MMM — Assignment

Due date: 06 January 2021, 11:59 PM

The kinematic system K consists of the motions of the particles P_1 and P_2 in the plane O_1xy subject to these constraints: $|O_1P_1| = l_1$, $|O_2P_2| = l_2$, $|P_1P_2| = l_2$. The points O_1, O_2 , P_1, and P_2 have coordinates (0, 0), (l_1, 0), (x_1, y_1), and (x_2, y_2), respectively. (A four-bar linkage with fixed centers O_1 and O_2.)

Let $l_1 = 1$ and $l_2 = 2 + n/1000$, where n is given by the last three digits of your matricola.

1. Consider K as a system of motions of the point (x_1, y_1, x_2, y_2) in 4-dimensional Cartesian space, $Ox_1y_1x_2y_2$. Give a system of polynomial equations describing the configuration space, C, of the system.
2. Let C_{x1} be the image of C by the projection (x_1, y_1, x_2, y_2) ↦ (x_2, y_2, x_1). Is C_{x1} a bijective image of C? Prove your answer. Make a 3D plot of C_{x1} in $Ox_2y_2x_1$.
3. Let C_{y1} be the image of C by the projection (x_1, y_1, x_2, y_2) ↦ (x_2, y_2, y_1). Is C_{y1} a bijective image of C? Prove your answer.
4. Let C_{y2} be the image of C by the projection (x_1, y_1, x_2, y_2) ↦ (x_1, y_1, y_2). Is C_{y2} a bijective image of C? Prove your answer.
5. Let θ_i be the angle from the fixed O_1x axis to O_iP_i. Consider K as a system of motions of the point (θ_1, θ_2) on a torus, T^2. Derive the equation (in terms of θ_1 and θ_2) defining the configuration space C on T^2. Make a 2D plot of C on the periodic plane (θ_1, θ_2), as well as in 3D on a torus. In the 3D plot use l_1 and l_2 as the small and large radii of the torus.
6. Make a drawing of each feasible configuration which has either a θ_1 value in $\{0, \pi/2, \pi, 3\pi/2\}$ or a θ_2 value in $\{0, \pi/2, \pi, 3\pi/2\}$. Mark each configuration on each of your plots.
7. Assume that the input is θ_1 and output is θ_2. Find all singularities:
   (a) configuration-space singularities;
   (b) configurations allowing nonzero instantaneous motion for zero input velocity;
   (c) configurations allowing nonzero instantaneous motion for zero output velocity.
   Illustrate each singularity with a drawing and identify it on all plots.

   Remark: In all cases try to simplify the system of equations as much as possible without losing solutions.

Note.

1. Simulations must be performed in MATLAB *only*.
2. Submit both PDF file and MATLAB file for the plot. The PDF images must be clear, with proper aspect ratio. Upon running the MATLAB file, the plots should be generated

3. If the solution is written in LaTeX or Word file, place the plots (clear plots) inside the solution pdf in order of the question and the MATLAB files.
4. If the solution is hand-written, a PDF of the can of the solution, all the PDF files of images for the plots, and the MATLAB files for the simulation.