

Matrix operations

In NumPy, a matrix is simply a two-dimensional array. Operations on matrix are similar to those for 1-dimensional operations.

Correction!!!

1-dimensional array is **NOT** a column vector!!!
It is simply a sequence of numbers. There is
no concept of rows or columns in 1-D array in
Numpy.

This is different in other computational
languages such as Matlab.

Creating (initializing) and indexing matrices

```
data = np.array([[1,2],[3,4],[5,6]])
```

```
b = data[0,1]
```

```
c = data[1:3]          # along the 1st dimension
```

```
d = data[0:2, 0]
```

```
e = data[[1,2,0], [1,0,1]]
```

| | data | |
|---|------|---|
| | 0 | 1 |
| 0 | 1 | 2 |
| 1 | 3 | 4 |
| 2 | 5 | 6 |

| | data[0,1] | |
|---|-----------|---|
| | 0 | 1 |
| 0 | 1 | 2 |
| 1 | 3 | 4 |
| 2 | 5 | 6 |

| | data[1:3] | |
|---|-----------|---|
| | 0 | 1 |
| 1 | 3 | 4 |
| 2 | 5 | 6 |

| | data[0:2,0] | |
|---|-------------|---|
| | 0 | 1 |
| 0 | 1 | |
| 1 | 3 | |
| 2 | 5 | |

```
x = np.array([[0,1,2],[3,4,5],[6,7,8],[9,10,11]])
```

```
y = x[      [0,1]      , [1,2]      ]
```

```
z = x[    [ [1],[2] ] , [1,0]      ]
```

cf)

```
q = x[    [ [0,1]      ], [1,2]      ]      ???
```

x =

| | | |
|---|----|----|
| 0 | 1 | 2 |
| 3 | 4 | 5 |
| 6 | 7 | 8 |
| 9 | 10 | 11 |

$x \left[\begin{bmatrix} 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \end{bmatrix} \right] \rightarrow [x[0,1], x[1,2]] \rightarrow \begin{bmatrix} 1 & 5 \end{bmatrix}$ with **arrays**

$x \left[\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \end{bmatrix} \right] \rightarrow x \left[\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \right] \rightarrow \begin{bmatrix} 4 & 3 \\ 7 & 6 \end{bmatrix}$ with **arrays**
with **broadcasting**

Information about a matrix

`ndarray.ndim` # number of axis (number of dimensions)

`ndarray.shape` # number of elements in each dimension

`ndarray.size` # total number of elements

`np.unique()`

```
a_2d = np.array([[1, 2, 3, 4], [5, 6, 7, 8], [9, 10, 11, 12], [1, 2, 3, 4]])
unique_rows = np.unique(a_2d, axis=0)
unique_rows, indices, occurrence_count = np.unique(
    a_2d, axis=0, return_counts=True, return_index=True)
```

Changing the shape of an array

`ndarray.reshape(m, n)` # change the dimension to m x n

`np.newaxis` `b = a[np.newaxis, :]`
 `b = a[:, np.newaxis]`

`np.expand_dims` `b = np.expand_dims(a, axis = 1)`
 `b = np.expand_dims(a, axis = 0)`

`np.squeeze()` `b = np.squeeze(a)`

`np.sort()` `b = np.sort(a)`

`np.flip()` `b = np.flip(a)` # reverse

Creating (initializing) a new array from existing arrays

```
a = a[3:]          # removing first 3 elements
```

```
a1 = np.array([[1,1],[2,2]])
```

```
a2 = np.array([[3,3],[4,4]])
```

```
np.vstack((a1,a2))
```

