

2. The underlying map  $\mathcal{F}$  defines a tautological map  $\mathcal{F} \longrightarrow \text{Id}_{t(\mathcal{F})}$  in  $\mathbf{sSet}^{\mathbb{I}} = \text{Path}(\mathbf{sSet})$ . This map corresponds to the tautological commutative square:

$$\begin{array}{ccc} s(\mathcal{F}) & \xrightarrow{\mathcal{F}} & t(\mathcal{F}) \\ \downarrow \mathcal{F} & & \downarrow \text{Id}_{t(\mathcal{F})} \\ t(\mathcal{F}) & \xrightarrow{\text{Id}_{t(\mathcal{F})}} & t(\mathcal{F}) \end{array}$$

3. Since every object in  $\mathbf{sSet}_Q$  is cofibrant, then there is a *Voevodsky section*

$$\pi : t(\mathcal{F}) \longrightarrow s(\mathcal{F})$$

which is necessarily a weak equivalence. And by definition of a section we have an equality (proof):

$$\pi \circ \mathcal{F} = \text{Id}_{t(\mathcal{F})}.$$

4. The section  $\pi$  also determines a tautological map

$$\overline{\pi} : \text{Id}_{t(\mathcal{F})} \xrightarrow{\sim} \mathcal{F}, \tag{2.1.1}$$

that corresponds to the commutative square:

$$\begin{array}{ccc} t(\mathcal{F}) & \xrightarrow{\pi} & s(\mathcal{F}) \\ \downarrow \text{Id}_{t(\mathcal{F})} & & \downarrow \mathcal{F} \\ t(\mathcal{F}) & \xrightarrow{\text{Id}_{t(\mathcal{F})}} & t(\mathcal{F}) \end{array}$$

5. If we look at the respective type of the target and source of the map  $\overline{\pi}$ , we would like to write something like:

$$(\quad = \quad) \xrightarrow{\sim} (\quad \simeq \quad).$$

Although we're not sure, it seems that this map somehow outlines the universality of the identity type which is the idea behind the Univalence axiom. If we abstract the Voevodsky section, then we are tempted to rewrite (2.1.1) as:

$$V : Eq(\text{Id}, QS).$$

## 2.2 Connection with the Riehl-Verity program

As mentioned before there are two cases that are very interesting and that lead directly to the work of Riehl and Verity.

1. If we consider the embedding  $\mathcal{U}_1 : \mathbf{2-Cat} \hookrightarrow \mathbf{sCat}_B$  (see [11]). This leads somehow to the *2-category theory of quasicategories* as named by the authors.
2. And if we consider the coherent nerve  $\mathcal{U}_2 : \mathbf{sCat}_B \longrightarrow \mathbf{sSet}_J$ , this leads to [10].