Optimizing Quantum Oracle for Standard Hash Algorithms

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Table of Contents

Introduction

Previous Work

- Studied the basics of quantum computing and Grover's Algorithm
- Analysis of reversible quantum S-DES oracle and its construction
- Implementation of quantum S-DES oracle with Grover's Algorithm in Microsoft Q#

Motivation

- Targeting SHA-2/3
  - Current de facto standard hash algorithm
- Goal is to perform a preimage attack.
  - The most straightforward to apply Grover's Algorithm.
  - It will be assumed that Grover's Algorithm(or some variation of it) will be applied
  - Therefore target is to optimize the quantum oracle for SHA-2/3.

Recap: Grover's Algorithm

- Searching in an unordered database among 2^n elements takes $\tilde{\mathcal{O}}(2^{n/2})$ time complexity and $\tilde{\mathcal{O}}(n)$ quantum space complexity
- Proved to be optimal in general
- Specifically, improved Grover's Algorithm for collision search yields $\tilde{\mathcal{O}}(2^{n/3})$ time complexity and (quantum) space complexity [BHT98]
  - However, quantum queries are costly in this algorithm

Chailloux's Algorithm [CNPS17]

- Use poly-time qubits and reduce time complexity to $\tilde{\mathcal{O}}(2^{2n/5})$ for collision search and $\tilde{\mathcal{O}}(2^{3n/7})$ for multi-target preimage attacks with additional classic memory

SHA-2/3 Pre-image Attack Cost Estimation [ADMG+16]

- Suggest cost metric for quantum computation based on surface code
- Theoretically implement reversible SHA-2/3 quantum circuits
- Estimate required physical resources and its scale

Expectations

- Focus on micro-scale improvement(regarding quantum oracle)
- May be able to reduce the cost in at least one of the following metrics:
  - The cost metric from before
  - The raw number of gates
  - The number of levels after achieving parallelization
- Main target will be the cost metric, not the gates or the levels

In-depth Topics

- Surface code(Toric code) : topological quantum error correcting code
- Cost metric based on surface code
- T-par [AMM14] : an quantum circuit optimization tool
- Advanced quantum circuit (in-place adder, *etc.*)

Theoretical Background

Why Clifford + T?

- By Gottesman-Knill Theorem, Clifford gates can be simulated efficiently on a classical computer.
- T-gate is eqal to $R_{\pi/4} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{i} \end{pmatrix}$, a $\frac{\pi}{4}$ rotation.
- Clifford+T gate utilizes only one inefficient gate, while is universal.
- Since Clifford gates can be efficiently simulated, we need to minimize the number(or the depth once parallized) of the T-gates.

Ripple-Carry Adder

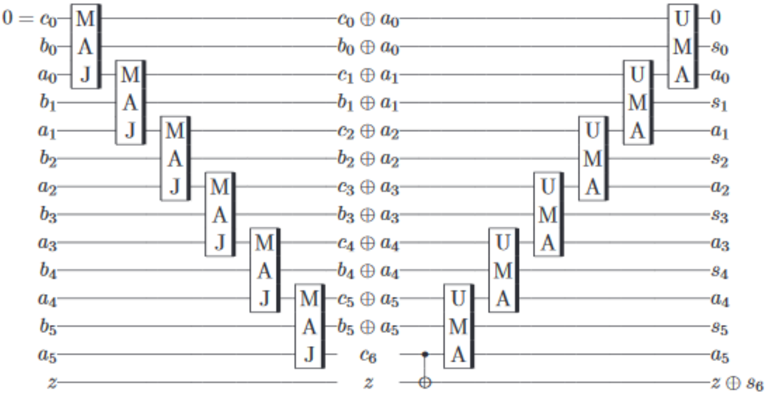


Figure 4: A simple ripple-carry adder for $n = 6$.

