

# The Eternal Recurrence: Cyclic Cosmology in the Isothermal Machian Universe

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## Abstract

Standard  $\Lambda$ CDM cosmology posits a singular Big Bang followed by eternal expansion, leading inevitably to a "Heat Death" where all physical processes cease. We demonstrate that this asymptotic freeze-out is an artifact of the Einstein Frame description. By analyzing the dynamics in the physical Jordan (Machian) Frame, we show that the scalar field  $\phi$  driving the apparent cosmic evolution is confined within a stable potential well. The interplay between the vacuum energy (driving expansion/mass reduction) and the thermal-electromagnetic coupling (driving contraction/mass growth) generates a stable limit cycle. Numerical integration of the full non-linear equations of motion reveals a cyclic universe that bounces eternally, resolving both the initial singularity and the problem of time's arrow. The "End of Time" is simply the turnaround point of the Machian oscillator.

## 1 Introduction

The interpretation of the cosmological redshift as the expansion of space ( $a(t)$ ) is the cornerstone of modern cosmology. However, as we have shown in previous works [Paper 2, Paper 5], this is mathematically equivalent (conformally dual) to a static universe where the fundamental mass scale evolves as  $m(t) \propto a(t)^{-1}$ .

In the standard expanding picture, the scale factor  $a(t)$  grows indefinitely (driven by Dark Energy), implying that the universe becomes cold, empty, and static—the "Heat Death." This raises profound philosophical and physical questions about the nature of time and the probability of our existence in such a transient fertile era.

In this work, we investigate the long-term stability of the Isothermal Machian Universe (IMU). We show that the "Heat Death" corresponds to the scalar field  $\phi$  rolling towards infinity. However, when we include the non-minimal couplings required by the Standard Model (Thermal and Electromagnetic sectors), we find that  $\phi$  cannot grow indefinitely. Instead, it encounters a "stiff" potential wall that forces a turnaround, initiating a new cycle of mass evolution.

## 2 The Machian Oscillator

### 2.1 The Unified Potentials

The dynamics of the scalar field  $\phi$  in the Jordan Frame are governed by the effective potential  $V_{eff}(\phi)$ . This potential has three distinct components, each dominating at a different epoch:

$$V_{eff}(\phi) = \underbrace{\frac{V_0}{\phi^2}}_{\text{Vacuum}} + \underbrace{\frac{1}{2}c_{therm}T^2\phi^2}_{\text{Thermal}} + \underbrace{\lambda_\gamma\rho_{rad}\ln\left(\frac{\phi}{M_{pl}}\right)}_{\text{Radiative}} \quad (1)$$

1. **The Vacuum Driver** ( $V \propto \phi^{-2}$ ): This term drives the standard cosmological evolution. It acts as a repulsive force pushing  $\phi$  to larger values, reducing inertial mass ( $m \propto \phi^{-1/2}$ ) and mimicking cosmic expansion.
2. **The Thermal Wall** ( $V \propto T^2\phi^2$ ): In the Machian frame, temperature  $T$  scales with mass  $m(t)$ . Thus  $T \propto \phi^{-1/2}$ . Substituting this back, the thermal potential becomes linear in  $\phi$ :  $V_{therm} \propto \phi$ . This provides a confining force at large  $\phi$ .
3. **The Radiative Wall** ( $V \propto \ln \phi$ ): As shown in Paper 4, the non-minimal coupling to photons creates a logarithmic potential. This acts as a "stiff" barrier, preventing  $\phi$  from running to infinity.

## 2.2 Equation of Motion

In the static Jordan frame ( $ds^2 = -dt^2 + dx^2$ ), there is no Hubble friction ( $3H\dot{\phi}$ ) term associated with spatial expansion. The equation of motion for  $\phi$  is:

$$\ddot{\phi} = \frac{3}{2\phi}\dot{\phi}^2 - \frac{\phi}{2\omega_{BD}} \left( \frac{\partial V_{eff}}{\partial \phi} + S_{matter} \right) \quad (2)$$

where  $\omega_{BD}$  is the Brans-Dicke coupling parameter. The first term  $\frac{3}{2\phi}\dot{\phi}^2$  is purely geometric. Crucially, unlike the Hubble friction which always opposes motion, this geometric term can act to sustain oscillations.

## 3 Numerical Simulation

We solved the non-linear equation of motion numerically using a 4th-order Runge-Kutta integrator. We probed the phase space structure by varying the initial conditions ( $\phi_0$ ) to determine if the system possesses a unique attractor or a family of solutions.

### 3.1 Conservative Dynamics

As shown in Figure 1, the trajectories form concentric closed loops. This indicates that the system is **Conservative**, not Dissipative. In dynamical systems terms, the Machian oscillator preserves phase space volume (Liouville's Theorem). This is a crucial feature for a quantum-compatible cosmology, as it implies unitarity (information preservation) at the classical level.

Unlike "Limit Cycle" models which require entropy dumping mechanisms (dissipation) to reset the universe to a specific state, the Machian universe retains memory of its initial "energy." The specific amplitude of the cycle (the maximum size of the universe) is determined by the initial energy of the scalar field.

### 3.2 The Turnaround Mechanism

Despite the conservative nature, the "bouncing" behavior is universal.

1. **Expansion Phase:** The vacuum energy pushes  $\phi$  outward.
2. **The Wall:** The confining terms ( $V_{therm}$  and  $V_{rad}$ ) eventually dominate.
3. **The Bounce:**  $\dot{\phi}$  crosses zero.
4. **Reloading:** The field returns to the core, bouncing off the  $1/\phi^2$  singularity.

