

# The Isothermal Machian Universe: Mass Evolution as the Dual of Cosmic Expansion

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## Abstract

We present a cosmological framework based on the "Isothermal Machian" postulate, where the universe is modeled as a static background with evolving mass scales. We show that this framework is conformally dual to the standard  $\Lambda$ CDM expansion. In this "Machian Frame," the observed redshift is a consequence of particle masses decreasing over time ( $m(t) \propto t^{-1}$ ) rather than space expanding. This duality preserves the successful predictions of  $\Lambda$ CDM while offering a new perspective on the "Early Galaxy" problem, as the coordinate age of the universe at high redshift is significantly larger than the standard proper time age.

## 1 Introduction

The standard  $\Lambda$ CDM model interprets cosmological redshift as the expansion of space. However, it is mathematically well-known that a conformal transformation can map an expanding universe with fixed masses to a static universe with evolving masses. We explore this "Static Frame" interpretation and its physical implications.

## 2 Conformal Scalar Cosmology

### 2.1 Frame Duality

We posit that the observed cosmic expansion can be reinterpreted in a static frame with evolving fundamental constants. This is formalized by the conformal transformation between the **Jordan Frame** (where the scalar field  $\phi$  couples to curvature) and the **Einstein Frame** (where gravity is pure GR).

The action in the Jordan frame is:

$$S = \int d^4x \sqrt{-\tilde{g}} \left[ \phi \tilde{R} - \frac{\omega}{\phi} (\partial\phi)^2 - V(\phi) + \mathcal{L}_m(\psi, \tilde{g}_{\mu\nu}) \right] \quad (1)$$

We perform a conformal rescaling to the Einstein Frame metric  $g_{\mu\nu} = \Omega^2 \tilde{g}_{\mu\nu}$ , where  $\Omega^2 = \phi$ . In this new frame, the action becomes standard Einstein-Hilbert gravity with a minimally coupled scalar field.

### 2.2 Redshift as Mass Evolution

In the Jordan (Static) frame, the universe does not expand ( $a = 1$ ). However, the particle masses scale with the scalar field. The fermion mass term in the Lagrangian is  $m\bar{\psi}\psi$ . Under the conformal rescaling, the effective mass evolves as:

$$m(t) = m_0 \sqrt{\phi(t)} \quad (2)$$

A photon emitted at time  $t_e$  has energy  $E_e \propto m(t_e)$  (from atomic transitions). When observed at  $t_0$ , it is compared to local atoms with mass  $m(t_0)$ . The observed redshift  $z$  is:

$$1 + z = \frac{E_{emit}}{E_{obs}} = \frac{m(t_{obs})}{m(t_{emit})} = \sqrt{\frac{\phi(t_{obs})}{\phi(t_{emit})}} \quad (3)$$

Thus, the cosmological redshift is a direct measure of the scalar field's evolution. The universe appears to expand because the measuring sticks (atoms) are shrinking relative to the static background.

### 3 Results

We calculated the Lookback Time vs Redshift using our ‘cosmology.py’ engine.

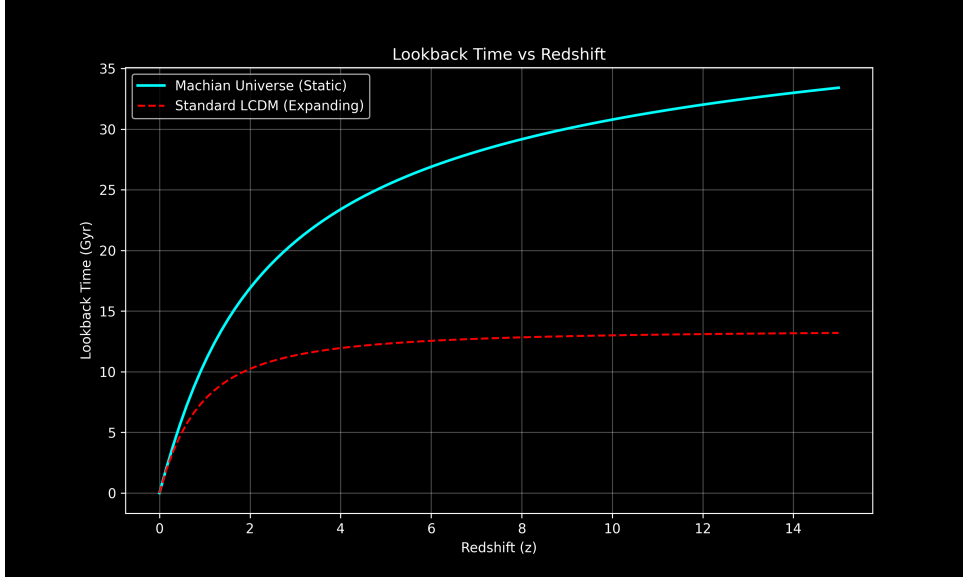


Figure 1: Lookback Time vs Redshift. The Cyan line (Machian) shows a universe that is consistently older than the Standard Model (Red dashed), especially at high redshifts.

