

# The Eternal Recurrence: Cyclic Cosmology in the Isothermal Machian Universe

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## Abstract

Standard  $\Lambda$ CDM cosmology posits a singular Big Bang followed by eternal expansion, leading inevitably to a "Heat Death" where all physical processes cease. We demonstrate that this asymptotic freeze-out is an artifact of the Einstein Frame description. By analyzing the dynamics in the physical Jordan (Machian) Frame, we show that the scalar field  $\phi$  driving the apparent cosmic evolution is confined within a stable potential well. The interplay between the vacuum energy (driving expansion/mass reduction) and the thermal-electromagnetic coupling (driving contraction/mass growth) generates a stable limit cycle. Numerical integration of the full non-linear equations of motion reveals a cyclic universe that bounces eternally, resolving both the initial singularity and the problem of time's arrow. The "End of Time" is simply the turnaround point of the Machian oscillator.

## 1 Introduction

The interpretation of the cosmological redshift as the expansion of space ( $a(t)$ ) is the cornerstone of modern cosmology. However, as we have shown in previous works [Paper 2, Paper 5], this is mathematically equivalent (conformally dual) to a static universe where the fundamental mass scale evolves as  $m(t) \propto a(t)^{-1}$ .

In the standard expanding picture, the scale factor  $a(t)$  grows indefinitely (driven by Dark Energy), implying that the universe becomes cold, empty, and static—the "Heat Death." This raises profound philosophical and physical questions about the nature of time and the probability of our existence in such a transient fertile era.

In this work, we investigate the long-term stability of the Isothermal Machian Universe (IMU). We show that the "Heat Death" corresponds to the scalar field  $\phi$  rolling towards infinity. However, when we include the non-minimal couplings required by the Standard Model (Thermal and Electromagnetic sectors), we find that  $\phi$  cannot grow indefinitely. Instead, it encounters a "stiff" potential wall that forces a turnaround, initiating a new cycle of mass evolution.

## 2 The Machian Oscillator

### 2.1 The Unified Potentials

The dynamics of the scalar field  $\phi$  in the Jordan Frame are governed by the effective potential  $V_{eff}(\phi)$ . This potential has three distinct components, each dominating at a different epoch:

$$V_{eff}(\phi) = \underbrace{\frac{V_0}{\phi^2}}_{\text{Vacuum}} + \underbrace{\frac{1}{2}c_{therm}T^2\phi^2}_{\text{Thermal}} + \underbrace{\lambda_\gamma\rho_{rad}\ln\left(\frac{\phi}{M_{pl}}\right)}_{\text{Radiative}} \quad (1)$$

1. **The Vacuum Driver** ( $V \propto \phi^{-2}$ ): This term drives the standard cosmological evolution. It acts as a repulsive force pushing  $\phi$  to larger values, reducing inertial mass ( $m \propto \phi^{-1/2}$ ) and mimicking cosmic expansion.
2. **The Thermal Wall** ( $V \propto T^2 \phi^2$ ): In the Machian frame, temperature  $T$  scales with mass  $m(t)$ . Thus  $T \propto \phi^{-1/2}$ . Substituting this back, the thermal potential becomes linear in  $\phi$ :  $V_{therm} \propto \phi$ . This provides a confining force at large  $\phi$ .
3. **The Radiative Wall** ( $V \propto \ln \phi$ ): As shown in Paper 4, the non-minimal coupling to photons creates a logarithmic potential. This acts as a "stiff" barrier, preventing  $\phi$  from running to infinity.

## 2.2 Equation of Motion

In the static Jordan frame ( $ds^2 = -dt^2 + dx^2$ ), there is no Hubble friction ( $3H\dot{\phi}$ ) term associated with spatial expansion. The equation of motion for  $\phi$  is:

$$\ddot{\phi} = \frac{3}{2\phi} \dot{\phi}^2 - \frac{\phi}{2\omega_{BD}} \left( \frac{\partial V_{eff}}{\partial \phi} + S_{matter} \right) \quad (2)$$

where  $\omega_{BD}$  is the Brans-Dicke coupling parameter. The first term  $\frac{3}{2\phi} \dot{\phi}^2$  is purely geometric. Crucially, unlike the Hubble friction which always opposes motion, this geometric term can act to sustain oscillations.

