Fizyka - Wzory

1. Iloczyn wektorowy:

$$\vec{a} \times \vec{b} = (a_y b_z - a_z b_y) \vec{i} + (a_z b_x - a_x b_z) \vec{j} + (a_x b_y - a_y b_x) \vec{k}$$

2. Prędkość wektorowa:

$$\vec{v}_{
m sr} = rac{\Delta \vec{r}}{\Delta t} = rac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t} \quad \left[rac{
m m}{
m s}
ight]$$

3. Prędkość średnia:

$$v_{\text{sr}} = \frac{x_{\text{całk}}}{t_{\text{całk}}} = \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} t_i}$$

4. Prędkość chwilowa:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}$$

5. Przyspieszenie:

$$\vec{a}_{\text{sr}} = \frac{\Delta \vec{v}}{\Delta t}, \quad \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{r}}{dt^2}$$

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt}, \quad a_s = \frac{d|\vec{v}|}{dt}, \quad a_n = \frac{v^2}{R}$$

6. Ruch jednostajny prostoliniowy:

$$\vec{a} = \vec{0} \implies \vec{v} = \overrightarrow{const} \implies \vec{r}(t) = \vec{r}_0 + \vec{v} \cdot t$$

Dla ruchu wzdłuż osi x:

$$x(t) = x_0 + v \cdot t$$

7. Ruch zmienny wzdłuż prostej:

$$\vec{a} = \overrightarrow{const} \implies \vec{v}(t) = \vec{v}_0 + \vec{a} \cdot t, \quad \vec{r}(t) = \vec{r}_0 + \vec{v}_0 \cdot t + \frac{\vec{a} \cdot t^2}{2}$$

8. I zasada dynamiki Newtona:

$$\vec{F}_w = \vec{0} \iff \vec{v} = \overrightarrow{const}$$

9. II zasada dynamiki Newtona:

$$\vec{a} = \frac{\vec{F_w}}{m}$$

10. **Pęd**:

$$\vec{p} = m \cdot \vec{v}$$

11. III zasada dynamiki Newtona:

$$\vec{F}_{A \to B} = -\vec{F}_{B \to A}$$

12. **Rzut poziomy**:

$$x(t) = x_0 + v_{0x} \cdot t = v_0 \cdot t$$
$$y(t) = y_0 + v_{0y} \cdot t - \frac{gt^2}{2} = H - \frac{gt^2}{2}$$

$$t = t_c \text{ dla } y = 0, \quad H - \frac{gt^2}{2} = 0, \quad H = \frac{gt^2}{2} \to t_c = \sqrt{\frac{2H}{g}} \left[\sqrt{\frac{m}{\frac{m}{s^2}}} = s \right]$$

$$x = z \text{ dla } y = 0, \quad z = v_0 \cdot t = v_0 \cdot \sqrt{\frac{2H}{g}} \left[\frac{m}{s} \sqrt{\frac{m}{\frac{m}{s^2}}} = \frac{m}{s} \cdot s = m \right]$$

$$v_x(t) = \frac{dx}{dt} = v_0 \cdot 1 \cdot t^{1-1} = v_0 = const$$

$$v_y(t) = \frac{dy}{dt} = -\frac{g}{2}2t = -gt \left[\frac{m}{s^2}s = \frac{m}{s} \right]$$

13. Rzut pionowy:

$$y(t) = 0 + v_0 t - \frac{gt^2}{2}$$

$$v_y(t) = \frac{dy}{dt} = v_0 - gt$$

$$t = t_c \text{ dla } y = 0 \to v_0 t_c = \frac{g}{2} t_c^2$$

14. Rzut ukośny:

$$v_{0x} = v_0 \cdot \cos \alpha$$

$$v_{0y} = v_0 \cdot \sin \alpha$$

$$x(t) = x_0 + v_{0x} \cdot t = v_0 \cdot t \cdot \cos \alpha$$

$$y(t) = y_0 + v_{0y} \cdot t - \frac{gt^2}{2} = v_0 \cdot t \cdot \sin \alpha - \frac{gt^2}{2}$$

$$v_x(t) = v_{0x} = v_0 \cdot \cos \alpha$$

$$v_y(t) = v_{0y} - g \cdot t = v_0 \cdot \sin \alpha - g \cdot t$$

$$t = t_c \text{ dla } y = 0$$

$$y(t) = v_0 \cdot t \cdot \sin \alpha - \frac{g \cdot t^2}{2}, \quad v_0 \cdot t_c \cdot \sin \alpha - \frac{g \cdot t_c^2}{2} = 0$$

$$3$$

$$\begin{split} t_c \left(v_0 \cdot \sin \alpha - \frac{g \cdot t_c}{2} \right) &= 0 \Rightarrow t_c = 0 \quad \lor \quad t_c = \frac{2 v_0 \cdot \sin \alpha}{g} \\ z &= x(t_c) \longrightarrow \text{z - zasięg} \\ x(t) &= v_0 \cdot t \cdot \cos \alpha \Rightarrow x(t_c) = v_0 \cdot \frac{2 v_0 \sin \alpha}{g} \cdot \cos \alpha = \frac{2 v_0^2 \sin \alpha \cos \alpha}{g} \\ y &= H_{max} \text{ dla } t = t_{wzn} \\ H_{max} &= y(t_{wzn}) = v_0 \cdot t_{wzn} \cdot \sin \alpha - \frac{g \cdot t_{wzn}^2}{2} = \\ v_0 \cdot \frac{v_0 \cdot \sin \alpha}{g} \cdot \sin \alpha - \frac{g}{2} \cdot \left(\frac{v_0 \cdot \sin \alpha}{g} \right)^2 = \frac{v_0^2 \cdot \sin^2 \alpha}{g} - \frac{v_0^2 \cdot \sin^2 \alpha}{2 \cdot g} \\ H_{max} &= \frac{v_0^2 \cdot \sin^2 \alpha}{2 \cdot g} \end{split}$$