Understanding higher structures through Quillen-Segal objects

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Abstract

If $\mathscr{M}$ is a model category and $\mathcal{U} : \mathfrak{A} \longrightarrow \mathscr{M}$ is a functor, we defined a Quillen-Segal $\mathcal{U}$-object as a weak equivalence $\mathcal{F} : s(\mathcal{F}) \xrightarrow{\sim} t(\mathcal{F})$ such that $t(\mathcal{F}) = \mathcal{U}(b)$ for some $b \in \mathfrak{A}$. If $\mathcal{U}$ is the nerve functor $\mathcal{U} : \mathbf{Cat} \longrightarrow \mathbf{sSet}_{\mathrm{J}}$, with the Joyal model structure on $\mathbf{sSet}$, then studying the comma category $(\mathbf{sSet}_{\mathrm{J}} \downarrow \mathcal{U})$ leads naturally to concepts, such as Lurie's ∞-operad. It also gives simple examples of presentable, stable ∞-category, and higher topos. If we consider the *coherent nerve* $\mathcal{U} : \mathbf{sCat}_{\mathrm{B}} \longrightarrow \mathbf{sSet}_{\mathrm{J}}$, then the theory of QS-objects directly connects with the program of Riehl and Verity. If we apply our main result when $\mathcal{U}$ is the identity $\mathrm{Id} : \mathbf{sSet}_{\mathrm{Q}} \longrightarrow \mathbf{sSet}_{\mathrm{Q}}$, with the Quillen model structure, the homotopy theory of QS-objects is equivalent to that of Kan complexes and we believe that this is an *avatar* of Voevodsky's *Univalence axiom*. This equivalence holds for any combinatorial and left proper $\mathscr{M}$. This result agrees with our intuition, since by essence the '*Quillen-Segal type*' is the *Equivalence type*.

1 Background and Motivations

This short discussion is motivated by our desire to have an understanding of the theory of quasicategories developed by Joyal and Lurie. And we also hope that our approach can be brought to a more general context of (∞, n)-categories.

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