

# Theoretical Addendum: The Dilaton Interpretation of the Machian Scalar

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## Abstract

We provide a theoretical justification for the "Universal Conformal Coupling" assumed in the Isothermal Machian Universe. We identify the scalar field  $\phi$  as the **Dilaton** of a spontaneously broken scale invariance. This symmetry ensures that all mass scales (Higgs VEV, QCD confinement) scale uniformly with  $\phi$ , protecting the Weak Equivalence Principle (WEP) from violations.

## 1 Introduction

The Isothermal Machian Universe relies on a scalar field  $\phi$  that couples to all matter with a universal strength  $\beta$ . Critics argue that such a coupling is fine-tuned and should violate the Weak Equivalence Principle (WEP) due to composition-dependent forces (e.g., binding energy differences). Here, we show that if  $\phi$  is the Goldstone boson of Scale Invariance (the Dilaton), this universality is a consequence of the underlying symmetry.

## 2 The Dilaton Hypothesis

Assume the fundamental Lagrangian of the universe is Scale Invariant.

$$x^\mu \rightarrow \lambda x^\mu, \quad \Phi \rightarrow \lambda^{-\Delta} \Phi \quad (1)$$

If this symmetry is spontaneously broken at a scale  $f$ , a Goldstone boson  $\chi$  (the Dilaton) emerges. The effective action for matter fields  $\psi$  coupled to the Dilaton is constrained by the non-linear realization of the symmetry:

$$S_{matter} = \int d^4x \sqrt{-g} \mathcal{L}_{SM}(\psi, g_{\mu\nu} e^{2\beta\phi/M_{pl}}) \quad (2)$$

This implies that the metric  $g_{\mu\nu}$  always appears in the combination  $\hat{g}_{\mu\nu} = e^{2\beta\phi} g_{\mu\nu}$ . This is exactly the **Universal Conformal Coupling** ansatz used in our papers.

## 3 Protection of the Equivalence Principle

### 3.1 Tree Level Universality

Because the coupling is geometric (via the metric), it couples to the trace of the energy-momentum tensor  $T^\mu_\mu$ . Thus, the force on a test body is proportional to its mass  $M$ , regardless of its composition.

$$\vec{F} = -\beta \frac{\vec{\nabla} \phi}{M_{pl}} M \quad (3)$$

This implies that the acceleration  $\vec{a} = \vec{F}/M$  is independent of mass, satisfying the WEP.

## 3.2 Quantum Consistency (The "Final Boss")

### 3.2.1 The QCD Trace Anomaly and Loop Corrections

A subtle but critical challenge arises at the quantum level. The proton mass is dominated not by the quark masses (which couple to the Higgs/Dilaton), but by the QCD binding energy, determined by the confinement scale  $\Lambda_{QCD}$ . If  $\Lambda_{QCD}$  does not scale exactly like the Higgs VEV  $v$ , the proton-to-electron mass ratio  $\mu = m_p/m_e$  would vary, leading to catastrophic WEP violations.

In standard QFT, the scale  $\Lambda_{QCD}$  is generated by **\*\*dimensional transmutation\*\*** of the running coupling  $\alpha_s(\mu)$ . The running is governed by the beta function:

$$\mu \frac{dg_s}{d\mu} = \beta(g_s) = -bg_s^3 + \dots \quad (4)$$

Integration yields the scale  $\Lambda_{QCD}$ :

$$\Lambda_{QCD} = \mu \exp\left(-\frac{1}{2bg_s^2(\mu)}\right) \quad (5)$$

In a generic theory,  $\mu$  is an arbitrary subtraction scale. However, in a **\*\*Spontaneously Broken Scale Invariant Theory\*\***, there are no explicit mass scales in the Lagrangian. The only physical scale is the Dilaton VEV  $\chi$ . Therefore, the running coupling  $g_s$  can only depend on the dimensionless ratio of the energy scale to the symmetry breaking scale:

$$g_s(\mu) \rightarrow g_s\left(\frac{\mu}{\chi}\right) \quad (6)$$

Substituting this back into the definition of  $\Lambda_{QCD}$ , we find:

$$\Lambda_{QCD}(\chi) = \chi \cdot \mathcal{F}(g_{UV}) \propto \chi \quad (7)$$

Thus, the QCD confinement scale is **directly proportional** to the Dilaton field  $\chi$ . Since the electron mass also scales as  $m_e \propto v \propto \chi$ , we conclude that:

$$\frac{m_{proton}(\chi)}{m_{electron}(\chi)} \propto \frac{\Lambda_{QCD}(\chi)}{v(\chi)} \propto \frac{\chi}{\chi} = \text{const} \quad (8)$$

This proves that **\*\*Universal Conformal Coupling holds at the quantum loop level\*\***. The QCD trace anomaly, rather than breaking the equivalence principle, naturally tracks the Dilaton field, ensuring that all bound states scale uniformly. The "Fifth Force" remains composition-independent even when strong interaction contributions are included.

## 4 Prediction for Beta

The Dilaton coupling strength  $\beta$  is fixed by the geometry of the symmetry breaking. For a standard scalar-tensor theory (e.g., 5D Kaluza-Klein or String Theory), the coupling is often:

$$\beta = \frac{1}{\sqrt{6}} \approx 0.408 \quad (9)$$

Our phenomenological fit to Galaxy Rotation Curves (Paper 1) yields a survey mean of  $\beta \approx 0.60 \pm 0.33$ , which is consistent with the theoretical prediction of  $\beta \approx 0.41$ . This discrepancy suggests that the underlying theory might be a **\*\*Scalar-Tensor-Vector\*\*** theory or involve non-minimal couplings to the curvature (e.g.,  $R^2$  terms). However, the order of magnitude is consistent ( $\mathcal{O}(1)$ ), supporting the identification of the Machian scalar as a gravitational modulus.