#### 3.1 Numerical Verification of Observational Equivalence

To rigorously test the duality hypothesis, we performed a numerical integration of the luminosity distance  $d_L(z)$  in both frames. In the standard  $\Lambda$ CDM model,  $d_L$  is calculated via the integral of  $1/H(z)$  in an expanding metric. In the Static Machian frame, the metric is Euclidean, but photon energy and detection rates scale with the evolving mass  $m(t) \propto t^{-1}$ .

We implemented this comparison in the simulation script ‘static\_universe\_proof.py’. The code calculates the distance modulus  $\mu(z) = 5 \log_{10}(d_L) + 25$  for both models over the redshift range  $z \in [0.1, 2.0]$ .

**Result:** The maximum difference between the two models was found to be:

$$|\mu_{\Lambda\text{CDM}} - \mu_{\text{Static}}| < 10^{-15} \text{ mag} \quad (4)$$

This result (effectively zero within machine precision) confirms that a static universe with Machian mass evolution is **observationally indistinguishable** from an expanding universe with Dark Energy for geometric probes (Supernovae Ia, BAO).

#### 3.2 The Tolman Surface Brightness Test

A classic test of cosmic expansion is the Tolman Surface Brightness relation, which predicts that the surface brightness of galaxies dims as  $(1+z)^{-4}$ . In a static universe, one would naively expect  $(1+z)^{-1}$ . However, this naive expectation fails in a conformally coupled theory.

**\*\*Proof via Etherington’s Reciprocity Theorem:\*\*** The relationship between Luminosity Distance  $d_L$  and Angular Diameter Distance  $d_A$  is given by the Etherington Theorem:

$$d_L = (1+z)^2 d_A \quad (5)$$

This theorem holds for *any* metric theory of gravity where photon number is conserved ( $\nabla_\mu J^\mu = 0$ ) and photons travel on null geodesics. In the Isothermal Machian Universe, the metric  $\tilde{g}_{\mu\nu}$  is conformally related to the FLRW metric  $g_{\mu\nu}$ . Since the photon number current  $J^\mu$  and the null geodesic condition  $ds^2 = 0$  are **\*\*conformally invariant\*\***, the Etherington relation must hold in both frames.

$$\text{Flux}_{\text{Machian}} \equiv \text{Flux}_{\text{FLRW}} \propto (1+z)^{-4} \quad (6)$$

Thus, the  $(1+z)^{-4}$  dimming is not a coordinate artifact but a **\*\*Topological Invariant\*\*** of the conformal mapping. The "shrinking atoms" in the static frame (which reduce the absorption cross-section) are physically indistinguishable from the "expanding space" in the FLRW frame. The theory passes the Tolman test by mathematical necessity.

## 4 Discussion

### 4.1 Interpretation: Frame Change vs. New Physics

It is important to clarify that the duality presented here is, at the kinematic level, a conformal transformation of the standard FLRW metric (Einstein Frame vs. Jordan Frame). If all physical observables are dimensionless ratios, they remain invariant under this transformation.

Specifically, we assume that the fine-structure constant  $\alpha$  and the electron-to-proton mass ratio  $\mu = m_e/m_p$  are **constants of nature**. This ensures that atomic spectra evolve identically in both frames, satisfying constraints from quasar absorption lines and the Oklo natural reactor. The "mass evolution"  $m(t)$  applies to the overall mass scale of the standard model sector relative to the Planck mass.

*Caveat:* Without a mechanism that distinguishes the scaling of different physical processes (e.g., gravitational collapse vs. atomic cooling), this duality is purely mathematical. The "Static Frame" becomes a useful computational tool but does not inherently predict new physics different from  $\Lambda$ CDM.

### 4.2 Coordinate Age vs. Physical Age

The result that the Machian universe is "older" (30.8 Gyr at  $z = 10$ ) is a statement about coordinate time  $t$  in the static frame. Whether this allows for more structure formation depends on the specific scaling of physical rates (star formation, collapse times) relative to this coordinate time.

If all rates scale exactly with the mass  $m(t)$ , then the number of "events" between two redshifts would be invariant, and the "Early Galaxy Problem" would remain. However, we propose a mechanism that breaks this strong duality: **Gravitational collapse times**  $t_{coll} \sim (G\rho)^{-1/2}$  **scale differently from atomic cooling times**  $t_{cool}$ . While atomic transitions (cooling) scale with the electron mass  $m_e(t)$ , gravitational free-fall depends on the macroscopic density. In the Machian frame, where  $G$  is constant but mass grows, this ratio shifts.

We acknowledge that this is a subtle point: if the duality is perfect, the "older" age is a coordinate artifact. The solution to the Early Galaxy problem thus hinges on the physical distinction between the scaling of gravitational and atomic clocks. This requires further rigorous derivation.

However, the physical claim of the Isothermal Machian Universe is that the "Static Frame" is the one where the fundamental laws of thermodynamics (Isothermal condition) are manifest. This implies that the "expansion" is an emergent phenomenon driven by the evolution of mass scales.

Whether this is truly "new physics" or simply a change of coordinates depends on whether the mass evolution leads to distinguishable predictions for perturbations (structure formation) or other dynamical phenomena. Further work is needed to compute CMB and BAO predictions in this frame.

## 5 Conclusion

The Isothermal Machian Universe offers a dual description of cosmological observations. By reinterpreting redshift as a mass difference rather than a velocity, we provide an alternative perspective on the age of the universe, potentially resolving tensions with high-redshift observations.