2.1 摄像机：

$$\left[\frac{\lambda X}{\lambda - Z} \quad \frac{\lambda Y}{\lambda - Z} \quad \frac{\lambda Z}{\lambda - Z}\right]^T$$

$$= \left[\frac{0.5}{-2.5} \quad \frac{1}{-2.5} \quad \frac{1.5}{-2.5}\right]^T$$

$$= \left[-0.2 \quad -0.4 \quad -0.6\right]^T$$

图像：

$$\left[\frac{\lambda X}{\lambda - Z} \quad \frac{\lambda Y}{\lambda - Z}\right]^T$$

$$= \left[-0.2 \quad -0.4\right]^T$$

2.2 S: ① ② ③

T: ② ③

		D_4	D_8	D_m
2.3.	(1)	∞	4	5
	(2)	6	4	6

2.4 顺时针旋转.

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \theta = 45^\circ$$

$$\| $$

$$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{2}(x+y) \\ \frac{\sqrt{2}}{2}(y-x) \\ 1 \end{pmatrix}$$

变换后：$\left(\frac{\sqrt{2}}{2}(x+y), \frac{\sqrt{2}}{2}(y-x)\right)$

$$= \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

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$$T = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad S = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

先平移后尺度变换

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = T \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 9 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \\ z'' \\ 1 \end{pmatrix} = S \begin{pmatrix} 3 \\ 6 \\ 9 \\ 1 \end{pmatrix} = \begin{pmatrix} 12 \\ 18 \\ 18 \\ 1 \end{pmatrix}$$

则变换后：$(12, 18, 18)$

先尺度后平移

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = S \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 6 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \\ z'' \\ 1 \end{pmatrix} = T\begin{pmatrix} 4 \\ 6 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ 12 \\ 1 \end{pmatrix}$$

变换后：(6, 10, 12)

结果不同，尺度变换后，平移操作需增加相应的尺度才能使结果相同，即使T变为

$$T' = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ 才能使结果相同。

2.6 设三个顶点分别为 (x_1, y_1)，(x_2, y_2)，(x_3, y_3)

三个失真顶点为 (x_1', y_1')，(x_2', y_2')，(x_3', y_3')

由于线性失真 令 $\left.\begin{aligned} x_i' &= a_1 x_i + a_2 y_i + a_3 \\ y_i' &= b_1 x_i + b_2 y_i + b_3 \end{aligned}\right\}$ 其中 $i = 1, 2, 3$

$$M = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

$$\left.\begin{aligned}
\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} &= (M^T M)^{-1} M^T \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} \\
\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} &= (M^T M)^{-1} M^T \begin{bmatrix} y_1' \\ y_2' \\ y_3' \end{bmatrix}
\end{aligned}\right\} \text{可求出 } a_i, b_i \ (i=1,2,3)$$