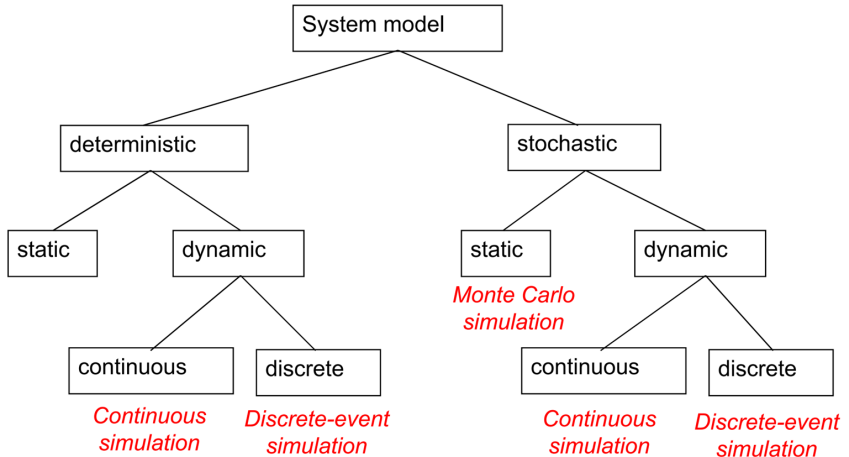


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Verification and Validation of Model (1)

Verification

The process of ensuring that the model is implemented correctly and operates as intended according to its specifications and requirements.

Did I build the model right?

Activities

- Checking the model code or implementation against its design specifications to identify errors or discrepancies.
- Conducting unit tests to verify individual components or modules of the model.
- Performing code reviews and inspections to identify coding errors, logic flaws, or inconsistencies.
- Using automated testing tools and techniques to detect errors and ensure consistency in the model implementation.

Verification and Validation of Model (2)

Purpose

Verification aims to confirm that the model accurately represents the conceptual design and follows the intended behavior as specified by the requirements. It focuses on identifying and correcting errors in the model implementation to improve its quality and reliability.

Validation

The process of assessing whether the model accurately represents the real-world system it is intended to simulate or analyze.

Did I build the right model?

Activities

- Comparing the model's predictions or outputs with empirical data, experimental results, or observations from the real system.
- Conducting sensitivity analyses to evaluate the model's response to changes in input parameters or assumptions.

Verification and Validation of Model (3)

- Performing uncertainty analyses to quantify and understand the uncertainties and limitations of the model predictions.
- Validating the model against known benchmarks or established standards in the field.

Purpose

Validation aims to assess the model's credibility and reliability by confirming that it produces accurate and meaningful results consistent with real-world observations. It provides confidence in the model's ability to support decision-making and its suitability for its intended application.

Verification and validation are iterative processes that are often conducted in parallel throughout the model development lifecycle.