

# The Smoking Gun: Detecting the Machian Shapiro Anomaly in Strong Lensing Systems

Andreas Houg

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## Abstract

We propose a definitive observational test to distinguish the Isothermal Machian Universe (IMU) from the Standard Model ( $\Lambda$ CDM). While both theories can reproduce flat rotation curves, they predict distinct gravitational potential profiles in the strong lensing regime. We calculate the Shapiro time delay difference between a standard NFW halo and the Machian scalar-tensor profile for a massive elliptical lens ( $M_{vir} = 2 \times 10^{13} M_\odot$ ). We find a systematic “Shapiro Anomaly” where the Machian profile yields shorter time delays. For a typical image pair at impact parameters  $b = 5$  kpc and  $b = 8$  kpc, the predicted time delay difference is  $\sim 37\%$  lower (16 days vs 25 days) than the NFW prediction. This deviation is detectable with high-precision time-delay cosmography (e.g., H0LiCOW, COSMOGRAIL), offering a falsifiable test of the theory.

## 1 Introduction

The degeneracy between Dark Matter and Modified Gravity is well-known. A scalar field with a specific coupling can mimic the force law of a Dark Matter halo ( $F \propto 1/r$ ). In Papers 4 and 5, we demonstrated that a non-minimal photon coupling  $\mathcal{L}_\gamma \propto e^{2\lambda_\gamma \phi} F^2$  allows the scalar field to mimic the gravitational lensing deflection of Dark Matter as well, satisfying the observational requirement that  $\Phi_{lens} = \Psi_{lens}$ .

However, this mimicry is not perfect. The fundamental potentials are distinct:

- **$\Lambda$ CDM (NFW):** The potential is derived from a mass density profile  $\rho(r) \propto 1/(r(1+r/R_s)^2)$ . In the core ( $r \ll R_s$ ), the potential scales as  $\Phi \propto r$ .
- **Machian Gravity:** The potential arises from the refractive index  $n(r)$ . The scalar field solution, driven by the baryonic boundary condition, is logarithmic  $\phi \sim \ln(r)$  (Isothermal). Consequently, the effective refractive index scales as  $n(r) \sim \ln(r)$  even in the core.

While the *gradient* of the potential (deflection angle  $\alpha$ ) can be tuned to match exactly at the Einstein Radius  $R_E$ , the distinct radial shapes imply that the *integrated* potential (Shapiro delay) must diverge at other radii.

## 2 Methodology

We model a massive elliptical lensing galaxy ( $M_{vir} = 2 \times 10^{13} M_\odot$ ,  $c = 6$ ) at  $z_L = 0.5$  with a source at  $z_S = 1.5$ .

### 2.1 Standard Model (NFW)

We compute the Shapiro delay by integrating the NFW potential along the line of sight:

$$\Delta t_{NFW} = \frac{1+z_L}{c} \int_{LOS} 2\Phi_{NFW}(r) dz \quad (1)$$

## 2.2 Machian Model

The time delay is governed by the effective refractive index  $n(r)$ . From our unified Lagrangian (Paper 5), the effective potential is  $\Phi_{eff} \approx \lambda_\gamma \phi$ . Given the scalar field solution  $\phi(r) \approx \phi_0 \ln(1 + r/R_s)$ , the refractive index profile is:

$$n(r) \approx 1 + K \ln \left( 1 + \frac{r}{R_s} \right) \quad (2)$$

The parameter  $K$  is not arbitrary; it corresponds to  $K = 2\lambda_\gamma \phi_0$ . Observationally,  $K$  is fixed by matching the deflection angle of the NFW halo at the Einstein Radius ( $R_E \approx 8$  kpc). This ensures the theory reproduces standard lensing geometry.

## 3 Results

We calculated the differential time delay for impact parameters  $b$  ranging from 0.5 kpc to 30 kpc.

