(1). By the factorization theorem,

$T(X) = \min_i (X_i/i)$ is sufficient,

as, the joint pdf of $X_1, \ldots, X_n$

$$f(x_1, \ldots, x_n | \theta) = \prod_{i=1}^{n} e^{i\theta - x_i} I_{(i\theta, +\infty)}(x_i)$$

$$= e^{\theta \sum_{i=1}^{n} i} \cdot e^{-\sum_{i=1}^{n} x_i} I_{(\theta, +\infty)}\left[\min_i (x_i/i)\right]$$

$$= \underbrace{e^{\theta \sum_{i=1}^{n} i} \cdot I_{(\theta, +\infty)}(T(x))}_{g(T(x)|\theta)} \cdot \underbrace{e^{-\sum_{i=1}^{n} x_i}}_{h(x)}$$

Here, we use the fact that, $i > 0$ and all $x_i > i\theta$ if and only if $\min(x_i/i) > \theta$.

(Proved)

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(2) Let $X_1, \ldots, X_n$ be iid with pdf or pmf $f(x|\theta)$.

Now, if we take the joint pdf of $X = (X_1, \ldots, X_n)$,

$$f_\theta(x) = \prod_{i=1}^{n} f_\theta(x_i)$$
$$= \prod_{i=1}^{n} f_\theta(x_{(i)})$$

Here, given the $X_1, \ldots, X_n$ are i.i.d., $x_{(1)}, \ldots, x_{(n)}$ are the ordered values of $x_1, \ldots, x_n$.

Now, By the factorization theorem, $(X_{(1)}, \ldots, X_{(n)})$ is sufficient for θ where $h(x) = 1$.

Here, we used the fact that $X_1, \ldots, X_n$ are iid as the order statistics is sufficient only when positions of x_i's are not of interest.

So, given, $X_1, \ldots, X_n$ are iid,
$T(X_1, \ldots, X_n) = (X_{(1)}, \ldots, X_{(n)})$ are sufficient for θ.