Simulation and Modeling

System Simulation

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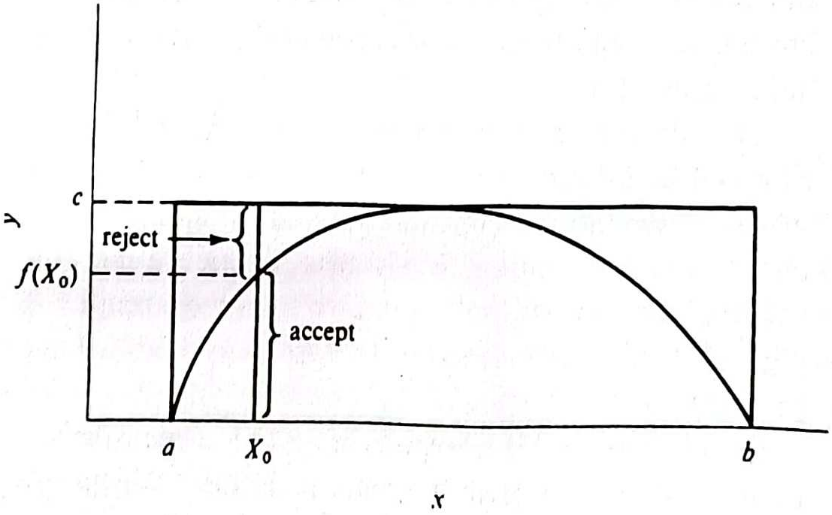
The Techniques of Simulation-Monte Carlo Method (1)

The Monte Carlo Method is a computational technique that uses randomness to solve problems which might have deterministic solutions. In this context, Monte Carlo integration estimates the value of definite integrals using random sampling. We use this method to estimate the area under a curve (the integral) over a given range.

Suppose we have a function $f(x)$ that is positive and bounded within the interval $[a, b]$, and the function is bounded above by some value c. The graph of the function lies within a rectangle of width $(b - a)$ and height c. We can estimate the integral of $f(x)$ by randomly sampling points within this rectangle.

The Techniques of Simulation-Monte Carlo Method (2)

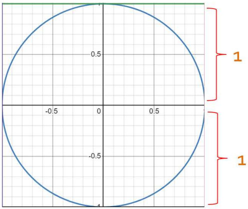


Problems Depicting Monte Carlo Method (1)

Calculate the value of π using Monte Carlo Method. Use any suitable integral.

- We use the following Integral:

$$\int_{-1}^{1} \sqrt{1 - x^2}\, dx$$



Problems Depicting Monte Carlo Method (2)

Sample No.	x_i	y_i	$x_i^2 + y_i^2$	Inside Circle? $x^2 + y^2 \leq 1$
1	0.12	0.98	0.96	Yes
2	-0.45	0.53	0.50	Yes
3	0.77	-0.22	0.63	Yes
4	-0.30	-0.40	0.25	Yes
5	0.68	0.99	1.40	No
6	-0.98	-0.14	0.97	Yes
7	0.44	0.58	0.50	Yes
8	-0.22	-0.45	0.23	Yes
9	0.15	-0.75	0.58	Yes
10	-0.10	0.89	0.80	Yes

$$A_{\text{rect}} = c(b - a) = 2 \times 2 = 4$$

Problems Depicting Monte Carlo Method (3)

$$\text{Area Under the curve} = \int_{-1}^{1} \sqrt{1 - x^2}\, dx = \frac{n}{N} * A_{\text{rect}} = \frac{9}{10} * 4 = 3.6$$

$$\text{Area Under the curve} = \text{Area of Circle} = \pi \times r^2 = 3.6$$

$$\text{Area Under the curve} = \text{Area of Circle} = \pi \times 1^2 = 3.6$$

Problems Depicting Monte Carlo Method (4)

Calculate the value of integral using Monte Carlo method of Integration.

$$I = \int_{-2}^{3} x^3 \, dx$$

Given:

- The function is $f(x) = x^3$,
- The bounds are $a = -2$ and $b = 3$,
- The maximum value of $f(x)$ in the interval $[-2, 3]$ is $c = 27$, since $f(3) = 3^3 = 27$.

