Quantum Teleportation: Simulation on IBM QE Cloud Quantum Computer

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With rapid progress of quantum information science, quantum teleportation is becoming increasingly useful. The protocol is applicable in quantum communication for transferring quantum states between distant locations and for teleporting qubits within a quantum device as a means of enabling computation. Generally, entanglement between two qubits at the sending and receiving ends must be established first, with a classical communication channel used to inform the receiver about the sender's measurement outcomes. The basic protocol can be realized on at least three-qubit device, such as the IBM Quantum Experience cloud computer. Here, teleportation is performed for various starting entangled states and fidelity of transmission is computed by quantum state tomography. It is found that $+Z$ eigenstate teleports with higher fidelity than a superposition state.

INTRODUCTION

The term Quantum Teleportation (QT) refers to a protocol by which a quantum state (qubit) is destroyed at the input and a perfect copy of it is created at the output. The information transfer happens in two parts. First, quantum mechanical information is transferred through a pair of entangled states. Then, sender's measurement outcome is communicated via a classical channel, which allows the receiver to recover an exact copy. The protocol relies on the phenomenon of entanglement formally established in the second half of the 19th century [1, 2]. QT was initially proposed by Bennet et al. in 1993 [3] and was first experimentally realized in 1997 [4]. Since then, the value of QT has been recognized in domains of quantum computation [5, 6] and quantum communication [7, 8]. Long-range demonstrations have been successfully performed [9, 10], with a record distance of 1400km [11]. Among many others, QT of squeezed photons [12] and multi-qubit composite systems [13, 14] has also been demonstrated. However, the basic protocol can be comprehensively simulated on a three-qubit device. Here, quantum theory underlying QT will be presented, followed by a simulation carried out on IBM's Quantum Experience cloud quantum computer and a discussion of results and limitations of the protocol.

FROM ENTANGLEMENT TO TELEPORTATION

To perform QT, the sender "Alice" and the receiver "Bob" must share a pair of qubits in a maximally entangled state, such that $|\Psi_A\rangle \otimes |\Psi_B\rangle$ is one of the four Bell states in the basis $\{|00\rangle_{AB}, |01\rangle_{AB}, |10\rangle_{AB}, |11\rangle_{AB}\}$:

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle_{AB} \pm |10\rangle_{AB}), \tag{1}$$

$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle_{AB} \pm |11\rangle_{AB}). \tag{2}$$

Importantly, these mutually orthogonal states can be transformed into one another by an operation on a single qubit. For example,

$$(\hat{X} \otimes \hat{1})|\Phi_+\rangle = |\Psi_+\rangle \tag{3}$$

$$(\hat{Y} \otimes \hat{1})|\Phi_+\rangle = i|\Psi_-\rangle \tag{4}$$

$$(\hat{Z} \otimes \hat{1})|\Phi_+\rangle = |\Phi_-\rangle \tag{5}$$

where $\hat{X}, \hat{Y}$ and $\hat{Z}$ are standard rotations around the Bloch sphere, corresponding to Pauli operators. In addition to one of the entangled qubits $|\Psi_A\rangle$, Alice owns the initial qubit that will be teleported:

$$\varphi_i = \alpha|0_i\rangle + \beta|1_i\rangle, \tag{6}$$

where α and β are arbitrary complex numbers, normalized such that $|\alpha|^2 + |\beta|^2 = 1$. The goal is to transfer the $|\varphi_i\rangle$ state onto Bob's $|\Psi_B\rangle$, which is otherwise forbidden for nonzero α and β due to the no-cloning theorem. To do that, Alice first applies a CNOT gate to her two qubits, controlled by the qubit i, that is, the state of qubit A changes if qubit i is in state $|1_i\rangle$ and remains the same if qubit i is in state $|0_i\rangle$. The CNOT gate is a unitary transformation that can be written in the basis $\{|00\rangle_{iA}, |01\rangle_{iA}, |10\rangle_{iA}, |11\rangle_{iA}\}$ as

$$\hat{U}_{CNOT} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{7}$$

Following this gate operation, the three qubits have to be described as a whole,

$$\hat{U}_{CNOT}|\varphi_i\rangle \otimes (|\Psi_A\rangle \otimes |\Psi_B\rangle) = \hat{U}_{CNOT}|\varphi_i\rangle|\Psi_{AB}\rangle = |\Psi_{iAB}\rangle \tag{8}$$