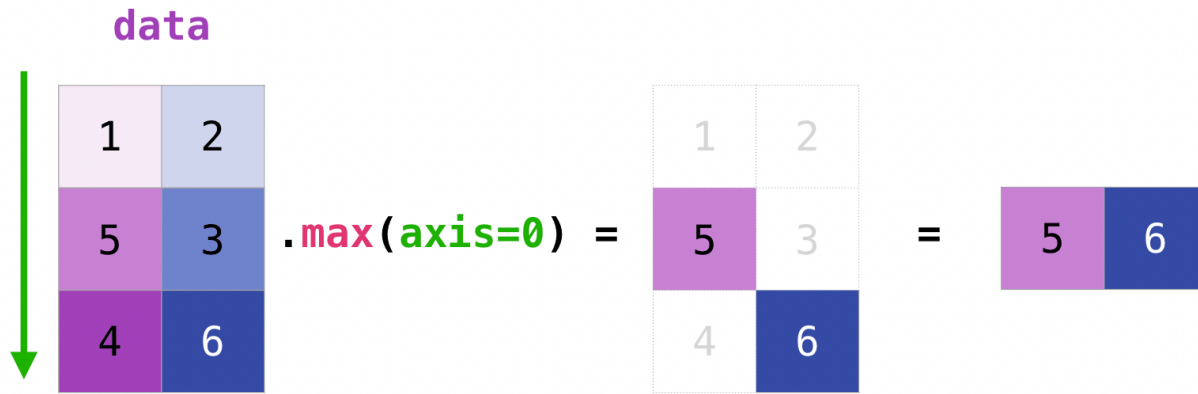
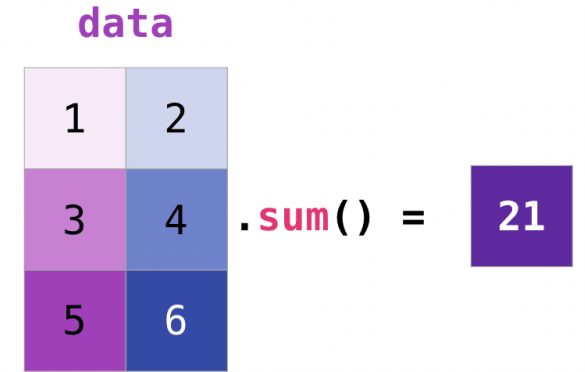
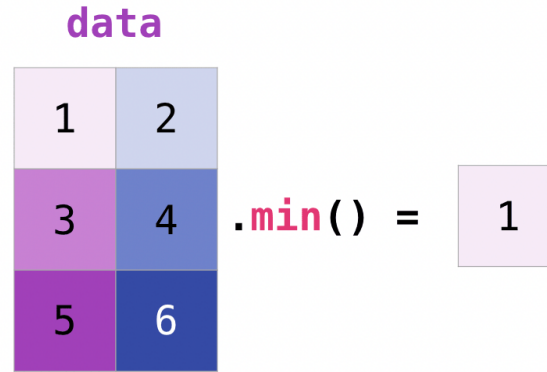
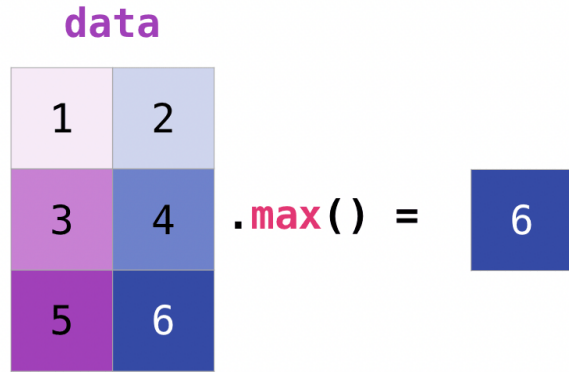
```
np.hstack((a1,a2))
```

```
x = np.arange(1,25).reshape(2,12)
```

```
a,b,c = np.hsplit(x,3)
```

