CSC 225 FALL 2014
ALGORITHMS AND DATA STRUCTURES I
ASSIGNMENT 1
UNIVERSITY OF VICTORIA

1. Order the following functions by order of growth starting with the slowest.
   $n^{0.4}, 2^{2^n}, 4n, (\log n)^4, n^4, 4, 4^n, n!, 4^{\log n}, 4n \log\log n$.

2. Consider the following sum: $S(n) = \sum_{i=1}^{n} \log i$. Give a simple function $f(n)$ so that the sum $S(n)$ is $O(f(n))$. Explain why.

3. (a) Show that $f(n) = 4\log n + \log\log n$ is $\Theta(log\ n)$ by finding a real numbers $c_1, c_2 > 0$ and an integer $n_0 > 0$, such that
   $$c_1 \log n \leq 4\log n + \log\log n \leq c_2 \log n,$$
   for all $n \geq n_0$.

   (b) Show that $f(n) = 4\log n + \log\log n$ is $\Theta(log\ n)$ by showing that there exists a real number $c > 0$, such that
   $$\lim_{n\to\infty} \frac{f(n)}{\log n} = c$$

4. Consider the recurrence equation,
   $$T(n) = \begin{cases} 1 & \text{if } n = 0 \\ T(n-1) + 2^n & \text{otherwise.} \end{cases}$$
   Show, by induction, that $T(n) = 2^{n+1} - 1$.

5. Consider the Algorithm `arrayFind`, given below, which searches an array A for an element x.

   Algorithm `arrayFind`(x, A):
   Input: An element x and an n-element array, A.
   Output: The index i such that $x = A[i]$ or -1 if no element of A is equal to x.

   $i \leftarrow 0$
   while $i < n$ **do**
   if $x = A[i]$ **then**
   return i
   else
   $i \leftarrow i + 1$
   return -1

   (a) Counting assignments, comparisons, and returns only, calculate the worst-case, $T(n)$, and best-case, $T_b(n)$, running times of `arrayFind`.

   (b) Prove by induction (loop invariants) that `arrayFind` is correct.