The last step necessary to entangle $|\varphi_i\rangle$ and $|\Psi_A\rangle$ is the application of a Hadamard gate on the qubit i. The Hadamard gate in $\{|0_i\rangle, |1_i\rangle\}$ basis is

$$\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{9}$$

Supposing the entangled pair $|\Psi_{AB}\rangle$ starts out in $|\Psi_+\rangle$ Bell state, this final state becomes:

$$\begin{aligned}\hat{H}|\Psi_{iAB}\rangle = \frac{1}{\sqrt{2}}|00\rangle_{iA}(\alpha|0_B\rangle + \beta|1_B\rangle)+ \\ +\frac{1}{\sqrt{2}}|01\rangle_{iA}(\alpha|1_B\rangle + \beta|0_B\rangle)+ \\ +\frac{1}{\sqrt{2}}|10\rangle_{iA}(\alpha|0_B\rangle - \beta|1_B\rangle)+ \\ +\frac{1}{\sqrt{2}}|11\rangle_{iA}(\alpha|1_B\rangle - \beta|0_B\rangle).\end{aligned} \tag{10}$$

In order to realize the teleportation, Alice measures her two qubits in, for example, the Z basis, collapsing her original $|\varphi_i\rangle$ state, and gaining knowledge as to which of the four equally probable states Bob's qubit is in (Eq.10). Only one of the four possible final $|\Psi_B\rangle$ corresponds to the original $|\varphi_i\rangle$, hence, Alice needs to classically communicate the results of her measurements to Bob. Based on that information, Bob (receiver) applies a unitary transformation to his $|\Psi_B\rangle$ to recover $|\varphi_i\rangle$. In this example $\hat{1}, \hat{X}, \hat{Z}$ or $i\hat{Y}$, respectively (Fig. 1).

SMO	$\|\Psi_B\rangle$ final state	Receiver
$\|00\rangle_{iA}$	$\alpha\|0_B\rangle + \beta\|1_B\rangle$	I
$\|01\rangle_{iA}$	$\alpha\|1_B\rangle + \beta\|0_B\rangle$	X
$\|10\rangle_{iA}$	$\alpha\|0_B\rangle - \beta\|1_B\rangle$	Z
$\|11\rangle_{iA}$	$\alpha\|1_B\rangle - \beta\|0_B\rangle$	iY

FIG. 1. Gates necessary to recover the teleported state for the starting entangled state $|\Psi_+\rangle$.

In the case, where Bob is ignorant of Alice's measurement results, no information is transferred because, on average, his qubit is in equal superposition of $|0_B\rangle$ and $|1_B\rangle$ [15]. Even though it was assumed that the entangled pair was in the $|\Psi_+\rangle$ Bell state, the results of the procedure are analogous for any of $|\Psi_\pm\rangle, |\Phi_\pm\rangle$. Depending on the starting Bell state, the necessary gates applied ultimately by Bob will change with correspondence to Eqs. 3-5. Fig. 2 summarizes the necessary gates applied by the receiver, based on the starting entangled pair and sender's measurement outcome (SMO) in the Z basis.

Entangled state shared by Alice and Bob

	$\|\Psi_+\rangle$	$\|\Psi_-\rangle$	$\|\Phi_+\rangle$	$\|\Phi_-\rangle$
SMO		Receiver		
$\|00\rangle_{iA}$	I	Z	X	iY
$\|01\rangle_{iA}$	X	iY	I	Z
$\|10\rangle_{iA}$	Z	I	iY	X
$\|11\rangle_{iA}$	iY	X	Z	I

FIG. 2. Gates necessary to recover the teleported state for all possible initially shared entangled states.

QT SIMULATION ON IBM QE

IBM's Quantum Experience (QE) is a 5-qubit cloud computer available to the public via a graphical interface. The processor is based on transmon qubits developed in 2007 [16]. The star-shape-connected circuit comprises superconducting electrical circuits linked by Josephson junctions. Transmons are superconducting charge qubits operating in a regime of high ratio of Joshepson energy to charging energy, which decreases the sensitivity to charge noise in the island.

State Preparation

Following the procedure outlined by Fedortchenko [Fedortchenko, 2016], an initial state $|\varphi_i\rangle = \alpha|0_i\rangle + \beta|1_i\rangle$ is prepared using the gate sequence shown in Fig. 3(a). The sequence of Hadamard, phase gate $T(\frac{\pi}{4}$rotation around $Z)$ and phase gate S(mapping X to Y, Z to Z) outputs a state $|\varphi_i\rangle = \cos\frac{\theta}{2}|0_i\rangle + \sin\frac{\theta}{2}|1_i\rangle$, with $\theta = \frac{\pi}{4}$. Theoretically, then, $|\alpha|^2 = |\cos\frac{\pi}{8}|^2 = 0.854$ and $|\beta|^2 = |\sin\frac{\pi}{8}|^2 = 0.146$. Fig. 3(b) displays probabilistic measurement outcome on QE, based on 8192 repetitions.

