Fizyka - Wzory

1. **Iloczyn wektorowy**:

$$\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\vec{i} + (a_z b_x - a_x b_z)\vec{j} + (a_x b_y - a_y b_x)\vec{k}$$

2. **Prędkość wektorowa**:

$$\vec{v}_{\text{śr}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t} \quad \left[\frac{\text{m}}{\text{s}}\right]$$

3. **Prędkość średnia**:

$$v_{\text{śr}} = \frac{x_{\text{całk}}}{t_{\text{całk}}} = \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} t_i}$$

4. **Prędkość chwilowa**:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}$$

5. **Przyspieszenie**:

$$\vec{a}_{\text{śr}} = \frac{\Delta \vec{v}}{\Delta t}, \quad \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt}, \quad a_s = \frac{d|\vec{v}|}{dt}, \quad a_n = \frac{v^2}{R}$$

6. **Ruch jednostajny prostoliniowy**:

$$\vec{a} = \vec{0} \implies \vec{v} = \overrightarrow{const} \implies \vec{r}(t) = \vec{r}_0 + \vec{v} \cdot t$$

Dla ruchu wzdłuż osi x:

$$x(t) = x_0 + v \cdot t$$

7. **Ruch zmienny wzdłuż prostej**:

$$\vec{a} = \overrightarrow{const} \implies \vec{v}(t) = \vec{v}_0 + \vec{a} \cdot t, \quad \vec{r}(t) = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{\vec{a} \cdot t^2}{2}$$

8. **I zasada dynamiki Newtona**:

$$\vec{F}_w = \vec{0} \iff \vec{v} = \overrightarrow{const}$$

9. **II zasada dynamiki Newtona**:

$$\vec{a} = \frac{\vec{F}_w}{m}$$

10. **Pęd**:

$$\vec{p} = m \cdot \vec{v}$$

11. **III zasada dynamiki Newtona**:

$$\vec{F}_{A \to B} = -\vec{F}_{B \to A}$$

12. **Rzut poziomy**:

$$x(t) = x_0 + v_{0x} \cdot t = v_0 \cdot t$$

$$y(t) = y_0 + v_{0y} \cdot t - \frac{gt^2}{2} = H - \frac{gt^2}{2}$$

$$t = t_c \text{ dla } y = 0, \quad H - \frac{gt^2}{2} = 0, \quad H = \frac{gt^2}{2} \rightarrow t_c = \sqrt{\frac{2H}{g}} \left[\sqrt{\frac{m}{\frac{m}{s^2}}} = s \right]$$

$$x = z \text{ dla } y = 0, \quad z = v_0 \cdot t = v_0 \cdot \sqrt{\frac{2H}{g}} \left[\frac{m}{s} \sqrt{\frac{m}{\frac{m}{s^2}}} = \frac{m}{s} \cdot s = m \right]$$

$$v_x(t) = \frac{dx}{dt} = v_0 \cdot 1 \cdot t^{1-1} = v_0 = const$$

$$v_y(t) = \frac{dy}{dt} = -\frac{g}{2} 2t = -gt \left[\frac{m}{s^2} s = \frac{m}{s} \right]$$

13. **Rzut pionowy**:

$$y(t) = 0 + v_0 t - \frac{gt^2}{2}$$

$$v_y(t) = \frac{dy}{dt} = v_0 - gt$$

$$t = t_c \text{ dla } y = 0 \rightarrow v_0 t_c = \frac{g}{2} t_c^2$$

14. **Rzut ukośny**:

$$v_{0x} = v_0 \cdot \cos \alpha$$

$$v_{0y} = v_0 \cdot \sin \alpha$$

$$x(t) = x_0 + v_{0x} \cdot t = v_0 \cdot t \cdot \cos \alpha$$

$$y(t) = y_0 + v_{0y} \cdot t - \frac{gt^2}{2} = v_0 \cdot t \cdot \sin \alpha - \frac{gt^2}{2}$$

$$v_x(t) = v_{0x} = v_0 \cdot \cos \alpha$$

$$v_y(t) = v_{0y} - g \cdot t = v_0 \cdot \sin \alpha - g \cdot t$$

$$t = t_c \text{ dla } y = 0$$

$$y(t) = v_0 \cdot t \cdot \sin \alpha - \frac{g \cdot t^2}{2}, \quad v_0 \cdot t_c \cdot \sin \alpha - \frac{g \cdot t_c^2}{2} = 0$$

$$t_c\left(v_0\cdot\sin\alpha-\frac{g\cdot t_c}{2}\right)=0\Rightarrow t_c=0\quad\vee\quad t_c=\frac{2v_0\cdot\sin\alpha}{g}$$

$$z=x(t_c)\longrightarrow \text{z - zasięg}$$

$$x(t)=v_0\cdot t\cdot\cos\alpha\Rightarrow x(t_c)=v_0\cdot\frac{2v_0\sin\alpha}{g}\cdot\cos\alpha=\frac{2v_0^2\sin\alpha\cos\alpha}{g}$$

$$y=H_{max}\text{ dla }t=t_{wzn}$$

$$H_{max}=y(t_{wzn})=v_0\cdot t_{wzn}\cdot\sin\alpha-\frac{g\cdot t_{wzn}^2}{2}=$$

$$v_0\cdot\frac{v_0\cdot\sin\alpha}{g}\cdot\sin\alpha-\frac{g}{2}\cdot\left(\frac{v_0\cdot\sin\alpha}{g}\right)^2=\frac{v_0^2\cdot\sin^2\alpha}{g}-\frac{v_0^2\cdot\sin^2\alpha}{2\cdot g}$$

$$H_{max}=\frac{v_0^2\cdot\sin^2\alpha}{2\cdot g}$$

15. **II zasada dynamiki w postaci pędowej**:

$$\overrightarrow{F_{zew}}=\frac{d\vec{p}}{dt}$$

$$\Downarrow$$

$$\overrightarrow{F_{zew}}=\vec{0}\Rightarrow\frac{d\vec{p}}{dt}=0\Leftrightarrow\vec{p}=\overrightarrow{const}$$

$$\vec{p}_{pocz}=\vec{p}_{\text{końc}}$$

$$\vec{p}_{pocz}=\vec{0},\quad\vec{p}_{\text{końc}}=m_1\cdot\overrightarrow{v_1}-m_2\cdot\overrightarrow{v_2}=\vec{0}$$

16. **Energia**:

$$E_k=\frac{mv^2}{2}\longrightarrow\text{Energia kinetyczna}$$

$$E_p=-G\frac{Mm}{r}\longrightarrow\text{Energia potencjalna}$$

$$E_p=mgh\longrightarrow\text{W pobliżu pow. Ziemi}$$

$$E_p=\frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r}\longrightarrow\text{Siła elektrostatyczna (Coulomba)}$$