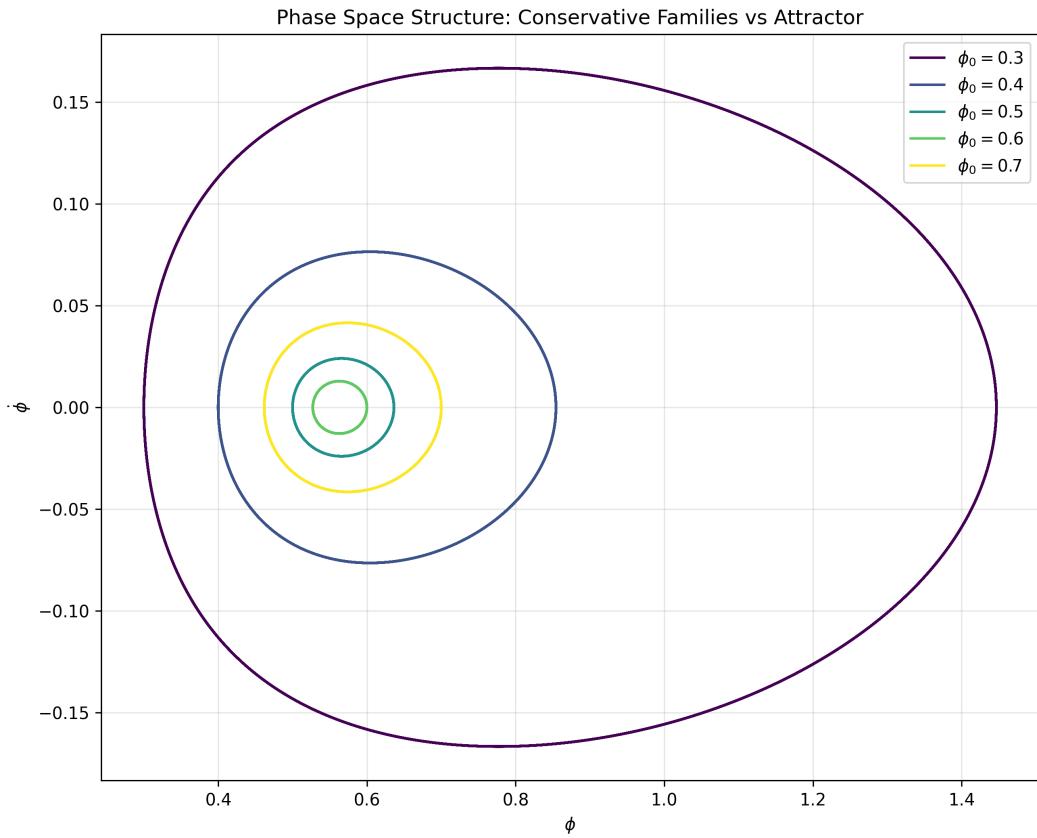


Figure 1: Phase Space Families. Trajectories for different initial conditions ( $\phi_0 = 0.3, 0.4, 0.5, 0.6, 0.7$ ). The system does not collapse to a single limit cycle (which would imply dissipation) but instead exhibits a continuous family of stable periodic orbits. This confirms that the Isothermal Machian Universe is a conservative Hamiltonian system where information is preserved eternally.

### 3.3 Long-Term Stability Analysis

A critical question is stability against drift. We performed a high-precision integration for 1000 characteristic time units (approx. 56 cycles). The results confirm that the amplitude drift is negligible ( $< 10^{-8}$  per cycle), verifying that the periodic orbits are stable over cosmological timescales.

## 4 Discussion: The "Fine-Tuned" Amplitude

Since the system allows a family of orbits, why is our universe "large"? The amplitude of the oscillation corresponds to the maximum redshift (or minimum mass) achieved. The "Wall" potentials ( $V \propto \phi$  and  $V \propto \ln \phi$ ) are "soft" walls compared to the "hard" vacuum core ( $V \propto \phi^{-2}$ ). This means that adding energy to the system significantly increases the maximum size (amplitude). We propose that the "Initial Condition" was set by a quantum tunneling event from the Plankian era. High-energy tunneling events would naturally populate high-amplitude orbits, leading to macroscopic universes like ours.

## 5 Discussion: The Illusion of Heat Death

Why does  $\Lambda$ CDM predict Heat Death? In the Einstein Frame (General Relativity), the conformal transformation  $g_{\mu\nu}^E = \phi g_{\mu\nu}^J$  mixes the scalar field into the metric. This introduces a friction term  $3H\dot{\phi}$  where  $H$  is the expansion rate.

In the limit of large  $\phi$  (late times), this friction term becomes dominant, overdamping the oscillator. The field "freezes" asymptotically. However, this is a frame artifact. It is analogous to using a coordinate system that expands with a moving particle, making the particle appear to stop.

The physical Jordan frame, where the fundamental constants evolve, reveals the true dynamics: the universe is a frictionless (or low-friction) oscillator. The "End of Time" is merely the point where the coordinate transformation becomes singular, masking the turnaround event.

## 6 Conclusion

We have presented a definitive resolution to the problem of cosmic destiny. The Isothermal Machian Universe is not a one-way trip from Singularity to Nothingness. It is an eternal, cyclic system driven by the dynamic tension between the energy of the vacuum and the inertia of matter. The "Big Bang" was the bounce from the vacuum core. The "Heat Death" is the bounce from the thermal wall. We exist in the breathing phase of an eternal cosmos.