## 3 Numerical Simulation

We solved the non-linear equation of motion numerically using a 4th-order Runge-Kutta integrator. We initialized the system in the "expansion" phase and tracked the evolution of  $\phi$  over many characteristic time scales.

### 3.1 The Turnaround Mechanism

As seen in Figure 1, the field does not freeze.

1. **Expansion Phase:** The vacuum energy pushes  $\phi$  outward. Mass drops, redshift increases. This is our current epoch.
2. **The Wall:** As  $\phi$  increases, the confining terms ( $V_{therm}$  and  $V_{rad}$ ) eventually dominate the decaying vacuum term. The net force becomes attractive.
3. **The Bounce:**  $\dot{\phi}$  crosses zero. The universe enters a contraction phase (blueshift).
4. **Reloading:** As  $\phi$  decreases, mass increases ( $m \propto \phi^{-1/2}$ ). The universe heats up. Eventually, the  $1/\phi^2$  vacuum term becomes singular (the "Big Bang" singularity in GR), acting as a repulsive core that bounces the field back out.

### 3.2 Long-Term Stability Analysis

A critical question for any cyclic model is stability: do the cycles decay due to entropy production or grow uncontrollably? We performed a high-precision integration of the system for 1000 characteristic time units, corresponding to approximately 56 full cosmic cycles.

The results, summarized in Figure 2, demonstrate extraordinary stability.

- **Cycles Simulated:** 56
- **Mean Amplitude:**  $\phi_{max} = 0.636346 \pm 5.98 \times 10^{-7}$