Problems Depicting Monte Carlo Method (5)

Step 1: Area of the Rectangle

The area of the rectangle formed by the bounds $a = -2$, $b = 3$, and the maximum value $c = 27$ is given by:

$$A_{\text{rectangle}} = c \times (b - a) = 27 \times (3 - (-2)) = 27 \times 5 = 135$$

Step 2: Monte Carlo Integration

The Monte Carlo method estimates the value of the integral by randomly sampling points within the rectangle and checking how many fall below the curve $f(x)$. The accuracy of the estimate improves as the number of sampled points increases.

Comparison of Simulation and Analytical Methods (1)

Drawback of Simulation

- Specific Solutions: Simulation gives specific solutions rather than general solutions. For example, in the study of an automobile wheel, an analytical solution provides all the conditions that could cause oscillation. However, each execution of a simulation only tells whether a particular set of conditions did or did not cause oscillation. To identify all such conditions, the simulation must be repeated under many different conditions.
- Computational Complexity: The step-by-step nature of the simulation technique means that the amount of computation increases very rapidly as the level of detail increases. Coupled with the need to run the simulation many times, the model results in an extensive amount of computing power and time.

Comparison of Simulation and Analytical Methods (2)

- Local vs Global Maximum: Many simulation runs may be required to find a maximum, but this may leave the question undecided as to whether it is a local or global maximum. The simulation may not guarantee that the optimal solution is found unless sufficiently many runs are conducted across the possible space.

Drawback of Analytical Technique

- Required Specific Format: Analytical techniques require that the model must be expressed in a particular format, such as linear algebraic equations or continuous linear differential equations. This makes it difficult to handle complex models or systems that do not fit neatly into these formats.

Comparison of Simulation and Analytical Methods (3)

- Limitations of Solvability: There are simple limitations on a system, such as physical constraints, finite time delays, or non-linear forces, which may make a mathematically soluble model unsolvable. In contrast, simulation removes these limitations, allowing more flexibility in modeling complex systems.
- Inability to Handle Uncertainty: The range of problems that can be solved mathematically is limited. Analytical methods may struggle to account for uncertainty and variability in real-world data or parameters. Uncertainty can arise from various sources, such as measurement errors, variability in inputs, or stochastic processes, and incorporating this uncertainty into analytical models is challenging.

Comparison of Simulation and Analytical Methods (4)

- Complexity of Real-World Problems: Analytical techniques are most effective for relatively simple systems with well-defined relationships between variables. However, many real-world problems are highly complex, involving numerous interrelated factors that are difficult to capture analytically. For such complex systems, simulation is often a better alternative.

Simulation vs Analytical Methods in Practice

The simulation technique does not make a specific attempt to isolate the relationships between any particular variables. Instead, it observes the way in which all variables of the model change with time. Relationships between the variables must be derived from these observations.

Comparison of Simulation and Analytical Methods (5)

For instance, a simulation study of the automobile wheel suspension system would proceed by following the motion of the wheel under different conditions. The relationship between damping (D), stiffness (K), and mass (M) to prevent oscillation, which was previously discovered analytically, would have to be discovered by observing the values that result in the motion being non-oscillatory.

Thus, simulation is essentially an experimental problem-solving technique. Many simulation runs are required to understand the relationships involved in the system. Therefore, the use of simulation in a study must be planned as a series of experiments, where each run provides new insights or refines the understanding of the system's behavior.

Experimental Nature of Simulation (1)

The experimental nature of simulation refers to the practice of using computer-generated models and simulations to replicate real-world systems or phenomena for the purpose of exploration, understanding, and testing hypotheses. This approach allows researchers, engineers, and scientists to experiment with complex systems in ways that would be difficult, costly, or unethical to perform in real life. In essence, simulations serve as virtual laboratories where various scenarios and conditions can be tested in a controlled and repeatable environment.

1. Hypothesis Testing

Just like in traditional experimental methods, simulations are often used to test hypotheses about the behavior of a system under specific conditions. In a real-world experiment, researchers might observe how a system behaves by physically manipulating its parameters or testing its response under controlled conditions. In a simulation, similar tests can be conducted

Experimental Nature of Simulation (2)

by adjusting the model's input variables, such as initial conditions, environmental factors, or system configurations. These controlled experiments allow researchers to test hypotheses without the constraints of real-world limitations, and the results can be used to validate theoretical models or predict outcomes under new, untested conditions.

