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$$\begin{aligned}
G_{t \ t+n} &= R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n Q_{t+n-1}(S_{t+n}, A_{t+n}) \\
&= R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - \cancel{\gamma Q_t(S_{t+1}, A_{t+1})} - Q_{t-1}(S_t, A_t) + Q_{t-1}(S_t, A_t) \\
&\quad + \gamma R_{t+2} + \gamma^2 Q_{t+1}(S_{t+2}, A_{t+2}) - \cancel{\gamma^2 Q_{t+1}(S_{t+2}, A_{t+2})} - \gamma Q_t(S_{t+1}, A_{t+1}) + \cancel{\gamma Q_t(S_{t+1}, A_{t+1})} \\
&\quad + \dots \\
&\quad + \gamma^{n-1} R_{t+n} + \gamma^n Q_{t+n-1}(S_{t+n}, A_{t+n}) - \cancel{\gamma^n Q_{t+n-1}(S_{t+n}, A_{t+n})} \\
&\quad - \gamma^{n-1} Q_{t+n-2}(S_{t+n-1}, A_{t+n-1}) + \cancel{\gamma^{n-1} Q_{t+n-2}(S_{t+n-1}, A_{t+n-1})} \\
&\quad + \cancel{\gamma^n Q_{t+n-1}(S_{t+n}, A_{t+n})} \\
&= Q_{t-1}(S_t, A_t) + \sum_{k=t}^{\min(t+n, T)-1} \gamma^{k-t} [R_{k+1} + \gamma Q_k(S_{k+1}, A_{k+1}) - Q_{k-1}(S_k, A_k)]
\end{aligned}$$