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Lab session # LAB2
ALTEGRAD 2023

10/24/23

Question 1

An undirected graph without self loops of n nodes can have maximum: $\frac{n(n-1)}{2}$ edges. Each node i is connected to (n-1) nodes of the graph and since it's an unidrected graph the edge (i,j) is the same as edge (i,j), so the maximum number of edges is $\sum_{i=1}^{n-1} n - i = \frac{n(n-1)}{2}$

The maximum number of trinagles on an undirected graph of nodes n is the number of combination of 3 nodes within a set of n nodes $\binom{n}{3} = \frac{n!}{3!(n-3)!} = \frac{n(n-1)(n-2)}{6}$

Question 2

Two graphs having the same degree distribution are not isomorphic. Let's take as an example an Hexagon and two separated triangles, these two graphs have the same degree distributions but they are not isomorphic.

Question 3

Let C_n denote a cycle graph on n vertices. The global clustering coefficient of a graph is defined as the number of triangles divided by the number of triplets of the graph.

- For n = 3, the graph C_3 is triangle and so the global clustering coefficient is 1.
- For $n >= 4$, the graph C_n is a cycle that doesn't contain any triangle, so its global clustering coefficient is 0.

Question 4

The Laplacia matrice is positive semi definite and has an eigenvalue zero, so the zero is the smallest one.
Let u_1 be the eigenvector associated to the smallest eigenvalue.

$$L_{rw} * u_1 = (I - D^{-1} * A)u_1 = 0$$

So:

$$(D - A)u_1 = 0$$

So:

$$(\sum_{j=1}^{n} A_{ij}) * (u_1)_i - \sum_{j=1}^{n} A_{ij} * (u_1)_j = 0, \quad \forall i \in [1, n]$$

$$(\sum_{j=1}^{n} A_{ij}) * (u_1)_i^2 - \sum_{j=1}^{n} A_{ij} * (u_1)_j * (u_1)_i = 0, \quad \forall i \in [1, n]$$

So:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} * (u_1)_i^2 - \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} * (u_1)_j * (u_1)_i = 0 \tag{1}$$

A is the matrix of adjencency and it's symmetric, so:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} * (u_1)_i^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} * (u_1)_j^2 \tag{2}$$

If we multiply 1 by 2, we can obtain by using 2 that:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} * ((u_1)_i^2 - (u_1)_j^2) = 0$$

Question 5

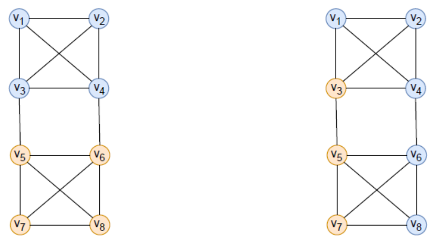

Figure 1: Two graphs where nodes have been assigned to 2 clusters.

We have: $m = number\_of\_edges = 14$.
For the first graph G_1, we have two identical clusters:

- We have: $l_c = 6$ and $d_c = 3 + 3 + 4 + 4 = 14$.

$$q_1 = \frac{6}{14} - (\frac{14}{2 * 14})^2 = 0.178$$

- So the modularity of G_1 is:

$$Q_1 = q_1 + q_2 = 2 * q_1 = 0.356$$

For the second graph G_2, we have two clusters:

- For the first cluster, we have $l_1 = 5$ and $d_1 = 3 + 3 + 4 + 4 + 3 = 17$. Then:

$$q_1 = \frac{5}{14} - (\frac{17}{2 * 14})^2 = -0.226$$

- For the second cluster, we have $l_2 = 2$ and $d_2 = 4 + 4 + 3 = 11$. Then:

$$q_2 = \frac{2}{14} - (\frac{11}{2 * 14})^2 = -0.012$$

- So the modularity of G_2 is:

$$Q_1 = q_1 + q_2 = -0.238$$

Question 6

For the Path Graph 2:

$$\Phi(P_4) = [3, 2, 1, 0, ..., 0]$$

For the Star Graph 3:

$$\Phi(S_4) = [3, 3, 0, 0, ..., 0]$$

Figure 2: Path Graph with 4 vertices.

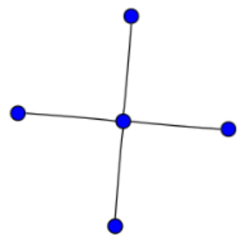

Figure 3: Star graph with 4 vertices.

So the shortest path for the pairs (P_4, P_4), (P_4, S_4) and (S_4, S_4) is:

$$k(P_4, P_4) = 9 + 4 + 1 = 14$$

$$k(P_4, S_4) = 9 + 6 = 15$$

$$k(S_4, S_4) = 9 + 9 = 18$$

Question 7

Let k denote the graphlet kernel that decomposes graphs into graphlets of size 3.

Let also G, G' two graphs for which:

$$k(G, G') = f_G^T f_G = 0.$$

Let

$$f_G = [a_1, a_2, a_3, a_4]$$

and

$$f'_G = [b_1, b_2, b_3, b_4]$$

So:

$$f_G^T f_G = 0 \equiv a_1 * b_1 + a_2 * b_2 + a_3 * b_3 + a_4 * b_4 = 0 \equiv a_i * b_i = 0 \quad \forall i \in [1, 4]$$

This means that the two graphs do not contain any commun graphlet of size 3.

Here is an example of these two graphs:

Figure 4: Example of Graphs G and G'.

Fot this example we have:

$$f_G = [0, 2, 0, 0]$$

and

$$f_{G'} = [0, 0, 3, 1]$$

So:

$$k(G, G') = 0$$