2. Scenario Exploration

One of the primary strengths of simulation is the ability to explore a wide range of scenarios that might be too costly, time-consuming, dangerous, or even impossible to replicate in reality. For example, simulating the impact of a natural disaster on an urban area, testing the performance of a new product prototype in a variety of conditions, or exploring the dynamics of biological systems under different interventions can be accomplished with simulations. By modifying input parameters—such as environmental conditions, system design, or user behavior—researchers can observe how these

Experimental Nature of Simulation (3)

changes affect the system's behavior and outcomes. This ability to experiment with different "what-if" scenarios is invaluable for decision-making, risk assessment, and optimization in fields ranging from engineering and economics to healthcare and climate science.

3. Iterative Process

Simulation is often an iterative process, where the model is refined and adjusted based on the results of previous runs. In this process, initial simulations might provide insights into potential behaviors, but the findings often prompt further questions or lead to modifications in the model. For example, if a simulation reveals unexpected behavior, researchers might adjust the model's parameters, incorporate additional variables, or even rethink the system's underlying assumptions. This continuous feedback loop allows researchers to improve the model's accuracy and increase its predictive power. As a result, simulations help in developing more robust models that

Experimental Nature of Simulation (4)

can account for real-world complexity, leading to more accurate and reliable predictions.

4. Risk-Free Testing

Unlike real-world experiments that may involve inherent risks or costs, simulations provide a risk-free environment where new ideas, designs, or policies can be tested without the danger of failure, injury, or financial loss. This makes simulations particularly useful in industries such as aviation, automotive design, medicine, and military operations, where the consequences of real-world testing could be catastrophic. For instance, in aerospace engineering, simulations of flight conditions can be used to test aircraft performance before physical prototypes are built. This significantly reduces the costs associated with trial-and-error approaches and accelerates the development process.

Experimental Nature of Simulation (5)

5. Complex Systems Analysis

Many real-world systems are highly complex, involving numerous interacting components, non-linear relationships, and unpredictable behaviors. Examples include climate systems, financial markets, and neural networks. Simulation provides a means to break down these complex systems into manageable models that can be studied and analyzed. Through the simulation of these systems, researchers can examine how changes in one component might affect the entire system and identify emergent behaviors that might not be apparent from a purely theoretical standpoint.

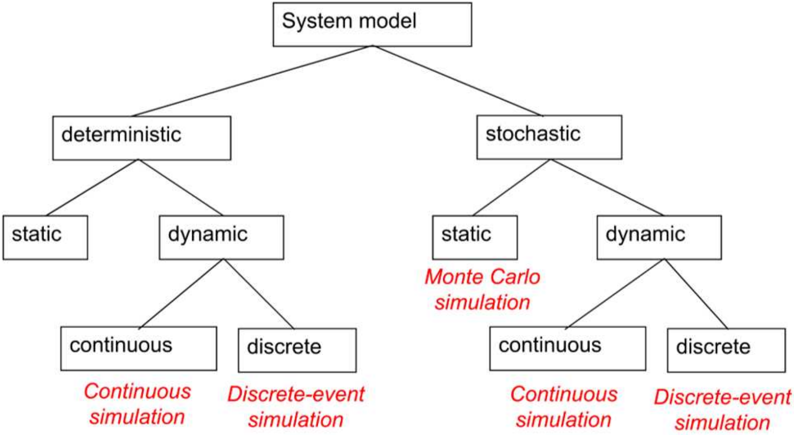
6. Optimization and Efficiency

The experimental nature of simulation allows for the optimization of systems and processes by testing different configurations to identify the most efficient and effective solutions. For example, in manufacturing, simulations can be used to optimize production lines, supply chain logistics, and inventory

Experimental Nature of Simulation (6)

management, helping companies save time and resources. In transportation, simulations can help optimize traffic flow, route planning, and scheduling to improve efficiency and reduce congestion. By iterating through different scenarios, simulations provide insights into how to improve the performance of systems and processes in ways that would be difficult to achieve through real-world experimentation alone.

Types of System Simulation (1)



Distributed Lag Models (1)

- A Distributed Lag Model (DLM) is a type of econometric model that explains the relationship between a dependent variable and one or more independent variables, where the effect of the independent variables is spread (or "lagged") over time.
- Essentially, a distributed lag model captures the delayed effect of a change in the independent variable(s) on the dependent variable.
- In simple terms, the impact of a change in an explanatory variable is not instantaneous but occurs over several periods (lags), and the model accounts for these delayed effects.
- In a DLM, past values of the independent variables (and sometimes past values of the dependent variable) are included in the model to explain the current value of the dependent variable.
- The lags refer to the number of periods (usually time periods) between the cause and the effect.