SHA-2 Quantum Circuit Experiment

SHA-256 Algorithm

Algorithm 1 SHA-256. All variables are 32-bit words.

for i=0 **to** 63 **do**

  $\Sigma_1 \leftarrow (\mathbf{E} \ggg 6) \oplus (\mathbf{E} \ggg 11) \oplus (\mathbf{E} \ggg 25)$

  $\mathbf{Ch} \leftarrow (\mathbf{E} \wedge \mathbf{F}) \oplus (\neg\mathbf{E} \wedge \mathbf{G})$

  $\mathrm{t}_1 \leftarrow \mathbf{H} + \Sigma_1 + \mathbf{Ch} + \mathbf{K}[i] + \mathbf{W}[i]$

  $\Sigma_0 \leftarrow (\mathbf{A} \ggg 2) \oplus (\mathbf{A} \ggg 13) \oplus (\mathbf{A} \ggg 22)$

  $\mathrm{Maj} \leftarrow (\mathbf{A} \wedge \mathbf{B}) \oplus (\mathbf{A} \wedge \mathbf{C}) \oplus (\mathbf{B} \wedge \mathbf{C})$

  $\mathrm{t}_2 \leftarrow \Sigma_0 + \mathrm{Maj}$

  $\mathbf{H} \leftarrow \mathbf{G}$

  $\mathbf{G} \leftarrow \mathbf{F}$

  $\mathbf{F} \leftarrow \mathbf{E}$

  $\mathbf{E} \leftarrow \mathbf{D} + \mathrm{t}_1$

  $\mathbf{D} \leftarrow \mathbf{C}$

  $\mathbf{C} \leftarrow \mathbf{B}$

  $\mathbf{B} \leftarrow \mathbf{A}$

  $\mathbf{A} \leftarrow \mathrm{t}_1 + \mathrm{t}_2$

end for

Embedded images and charts in the section are from [ADMG+16].

SHA-256 Stretching

Algorithm 2 SHA-256 Stretch. All variables are 32-bit words.

for $i = 16$ **to** 63 **do**

$\quad \sigma_0 \leftarrow (\mathbf{W}_{i-15} \ggg 7) \oplus (\mathbf{W}_{i-15} \ggg 18) \oplus (\mathbf{W}_{i-15} \gg 3)$

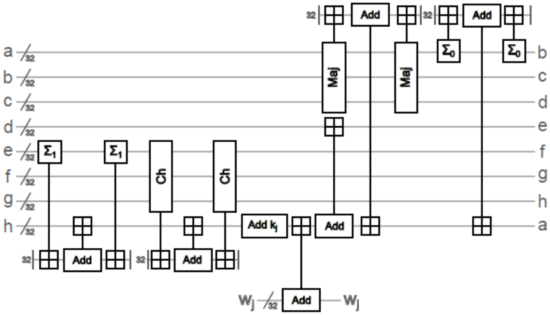
$\quad \sigma_1 \leftarrow (\mathbf{W}_{i-2} \ggg 17) \oplus (\mathbf{W}_{i-2} \ggg 19) \oplus (\mathbf{W}_{i-2} \gg 10)$

$\quad w[i] \leftarrow \mathbf{W}_{i-16} + \sigma_0 + \mathbf{W}_{i-7} + \sigma_1$

end for

- Both requires bitwise operations, bit shifting and addition
- Addition can be done by Ripple-Carry Adder

SHA-256 Quantum Circuit



- Ignore padding and precalculations
- Thesis used 2402 qubits
- But with our calculation, 801 explicit qubits were sufficient
  - 256 for hash, 32×16 for $w[i]$, $32+1$ for ancilla

SHA-256 Quantum Circuit Analysis

	$T/T^\dagger$	$P/P^\dagger$	Z	H	CNOT	T-Depth	Depth
Round	5278	0	0	1508	6800	2262	8262
Round (Opt.)	3020	931	96	1192	63501	1100	12980
Stretch	1329	0	0	372	2064	558	2331
Stretch (Opt.)	744	279	0	372	3021	372	2907
SHA-256	401584	0	0	114368	534272	171552	528768
SHA-256 (Opt.)	228992	72976	6144	94144	4209072	70400	830720

The literature has calculated the number of required gates.