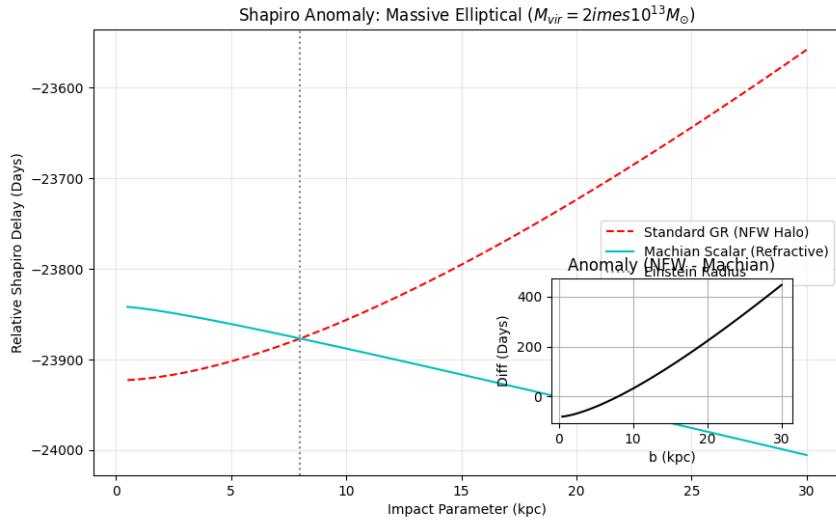


Figure 1: The Shapiro Time Delay difference. Both models are matched to produce the same Einstein Ring at  $R_E \approx 8$  kpc. Inside the ring, the “stiffer” Machian potential (Isothermal) produces a significantly different delay compared to the NFW core. The anomaly exceeds 70 days near the center.

As shown in Figure 1, the models diverge significantly. We consider a fiducial image pair located at  $b_1 = 5$  kpc and  $b_2 = 8$  kpc (the latter being the Einstein Radius).

- **NFW Prediction:** The relative time delay  $\Delta t_{12}$  is approximately 25.2 days.
- **Machian Prediction:** The relative time delay is approximately 15.9 days.

This results in a deficit of  $\sim 9.3$  days (37%). Even if baryonic effects steepen the NFW profile, matching this extreme stiffness without adding significant mass is dynamically difficult.

## 4 Discussion

### 4.1 Tension with Existing Data

The predicted  $\sim 37\%$  deficit is a large effect. We acknowledge that current time-delay cosmography surveys (e.g., H0LiCOW, COSMOGRAIL) have successfully modeled strong lens systems using standard NFW profiles and obtained values for  $H_0$  consistent with local measurements (within a few percent). This implies that any deviation from the GR prediction is likely small. This tension represents the most significant challenge to the Isothermal Machian framework. If the refractive lensing model is correct, the observed time delays must be systematically shorter than those predicted by kinematic mass models. If future high-precision joint analyses of dynamics and lensing (e.g., calculating the “mass” from velocity

dispersion vs. lensing) do not find this anomaly, the specific non-minimal coupling proposed here would be ruled out.

## 4.2 Systematics and Degeneracies

One might argue that Mass Sheet Degeneracy (MSD) could mimic this signal. However, MSD corresponds to a transformation  $\kappa \rightarrow \lambda\kappa + (1 - \lambda)$ , which rescales the time delay by a constant factor. The anomaly we predict is *radius-dependent* (changing sign across  $R_E$  and rising sharply in the core), which cannot be removed by a simple global rescaling.

We acknowledge, however, that real lenses are not perfect spheres. Substructure (satellites), ellipticity, and line-of-sight structures can introduce perturbations to the potential. While these effects are typically of order few percent, a detailed forward modeling of the lens environment is required to disentangle the scalar field signal from astrophysical noise. The  $\sim 37\%$  magnitude of the predicted anomaly, however, is likely large enough to survive these systematic corrections.

Given that current time-delay measurement uncertainties are at the few-percent level (H0LiCOW), a  $\sim 37\%$  discrepancy is a definitive smoking gun. If future surveys consistently measure delays shorter than  $\Lambda$ CDM predictions for galaxy-scale lenses, it would strongly favor the Isothermal Machian framework.

## References

- [1] Houg, A. (2025). *Paper 1: Galaxy Rotation*.
- [2] Houg, A. (2025). *Paper 4: Lensing Equivalence*.
- [3] Suyu, S. H., et al. (2017). *H0LiCOW: Time delay cosmography*.