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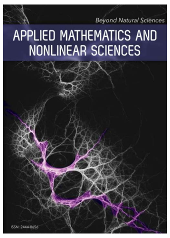

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Dynamic Optimization of Robot Automatic Control System Based on Differential Algebraic Equations

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Submission Info

Communicated by Z. Sabir
Received March 4, 2022
Accepted July 5, 2022
Available online March 17, 2023

Abstract

In order to understand the different performances of robots under different control systems, the author has carried out dynamic optimization research on the control system of robots combined with differential algebraic equations. In this study, the general form of the discrete differential-algebraic equation (DAE) optimization problem using the Orthogonal Configuration of Finite Element (OCFE) method is deeply analyzed, and the equivalent conditions of the direct discrete scheme and the indirect discrete scheme are obtained through rigorous proof. On this basis, a variety of common configuration methods are simulated and analyzed, and it is found that indirect Lobatto configuration can achieve better results in many aspects. The results show that the discrete algorithm using differential algebraic equations can effectively achieve dynamic optimization of the control system, thus achieving the author's research purpose.

Keywords: Differential algebraic equation; robot; automatic control system; dynamic optimization
AMS 2020 codes: 34A09

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ISSN 2444-8656
https://doi.org/10.2478/amns.2023.1.00027

1 Introduction

If there are only senses and muscles, human limbs still cannot move. On the one hand, there are no organs to receive and process signals from the senses, and on the other hand, there are no organs to send out nerve signals that drive muscles to contract or relax. Likewise, if the robot only has sensors and actuators, the robotic arm will not work properly. The reason is that the signal output by the sensor does not work, and the driving motor can not get the driving voltage and current, so the robot needs to have a controller, which is a control system composed of hardware and software [1]. The function of the robot control system is to receive the detection signal from the sensor, according to the requirements of the operation task, the motors in the robotic arm are driven. Just as our human activities depend on our senses, the motion control of robots is inseparable from sensors. Robots need sensors to detect various states. The internal sensor signals of the robot are used to reflect the actual motion state of the manipulator joints, and the external sensor signals of the robot are used to detect changes in the working environment. Therefore, the combination of the nerve and the brain of the robot can become a complete robot control system, as shown in Figure 1. A differential-algebraic equation is a system of several differential equations and pure algebraic equations (without derivatives) [2]. Similar to partial differential equations, differential algebraic equations are difficult to find exact results. Differential equation is a very interesting mathematical object, mainly because its structure contains analytical research objects such as derivatives and integrals, its solution also contains algebraic research objects such as symmetry and linear space. We can declare without exaggeration that differential equations are a mathematical perspective from which we see the world.

