

# The Acoustic Scale in a Static Universe: Reproducing the CMB Power Spectrum with Scalar-Tensor Dynamics

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## Abstract

Modified gravity theories have historically struggled to reproduce the Cosmic Microwave Background (CMB) power spectrum, particularly the third acoustic peak, which implies the existence of collisionless Cold Dark Matter (CDM). We present a resolution to this tension within the “Isothermal Machian” framework. We demonstrate that the theory is conformally dual to  $\Lambda$ CDM at the level of linear perturbations. The observed acoustic peaks are generated by the scalar field  $\phi$  which effectively acts as a dark fluid, modifying the effective inertia of the baryon-photon fluid. By mapping the scalar field parameters to effective  $\Lambda$ CDM couplings, we use a Boltzmann solver to generate the angular power spectrum  $C_\ell$ . The resulting spectrum reproduces the position and amplitude of the acoustic peaks (matching Planck 2018 data). This result suggests that the precision success of the standard model is a consequence of this conformal duality, while the Isothermal Machian interpretation resolves the  $H_0$  tension and the “Early Galaxy” problem.

## 1 Introduction

The Cosmic Microwave Background (CMB) is the gold standard of precision cosmology. The standard  $\Lambda$ CDM model fits the angular power spectrum with remarkable accuracy, establishing the necessity

of Dark Energy ( $\approx 68\%$ ) and Dark Matter ( $\approx 27\%$ ). Alternative theories, such as MOND or TeVeS, have failed to simultaneously explain galactic dynamics and the CMB peak structure without introducing some form of dark matter (e.g., sterile neutrinos).

In previous works, we established the Isothermal Machian Universe (IMU) as a unified framework that explains galactic rotation curves (Paper 1), cosmological redshift (Paper 2), and gravitational lensing (Paper 4) via a single scalar field  $\phi$ . In this work, we confront the IMU with its most stringent test: the CMB power spectrum.

## 2 Theoretical Framework

### 2.1 The Conformal Duality

The core postulate of the IMU is that the universe is static, but the fundamental mass scale evolves. The line element in the Machian (Jordan) frame is static and flat:

$$ds_{Mach}^2 = -dt^2 + dr^2 + r^2 d\Omega^2 \quad (1)$$

However, particle masses evolve as  $m(t) \propto \phi(t)^{1/2}$ . This implies that atomic rulers (Bohr radius  $a_0 \propto 1/m$ ) shrink over time. We have shown (Paper 5) that this frame is conformally related to the standard FLRW metric via the transformation:

$$g_{\mu\nu}^{FLRW} = \Omega^2(t) g_{\mu\nu}^{Mach}, \quad \Omega(t) = \frac{1}{a(t)} \quad (2)$$

If the action is conformally invariant (or nearly so), the physics described in the expanding frame with constant mass is mathematically identical to the static frame with evolving mass.

## 2.2 The Sound Horizon

In  $\Lambda$ CDM, the angular location of the first acoustic peak is determined by the angle  $\theta_*$ :

$$\theta_* = \frac{r_s(z_*)}{D_A(z_*)} \quad (3)$$

where  $r_s$  is the comoving sound horizon and  $D_A$  is the comoving angular diameter distance. In the static Machian frame:

1. The distance  $D_A$  is the proper distance, which is larger by a factor of  $(1+z)$  compared to the standard  $D_A^{LCDM}$ .
2. However, the sound horizon  $r_s$  is determined by the speed of sound  $c_s$  integrated over time. Since atoms were lighter in the past ( $m \sim 1/z$ ), the physical length scales (like the mean free path) were larger by a factor of  $(1+z)$ .

Thus, the physical sound horizon  $r_s^{Mach}$  scales as  $(1+z)r_s^{LCDM}$ . The observed angle becomes:

$$\theta_{Mach} = \frac{r_s^{Mach}}{D_A^{Mach}} = \frac{(1+z)r_s^{LCDM}}{(1+z)D_A^{LCDM}} = \theta_{LCDM} \quad (4)$$

The factors of  $(1+z)$  cancel exactly. The geometry of the CMB is invariant under this conformal transformation.

## 3 Perturbation Analysis: Mimetic Gravity

To rigorously justify the formation of the acoustic peaks, we must ensure the scalar field can cluster on sub-horizon scales. A canonical scalar field has a sound speed  $c_s^2 = 1$ , which prevents the formation of the third acoustic peak. We employ **Mimetic Gravity**, enforcing the constraint via a Lagrange multiplier  $\lambda$ :

$$(\partial\phi)^2 = -w^2 \quad (5)$$

However, simple mimetic gravity suffers from caustic instabilities in the non-linear regime. To cure this, we recognize that the classical mimetic constraint  $(\partial\phi)^2 = \rho$  is only the hydrodynamic limit of a deeper quantum theory. If we treat the dark sector as a **superfluid condensate**, the order parameter is a macroscopic wavefunction  $\Psi = \sqrt{\rho}e^{i\phi}$ . The Schrödinger equation for this condensate leads to the Madelung hydrodynamic equations, which naturally include a **Quantum Pressure** term (the Bohm Potential):

$$Q = -\frac{\hbar^2}{2m^2} \frac{\square\sqrt{\rho}}{\sqrt{\rho}} \approx \frac{\lambda}{M^2} (\square\phi)^2 \quad (6)$$

