BASIC MATHEMATICAL TOOLS USED IN DIGITAL IMAGE PROCESSING

NGUYỄN HẢI TRIỀU[1]

[1]Bộ môn Kỹ thuật phần mềm,

Khoa Công nghệ thông tin, Trường ĐH Nha Trang

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1 ELEMENTWISE VERSUS MATRIX OPERATIONS

2 ARITHMETIC OPERATIONS

3 SET AND LOGICAL OPERATIONS

The elementwise product

An elementwise operation involving one or more images is carried out on a pixel-by-pixel basis. For example, consider the following 2×2 images (matrices):

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ and } \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

The elementwise product (often denoted using the symbol $\bigodot$ or $\bigotimes$) of these two images is

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \odot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} \cdot b_{11} & a_{12} \cdot b_{12} \\ a_{21} \cdot b_{21} & a_{22} \cdot b_{22} \end{bmatrix}$$

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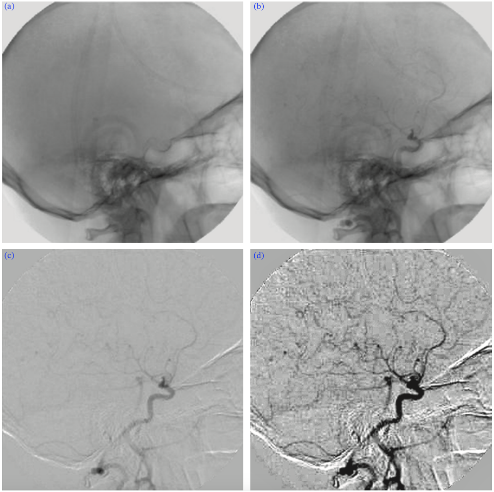
Arithmetic operations between two images $f(x, y)$ and $g(x, y)$ are denoted as

$$
\begin{aligned}
s(x, y) &= f(x, y) + g(x, y), \\
d(x, y) &= f(x, y) - g(x, y), \\
p(x, y) &= f(x, y) \times g(x, y), \\
v(x, y) &= f(x, y) \div g(x, y).
\end{aligned}
$$

These are **elementwise operations** which means that they are **performed between corresponding pixel pairs in f and g.**

Comparing images using subtraction operation -

Image subtraction is used routinely for enhancing differences between images.



Hình 1: Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image.

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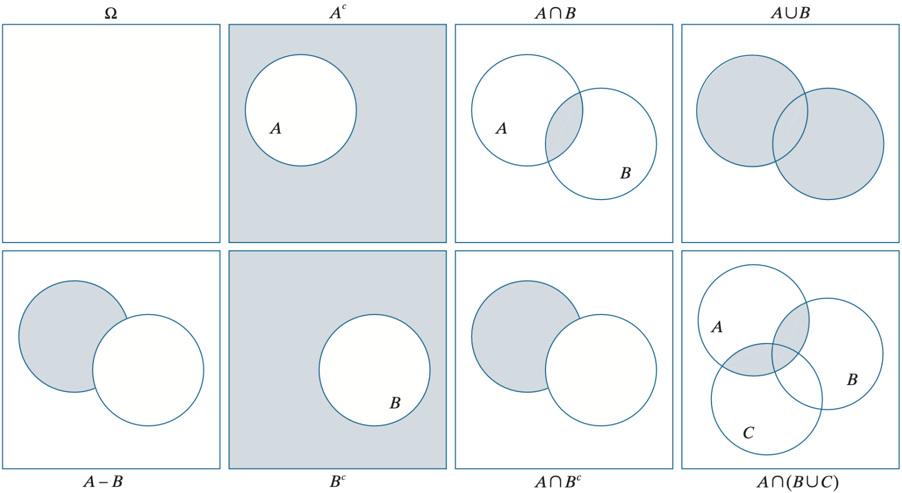
A *set* is a collection of distinct objects. If a is an element of set A, then we write $a \in A$. Similarly, if a is not an element of A we write $a \notin A$.

- The set with no elements is called the *null* or *empty set*, and is denoted by $\emptyset$.
- If every element of a set A is also an element of a set B, then A is said to be a *subset* of B, denoted as $A \subseteq B$.
- The *union* of two sets A and B, denoted as $C = A \cup B$.
- The intersection of two sets A and B, denoted by $D = A \cap B$.
- The *complement* (phần bù) of a set A is the set of elements that are not in A: $A^c = \{w | w \notin A\}$
- The difference of two sets A and B, denoted A - B, is defined as $A - B = \{w | w \in A,\ w \notin B\} = A \cap B^c$

Description	Expressions
Operations between the sample space and null sets	$\Omega^c = \varnothing;\ \varnothing^c = \Omega;\ \Omega \cup \varnothing = \Omega;\ \Omega \cap \varnothing = \varnothing$
Union and intersection with the null and sample space sets	$A \cup \varnothing = A;\ A \cap \varnothing = \varnothing;\ A \cup \Omega = \Omega;\ A \cap \Omega = A$
Union and intersection of a set with itself	$A \cup A = A;\ A \cap A = A$
Union and intersection of a set with its complement	$A \cup A^c = \Omega;\ A \cap A^c = \varnothing$
Commutative laws	$A \cup B = B \cup A$ $A \cap B = B \cap A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$
Distributive laws	$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
DeMorgan's laws	$(A \cup B)^c = A^c \cap B^c$ $(A \cap B)^c = A^c \cup B^c$

Hình 2: Some important set operations and relationships.



Hình 3: Venn diagrams.

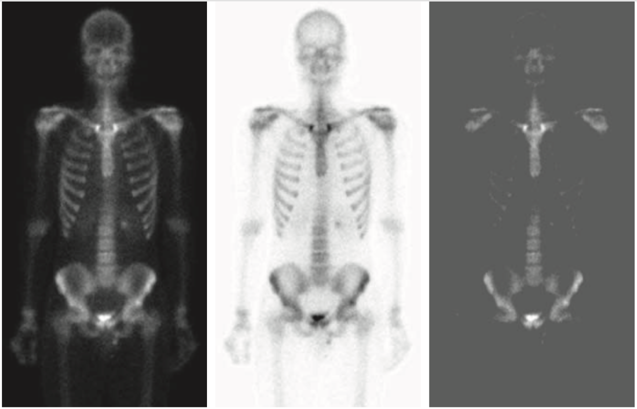
Set operations involving grayscale images

Let the elements of a grayscale image be represented by a set A whose elements are triplets of the form (x, y, z), where x and y are spatial coordinates, and z denotes intensity values. We define the complement of A as the set

$$A^c = \{(x, y, K - z) | (x, y, z) \in A\},$$

where, K is equal to the maximum intensity value in the image, $2k - 1$, where k is the number of bits used to represent z.

Let A denote the 8-bit grayscale image in Fig. 4 and suppose that we want to form the negative of A using set operations. The negative is the set complement $A^c = \{(x, y, 255 - z) | (x, y, z) \in A\}$



Hình 4: Set operations involving grayscale images. (a) Original image. (b) Image negative obtained using grayscale set complementation. (c) The union of image (a) and a constant image.

Tài liệu tham khảo

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