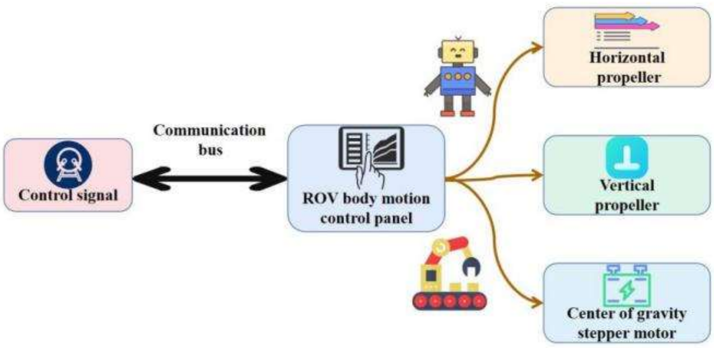

Figure 1. Robot control system

Chen Y et al. stated that the robot control system refers to a management system with its own goals and functions composed of control subjects, control objects and control media. A control system means by which any interesting or variable quantity within a machine, mechanism or other device can be maintained and changed in a desired manner [3]. Filipkovskaya M S et al think that the control system is simultaneously implemented in order to make the controlled object reach a predetermined ideal state. The control system makes the controlled object tend to a certain required stable state [4]. Li B and others believe that the controller is the core of the robot system, in recent years, with the development of microelectronics technology, the performance of microprocessors has become higher and higher, and the price has become cheaper and cheaper, at present, there have been 1-2 US dollar 32-bit microprocessor. Cost-effective microprocessors have brought new development opportunities for robot controllers, making it possible to develop low-cost, high-performance robot controllers [5]. Samoilenko A M et al. stated that in order to ensure that the system has sufficient computing and storage capabilities, at present, robot controllers are mostly composed of ARM series, DSP series, POWERPC series, Intel series and other chips with strong computing power. In addition, because the functions and performance of the existing general-purpose chips cannot fully meet the requirements of some robot systems in terms of price, performance, integration and interface, etc., this has created the demand for SoC (SystemonChip) technology for robotic systems, integrate a specific processor with the required interface, it can simplify the design of the peripheral circuit of the system, reduce the size of the system, and reduce the cost [6]. Liu, X. et al. stated that in terms of controller architecture, their research focuses on functional partitioning and specification of information exchange between functions. In the research of open controller architecture, there are two basic structures, one is the structure based on hardware hierarchy, this type of structure is relatively simple, and the other is a structure based on function division, which considers both software and hardware, which is the direction of research and development of robot controller architecture. Use the most common language to explain "dynamic optimization", that is, "dynamic optimization is the optimization of dynamic systems". This explanation is not completely meaningless, at least it specifies the object of dynamic optimization, namely "dynamic system" [7]. Ogunfeyitimi S E et al. stated that the dynamic system (or dynamical system) is an all-encompassing concept, because we exist in a world of "absolute change". Simply put, dynamic systems can almost exhaust all the rules of change in the fields of physics, chemistry, biology, and economics [8]. Erol H E et al. use scientific language for abstraction and induction: The so-called dynamic system is a kind of system whose motion state evolves with time according to a certain law or a certain statistical law; In other words, a dynamical system describes how the future state depends on the current state [9]. Although dynamic systems have various forms of existence, they can basically be described in the powerful language of mathematics, this is the mathematical model of dynamic systems—dynamic models, the research scope of dynamic optimization is shown in Figure 2.

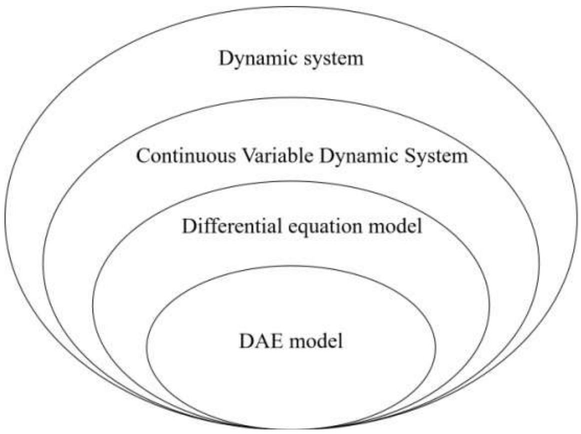

Figure 2. The research scope of dynamic optimization

2 Methods

Figure 3 is a schematic diagram of several main dynamic optimization solutions, which can be seen by comparison: In order to solve the variational method, the DAE optimization model must be converted into the DAE model according to the principle of maximum value, and then discretized into a nonlinear equation system to solve; The use of the joint method is to first discretize the DAE optimization model into an NLP model, and the commonly used solver also transforms the optimization problem into a system of equations for iterative solution based on optimality conditions [10]. It can be seen that the conventional dynamic optimization solution method needs to complete two transformations (obviously the order is different), that is, the optimization solution is transformed into an equation solution and the continuous problem is transformed into a discrete problem. In addition, we also find striking similarities between the variational principle and the optimality condition—the equivalence condition between the discretized form of the variational condition and the optimality condition of discretized NLP has been demonstrated.

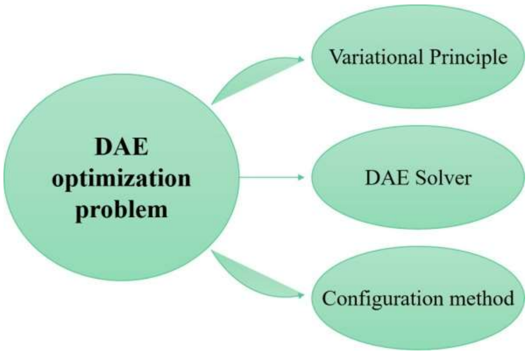

Figure 3. Schematic diagram of several main solutions of dynamic optimization

DAE optimization problem is a typical dynamic optimization problem, in our research, we always hope to have a model without loss of generality that can generalize as much as possible to all specific problems belonging to this class [11]. The description form of the dynamic optimization problem adopted by the author is as formula 1-6:

$$\min_{z(t),u(t),p} \varphi(z(l,f)) \tag{1}$$

$$s.t. \quad \dot{z}(t) = \int(z(t), y(t), u(t), p), \qquad z(t_0) = z_0 \tag{2}$$

