If we don't worry about size issues, the category $\mathbf{sCat}_{\mathrm{B}}$ is a combinatorial and left proper model category and each of $\mathcal{U}_1$ and $\mathcal{U}_2$ are right adjoint between locally presentable categories. Then by Theorem 1.2 we have the following.

Proposition 2.1. *1. There is a model structure on* $(\mathbf{sCat}_{\mathrm{B}} \downarrow \mathcal{U}_1)$ *in which any fibrant object is a trivial fibration of* $\mathcal{F} : s(\mathcal{F}) \longrightarrow \mathcal{U}_1(\mathcal{C})$ *with* $s(\mathcal{F})$ *fibrant in* $\mathbf{sCat}_{\mathrm{B}}$*. In particular* $s(\mathcal{F})$ *is enriched over Kan complexes and therefore the coherent nerve* $\mathcal{U}_2(s(\mathcal{F}))$ *is a quasicategory.*

2. *There is a model structure on* $(\mathbf{sSet}_{\mathrm{J}} \downarrow \mathcal{U}_2)$ *in which any fibrant object is a trivial fibration of* $\mathcal{F} : s(\mathcal{F}) \longrightarrow \mathcal{U}_1(\mathcal{C})$ *with* $s(\mathcal{F})$ *fibrant in* $\mathbf{sSet}_{\mathrm{J}}$*, that is a quasicategory.*

3. *The following chain of functor connects the two theories:*

$$\mathbf{2\text{-}Cat} \overset{\mathcal{U}_1}{\hookrightarrow} \mathbf{sCat}_{\mathrm{B}} \xrightarrow{\mathcal{U}_2} \mathbf{sSet}_{\mathrm{J}} .$$

4. *Since in the Joyal model structure every object is cofibrant, there exists a section* $\pi : \mathcal{U}_i(\mathcal{C}) \longrightarrow s(\mathcal{F})$ *which is automatically an equivalence of quasicategories. Thanks to this section we can lift concepts of* 2*-categories/simplicial categories to quasicategories and vice-versa.*

Warning. There is a potential conflict of terminology between ours and that of Riehl-Verity. If we let $\mathscr{M}$ be either $\mathbf{sSet}_{\mathrm{J}}$ or $\mathbf{sCat}_{\mathrm{B}}$, then with our notation, the two comma categories will be denoted by $\mathscr{M}_{\mathcal{U}_1}[\mathbf{2\text{-}Cat}]$ and $\mathscr{M}_{\mathcal{U}_2}[\mathbf{sCat}_{\mathrm{B}}]$. In [1], we've interpreted the notation $\mathscr{M}_{\mathcal{U}}[\mathfrak{A}]$ as the category of objects of $\mathscr{M}$ with *coefficients or coordinates* in $\mathfrak{A}$ (with respect to $\mathcal{U}$).

In particular we would say for example that: "$\mathscr{M}_{\mathcal{U}}[\mathbf{2\text{-}Cat}]$ is the quasicategory theory of 2-categories !". This terminology is justified by two reasons:

- The first reason is because the fibrant objects there are the quasicategories that are equivalent to 2-categories;
- The second reason is the language of *points* introduced by Grothendieck, as we consider the representable $\mathrm{Hom}(-, \mathscr{M})$. Moreover it's highly likely that the forgetful functor we've considered are geometric morphisms of higher topoi.

2.3 Producing examples of 'higher concepts'

In this part, we simply list some examples that follow from Theorem 1.2 applied to the usual nerve functor

$$\mathcal{U} : \mathbf{Cat} \longrightarrow \mathbf{sSet}_{\mathrm{J}} .$$

In this case the homotopy theory $\mathscr{M}_{\mathcal{U}}[\mathfrak{A}]$ is the *Quasicategory theory of* 1*-categories*. Indeed, just like previously, any fibrant object is a trivial fibration $\mathcal{F} : s(\mathcal{F}) \longrightarrow \mathcal{U}(\mathcal{C})$, such that $s(\mathcal{F})$ is fibrant i.e., a quasicategory and where $\mathcal{C}$ is a usual 1-category.

As usual, since we have a trivial fibration whose source is fibrant and the target is cofibrant, there is a section $\pi : \mathcal{C} \longrightarrow s(\mathcal{F})$. Given such section, it's an interesting exercise to determine 'by hands', the structure of the quasicategory $s(\mathcal{F})$ attached to $\mathcal{C}$, if: