

# Fizyka - Wzory

## 1. Iloczyn wektorowy:

$$\vec{a} \times \vec{b} = (a_y b_z - a_z b_y) \vec{i} + (a_z b_x - a_x b_z) \vec{j} + (a_x b_y - a_y b_x) \vec{k}$$

## 2. Prędkość wektorowa:

$$\vec{v}_{\text{sr}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t} \quad \left[ \frac{\text{m}}{\text{s}} \right]$$

## 3. Prędkość średnia:

$$v_{\text{sr}} = \frac{x_{\text{całk}}}{t_{\text{całk}}} = \frac{\sum_{i=1}^n x_i}{\sum_{i=1}^n t_i}$$

## 4. Prędkość chwilowa:

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$
$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}$$

## 5. Przyspieszenie:

$$\vec{a}_{\text{sr}} = \frac{\Delta \vec{v}}{\Delta t}, \quad \vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{r}}{dt^2}$$
$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt}, \quad a_s = \frac{d|\vec{v}|}{dt}, \quad a_n = \frac{v^2}{R}$$

6. Ruch jednostajny prostoliniowy:

$$\vec{a} = \vec{0} \implies \vec{v} = \overrightarrow{const} \implies \vec{r}(t) = \vec{r}_0 + \vec{v} \cdot t$$

Dla ruchu wzdłuż osi  $x$ :

$$x(t) = x_0 + v \cdot t$$

7. Ruch zmienny wzdłuż prostej:

$$\vec{a} = \overrightarrow{const} \implies \vec{v}(t) = \vec{v}_0 + \vec{a} \cdot t, \quad \vec{r}(t) = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{\vec{a} \cdot t^2}{2}$$

8. I zasada dynamiki Newtona:

$$\vec{F}_w = \vec{0} \iff \vec{v} = \overrightarrow{const}$$

9. II zasada dynamiki Newtona:

$$\vec{a} = \frac{\vec{F}_w}{m}$$

10. Pęd:

$$\vec{p} = m \cdot \vec{v}$$

11. III zasada dynamiki Newtona:

$$\vec{F}_{A \rightarrow B} = -\vec{F}_{B \rightarrow A}$$

12. Rzut poziomy:

$$x(t) = x_0 + v_{0x} \cdot t = v_0 \cdot t$$

$$y(t) = y_0 + v_{0y} \cdot t - \frac{gt^2}{2} = H - \frac{gt^2}{2}$$

$$t = t_c \text{ dla } y = 0, \quad H - \frac{gt^2}{2} = 0, \quad H = \frac{gt^2}{2} \rightarrow t_c = \sqrt{\frac{2H}{g}} \left[ \sqrt{\frac{m}{s^2}} = s \right]$$

$$x = z \text{ dla } y = 0, \quad z = v_0 \cdot t = v_0 \cdot \sqrt{\frac{2H}{g}} \left[ \frac{m}{s} \sqrt{\frac{m}{s^2}} = \frac{m}{s} \cdot s = m \right]$$

$$v_x(t) = \frac{dx}{dt} = v_0 \cdot 1 \cdot t^{1-1} = v_0 = \text{const}$$

$$v_y(t) = \frac{dy}{dt} = -\frac{g}{2} 2t = -gt \left[ \frac{m}{s^2} s = \frac{m}{s} \right]$$

13. Rzut pionowy:

$$y(t) = 0 + v_0 t - \frac{gt^2}{2}$$

$$v_y(t) = \frac{dy}{dt} = v_0 - gt$$

$$t = t_c \text{ dla } y = 0 \rightarrow v_0 t_c = \frac{g}{2} t_c^2$$

14. Rzut ukośny:

$$v_{0x} = v_0 \cdot \cos \alpha$$

$$v_{0y} = v_0 \cdot \sin \alpha$$

$$x(t) = x_0 + v_{0x} \cdot t = v_0 \cdot t \cdot \cos \alpha$$

$$y(t) = y_0 + v_{0y} \cdot t - \frac{gt^2}{2} = v_0 \cdot t \cdot \sin \alpha - \frac{gt^2}{2}$$

$$v_x(t) = v_{0x} = v_0 \cdot \cos \alpha$$

$$v_y(t) = v_{0y} - g \cdot t = v_0 \cdot \sin \alpha - g \cdot t$$

$$t = t_c \text{ dla } y = 0$$

$$y(t) = v_0 \cdot t \cdot \sin \alpha - \frac{g \cdot t^2}{2}, \quad v_0 \cdot t_c \cdot \sin \alpha - \frac{g \cdot t_c^2}{2} = 0$$

$$t_c \left( v_0 \cdot \sin \alpha - \frac{g \cdot t_c}{2} \right) = 0 \Rightarrow t_c = 0 \quad \vee \quad t_c = \frac{2v_0 \cdot \sin \alpha}{g}$$

$$z = x(t_c) \longrightarrow z - \text{zasieg}$$

$$x(t) = v_0 \cdot t \cdot \cos \alpha \Rightarrow x(t_c) = v_0 \cdot \frac{2v_0 \sin \alpha}{g} \cdot \cos \alpha = \frac{2v_0^2 \sin \alpha \cos \alpha}{g}$$

$$y = H_{max} \text{ dla } t = t_{wzn}$$

$$H_{max} = y(t_{wzn}) = v_0 \cdot t_{wzn} \cdot \sin \alpha - \frac{g \cdot t_{wzn}^2}{2} =$$

$$v_0 \cdot \frac{v_0 \cdot \sin \alpha}{g} \cdot \sin \alpha - \frac{g}{2} \cdot \left( \frac{v_0 \cdot \sin \alpha}{g} \right)^2 = \frac{v_0^2 \cdot \sin^2 \alpha}{g} - \frac{v_0^2 \cdot \sin^2 \alpha}{2 \cdot g}$$

$$H_{max} = \frac{v_0^2 \cdot \sin^2 \alpha}{2 \cdot g}$$

15. **II zasada dynamiki w postaci pędowej:**

$$\overrightarrow{F_{zew}} = \frac{d\vec{p}}{dt}$$

$$\Downarrow$$

$$\overrightarrow{F_{zew}} = \vec{0} \Rightarrow \frac{d\vec{p}}{dt} = 0 \Leftrightarrow \vec{p} = \overrightarrow{const}$$

$$\vec{p}_{pocz} = \vec{p}_{koc}$$

$$\vec{p}_{pocz} = \vec{0}, \quad \vec{p}_{koc} = m_1 \cdot \vec{v}_1 - m_2 \cdot \vec{v}_2 = \vec{0}$$

16. **Energia:**

$$E_k = \frac{mv^2}{2} \longrightarrow \text{Energia kinetyczna}$$

$$E_p = -G \frac{Mm}{r} \longrightarrow \text{Energia potencjalna}$$

$$E_p = mgh \longrightarrow \text{W pobliżu pow. Ziemi}$$

$$E_p = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r} \longrightarrow \text{Siła elektrostatyczna (Coulomba)}$$