$$g(z(t), y(t), u(t), p) = 0 \tag{3}$$

$$h(z(l_f)) = 0 \tag{4}$$

$$z^L \le z(t) \le z^U, y^L \le y(t) \le y^U \tag{5}$$

$$u^L \le u(t) \le u^U, p^L \le p(t) \le p^U \tag{6}$$

In the formula, $z(t) \in R^{n_z}$A is a differential variable (or called a state variable), $y(t) \in R^{n_y}$ is an algebraic variable, $u(t) \in R^{n_u}$ is a control variable, these variables are all functions of the time scalar $t \in [t_0, t_f]$, $p(t) \in R^{n_p}$ is the parameter to be optimized independent of t, here n_z, n_y, n_u, n_p respectively represents the number of corresponding variables. The equality constraints of the model are the DAEs. Without loss of generality, it is assumed that the DAE conditions here have continuous second-order partial derivatives for each variable, and the index (index) is 1 [12]. This model is a typical description of a DAE optimization problem, and most continuous dynamic optimization problems can be represented or transformed into this form. From the previous content about the development of dynamic optimization, it can be seen that the joint method is one of the most advanced means to solve the DAE optimization problem. In fact, joint law is more precisely a problem-solving idea or solution, which is a synthesis of multiple methods, so we prefer to call it a "strategy". Simultaneous solving strategies follow a fixed problem-solving pattern—discrete first and then solve. The so-called discrete is to convert the original DAE optimization problem into an NLP problem, that is, use a discrete dynamic model to approximate the continuous dynamic model. It can be said that the essence of discretization is the same as that of numerical solution of differential equations, that is to say, all numerical solutions of differential equations can be used to discretize dynamic problems [13].

The waveform relaxation method for nonlinear differential equations is shown in Equations 7 and 8:

$$F(\dot{z},z,u)=0 \tag{7}$$

$$E(z(0)-z_0)=0 \tag{8}$$

Where $z \in R^n$, $E \in R^{m\times n}(m \leq n)$ are matrices of rank m. Two basic algorithms of waveform relaxation methods for solving initial value problems are proposed, Gauss-Jacobi waveform relaxation method and Gauss-Seidel waveform relaxation method [14]. Consider converting the compatible waveform relaxation algorithm to the standard form, as shown in Equation 9:

$$\begin{cases} \dot{x}^{k+1} = f\left(x^{k+1}, x^k, \dot{x}^k, y^k, u\right) \\ y^{k+1} = g\left(x^{k+1}, x^k, \dot{x}^k, y^k, u\right) \\ x^{k+1}(0) = x(0) \end{cases} \tag{9}$$

Among them on $x \in R^{n_1}$, $y \in R^{n_2}$, $n_1 + n_2 = n$, $u \in R^r$. The convergence of the Gauss-Jacobi waveform relaxation method and the Gauss-Seidel waveform relaxation method is demonstrated. For circuit simulations, the waveform relaxation method has been proved to be effective for the numerical solution of its corresponding system of differential-algebraic equations, and the algorithm is convergent under certain conditions, as shown in Equation 10:

$$\begin{cases} \dot{x} = f(x,x,\dot{x},y,u), x(0) = x_0 \\ y = g(x,x,\dot{x},y,u) \end{cases} \tag{10}$$

In the process of kinematic equation of discrete mechanical system, the discrete mechanics and optimal control (DMOC) method can avoid the problem of increasing or decreasing the energy of the system, and has the advantage of maintaining the inherent characteristics of the mechanical system, therefore, DMOC is an advantageous tool for solving optimal control problems of mechanical systems [15]. The main idea of this method is to use the Lagrange-D'Alembert principle and the Legendre transform, transform the dynamics-kinematic model and boundary conditions of the biped robot into discrete forms, specifically, see Figure 4. The optimal functional extremum problem with constraints is transformed into a finite-dimensional nonlinear constrained optimization problem, that is:

$$\min J_d(q_d, u_d) = \sum_{k=0}^{N-1} C_d(q_k, q_{k+1}, u_k, u_{k+1}) \tag{11}$$

$$s.t. \quad D_2 L(q(0), \dot{q}(0)) + D_1 L_d(q_0, q_1) + u_0^- = 0 \tag{12}$$

$$D_2 L_d(Q_{k-1}) + D_1 L_d(Q_k) + u_{k-1}^+ + u_k^- = 0 \tag{13}$$

$$D_1 L_d(Q_{N-1}) - D_2 L(Q_T) + u_{N-1}^{+} = 0 \tag{14}$$

Among

$$Q_{k-1} = (q_{k-1}, q_k), Q_k = (q_k, q_{k+1}) \tag{15}$$

$$Q_{N-1} = (q_{N-1}, q_N), Q_T = (q(T), \dot{q}(T)) \tag{16}$$

$k \in \{1,2,\dots,N-1\}$, C_d are the performance index functionals of the biped robot system, in general, the continuous trajectories $q(t)$ and $u(t)$ selected as the minimum energy consumption system are discretized into q_d and u_d respectively [16]. D_i represents the partial derivative corresponding to the i-th element, for example:

$$D_2 L_d(q_{k-1}, q_k) = \frac{\partial L_d}{\partial q_k}, D_1 L_d(q_k, q_{k+1}) = \frac{\partial L_d}{\partial q_k} \tag{17}$$

