Theoretical Addendum:
The Dilaton Interpretation of the Machian Scalar

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(Research aided by Gemini 3)

November 23, 2025

Abstract

We provide a theoretical justification for the "Universal Conformal Coupling" assumed in the Isothermal Machian Universe. We identify the scalar field ϕ as the **Dilaton** of a spontaneously broken scale invariance. This symmetry ensures that all mass scales (Higgs VEV, QCD confinement) scale uniformly with ϕ, protecting the Weak Equivalence Principle (WEP) from violations.

1 Introduction

The Isothermal Machian Universe relies on a scalar field ϕ that couples to all matter with a universal strength β. Critics argue that such a coupling is fine-tuned and should violate the Weak Equivalence Principle (WEP) due to composition-dependent forces (e.g., binding energy differences). Here, we show that if ϕ is the Goldstone boson of Scale Invariance (the Dilaton), this universality is a consequence of the underlying symmetry.

2 The Dilaton Hypothesis

Assume the fundamental Lagrangian of the universe is Scale Invariant.

$$x^\mu \to \lambda x^\mu, \quad \Phi \to \lambda^{-\Delta}\Phi \tag{1}$$

If this symmetry is spontaneously broken at a scale f, a Goldstone boson χ (the Dilaton) emerges. The effective action for matter fields ψ coupled to the Dilaton is constrained by the non-linear realization of the symmetry:

$$S_{matter} = \int d^4x\sqrt{-g}\mathcal{L}_{SM}(\psi, g_{\mu\nu}e^{2\beta\phi/M_{pl}}) \tag{2}$$

This implies that the metric $g_{\mu\nu}$ always appears in the combination $\hat{g}_{\mu\nu} = e^{2\beta\phi}g_{\mu\nu}$. This is exactly the **Universal Conformal Coupling** ansatz used in our papers.

3 Protection of the Equivalence Principle

3.1 Tree Level Universality

Because the coupling is geometric (via the metric), it couples to the trace of the energy-momentum tensor T^μ_μ. Thus, the force on a test body is proportional to its mass M, regardless of its composition.

$$\vec{F} = -\beta\frac{\vec{\nabla}\phi}{M_{pl}}M \tag{3}$$

This implies that the acceleration $\vec{a} = \vec{F}/M$ is independent of mass, satisfying the WEP.

3.2 Quantum Consistency (The "Final Boss")

3.2.1 The QCD Trace Anomaly and Loop Corrections

A subtle but critical challenge arises at the quantum level. The proton mass is dominated not by the quark masses (which couple to the Higgs/Dilaton), but by the QCD binding energy, determined by the confinement scale Λ_{QCD}. If Λ_{QCD} does not scale exactly like the Higgs VEV v, the proton-to-electron mass ratio $\mu = m_p/m_e$ would vary, leading to catastrophic WEP violations.

In standard QFT, the scale Λ_{QCD} is generated by **dimensional transmutation** of the running coupling $\alpha_s(\mu)$. The running is governed by the beta function:

$$\mu \frac{dg_s}{d\mu} = \beta(g_s) = -bg_s^3 + \dots \tag{4}$$

Integration yields the scale Λ_{QCD}:

$$\Lambda_{QCD} = \mu \exp\left(-\frac{1}{2bg_s^2(\mu)}\right) \tag{5}$$

In a generic theory, μ is an arbitrary subtraction scale. However, in a **Spontaneously Broken Scale Invariant Theory**, there are no explicit mass scales in the Lagrangian. The only physical scale is the Dilaton VEV χ. Therefore, the running coupling g_s can only depend on the dimensionless ratio of the energy scale to the symmetry breaking scale:

$$g_s(\mu) \to g_s\left(\frac{\mu}{\chi}\right) \tag{6}$$

Substituting this back into the definition of Λ_{QCD}, we find:

$$\Lambda_{QCD}(\chi) = \chi \cdot \mathcal{F}(g_{UV}) \propto \chi \tag{7}$$

Thus, the QCD confinement scale is **directly proportional** to the Dilaton field χ. Since the electron mass also scales as $m_e \propto v \propto \chi$, we conclude that:

$$\frac{m_{proton}(\chi)}{m_{electron}(\chi)} \propto \frac{\Lambda_{QCD}(\chi)}{v(\chi)} \propto \frac{\chi}{\chi} = \text{const} \tag{8}$$

This proves that **Universal Conformal Coupling holds at the quantum loop level**. The QCD trace anomaly, rather than breaking the equivalence principle, naturally tracks the Dilaton field, ensuring that all bound states scale uniformly. The "Fifth Force" remains composition-independent even when strong interaction contributions are included.

4 Prediction for Beta

The Dilaton coupling strength β is fixed by the geometry of the symmetry breaking. For a standard scalar-tensor theory (e.g., 5D Kaluza-Klein or String Theory), the coupling is often:

$$\beta = \frac{1}{\sqrt{6}} \approx 0.408 \tag{9}$$

Our phenomenological fit to Galaxy Rotation Curves (Paper 1) yields a survey mean of $\beta \approx 0.60 \pm 0.33$, which is consistent with the theoretical prediction of $\beta \approx 0.41$. This discrepancy suggests that the underlying theory might be a **Scalar-Tensor-Vector** theory or involve non-minimal couplings to the curvature (e.g., R^2 terms). However, the order of magnitude is consistent ($\mathcal{O}(1)$), supporting the identification of the Machian scalar as a gravitational modulus.