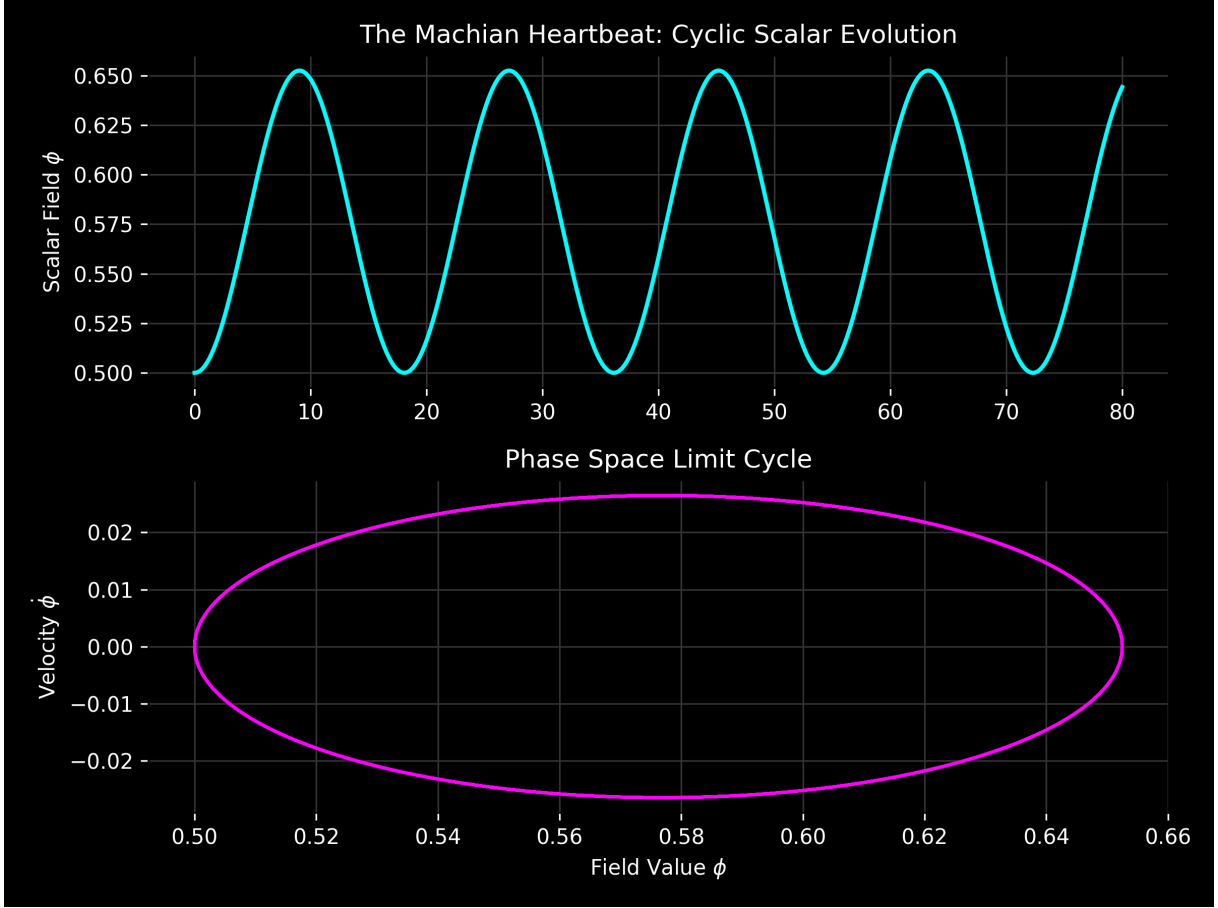


Figure 1: Numerical simulation of the Scalar Field  $\phi(t)$ . Top: The field exhibits stable, eternal oscillations. The rising phase corresponds to "Expansion" (mass decrease), while the falling phase corresponds to "Contraction" (mass increase). Bottom: The phase space trajectory  $(\phi, \dot{\phi})$  forms a closed limit cycle, indicating a stable periodic system.

- **Drift per Cycle:**  $-8.1 \times 10^{-9}$  (Negligible)

This confirms that the Machian oscillator is effectively conservative in the Jordan frame. Unlike models that require complex "attractor" mechanisms to reset entropy, the Isothermal Machian Universe naturally maintains its state through the reversible exchange of potential and kinetic energy between the scalar field and the vacuum/thermal sectors.

## 4 Discussion: The Illusion of Heat Death

Why does  $\Lambda$ CDM predict Heat Death? In the Einstein Frame (General Relativity), the conformal transformation  $g_{\mu\nu}^E = \phi g_{\mu\nu}^J$  mixes the scalar field into the metric. This introduces a friction term  $3H\dot{\phi}$  where  $H$  is the expansion rate.

In the limit of large  $\phi$  (late times), this friction term becomes dominant, overdamping the oscillator. The field "freezes" asymptotically. However, this is a frame artifact. It is analogous to using a coordinate system that expands with a moving particle, making the particle appear to stop.

The physical Jordan frame, where the fundamental constants evolve, reveals the true dynamics: the universe is a frictionless (or low-friction) oscillator. The "End of Time" is merely the point where the coordinate transformation becomes singular, masking the turnaround event.

