It has been proved by Bergner [2], Joyal [5], Lurie [7], Rezk [9], that simplicial categories, quasicategories, complete Segal spaces and Segal categories, are all models for $(\infty, 1)$ -categories.

There is an ongoing program developed by Riehl and Verity (see [10, 11, 12]), that aims to understand quasicategories through simplicial categories. And it turns out that this program coincide with the study of U-QS-objects for the *coherent nerve* of Cordier and Porter [4]:

$$\mathcal{U}: \mathbf{sCat}_{\mathrm{B}} \longrightarrow \mathbf{sSet}_{\mathrm{J}}$$
.

Here $\mathbf{sCat}_{\mathrm{B}}$ means that we consider the Bergner model structure on simplicial categories (see [3]). Our discussion is based on the previous paper [1], wherein we expose the general idea of Quillen-Segal objects.

When we consider the comma category ($\mathbf{sSet}_J \downarrow \mathcal{U}$) we keep a control of what happens between simplicial categories and quasicategories. In other words we 'temper' quasicategories by linking them with rigid structures that are known to be equivalent. An object of this category is a morphism $\mathcal{F}: s(\mathcal{F}) \longrightarrow t(\mathcal{F})$, where $t(\mathcal{F}) = \mathcal{U}(\mathcal{C})$ is a simplicial category. A morphism $\sigma: \mathcal{F} \longrightarrow \mathcal{G}$ is given by a morphism of quasicategories $\sigma_0: s(\mathcal{F}) \longrightarrow s(\mathcal{G})$ and a morphism of simplicial categories $\sigma_1: \mathcal{C} \longrightarrow \mathcal{D}$ such that we have a commutative square:

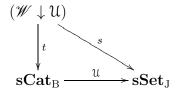
$$s(\mathcal{F}) \xrightarrow{\sigma_0} s(\mathcal{G})$$

$$\downarrow^{\mathcal{F}} \qquad \qquad \downarrow^{\mathcal{G}}$$

$$t(\mathcal{F}) \xrightarrow{\mathcal{U}(\sigma_1)} t(\mathcal{G})$$

We are not really interested in this big category, but rather the full subcategory $(W_J \downarrow U)$ of QS-objects $\mathcal{F}: s(\mathcal{F}) \xrightarrow{\sim} t(\mathcal{F})$. We can define the weak equivalences as the morphism $\sigma: \mathcal{F} \longrightarrow \mathcal{G}$ such that σ_0 is a weak equivalence in the Joyal model structure. The 3-for-2 property will force σ_1 to be also a weak equivalence.

Now since $\mathcal{U}: \mathbf{sCat}_B \longrightarrow \mathbf{sSet}_J$ is part of a Quillen equivalence it's not hard to show that the commutative triangle below descends to a triangle of equivalences between the homotopy categories. In fact we have a better statement as we shall see.



With this equivalence of homotopy categories, we can pretend that the theory of quasicategories and simplicial categories can complete each other in the following sense.

1. We can take advantage of the material developed by Joyal and Lurie for quasi-categories and transpose everything in term of simplicial categories, e.g ∞ -limit and ∞ -colimit, higher topos, etc.