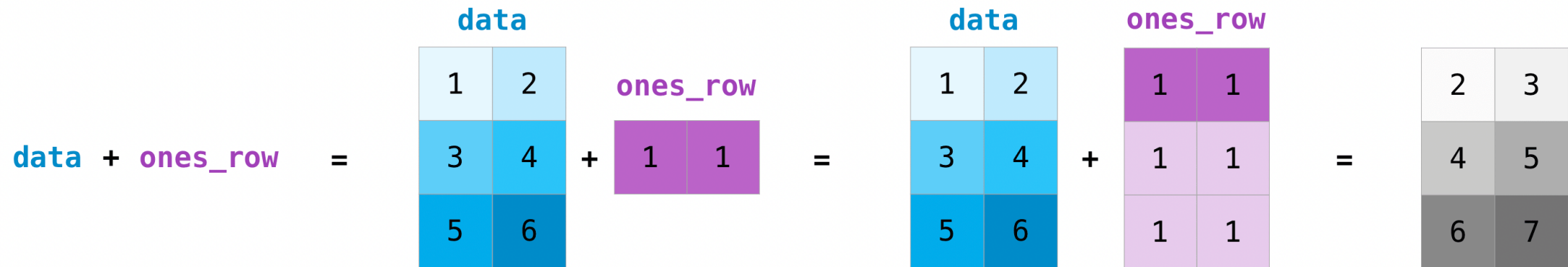
Operations on matrices: max(), min(), sum()

also: cumsum(), mean(), median(), std(), etc...



Operations on matrices (broadcasting)

```
data = np.array([[1,2],[3,4],[5,6]])  
ones_row = np.array([[1,1]])    # Note: a nested list  
b = data + ones_row
```



Operations on matrices: transposing, multiplication

```
data = np.array([[1,2],[3,4],[5,6]])  
b = data.T          # equivalent to → b = data.transpose()  
c = data @ data.T    # Matrix multiplication
```

data

| | |
|---|---|
| 1 | 2 |
| 3 | 4 |
| 5 | 6 |

data.T

| | | |
|---|---|---|
| 1 | 3 | 5 |
| 2 | 4 | 6 |

```
# Advanced transpose  
x_test = np.arange(30).reshape(3, 2, 5)  
print(x_test)  
print(x_test.shape)  
y = np.transpose(x_test, (0, 2, 1)) # axis change  
print(y)  
print(y.shape)
```

Operations on matrices: reshaping

```
data = np.arange(1,7)
b = data.reshape(2,3)
c = data.reshape(3,2)
e = data.reshape(3,-1)    # automatic # of cols
f = data.reshape(-1,3)    # automatic # of rows
```

data

| |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

data.reshape(2,3)

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |

data.reshape(3,2)

| | |
|---|---|
| 1 | 2 |
| 3 | 4 |
| 5 | 6 |

Operations on matrices: reverse

```
data = np.arange(1,7).reshape(2,3)
```

```
b = np.flip(data)
```

```
c = np.flip(data, axis = 1)  # reverses only the given dim
```

```
data[1] = np.flip(data[1])   # reverses a single row
```

```
data[:,1] = np.flip(data[:,1])  # reverses a single column
```

Tiling

```
data = np.arange(1,7).reshape(2,3)
```

```
b = np.tile(data, (3,1))
```

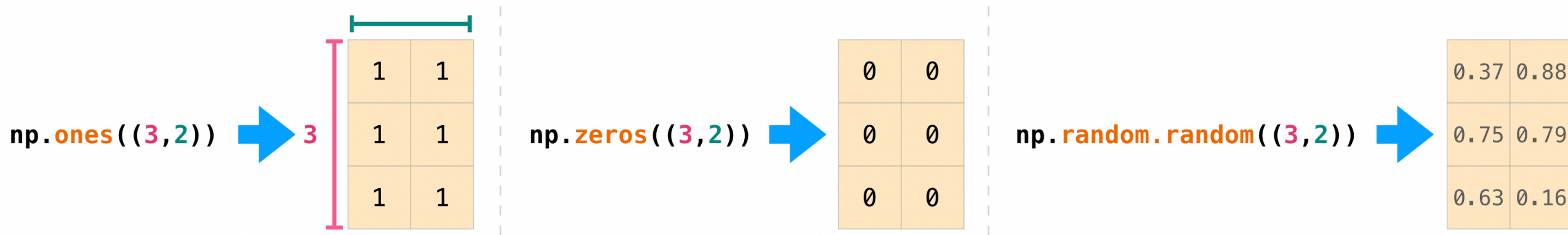
```
c = np.tile(data.reshape(6,1), (1,3))
```

