Fizyka - Wzory

1. **Iloczyn wektorowy**:

$$\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\vec{i} + (a_z b_x - a_x b_z)\vec{j} + (a_x b_y - a_y b_x)\vec{k}$$

2. **Prędkość wektorowa**:

$$\vec{v}_{\text{śr}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t} \quad \left[\frac{\text{m}}{\text{s}}\right]$$

3. **Prędkość średnia**:

$$v_{\text{śr}} = \frac{x_{\text{całk}}}{t_{\text{całk}}} = \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} t_i}$$

4. **Prędkość chwilowa**:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}$$

5. **Przyspieszenie**:

$$\vec{a}_{\text{śr}} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt}, \quad a_s = \frac{d|\vec{v}|}{dt}, \quad a_n = \frac{v^2}{R}$$

6. **Ruch jednostajny prostoliniowy**:

$$\vec{a} = \vec{0} \implies \vec{v} = \overrightarrow{const} \implies \vec{r}(t) = \vec{r}_0 + \vec{v} \cdot t$$

   Dla ruchu wzdłuż osi x:

$$x(t) = x_0 + v \cdot t$$

7. **Ruch zmienny wzdłuż prostej**:

$$\vec{a} = \overrightarrow{const} \implies \vec{v}(t) = \vec{v}_0 + \vec{a} \cdot t$$

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{\vec{a} \cdot t^2}{2}$$

8. **I zasada dynamiki Newtona**:

$$\vec{F}_w = \vec{0} \iff \vec{v} = \overrightarrow{const}$$

9. **II zasada dynamiki Newtona**:

$$\vec{a} = \frac{\vec{F}_w}{m}$$

10. **Pęd**:

$$\vec{p} = m \cdot \vec{v}$$

11. **III zasada dynamiki Newtona**:

$$\vec{F}_{A \to B} = -\vec{F}_{B \to A}$$

12. **Rzut poziomy**:

$$x(t) = x_0 + v_{0x} \cdot t = v_0 \cdot t$$

$$y(t) = y_0 + v_{0y} \cdot t - \frac{gt^2}{2} = H - \frac{gt^2}{2}$$

$$t = t_c \text{ dla } y = 0$$

$$H - \frac{gt^2}{2} = 0$$

$$H = \frac{gt^2}{2} \rightarrow t_c = \sqrt{\frac{2H}{g}} \left[\sqrt{\frac{m}{\frac{m}{s^2}}} = s\right]$$

$$x = z \text{ dla } y = 0$$

$$z = v_0 \cdot t = v_0 \cdot \sqrt{\frac{2H}{g}} \left[\frac{m}{s}\sqrt{\frac{m}{\frac{m}{s^2}}} = \frac{m}{s} \cdot s = m\right]$$

$$v_x(t) = \frac{dx}{dt} = v_0 \cdot 1 \cdot t^{1-1} = v_0 = const$$

$$v_y(t) = \frac{dy}{dt} = -\frac{g}{2}2t = -gt \left[\frac{m}{s^2}s = \frac{m}{s}\right]$$

13. **Rzut pionowy**:

$$y(t) = 0 + v_0 t - \frac{gt^2}{2}$$

$$v_y(t) = \frac{dy}{dt} = v_0 - gt$$

$$t = t_c \text{ dla } y = 0 \rightarrow v_0 t_c = \frac{g}{2}t_c^2$$

14. **Rzut ukośny**:

$$v_{0x} = v_0 \cdot \cos\alpha$$

$$v_{0y} = v_0 \cdot \sin\alpha$$

$$x(t) = x_0 + v_{0x} \cdot t = v_0 \cdot t \cdot \cos\alpha$$

$$y(t) = y_0 + v_{0y} \cdot t - \frac{gt^2}{2} = v_0 \cdot t \cdot \sin\alpha - \frac{gt^2}{2}$$

$$v_x(t) = v_{0x} = v_0 \cdot \cos\alpha$$

$$v_y(t) = v_{0y} - g \cdot t = v_0 \cdot \sin\alpha - g \cdot t$$

$$t = t_c \text{ dla } y = 0$$

$$y(t) = v_0 \cdot t \cdot \sin\alpha - \frac{g \cdot t^2}{2}, \quad v_0 \cdot t_c \cdot \sin\alpha - \frac{g \cdot t_c^2}{2} = 0$$

