- 2. We can alternatively embed  $\mathbf{Vect_k} \hookrightarrow (\mathbf{sSet_Q} \downarrow \mathcal{U})$ , and this should be the right direction toward type theory (Kan complexes).
- 3. We also have two possibilities  $\mathbf{Set} \hookrightarrow (\mathbf{sSet}_Q \downarrow \mathcal{U})$  or  $\mathbf{Set} \hookrightarrow (\mathbf{sSet}_Q \downarrow \mathcal{U})$ .
- 4. If  $\mathcal{M} = \mathbf{sSet}_J$ , and  $\mathfrak{A} = \mathbf{Set}$ , we get the quasicategory theory of sets. Here we regard the inclusion  $\mathbf{Set} \longrightarrow \mathbf{sSet}_J$  as a right adjoint, and indeed as geometric point of the higher topos  $\mathbf{sSet}_J$ . In particular it's a monoidal functor with the cartesian product.

We can put the relative pushout product on  $\mathcal{M}_{\mathfrak{U}}[\mathfrak{A}]$ . This way we get a monoidal category of quasicategory of sets. In particular we can determine the controlled quasicategories  $s(\mathcal{F}) \longrightarrow X$  that look like:

- The integers,
- The rationals.
- The real numbers;
- The complex numbers;
- a ring, field, groups, Hodge structure (...and maybe a prime number in a quasicategory)

It's reasonable to believe that this should join Lurie's program on *Higher Algebra* [6]. Morally we're enhancing the Grothendieck topos (**Set**,  $\times$ , 1) by a *minimal homotopical model*. And the advantage of this is that we keep track of everything since there is always an weak section (Voevodsky section)  $\pi: X \longrightarrow s(\mathfrak{F})$ .

- 5. Doing the same thing with the Quillen model structure  $\mathbf{sSet}_{\mathbf{Q}}$  should normally fits in type theory.
- 6. We can restrict to the right adjoint  $\mathbf{Vect_k} \longrightarrow \mathbf{Set} \longrightarrow \mathbf{sSet_J}$ , and do the relative lax pushout product. The category  $\mathscr{M}_{\mathfrak{U}}[\mathfrak{A}]$ , that is the quasicategory of vectors spaces will inherit a monoidal product where the unit is the map  $* \longrightarrow \mathbf{k}$  that selects the unit element of the field  $\mathbf{k}$ . We still have a functor that is monoidal  $\mathscr{M}_{\mathfrak{U}}[\mathfrak{A}] \longrightarrow \mathbf{Vect_k}$ .

Doing this is not only general nonsense. We can use for example the category  $\mathbf{sSet}_{J}[\mathbf{Vect}_{\mathbb{C}}]$  as coefficient category for general TQFT and so on.

- 7. The category ( $\mathbf{Set}, \times, 1$ ) is known as the Grothendieck topos of sheaves over the point. Given a general site  $\mathcal{C}$ , we can form a homotopical minimal enhancement of its category of sheaves by taking  $\mathcal{M}$  to be the model category of simplicial (pre)sheaves à la Jardine-Joyal. The category  $\mathcal{M}_{\mathcal{U}}[\mathfrak{A}]$  in this case is still a combinatorial and left proper and we can perform again Bousfield localization to converge to Morel-Voevodsky work [8].
- 8. Finally it's important to observe that if  $\mathcal{U}: \mathcal{A} \hookrightarrow Mod(\mathcal{A}^{op})$  is the Yoneda embedding of a dg-category, then a QS-object is exactly what Toën call quasi-representable (see [13]).

Keep enhancing!