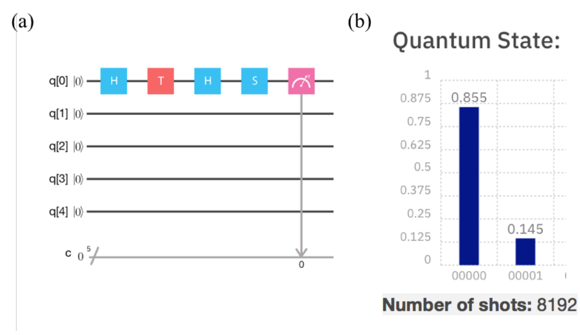

FIG. 3. (a) shows the gate sequence used to prepare $|\varphi_i\rangle$. (b) is the result obtained by 8192 repetitions of Z measurement on QE.

Teleportation

The gate sequence necessary to perform the protocol is shown in Fig. 4, where $q[0] \equiv |\varphi_i\rangle, q[1] \equiv |\Psi_A\rangle$ and $q[2] \equiv |\Psi_B\rangle$. In the first part of the implementation, qubits A and B become entangled. Application of the Identity, X, Y or $Z + Y$ gates on Alice's qubit lets her put $|\Psi_{AB}\rangle$ in one of the four Bell states. The results (Fig. 5) were then post selected, as to retrieve the α and β coefficients of the teleported state for each starting entangled state. All 4 experiments were repeated 8192 times.

Results

Fig. 5 shows an example of experimental outcomes of the gate sequence in Fig. 4.
Post-selection is used to recover the coefficients α and β. The data corresponding to all 4 starting Bell states is

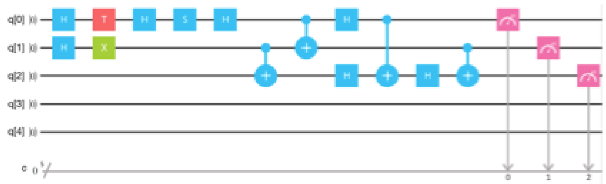

FIG. 4. QT gate sequence on QE.

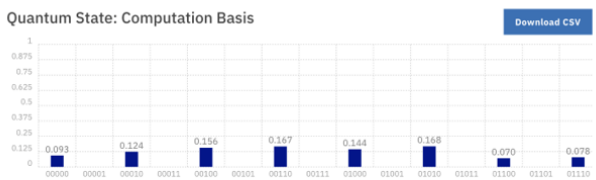

FIG. 5. Experimental output for the gate sequence in Fig. 4.

synthesized in Fig. 6 (bolded values correspond to α).

$\|BAi\rangle$	$\|000\rangle$	$\|001\rangle$	$\|010\rangle$	$\|011\rangle$	$\|100\rangle$	$\|101\rangle$	$\|110\rangle$	$\|111\rangle$	$\|\alpha\|^2$	$\|\beta\|^2$
Gate Applied				Theoretical	values					
I	**0.213**	**0.213**	0.037	0.037	0.037	0.037	**0.213**	**0.213**	**0.854**	0.146
X	0.037	0.037	**0.213**	**0.213**	**0.213**	**0.213**	0.037	0.037	**0.854**	0.146
Y	0.037	0.037	**0.213**	**0.213**	**0.213**	**0.213**	0.037	0.037	**0.854**	0.146
$Z+Y$	**0.213**	**0.213**	0.037	0.037	0.037	0.037	**0.213**	**0.213**	**0.854**	0.146
Gate Applied				Experimental	values					
I	**0.241**	**0.178**	0.049	0.078	0.054	0.047	**0.101**	**0.253**	**0.773**	0.228
X	0.093	0.124	**0.156**	**0.167**	**0.144**	**0.168**	0.070	0.078	**0.635**	0.365
Y	0.061	0.124	**0.105**	**0.283**	**0.194**	**0.171**	0.031	0.054	**0.753**	0.253
$Z+Y$	**0.177**	**0.158**	0.109	0.074	0.093	0.076	**0.165**	**0.147**	**0.647**	0.352

FIG. 6. Experimental output for all starting entanlged states, post-selected to recover the values of $|\alpha|^2$ and $|\beta|^2$.

