Theoretical Addendum:
The Machian Scalar as the Dilaton of Broken Scale Invariance

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Abstract

We propose a Microscopic UV Completion for the Isothermal Machian Universe. We identify the Machian scalar field ϕ as the pseudo-Nambu-Goldstone boson (Dilaton) of Spontaneously Broken Scale Invariance. We demonstrate that this identification naturally enforces the **Universal Conformal Coupling** required to satisfy Weak Equivalence Principle (MICROSCOPE) constraints. Furthermore, the theoretical prediction for the scalar-matter coupling strength, $\beta_{theory} = 1/\sqrt{6} \approx 0.41$, is in excellent agreement with the phenomenological value $\beta_{obs} \approx 0.60 \pm 0.33$ derived from galaxy rotation curves.

1 The Hierarchy Problem and Scale Invariance

The Standard Model of Particle Physics suffers from the Hierarchy Problem: why is the Higgs mass (125 GeV) so much smaller than the Planck mass (10^{19} GeV)? A popular solution is that the underlying theory is **Scale Invariant** (Conformal) in the UV, and this symmetry is spontaneously broken at some scale f.

In such a theory, the Planck mass is not a fundamental constant but the Vacuum Expectation Value (VEV) of a scalar field χ (the Dilaton).

$$S_{Jordan} = \int d^4x\sqrt{-\tilde{g}}\left[\frac{\chi^2}{12}\tilde{R} - \frac{1}{2}(\partial\chi)^2 - \lambda\chi^4 + \mathcal{L}_m(\psi,\chi)\right] \tag{1}$$

2 Derivation of Universal Coupling

To move to the Einstein Frame (where gravity is canonical), we perform the conformal transformation:

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = \frac{M_{pl}^2}{\chi^2} \tag{2}$$

The canonical scalar field ϕ is defined by the field redefinition:

$$\chi = f e^{\phi/(\sqrt{6}f)} \tag{3}$$

This yields the standard Einstein-Hilbert action plus a massless scalar.

2.1 Matter Sector

Crucially, in the UV scale-invariant theory, elementary particles (fermions) are massless. They acquire mass only via the Higgs mechanism. However, the Higgs VEV v itself must be proportional to the symmetry breaking scale χ:

$$v(\chi) \propto \chi \tag{4}$$

In the Einstein frame, the physical mass of a fermion f is:

$$m_f(\phi) = y_f v(\phi) = y_f v_0 e^{\phi/(\sqrt{6}M_{pl})} \tag{5}$$

This has the exact form of our **Universal Conformal Coupling** ansatz:

$$m(\phi) \propto e^{\beta\phi/M_{pl}} \tag{6}$$

Matching the exponents yields a precise theoretical prediction for the coupling constant:

$$\beta_{theory} = \frac{1}{\sqrt{6}} \approx 0.408 \tag{7}$$

3 Observational Verification

3.1 Universal Coupling (WEP)

Because all masses (quarks, electrons, W/Z bosons) derive their mass from the same Higgs VEV, they all couple to the Dilaton with the *exact same* strength β.

$$\Delta\beta = \beta_{electron} - \beta_{proton} = 0 \tag{8}$$

This creates a natural protection mechanism for the Weak Equivalence Principle. The stringent bounds from MICROSCOPE ($\eta < 10^{-15}$) are satisfied not by tuning parameters, but by the Ward Identities of the underlying symmetry.

3.2 Galaxy Rotation Curves

In Paper 1, we fitted the mass profile $m(r) \propto e^{\beta\phi(r)}$ to the SPARC galaxy NGC 6503. The phenomenological fit yielded:

$$\beta_{obs} \approx 0.98 \tag{9}$$

However, our subsequent survey of 20 galaxies (Paper 5) yielded a mean ensemble value:

$$\beta_{survey} \approx 0.60 \pm 0.33 \tag{10}$$

The theoretical prediction $\beta_{theory} \approx 0.41$ lies well within the 1σ error bar of the observational survey. This suggests that the "Dark Matter" observed in galaxies is indeed the gravitational footprint of the Dilaton field.

4 Conclusion

We have identified the physical origin of the Isothermal Machian scalar field. It is the Dilaton associated with the spontaneous breaking of scale invariance. This identification solves the fine-tuning problem of the model, protects it from WEP violations, and provides a quantitative prediction for the strength of the Fifth Force that matches astrophysical data.