Distributed Lag Models (2)

- The lag structure refers to how far back in time the model includes past values of the independent variable.
- For example, in a model with "k" lags, the current value of the dependent variable is explained by the independent variable values from $t-1, t-2, \ldots, t-k$ (where t is the current period).
- A distributed lag refers to the fact that the effects of the independent variable on the dependent variable are spread out over multiple time periods, rather than occurring all at once.

Distributed Lag Models (3)

As an example, consider the following simple dynamic mathematical model of the national economy. Let, C be consumption, I be investment, T be taxes, G be government expenditure and Y be national income. Then,

$$
\begin{aligned}
C &= 20 + 0.7(Y - T) \\
I &= 2 + 0.1Y \\
T &= 0.2Y \\
Y &= C + I + G
\end{aligned}
\tag{1}
$$

This is a static model, but it can be made dynamic by picking a fixed time interval, say one year, and expressing the current values of the variables in terms of values of the previous year. Any variable that appears in the form of its current value and one or more previous year's values is called lagged variables. Value of the previous year is denoted by the suffix with-1.

Distributed Lag Models (4)

The static model can be made dynamic by lagging all the variables, as follows

$$
\begin{aligned}
I &= 2 + 0.1Y_{-1} \\
T &= 0.2Y_{-1} \\
Y &= C_{-1} + I_{-1} + G_{-1} \\
C &= 20 + 0.7(Y_{-1} - T_{-1})
\end{aligned} \tag{2}
$$

In these equations if values for the previous year (with –1 subscript) is known then values for the current event can be computed. Taking these values as the input, values for the next year can also be computed. In equation (2) we have four equations in five unknown variables.

Distributed Lag Models (5)

It is however not necessary to lag all the variable like it is done in equation (2). Only one of the variable can be lagged and others can be expressed in terms of this variable. We solve equation for Y in equation (1) as,

$$Y = 45.45 + 2.27(I + G)$$

The set of equations can be then written in the form:

$$\begin{aligned} I &= 20 + 0.1Y_{-1} \\ Y &= 45.45 + 2.27(I + G) \\ T &= 0.2Y \\ C &= 20 + 0.7(Y - T) \end{aligned} \tag{3}$$

Thus we have, In equations (3) only lagged parameter is Y. Assuming that government expenditure is known for the current year, we first compute I. Knowing I and G, Y and T for the current year is known, and thus C is computed from the last equation.

In this problem, lagged model is quite simple and can be computed with hand calculator. But national economic models are generally not that simple and require long computations with number of parameters.

Find the growth in national consumption for five years using the model given in Sec. 3-8. Assume the initial income Y_{-1} is 80 and take the government expenditure in the 5 years to be as follows:

Year	G
1	20
2	25
3	30
4	35
5	40

Distributed Lag Models (7)

If demand and supply of a product obey following equations. $D = a - bP$, $S = c + dP$ and $D = S$. Here a, b, c, and d are given numbers, convert this model to distributed lagged model.

Cobweb Models (1)

A simple static model of marketing a product had been discussed. In that model two linear equations for demand D and supply S were considered. Aim was to compute the probable price and demand of a product in the market subject to a condition that supply and demand should be equal.

$$Q_d = a - bP$$

where Q_d is the quantity demanded and P is the price.

$$Q_s = c + dP$$

where Q_s is the quantity supplied.
At equilibrium, $Q_d = Q_s$, leading to:

$$P = \frac{a - c}{b + d}$$

Cobweb Models (2)

But supply of the product in the market depends on the previous year price, and that can be taken as lagged parameter. Thus above equations become

$$
\begin{aligned}
Q_d &= a - bP_{-1} \\
Q_s &= c + dP_{-1} \\
Q_d &= Q_s
\end{aligned}
\tag{4}
$$

In the above equations, values of parameters a, b, c, and d can be obtained by fitting a linear curve to the data from the past history of the product. We assign some initial value to the product, say P_0, and find S from the second equation of (2).

Thus, S and D are known, and the first equation of (2) gives us a new value of P. Using this value of P as the initial value, we repeat the calculations and again compute P for the next period. If the price converges, we say

Cobweb Models (3)

that model (2) is stable. Let us take two examples and test whether these models converge or not.