Where $q(t)$ is the flow of the dynamical system, $q_d(0)$ and $q_d(T)$ are the initial and final boundary conditions within a walking cycle, respectively, G_r is the collision switching surface function, $h(q(t))$ is the one-sided constraint, and $R(\alpha)q(T)$is the collision switching map [17].

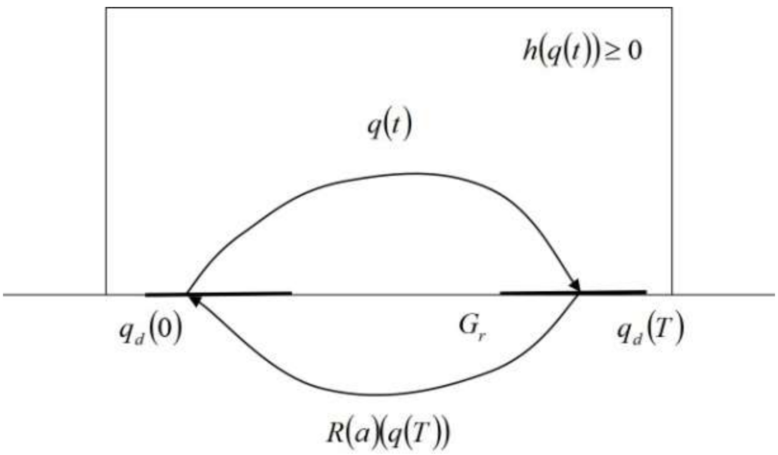

Figure 4. Schematic diagram of DMOC technology

3 Results and Analysis

Robot technology is widely used in industrial production, and the control modes used mainly include open-loop control and closed-loop control, the principle of the open-loop control system is shown in Figure 5.

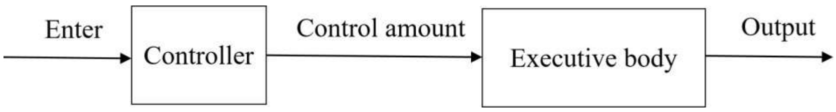

Figure 5. Open loop control system

For the control system using the open-loop method, it is difficult to reflect the information of the output to the input, that is, the input cannot be adjusted according to the output, so the control of the control system is poor. In actual production, the control system needs to adopt different control methods to meet the application requirements in different occasions [18]. For example, the system can either automatically cycle through a complete action process, or use manual operation to run a specific action. Electromechanical products can be decomposed into electromechanical control subsystems and electromechanical actuator subsystems according to different functions, among them, the electromechanical actuator subsystem can actually control the electromechanical production according to the control instructions of the control system, and the open-loop control method can be used in the occasions where the requirements for the electromechanical manufacturing products are not high. In the control system of the robot, if the control performance of the product is required to be high, the closed-loop control system can be used. In the closed-loop control system in the robot, when the sensor in the control system detects the feedback signal transmitted by the actuator, the feedback signal is input into the information processing and control system to regenerate a new control signal, then input it into the actuator subsystem, so that the control performance of the electromechanical control system is more superior. Generally, closed-loop control systems are mostly used in situations where the control system of the robot is required to be high.

A simple example is solved by the fully discrete joint method. Combined with the research purpose, this example removes the constraints on the state variables in the original problem:

$$\min_u z_3(\pi/2) \tag{18}$$

$$s.t. \quad \dot{z}_1(t) = z_2(t), \quad z_1(0) = 0 \tag{19}$$

$$\dot{z}_2(t) = u(t), \quad z_2(0) = 1 \tag{20}$$

$$\dot{z}_3(t) = \left(z_2^2(t) - z_1^2(t)\right)/2, \quad z_3(0) = 0 \tag{21}$$

The constraint of the model is a singular differential equation, it is easy to see that the control law $u(t)$ is the rate of change of the state variable $z_2(t)$, that is, the numerical solution of $z_2(t)$ has a slight change, it is also further amplified by the value of $u(t)$. Because the model is relatively simple, the optimal control law $u^*(t) = -\sin t$ can be analyzed by the variational method, and the minimum value of the objective function is 0. Here we discretize the model using 3-level Gauss, 3-level Radau and 4-level Lobatto orthogonal configurations, respectively, and transform it into a shape NLP problem. Take the number of finite elements as n = 50, and then use the IPOPT solver to solve [19]. The numerical solution of the optimal control law obtained by each configuration method is shown in Figure 6. The horizontal axis is the optimization time domain, the vertical axis represents the numerical value of the control law, the solid curve is the analytical solution of the optimal control law, and the dots are the size of the $u^*(t)$ numerical solution corresponding to each configuration point.

