It has been proved by Bergner [2], Joyal [5], Lurie [7], Rezk [9], that simplicial categories, quasicategories, complete Segal spaces and Segal categories, are all models for $(\infty, 1)$-categories.

There is an ongoing program developed by Riehl and Verity (see [10, 11, 12]), that aims to understand quasicategories through simplicial categories. And it turns out that this program coincide with the study of $\mathcal{U}$-QS-objects for the *coherent nerve* of Cordier and Porter [4]:

$$\mathcal{U} : \mathbf{sCat}_{\mathrm{B}} \longrightarrow \mathbf{sSet}_{\mathrm{J}}.$$

Here $\mathbf{sCat}_{\mathrm{B}}$ means that we consider the Bergner model structure on simplicial categories (see [3]). Our discussion is based on the previous paper [1], wherein we expose the general idea of Quillen-Segal objects.

When we consider the comma category $(\mathbf{sSet}_{\mathrm{J}} \downarrow \mathcal{U})$ we keep a control of what happens between simplicial categories and quasicategories. In other words we *'temper' quasicategories* by linking them with rigid structures that are known to be equivalent. An object of this category is a morphism $\mathcal{F} : s(\mathcal{F}) \longrightarrow t(\mathcal{F})$, where $t(\mathcal{F}) = \mathcal{U}(\mathcal{C})$ is a simplicial category. A morphism $\sigma : \mathcal{F} \longrightarrow \mathcal{G}$ is given by a morphism of quasicategories $\sigma_0 : s(\mathcal{F}) \longrightarrow s(\mathcal{G})$ and a morphism of simplicial categories $\sigma_1 : \mathcal{C} \longrightarrow \mathcal{D}$ such that we have a commutative square:

$$\begin{tikzcd}
s(\mathcal{F}) \arrow[r, "\sigma_0"] \arrow[d, "\mathcal{F}"] & s(\mathcal{G}) \arrow[d, "\mathcal{G}"] \\
t(\mathcal{F}) \arrow[r, "\mathcal{U}(\sigma_1)"] & t(\mathcal{G})
\end{tikzcd}$$

We are not really interested in this big category, but rather the full subcategory $(\mathscr{W}_J \downarrow \mathcal{U})$ of QS-objects $\mathcal{F} : s(\mathcal{F}) \xrightarrow{\sim} t(\mathcal{F})$. We can define the weak equivalences as the morphism $\sigma : \mathcal{F} \longrightarrow \mathcal{G}$ such that σ_0 is a weak equivalence in the Joyal model structure. The 3-for-2 property will force σ_1 to be also a weak equivalence.

Now since $\mathcal{U} : \mathbf{sCat}_{\mathrm{B}} \longrightarrow \mathbf{sSet}_{\mathrm{J}}$ is part of a Quillen equivalence it's not hard to show that the commutative triangle below descends to a triangle of equivalences between the homotopy categories. In fact we have a better statement as we shall see.

$$\begin{tikzcd}
(\mathscr{W} \downarrow \mathcal{U}) \arrow[d, "t"] \arrow[dr, "s"] & \\
\mathbf{sCat}_{\mathrm{B}} \arrow[r, "\mathcal{U}"] & \mathbf{sSet}_{\mathrm{J}}
\end{tikzcd}$$

With this equivalence of homotopy categories, we can pretend that the theory of quasi-categories and simplicial categories can complete each other in the following sense.

1. We can take advantage of the material developed by Joyal and Lurie for quasi-categories and transpose everything in term of simplicial categories, e.g ∞-limit and ∞-colimit, higher topos, etc.