$$t_c \left(v_0 \cdot \sin\alpha - \frac{g \cdot t_c}{2} \right) = 0 \Rightarrow t_c = 0 \quad \vee \quad t_c = \frac{2v_0 \cdot \sin\alpha}{g}$$

$$z = x(t_c) \longrightarrow \text{z - zasięg}$$

$$x(t) = v_0 \cdot t \cdot \cos\alpha \Rightarrow x(t_c) = v_0 \cdot \frac{2v_0 \sin\alpha}{g} \cdot \cos\alpha = \frac{2v_0^2 \sin\alpha \cos\alpha}{g}$$

$$y = H_{max} \text{ dla } t = t_{wzn}$$

$$H_{max} = y(t_{wzn}) = v_0 \cdot t_{wzn} \cdot \sin\alpha - \frac{g \cdot t_{wzn}^2}{2} =$$

$$v_0 \cdot \frac{v_0 \cdot \sin\alpha}{g} \cdot \sin\alpha - \frac{g}{2} \cdot \left(\frac{v_0 \cdot \sin\alpha}{g} \right)^2 = \frac{v_0^2 \cdot \sin^2\alpha}{g} - \frac{v_0^2 \cdot \sin^2\alpha}{2 \cdot g}$$

$$H_{max} = \frac{v_0^2 \cdot \sin^2\alpha}{2 \cdot g}$$

15. **II zasada dynamiki w postaci pędowej**:

$$\overrightarrow{F_{zew}} = \frac{d\vec{p}}{dt}$$

$$\Downarrow$$

$$\overrightarrow{F_{zew}} = \vec{0} \Rightarrow \frac{d\vec{p}}{dt} = 0 \Leftrightarrow \vec{p} = \overrightarrow{const}$$

$$\vec{p}_{pocz} = \vec{p}_{\text{końc}}$$

$$\vec{p}_{pocz} = \vec{0}, \quad \vec{p}_{\text{końc}} = m_1 \cdot \overrightarrow{v_1} - m_2 \cdot \overrightarrow{v_2} = \vec{0}$$

16. **Energia**:

$$E_k = \frac{mv^2}{2} \longrightarrow \text{Energia kinetyczna}$$

$$E_p = -G\frac{Mm}{r} \longrightarrow \text{Energia potencjalna}$$

$$E_p = mgh \longrightarrow \text{W pobliżu pow. Ziemi}$$

$$E_p = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \longrightarrow \text{Siła elektrostatyczna (Coulomba)}$$

$$E_p = \frac{kx^2}{w} \longrightarrow \text{Siła sprężystości}$$

17. **Praca**:

$$W = \vec{F} \circ \vec{s} = F \cdot s \cdot \cos a$$

$$\Delta W_i = F_i \cdot \Delta x$$

$$W = \sum_{i=1}^{n} F_i \cdot \Delta x$$

$$W = \lim_{\Delta x \to 0} \sum_{i=1}^{n} F_i \cdot \Delta x = \int_{x_1}^{x_2} F(x)dx$$

$$x = v_0 \cdot t + \frac{a \cdot t^2}{2}$$

$$v = v_0 + a \cdot t \Rightarrow a = \frac{v - v_0}{t}$$

$$x = \frac{v + v_0}{2} \cdot t$$

$$W = F \cdot x = m \cdot a \cdot x = m \left(\frac{v - v_0}{t}\right) \left(\frac{v + v_0}{2}\right) t = \frac{mv^2}{2} - \frac{mv_0^2}{2}$$

$$W = E_k - E_{k_0} \longrightarrow \text{Równoważność pracy i energii}$$

18. **Moc**:

$$P_r = \frac{\Delta W}{\Delta t}$$

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

$$P = \frac{dW}{dt} = \frac{\vec{F} \circ d\vec{s}}{dt} = \vec{F} \circ \vec{v}$$