- How to calculate these?
  - Implement digital SHA-256 quantum circuit in OpenQASM[CBSG17]
- How to optimize these?
  - T-par algorithm [AMM14]
  - Implemented in Feynman toolkit[1]

[1] https://github.com/meamy/feynman

Implementing SHA-256 in OpenQASM 2.0

OpenQASM : *an imperative programming language for describing quantum circuits*

- Latest version : 3.0(Pre-release), 2.0
- Feynman accetps 2.0 format
- Problem: 2.0 lacks basic operations
    - Only qubits/bits available, no loops, no array slicing

Implementing SHA-256 in OpenQASM 2.0

```python

ind = lambda i, r : (i + 8 - (r & 7)) & 7
wind = lambda x : '{:02d}'.format(x & 15) # x >= 0
prepare()
for round_num in range(64):
    print(f'// Round {round_num+1}')
    print('// [+] Calculating s1')
    for rsh in [6, 11, 25]:
        q_xor(qregs[ind(4, round_num)], qregs[-1], rsh)
    q_add(qregs[-1], qregs[ind(7, round_num)])
    for rsh in [6, 11, 25]:
        q_xor(qregs[ind(4, round_num)], qregs[-1], rsh)

    print('// [+] Calculating Ch')
    q_ch(qregs[ind(4, round_num)], qregs[ind(5, round_num)],
            qregs[ind(6, round_num)], qregs[-1])

    q_add(qregs[-1], qregs[ind(7, round_num)])

    q_ch_a(qregs[ind(4, round_num)], qregs[ind(5, round_num)],
            qregs[ind(6, round_num)], qregs[-1])
```

We made a code to generate SHA-256 OpenQASM code in Python 3.6.

- Excluding preambles like hardcoded gate definitions, 100 lines were sufficient.

Optimizing SHA-256

Generated OpenQASM code is quite long (36k lines)

- Gate calculation was successful
- However, optimizing with T-par as a whole failed due to memory limit
- Workaround: Split code into 32-64 parts

Our experiment has two criteria: optimization algorithm and round per code.

- Algorithms: `tpar`[AMM14] and `cnotmin`[AAM18]
- Code size : 1/2 round per code (64/32 parts)

Results

SHA-256 Type	T	H	X	$CNOT$	Misc.
Literature, naïve	401584	114368	0	534272	
Ours, naïve	351232	100352	1986	465088	
Ours, T-Par, 1r	343040	100352	10730	661376	8192 $SWAP$
Ours, CNOTmin, 1r	343040	100352	10730	381824	
Ours, T-Par, 2r	343040	100352	10730	664832	8192 $SWAP$
Ours, CNOTmin, 2r	343040	100352	10730	385280	
Literature, T-Par	228992	94144	0	4209072	72976 P, 6144 Z

'1r' and '2r' denotes 1 round per code and 2 rounds per code, respectively.

Analysis: Incomplete Business

Three strange discrepancies...

- Gate differences of naïve ones
- Smaller decrement of T-gate
- Increased $CNOT$ gate when applied 2 rounds per code compared to 1 rounds of code

Analysis: Naïve Gate Difference

Why did the Naïve gate differ?

- Different way of implementation
- Specifically, we utilized X-gates; the literature did not.
  - The reason for non-utilization is unknown
- The overall gate would have decreased due to the usage of X-gates, but its difference is too large.

Analysis: *T*-gate and *CNOT*-gate Discrepancies

Why did *T*-gate decrease less, even after considering the reduced gate count?
In the 2r case, the number of *CNOT* gates decreased less than 1r case. Why?

- Cannot be sure
- Possibility 1: Our implementation of SHA is wrong

⇒ Possible, since we couldn't test it, but even after we searched for incorrect result multiple times in the source and its generator, we couldn't find the error.

- Possibility 2: The software we used is wrong or is only partially implemented

⇒ Possible, and most probable due to the *CNOT* gate discrepancy.

References

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