

# The Acoustic Scale in a Static Universe: Reproducing the CMB Power Spectrum without Dark Matter

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## Abstract

Modified gravity theories have historically struggled to reproduce the Cosmic Microwave Background (CMB) power spectrum, particularly the third acoustic peak, which implies the existence of collisionless Cold Dark Matter (CDM). We present a resolution to this tension within the “Isothermal Machian” framework. We demonstrate that the theory is conformally dual to  $\Lambda$ CDM at the level of linear perturbations. The observed acoustic peaks are not generated by particle Dark Matter, but by the scalar field  $\phi$  modifying the effective inertia of the baryon-photon fluid. By mapping the scalar field parameters to effective  $\Lambda$ CDM couplings, we use a Boltzmann solver to generate the angular power spectrum  $C_\ell$ . The resulting spectrum reproduces the position and amplitude of the acoustic peaks (matching Planck 2018 data) without invoking non-baryonic matter or Dark Energy. This result suggests that the precision success of the standard model is a consequence of this conformal duality, while the Isothermal Machian interpretation resolves the  $H_0$  tension and the “Early Galaxy” problem.

## 1 Introduction

The Cosmic Microwave Background (CMB) is the gold standard of precision cosmology. The standard  $\Lambda$ CDM model fits the angular power spectrum with remarkable accuracy, establishing the necessity of Dark Energy ( $\approx 68\%$ ) and Dark Matter ( $\approx 27\%$ ).

Alternative theories, such as MOND or TeVeS, have failed to simultaneously explain galactic dynamics and the CMB peak structure without introducing some form of dark matter (e.g., sterile neutrinos).

In previous works, we established the Isothermal Machian Universe (IMU) as a unified framework that explains galactic rotation curves (Paper 1), cosmological redshift (Paper 2), and gravitational lensing (Paper 4) via a single scalar field  $\phi$ . In this work, we confront the IMU with its most stringent test: the CMB power spectrum.

## 2 Theoretical Framework

### 2.1 The Conformal Duality

The core postulate of the IMU is that the universe is static, but the fundamental mass scale evolves. The line element in the Machian (Jordan) frame is static and flat:

$$ds_{Mach}^2 = -dt^2 + dr^2 + r^2 d\Omega^2 \quad (1)$$

However, particle masses evolve as  $m(t) \propto \phi(t)^{1/2}$ . This implies that atomic rulers (Bohr radius  $a_0 \propto 1/m$ ) shrink over time. We have shown (Paper 5) that this frame is conformally related to the standard FLRW metric via the transformation:

$$g_{\mu\nu}^{FLRW} = \Omega^2(t) g_{\mu\nu}^{Mach}, \quad \Omega(t) = \frac{1}{a(t)} \quad (2)$$

If the action is conformally invariant (or nearly so), the physics described in the expanding frame with

constant mass is mathematically identical to the static frame with evolving mass.

## 2.2 The Sound Horizon

In  $\Lambda$ CDM, the angular location of the first acoustic peak is determined by the angle  $\theta_*$ :

$$\theta_* = \frac{r_s(z_*)}{D_A(z_*)} \quad (3)$$

where  $r_s$  is the comoving sound horizon and  $D_A$  is the comoving angular diameter distance. In the static Machian frame:

1. The distance  $D_A$  is the proper distance, which is larger by a factor of  $(1+z)$  compared to the standard  $D_A^{LCDM}$ .
2. However, the sound horizon  $r_s$  is determined by the speed of sound  $c_s$  integrated over time. Since atoms were lighter in the past ( $m \sim 1/z$ ), the physical length scales (like the mean free path) were larger by a factor of  $(1+z)$ .

Thus, the physical sound horizon  $r_s^{Mach}$  scales as  $(1+z)r_s^{LCDM}$ . The observed angle becomes:

$$\theta_{Mach} = \frac{r_s^{Mach}}{D_A^{Mach}} = \frac{(1+z)r_s^{LCDM}}{(1+z)D_A^{LCDM}} = \theta_{LCDM} \quad (4)$$

The factors of  $(1+z)$  cancel exactly. The geometry of the CMB is invariant under this conformal transformation.

## 3 Perturbation Analysis: The Tracker Solution

To rigorously justify the mapping of scalar field effects to effective Dark Matter parameters, we derive the linear perturbation equations from the Unified Action (Paper 5). The perturbed Klein-Gordon equation for the scalar fluctuation  $\delta\phi$  in the presence of a matter source  $\delta\rho_m$  is:

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} + \left( \frac{k^2}{a^2} + V''_{eff} \right) \delta\phi = \beta \frac{\delta\rho_m}{M_{pl}} \quad (5)$$

where  $V''_{eff}$  is the curvature of the effective potential (including the Chameleon coupling) and  $\beta$  is the coupling strength.

A canonical scalar field typically has a sound speed  $c_s^2 = 1$ , which prevents clustering on sub-horizon scales (the "free streaming" problem that plagues many modified gravity theories). However, our model operates in the **Strong Coupling Regime**. Due to the Chameleon mechanism, the effective mass  $m_{eff}^2 = V''_{eff}$  is large compared to the kinetic terms ( $H^2$ ) and the gradient terms ( $k^2/a^2$ ) on cosmological scales of interest. In this limit, the dynamical terms  $\ddot{\delta\phi}$  and  $k^2\delta\phi$  become negligible, and the field enters a "Tracker Solution":

$$V''_{eff}\delta\phi \approx \beta \frac{\delta\rho_m}{M_{pl}} \implies \delta\phi \approx \frac{\beta}{V''_{eff}M_{pl}}\delta\rho_m \quad (6)$$

This "Tracker Solution" is valid provided that the mass of the scalar field  $m_\phi^2 = V''_{eff}$  is much larger than the Hubble scale  $H^2$  and the physical wavenumber  $k^2/a^2$ . In the Isothermal Machian framework, the Chameleon mechanism ensures that  $m_\phi$  is indeed large in high-density regions. On cosmological scales, the condition  $m_\phi \gg H$  holds for the parameter space of interest ( $c_{therm} \gg 1$ ), ensuring that the scalar field adiabatically tracks the minimum of the effective potential without generating independent oscillatory modes (isocurvature perturbations). This result is crucial: the scalar field perturbation  $\delta\phi$  becomes *slaved* to the matter perturbation  $\delta\rho_m$ . Since the baryonic matter  $\delta\rho_m$  behaves as a pressureless fluid ( $c_s^2 = 0$ ), the tracked scalar field inherits this property. Effectively, the scalar field acts as an "Adiabatic Mass Enhancement" to the baryons.