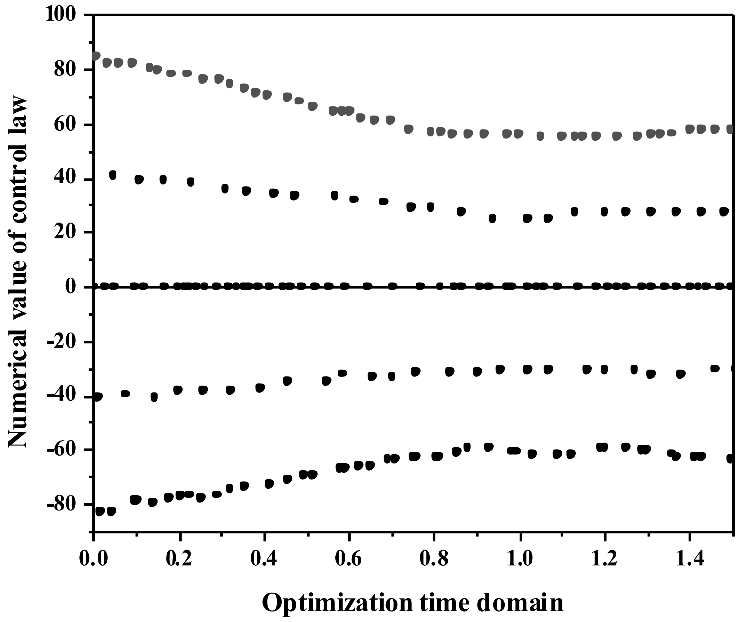

Figure 6. Numerical solution of discrete optimal control law of Gauss orthogonal configuration method

Robot-based dynamic grasping experimental platform, after many debugs and corrections of software and hardware, the experimental system successfully completed the "detection-tracking-grabbing-handling" experiment. Before starting the robot for automatic grasping work, related preparations must be carried out first, including the calibration and setting of the mechanical origin position of the robot, the calibration of the position coordinates of the visual inspection system, and the setting of the user coordinate system, actually measure the working stroke of the workpiece input conveyor belt motor for one revolution, and jog the manipulator to obtain and set the relevant motion positions, including the initial grasping position, placement position, waiting position and the height of the workpiece input conveyor belt [20]. After starting the robot automatic grasping control system, the robot will simulate grasping action according to the set initial grasping position to test whether the grasping motion trajectory is correct. Then, the belt conveyor is turned on and the experimental workpieces are randomly and continuously placed at the starting end, the trajectory of the robot is as follows: When the first workpiece enters the graspable area of the robot, the robot moves synchronously with the conveyor belt, and the robot follows the workpiece from the current position, after catching up, grab the workpiece and continue to synchronize with the conveyor belt, after the track target time has passed, the synchronization is released by synchronous stop. Then, a door-like motion is made to the set placement position, and when the robot reaches the placement point within the specified time, the robot's TCP becomes False. Next, perform absolute position movement to the set waiting position, and repeat the above process when the second workpiece enters the graspable area of the robot.

4 Conclusions

Due to the complexity of the dynamic optimization problem of continuous systems, there is no very effective direct analytical solution method so far, the most commonly used solution method is to convert the continuous system into a discrete system, and then use a computer to solve it numerically. In the discrete process, the distortion caused by the discrete should be reduced as much as possible, and the calculation efficiency and the accuracy of the results should be maintained when solving, therefore, both discrete methods and solving techniques are the contents that have received much attention in dynamic optimization. The content of the author's research involves both discrete and solution, including the analysis of discrete methods, the design of solution framework and the strategy of generating initial value points. Use Sysmac Studio to realize the integrated control of the machine, through simple settings, you can complete the configuration of the controller, network, servo and other field devices, the complex motion control can be completed through the internal function blocks, and the NS integrated simulation and 3D trajectory simulation functions can be used to improve the debugging efficiency of the program. At the same time, due to the realization of integrated control, the programming language has been unified, and the productivity and maintenance have been improved from the perspective of development and maintenance.

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