1 Vectors

Dot product

$$\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + ... + v_n w_n$$
$$\vec{v} \cdot \vec{w} = |v||u| \cos\theta$$

Dot Product Properties:

$$\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$$
$$\vec{u} \cdot (\vec{w} + \vec{v}) = \vec{u} \cdot \vec{w} + \vec{u} \cdot \vec{v}$$
$$\vec{v} \perp \vec{w}, \vec{v} \cdot \vec{w} = 0$$

Projection:

$$proj_{\vec{v}} \vec{u} = \frac{\vec{u}}{|\vec{v}|^2} \vec{v}$$

Cross product:

$$\vec{v} \times \vec{w} = \text{some garbage}$$
$$\vec{v} \times \vec{w} = |v||u| \sin\theta$$

Triple Scalar Product:

$$\vec{w} \cdot (\vec{v} \times \vec{u})$$

2 Lines and Surfaces

Lines in space:

$$\vec{r}(t) = r_0 + t\vec{v}$$

Planes:

$$\vec{r}(t, s) = r_0 + t\vec{v} + s\vec{w}$$
$$Ax + By + Cz = D$$

Normal to a Plane:

$$\vec{n} = \langle A, B, C \rangle$$

Level Sets:

$$\{(x_1, x_2, ...x_n) | f(x_1, x_2...x_n) = c\}$$

Circle:

$$x^2 + y^2 = r^2$$

Ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Parabola:

$$x^2 = 2py$$

Hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \pm 1$$

Ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Cone:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

Hyperboloid of 1 sheet:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Hyperboloid of 2 sheets:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

Saddle:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - z = 0$$

Elliptical Paraboloid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - z = 0$$

3 Vector Valued Functions

Parametric curves:

$$r(t) = \langle x(t), y(t), z(t) \rangle$$
$$r'(t) = \langle x'(t), y'(t), z'(t) \rangle$$
$$\int_a^b r(t) = \left\langle \int_a^b x(t), \int_a^b y(t), \int_a^b z(t) \right\rangle$$

Arc length:

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$
$$s = \int_a^b |v|\, dt$$

4 Partial Derivatives

Mixed derivative theorem:

$$f_{xy}(a,b) = f_{yx}(a,b)$$

Chain rule:

$$\frac{d}{dt}f(x,y,z) = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

Gradient:

$$\nabla f(x,y,z) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

Directional derivative:

$$D_u f = \left(\frac{df}{ds}\right)_{u,P_0} = \nabla f\Big|_{P_0} \cdot u$$

Path derivative:

$$\frac{d}{dt}f(r(t)) = \nabla f(r(t)) \cdot r'(t)$$

Tangent plane:

$$f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) + f_z(P_0)(z - z_0) = 0$$

Normal line:

$$\langle x_0 + f_x(P_0)t, y_0 + f_y(P_0)t, z_0 + f_z(P_0)t \rangle$$

Linearization:

$$L(x,y,z) = f(P_0) + f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) + f_z(P_0)(z - z_0)$$

5 Multiple Integrals

Fubini's Theorem:

$$\iint_R f(x,y)dA = \int_c^d \int_a^b f(x,y)\; dx\; dy = \int_a^b \int_c^d f(x,y)\; dy\; dx$$

Polar coordinates:

$$\iint_R r\; dr\; d\theta$$

Cylindral coordinates:

$$\iiint_U r\; dz\; dr\; d\theta$$

Spherical coordinates:

$$\iiint_U r^2 \sin\phi\; dr\; d\phi\; d\theta$$

Center of mass on the x direction:

$$\frac{\iiint_U x f(x,y,z)\; dx\; dy\; dz}{\iiint_U f(x,y,z)\; dx\; dy\; dz}$$

6 Integrals and Vector Fields

Line integrals of scalar functions:

$$\int_C f(x,y,z)\ ds = \int_a^b f(r(t))\ |r'(t)|\ dt$$

Line integrals along vector fields:

$$\int_C F \cdot dr = \int_a^b F(r(t)) \cdot \frac{dr}{dt}\ dt$$

Surface Integral:

$$\iint_S f(x,y) dS = \iint_{D_{uv}} f(u,v)\ |r_u \times r_v|\ du\ dv$$

Flux Integral:

$$\iint_F F \cdot dS = \iint_D F \cdot \hat{n} = \iint_D F(r(u,v)) \cdot (r_u \times r_v)\ du\ dv$$

Conditions for conversative vector fields:

$$\int_{C_1} F \cdot dr = \int_{C_2} F \cdot dr \iff F = \nabla f \iff \oint_C F \cdot dr = 0 \iff Curl = 0 \wedge F \text{ is simply connected}$$

2D Curl:

$$Q_x - P_y$$

3D Curl:

$$\nabla \times F = \langle R_y - P_z, P_z - R_x, Q_x - P_y \rangle$$

Divergence:

$$Div\ F = \nabla \cdot F$$

Green's Theorem:

$$\oint_C F \cdot dr = \iint_D (Q_x - P_y)\ dA$$

Stokes' Theorem:

$$\oint_C F \cdot dr = \iint_S \nabla \times F \cdot \hat{n}\ d\sigma$$

Gauss' Theorem:

$$\iint_S F \cdot dS = \iiint_E Div\ F\ dV$$

Unified Theorem:

$$\int_G dF = \int_{\partial G} F$$

Second derivative test:

$$D(x,y) + f_{xx} f_{yy} - (f_{xy})^2$$
$$D(x,y) > 0 \quad \wedge \quad f_{xx} > 0 \implies \text{Local min}$$
$$D(x,y) > 0 \quad \wedge \quad f_{xx} < 0 \implies \text{Local max}$$
$$D(x,y) < 0 \implies \text{Saddle point}$$
$$D(x,y) = 0 \implies \text{Inconclusive}$$

Method of Lagrange Multipliers:

$$\nabla f = \lambda \nabla g$$