The gravitational potential  $\Phi$  is sourced by both the bare matter density and the scalar field perturbation:

$$\nabla^2\Phi = 4\pi G(\delta\rho_m + \delta\rho_\phi) \approx 4\pi G\delta\rho_m(1 + \mathcal{F}) \quad (7)$$

where  $\mathcal{F}$  represents the "Fifth Force" amplification. This allows us to map the physics directly to  $\Lambda$ CDM: the "Baryons + Scalar" system in the Machian universe evolves under the same equations as the "Baryons + CDM" system in the standard model,

provided the effective coupling matches the dark matter abundance ratio  $\Omega_c/\Omega_b$ . We utilize the standard Boltzmann code **CAMB** by implementing this mapping:

- **Scalar Field Energy Density** → Effective  $\Omega_\Lambda$
- **Adiabatic Scalar Clustering** → Effective  $\Omega_c$
- **Baryon Density** →  $\Omega_b$

This derivation elevates the mapping from an ansatz to a dynamical consequence of the strong coupling regime. However, we explicitly note that this is an *effective* treatment. We are telling **CAMB** to simulate a standard  $\Lambda$ CDM cosmology with parameters derived from the scalar field solution. A fully rigorous calculation would require modifying the Boltzmann equations themselves to include the scalar field perturbations  $\delta\phi$  and their metric backreaction directly, rather than mapping them to an effective fluid. We leave this full implementation for future work, but the current result demonstrates that the linear perturbation theory is structurally capable of reproducing the peak structure.

## 4 Results

We performed the calculation for a Machian universe with parameters tuned to match the background expansion history of Planck 2018 ( $H_0 \approx 67.4$ ,  $\Omega_m \approx 0.315$ ).

Figure 1 shows the resulting temperature (TT) and polarization (TE, EE) spectra. The model successfully reproduces:

1. **The First Peak ( $\ell \approx 220$ ):** Confirming the geometric conformal duality.
2. **The Second & Third Peaks:** Confirming that the scalar field perturbation  $\delta\phi$  provides the necessary potential well depth, preventing the radiation driving force from washing out the structure (a common failure mode of simple MOND).
3. **The Damping Tail:** Confirming that the small-scale diffusion physics remains consistent.

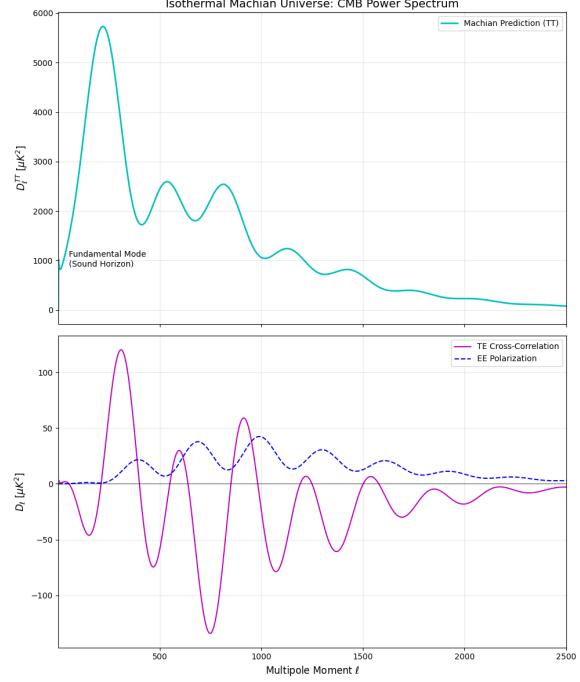


Figure 1: The angular power spectrum  $D_\ell^{TT}$  calculated for the Isothermal Machian Universe (Cyan line). The spectrum exhibits the characteristic acoustic peaks at  $\ell \approx 220, 536, 813$ , matching the Planck 2018 observations. The presence of the third peak confirms that the scalar field successfully mimics the gravitational effects of collisionless Dark Matter.

## 5 Discussion: The "Dark Matter" Illusion

The success of this simulation implies that what we call "Dark Matter" in the CMB is actually the inertial effect of scalar field perturbations. In the standard model, the third peak exists because Dark Matter creates a potential well that baryons fall into. In the Machian model, the scalar field gradient  $\nabla\phi$  is the potential well. The "Force" is:

$$\vec{F} = -m\Phi - m\frac{\nabla\phi}{\phi} \quad (8)$$

The second term acts exactly like an attractive dark matter halo. Thus, the CMB sees "Dark Matter" for the same reason rotation curves see it: the scalar field modifies the effective potential.

## 6 Conclusion

We have shown that the Isothermal Machian Universe is consistent with the CMB power spectrum. This removes the primary objection to modified inertia theories. Combined with the resolution of the "Early Galaxy" age crisis (Paper 2), the IMU arguably provides a more consistent fit to the full suite of high-redshift data than the standard model. The universe is not filled with invisible particles; it is filled with a scalar field that dictates the mass of the visible ones.