Model 1	**Model 2**
P_0 = 30	P_0 = 5
a = 12	a = 10.0
b = 30	b = 0.9
c = 1.0	c = –2.4
0.9	1.2

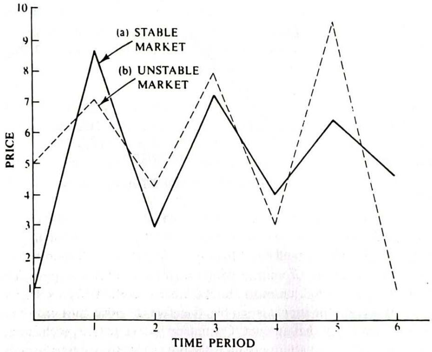
Table 1.1: Cobweb model for marketing a product

Model 1		**Model 2**	
i	P	i	P
0	–0.533333	0	7.11111
1	0.382667	1	4.2963
2	0.355187	2	8.04938
3	0.356011	3	3.04527
4	0.355986	4	9.71742
5	0.355987	5	0.821216
6	0.355987	6	12.6828
7	0.355987	7	–3.13265
8	0.355987	8	17.9546
9	0.355987	9	–10.1618
10	0.355987	10	27.3268

We can see from the table that results in the case of first model converge even in five steps where as in second model they do not converge at all.

Cobweb Models (4)

Data a, b, c, and d for model 2 is such that it does not converge. Thus data of second model is not realistic. These parameters can be calculated from the past history of the product by regression method. This model is called cobweb as it can be graphically expressed as shown in Figure.



Cobweb Models (5)

In Fig , we have first drawn supply and demand curves. A line parallel to quantity axis shows that for price equal to one unit, supply is 2 units. If we draw a line parallel to price axis so that it meets demand curve at point marked 1. Thus for the same quantity of supply and demand, price immediately shoots up to more than eight units, due to short supply of product. With this high price, supply shoots up to nine units. Again vertical line equating supply with demand reduces the price to three. We repeat the process and ultimately find that curve converges to optimum value.

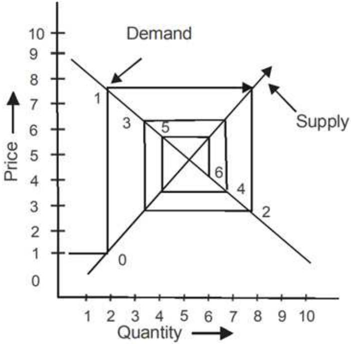


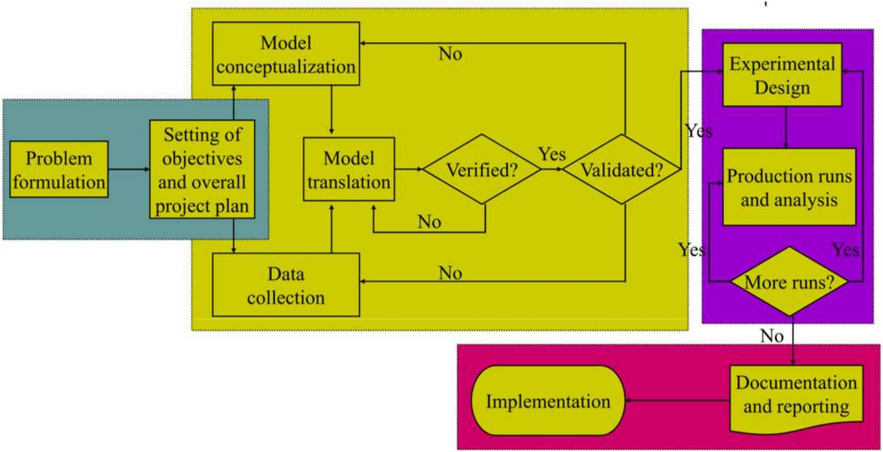
Fig. 1.7: *Cobweb model.*

Cobweb Models (6)

Draw the Cobweb Models for the following market:

$$
\begin{aligned}
Q_d &= 12.4 - 1.2P \\
Q_s &= 8 - 0.6P_{-1} \\
P_0 &= 1.0
\end{aligned}
\tag{5}
$$

Steps of Simulation Study (1)



Steps of Simulation Study (2)

1. Problem Formulation: Every simulation study begins with a clear statement of the problem. If the problem is presented by policymakers or stakeholders, the analyst must ensure that the issue is accurately understood. If the problem formulation is done by the analyst, it's critical that the policymakers understand and agree with the formulation. While not always explicitly shown, it is often necessary to revisit and reformulate the problem as the study progresses. Frequently, policymakers and analysts identify a problem long before its full nature is understood.