Quantum State Tomography

To probe the fidelity of the teleported state, Quantum State Tomography is performed as outlined in [17–19]. In short, the theoretically obtained density matrix of the initial state can be compared to that of the final, teleported state. The theoretical density matrix is given by equation

$$\rho_i^{TH} = |\varphi_i\rangle\langle\varphi_i| \tag{11}$$

which for $|\varphi_i\rangle = \cos\frac{\pi}{8}|0_i\rangle + \sin\frac{\pi}{8}|1_i\rangle$ comes out to

$$\rho_i^{TH} = \begin{pmatrix} 0.854 & 0.353 \\ 0.353 & 0.146 \end{pmatrix}. \tag{12}$$

The experimental density matrix can be obtained using the same post-selection methods established in earlier sections of this article. However, to get a full, density matrix picture one needs to measure the teleported state in X, Y and Z bases, which corresponds to rotating the state prior to Z measurement. A Hadamard gate is applied for measurement in X basis, and Hadamard followed by $S^\dagger$ gates for Y. Then, the average value of observable ⟨X⟩, can be evaluated by $\langle \mathrm{X}\rangle = tr(|\langle\varphi_i|\varphi_i\rangle)X$, and likewise for ⟨Y⟩, ⟨Z⟩. Finally, the experimentally determined density matrix is calculated with

$$\rho_i^{EXP} = \frac{1}{2}(I + \langle X\rangle X + \langle Y\rangle Y + \langle Z\rangle Z). \tag{13}$$

For the state teleported with circuit in Fig. 4, the following matrix was obtained:

$$\rho_i^{EXP} = \begin{pmatrix} 0.775 & 0.047 \\ 0.047 & 0.225 \end{pmatrix} + i\begin{pmatrix} 0.000 & -0.034 \\ 0.034 & 0.000 \end{pmatrix}. \tag{14}$$

Given these two density matrices, the closeness of the two states can be calculated with a fidelity metric

$$F(\rho_i^{TH}, \rho_i^{EXP}) = Tr\left(\sqrt{\sqrt{\rho_i^{TH}}\rho_i^{EXP}\sqrt{\rho_i^{TH}}}\right)^2 = 0.695 \tag{15}$$

To probe the extent of the compounding decoherence and gate error rates when preparing, teleporting and measuring a superposition state, a separate experiment using a $+Z$ eigenstate was performed. The density matrix for the state is

$$\rho_{+Z}^{TH} = \begin{pmatrix} 1.000 & 0.000 \\ 0.000 & 0.000 \end{pmatrix}, \tag{16}$$

and the matrix obtained by performing state tomography of the teleported qubit was found to be

$$\rho_{+Z}^{EXP} = \begin{pmatrix} 0.858 & 0.063 \\ 0.063 & 0.142 \end{pmatrix} + i\begin{pmatrix} 0.000 & -0.024 \\ 0.024 & 0.000 \end{pmatrix}. \tag{17}$$

Hence, the fidelity of teleportation for $+Z$ eigenstate is

$$F(\rho_{+Z}^{TH}, \rho_{+Z}^{EXP}) = 0.858, \tag{18}$$

which is substantially higher than for the superposition state $|\varphi_i\rangle$ (Eq. 15).

DISCUSSION

While the experimental QT protocol clearly suffers from various sources of decoherence, the results still follow the theoretical predictions and even beat the maximum fidelity of $\frac{2}{3}$ achievable by classical schemes that don't use entanglement [3, 20]. Previous research on the subject found four sources of noise in teleportation channels: decoherence of initial state prior to teleportation, decoherence of the entangled qubit pair held by Alice and Bob, noise in the measurements performed by Alice and that in the final gate operation performed by Bob [20]. All of these, except for the last one, contribute to decoherence of the qubits here, with overall greater impact on the superposition state than the eigenstate. Due to the nature of this implementation and a need for post-selection of data, the noise in Bob's gate operation is not relevant here. However, the system does suffer from gate errors in the quantum circuit, with more operations translating to lower fidelity. Thus, an interesting topic to explore, would be the extent by which each additional gate decoheres the qubits at hand.

It worth noting that the QT performed here differs from the full-fledged protocol also in other ways. Firstly, qubits are sitting on the same chip, with minimal spatial separation. Additionally, they are measured all at the same time, with post selection needed to recover the teleported state. Because of the simultaneous measurement, the gate operation on Bob's qubit is not feasible, which makes it more involved to perform quantum state tomography of the final state. It would be worthy and interesting to perform quantum state tomography for all four starting entangled states, which I was unable to do due to shortage of time.

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Appendix - Quantum Codes

State Preparation

```
include "qelib1.inc";
qreg q[5];
creg c[5];
h q[0];
t q[0];
h q[0];
s q[0];
```

Teleportation circuit used to obtain data in Fig. 6.; same circuit with different initial state and rotations prior to measurement was used for density matrix calculations.

```
include "qelib1.inc";
qreg q[5];
creg c[5];
h q[0];
t q[0];
h q[0];
s q[0];
cx q[1],q[2];
cx q[0],q[1];
h q[0];
h q[2];
cx q[0],q[2];
h q[2];
cx q[1],q[2];
measure q[2] - c[2];
```