1. $\mathcal{C}$ is usual abelian category;
2. $\mathcal{C}$ is the derived category of an abelian category;
3. $\mathcal{C}$ is (pre)triangulated
4. $\mathcal{C}$ is a Grothendieck topos,
5. $\mathcal{C}$ is complete/cocomplete;
6. $\mathcal{C}$ is the stable homotopy category,
7. $\mathcal{C}$ is a symmetric monoidal category, Tannakian;
8. $\mathcal{C}$ is a (stable) model category, combinatorial model category and so on.

We know that we should land with examples of the notions introduced by Joyal [5] and Lurie [6, 7].

Even if we don't consider the fibrant objects, but just objects $\mathcal{F} : s(\mathcal{F}) \longrightarrow \mathcal{U}(\mathcal{C})$ satisfying some conditions, we end up with concepts such as Lurie's ∞-operads.

The category of fibrant objects is the category of quasicategories that are the most close to 1-categories. For example if we have any functor of 1-category such as a localization functor going to the derived category:

$$L : \mathcal{A} \longrightarrow D(\mathcal{A}),$$

then a fibrant replacement of $D(\mathcal{A})$ is a trivial fibration $\mathcal{F} : s(\mathcal{F}) \longrightarrow \mathcal{U}(D(\mathcal{A}))$. And because $\mathcal{A}$ is cofibrant in the Joyal model structure, we can find a lift $\mathcal{A} \longrightarrow s(\mathcal{F})$. This means that $s(\mathcal{F})$ is the best quasicategory that approaches the derived category $D(\mathcal{A})$. In fact, homing a trivial fibration between quasicategories yields an equivalence of quasicategories.

2.4 General picture: Minimal Homotopy Enhancement

There are many categories such as **Set** or $\mathbf{Vect_k}$, that don't have an interesting homotopy theory. The general idea of Quillen-Segal objects is precisely to enhance these category by embedding them into a better category where there is a good homotopy theory: this is what we call *homotopy enhancement*.

Classically we would just embed $\mathcal{U} : \mathbf{Set} \hookrightarrow \mathbf{sSet}$ and similarly we would embed $\mathcal{U} : \mathbf{Vect_k} \hookrightarrow C(\mathbf{k})$ or to simplicial vector spaces. But these embedding are too big. Moreover, there are many reasons why we would like to embed them directly to quasicategories or to Kan complexes, rather than passing for example through Dold-Kan, etc.

With the Quillen-Segal formalism we get a *minimal embedding* each time. Here is how we would interpret these embeddings.

1. The embedding $\mathbf{Vect_k} \hookrightarrow (\mathbf{sSet}_{\mathrm{J}} \downarrow \mathcal{U})$ gives the quasicategory theory of vector spaces. Any fibrant object there is a quasicategory $s(\mathcal{F})$ that looks like a vector space !