Other ways of initializing matrices

```
a = np.ones( (3,2) )
```

```
a = np.zeros( (3,2) )
```

```
a = np.random.random( (3,2) )
```



```
a = np.random.randint(3, 10, size=(5,10))
```

```
a = np.random.normal(size=(6,6))
```

Let's use NumPy to evaluate a math formula

$$\text{MeanSquareError} = \frac{1}{n} \sum_{i=1}^n (Y_{\text{prediction}_i} - Y_i)^2$$

`Y_prediction` \rightarrow `y_pred`
`Y_i` \rightarrow `yi`

Then, `Y_prediction - Y_i` can be written as \rightarrow `y_pred - yi`

The square term can be broadcasted \rightarrow `(y_pred-yi)**2`

Summation \rightarrow `((y_pred-yi)**2).sum()`

`1/n` \rightarrow `((y_pred-yi)**2).sum()/yi.size`

Therefore \rightarrow `MSE = ((y_pred-yi)**2).sum()/yi.size`

Save and load NumPy arrays

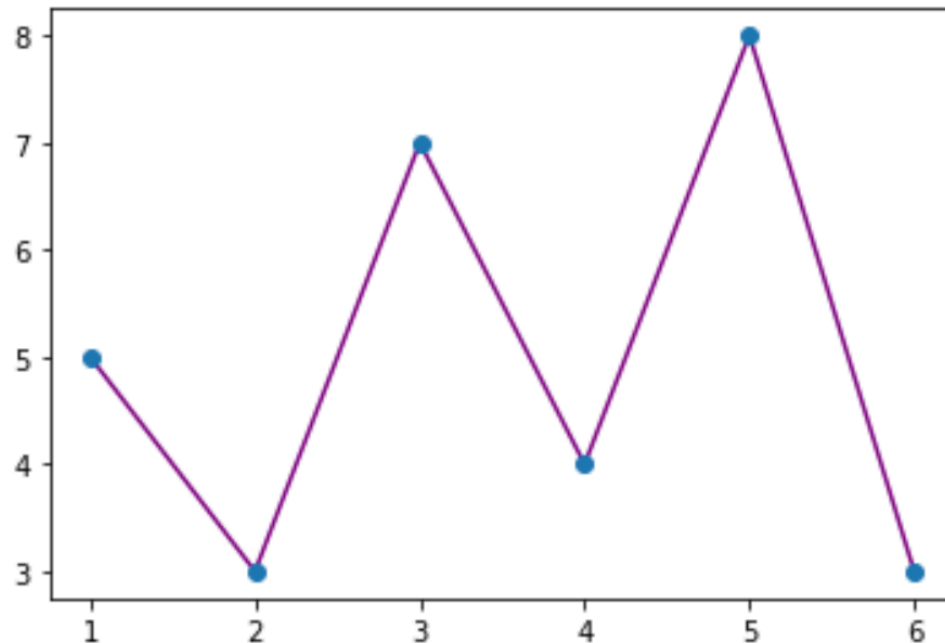
```
a = np.array([1,2,3,4,5,6])  
np.save('filename', a)  
m = np.load('filename.npy')
```

```
a = np.array([1,2,3,4,5,6])  
b = np.array([5,3,7,4,8,3, 1.0])  
np.savez('filename', abc=a, qqq=b)  
m = np.load('filename.npz')  
m['abc']  
m['qqq']
```

```
# Use 'savez_compressed' to compress the data  
# (my take more time to save)  
np.savez_compressed('filename', abc=a, qqq=b)  
m = np.load('filename.npz')
```

Plot

```
%matplotlib widget # use this for interactive plots
import matplotlib.pyplot as plt
a = np.array([1,2,3,4,5,6])
b = np.array([5,3,7,4,8,3])
plt.plot(a,b, 'purple') # line color
plt.plot(a,b, 'o')      # dots
```



Practice, practice practice!!!

The concepts and basic operations of Numpy we covered in the last and this week is essential. You HAVE to practice every single operations I mentioned in the lectures by yourself. Otherwise, you will quickly forget them.