2. Setting Objectives and Overall Project Plan: The objectives specify the key questions that the simulation aims to answer. At this stage, the analyst must assess whether simulation is the appropriate methodology for addressing the problem and achieving the stated objectives. If simulation is chosen, the project plan should outline the alternative systems to be considered and how the effectiveness of these alternatives will be evaluated. The plan

Steps of Simulation Study (3)

should also provide details about the number of people involved, the study's budget, the expected timeline, and the desired outcomes at each stage of the work.

3. Model Conceptualization: Building a model for a system requires both art and science. While no set of instructions guarantees success in every case, general guidelines exist for creating effective models. Successful modeling involves abstracting the key features of the problem, selecting and adjusting assumptions, and gradually increasing complexity as needed. It is often best to start with a simple model and add complexity only if necessary to meet the study's goals. The model should focus on capturing the essential elements of the system, not a one-to-one replication, to avoid unnecessary complexity and additional costs.

Steps of Simulation Study (4)

4. Data Collection: Data collection is an iterative process that runs parallel to model development. As the model evolves, the data needed may change. Given that data collection often represents a significant portion of the study's time and cost, it should begin early, typically alongside the initial stages of model building. The type of data collected depends on the study's objectives. For instance, in a bank simulation study, data on inter-arrival times, teller service times, and queue lengths at different times of day may be required. Historical data is also vital for validating the simulation model.

5. Model Translation: Real-world systems often lead to models that require substantial computational resources and data storage. Therefore, the model must be converted into a format suitable for a computer. This translation may or may not require significant coding, depending on the approach used.

Steps of Simulation Study (5)

The modeler must decide whether to use a simulation language (which offers flexibility) or specialized simulation software (which may reduce development time). While simulation software can simplify model development, its flexibility varies depending on the software chosen.

6. Verification: Verification ensures that the computer program accurately represents the simulation model. This step involves checking that the program is performing correctly, i.e., that it computes as expected. Verification is essential, particularly for complex models, where translating the entire model to a computer may introduce errors. If the input parameters and logical structure of the model are correctly translated, the program is considered verified. Common sense, along with debugging techniques, plays a crucial role in this phase.

Steps of Simulation Study (6)

7. Validation: Validation involves ensuring that the simulation model accurately reflects real-world behavior. This is typically achieved through calibration, which is an iterative process of comparing the model's output with actual system behavior. Discrepancies are identified and used to improve the model, and this process is repeated until the model's accuracy is deemed acceptable. Validation is crucial for ensuring the credibility of the model's results and conclusions.

8. Experimental Design: Once the system alternatives to be simulated are identified, decisions must be made about key aspects of the simulation, such as the length of the initialization period, the duration of simulation runs, and the number of replications required for each run. These decisions will often evolve as analysis of the initial runs provides more insight into the system's behavior.

Steps of Simulation Study (7)

9. Production Runs and Analysis: Production runs involve executing the simulation and analyzing the results to estimate the system's performance. Following the initial analysis, the analyst determines whether more runs are necessary to refine the results. The design of any additional experiments should be informed by the insights gained from previous runs.

10. Documentation and Reporting: There are two types of documentation: program documentation and progress documentation. Program documentation is vital for understanding the model, especially if it will be reused or modified by other analysts. It builds confidence in the model, helping policymakers and other stakeholders make informed decisions. Additionally, it enables users to experiment with different input parameters to understand how they influence the model's output. Progress documentation tracks the project's timeline, decisions, and milestones, helping ensure the project stays

Steps of Simulation Study (8)

on course.

11. Implementation: The success of the implementation phase depends heavily on how well the previous steps have been executed. Successful implementation also relies on how effectively the analyst has involved the model's end users throughout the simulation process. If users understand the model and its assumptions, and have been actively engaged during the model-building process, implementation is more likely to be successful. Conversely, poor communication of the model and its assumptions can lead to implementation challenges, even if the model itself is valid.

Time Advancement Mechanism (1)

Queuing Models and its Characteristics (1)

Queuing Discipline (1)

Measures of Queues, Single Server Queuing System (1)