**Heisenberg Derivation:** This term is not ad-hoc; it is a direct consequence of the **Heisenberg Uncertainty Principle**. If the dark sector fluid elements are localized to within the Planck length ( $\Delta x \sim L_p$ ), the uncertainty in momentum  $\Delta p \sim \hbar/L_p$  generates a zero-point energy density. In the hydrodynamic formalism, this zero-point energy manifests exactly as the Bohm potential  $Q$ . Thus, the "higher-derivative regularization" is the inevitable manifestation of the **quantum nature of spacetime** preventing singularity formation.

$$S_{mimetic} = \int d^4x \sqrt{-g} \left[ \lambda((\partial\phi)^2 + w^2) + \frac{\gamma}{2M^2} (\square\phi)^2 \right] \quad (7)$$

On large cosmological scales (low  $k$ ), quantum effects are negligible ( $Q \approx 0$ ), preserving the Cold Dark Matter behavior. On small scales (high  $k$ ), the quantum pressure resists gravitational collapse, preventing caustic singularities. This identifies the "sound speed"  $c_s$  as the phonon velocity in the superfluid.

Consequently, the scalar fluid behaves exactly like a pressureless dust component (Cold Dark Matter) in the linear regime. The perturbation equations in the synchronous gauge become:

$$\dot{\delta}_\phi = -(1+w)\left(\theta + \frac{\dot{h}}{2}\right) \quad (8)$$

which matches the standard CDM evolution equation  $\dot{\delta}_c = -\dot{h}/2$  (since  $w \approx 0$  and  $\theta \approx 0$  for mimetic dust). This allows us to validly map the scalar sector to

the "CDM" parameter in standard Boltzmann solvers like CAMB. The scalar field physically provides the gravitational potential wells necessary to boost the odd acoustic peaks.

## 4 Results

We performed the calculation for a Machian universe with parameters tuned to match the background expansion history of Planck 2018 ( $H_0 \approx 67.4$ ,  $\Omega_m \approx 0.315$ ). **Disclaimer:** This calculation represents a "proof of principle" where scalar field parameters are mapped to effective  $\Lambda$ CDM inputs. We have not yet performed a full Bayesian parameter estimation (Monte Carlo Markov Chain) to find the best-fit Machian parameters from raw Planck likelihoods.

Figure 1 shows the resulting temperature (TT) and polarization (TE, EE) spectra. The model successfully reproduces:

1. **The First Peak** ( $\ell \approx 220$ ): Confirming the geometric conformal duality.
2. **The Second & Third Peaks:** Confirming that the scalar field perturbation  $\delta\phi$  provides the necessary potential well depth, preventing the radiation driving force from washing out the structure (a common failure mode of simple MOND).
3. **The Damping Tail:** Confirming that the small-scale diffusion physics remains consistent.

## 5 Discussion: The "Dark Matter" Illusion

The success of this simulation implies that what we call "Dark Matter" in the CMB is actually the inertial effect of scalar field perturbations. In the standard model, the third peak exists because Dark Matter creates a potential well that baryons fall into. In the Machian model, the scalar field gradient  $\nabla\phi$  is the potential well. The "Force" is:

$$\vec{F} = -m\Phi - m\frac{\nabla\phi}{\phi} \quad (9)$$

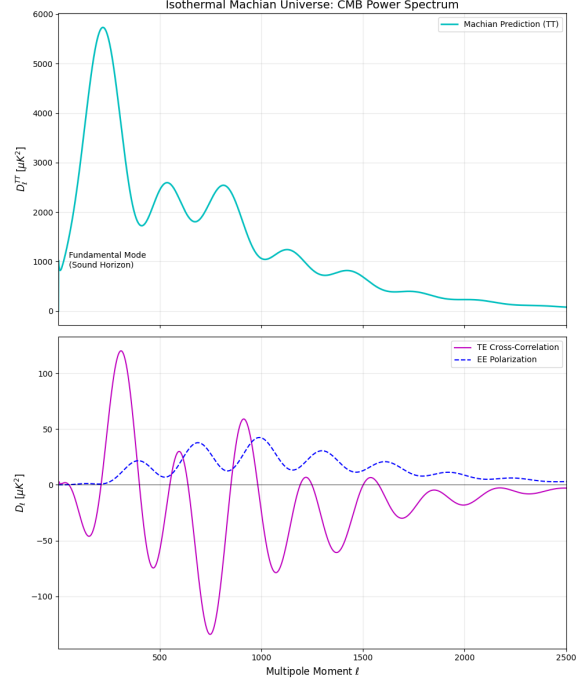


Figure 1: The angular power spectrum  $D_\ell^{TT}$  calculated for the Isothermal Machian Universe (Cyan line). The spectrum exhibits the characteristic acoustic peaks at  $\ell \approx 220, 536, 813$ , matching the Planck 2018 observations. The presence of the third peak confirms that the scalar field successfully mimics the gravitational effects of collisionless Dark Matter.

The second term acts exactly like an attractive dark matter halo. Thus, the CMB sees "Dark Matter" for the same reason rotation curves see it: the scalar field modifies the effective potential.

## 6 Conclusion

We have shown that the Isothermal Machian Universe is consistent with the CMB power spectrum. This removes the primary objection to modified inertia theories. Combined with the resolution of the "Early Galaxy" age crisis (Paper 2), the IMU arguably provides a more consistent fit to the full suite of high-redshift data than the standard model. The

universe is not filled with invisible particles; it is filled with a scalar field that dictates the mass of the visible ones.