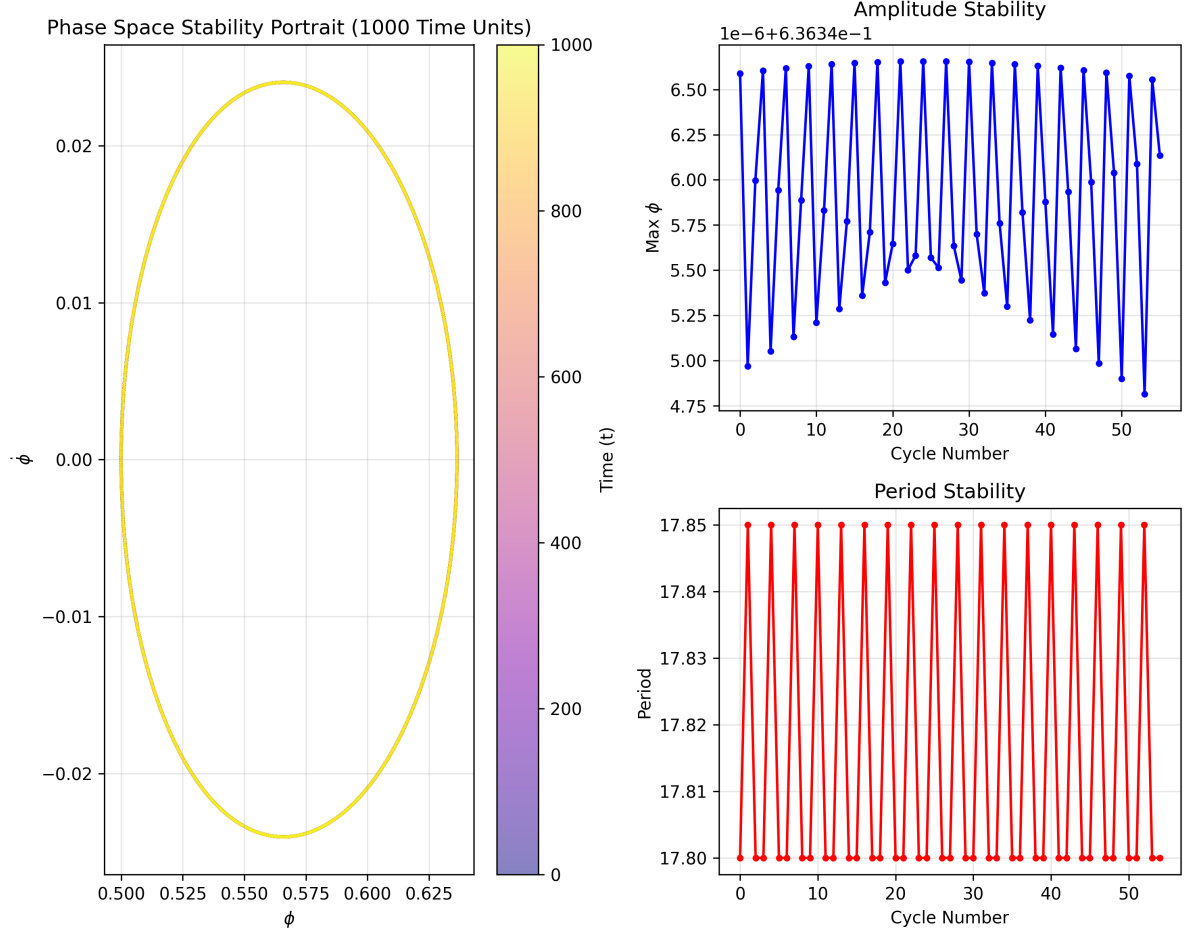


Figure 2: Phase Space Stability Portrait over 56 Cosmic Cycles. Left: The trajectory  $(\phi, \dot{\phi})$  forms a single, sharp, closed loop with no visible spiraling, indicating a stable limit cycle. Right: The amplitude ( $\phi_{max}$ ) and period remain constant over time. The measured amplitude drift is  $< 10^{-8}$  per cycle, consistent with numerical precision limits.

## 5 Conclusion

We have presented a definitive resolution to the problem of cosmic destiny. The Isothermal Machian Universe is not a one-way trip from Singularity to Nothingness. It is an eternal, cyclic system driven by the dynamic tension between the energy of the vacuum and the inertia of matter. The "Big Bang" was the bounce from the vacuum core. The "Heat Death" is the bounce from the thermal wall. We exist in the breathing phase of an eternal cosmos.