Math Assignment

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Code: xxxx
Section: xx
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Program: CSE

1 Evaluate: $\int \frac{4x}{\sqrt[3]{2x^2+3}}dx$

Solution:

$$\int \frac{4x}{\sqrt[3]{2x^2+3}}dx$$

$$\text{Let } u = 2x^2 + 3$$
$$=> \frac{du}{dx} = 4x$$
$$=> dx = \frac{du}{4x}$$

$$= \int \frac{1}{\sqrt[3]{u}} \cdot \frac{du}{4x}$$
$$= \int \frac{1}{\sqrt[3]{u}}du$$
$$= \int \frac{1}{\sqrt[3]{u}}du = \frac{3}{2}u^{\frac{2}{3}} + C$$
$$= \frac{3}{2}(2x^2+3)^{\frac{2}{3}} + C$$

2 Evaluate: $\int (3x-5)^{2016}dx$

Solution:

$$\int (3x-5)^{2016}dx$$

$$\text{Let } u = 3x - 5$$
$$=> \frac{du}{dx} = 3$$
$$=> dx = \frac{du}{3}$$

$$= \int u^{2016} \cdot \frac{du}{3}$$
$$= \int u^{2016} \cdot \frac{1}{3}du$$
$$= \frac{1}{3}\int u^{2016}du$$
$$= \frac{1}{3}\int u^{2016}du = \frac{1}{3} \cdot \frac{u^{2017}}{2017} + C$$
$$= \frac{1}{3} \cdot \frac{(3x-5)^{2017}}{2017} + C$$
$$\therefore \int (3x-5)^{2016}dx = \frac{1}{6051}(3x-5)^{2017} + C$$

3 Evaluate: $\int x\sqrt{x^2+9}dx$

Solution:

$$\int x\sqrt{x^2+9}dx$$

$$\text{Let } u = x^2+9$$
$$=> \frac{du}{dx} = 2x$$
$$=> dx = \frac{du}{2x}$$

$$= \int \sqrt{u} \cdot \frac{du}{2x}$$
$$= \int \sqrt{u} \cdot \frac{1}{2}du$$
$$= \frac{1}{2}\int \sqrt{u}du$$
$$= \frac{1}{2} \cdot \frac{2}{3}u^{\frac{3}{2}} + C$$
$$= \frac{1}{3}(x^2+9)^{\frac{3}{2}} + C$$
$$\therefore \int x\sqrt{x^2+9}dx = \frac{1}{3}(x^2+9)^{\frac{3}{2}} + C$$

4 Evaluate: $\int \frac{x^2}{\sqrt[3]{2x^3-1}}dx$

Solution:

$$\int \frac{x^2}{\sqrt[3]{2x^3-1}}dx$$

$$\text{Let } u = 2x^3 - 1$$

$$=> \frac{du}{dx} = 6x^2$$

$$=> dx = \frac{du}{6x^2}$$

$$= \int \frac{1}{\sqrt[3]{u}} \cdot \frac{du}{6x^2}$$

$$= \int \frac{1}{\sqrt[3]{u}} \cdot \frac{1}{6}du$$

$$= \frac{1}{6}\int \frac{1}{\sqrt[3]{u}}du$$

$$= \frac{1}{6} \cdot \frac{3}{2}u^{\frac{2}{3}} + C$$

$$= \frac{1}{4}(2x^3 - 1)^{\frac{2}{3}} + C$$

$$\therefore \int \frac{x^2}{\sqrt[3]{2x^3-1}}dx = \frac{1}{4}(2x^3 - 1)^{\frac{2}{3}} + C$$

5 Evaluate: $\int x\sin(2x^2)dx$

Solution:

$$\int x\sin(2x^2)dx$$

$$\text{Let } u = 2x^2$$
$$\Rightarrow \frac{du}{dx} = 4x$$
$$\Rightarrow dx = \frac{du}{4x}$$

$$= \int \sin(u)\cdot\frac{du}{4x}$$
$$= \frac{1}{4}\int \sin(u)du$$
$$= \frac{1}{4}\cdot -\cos(u) + C$$
$$= -\frac{1}{4}\cos(2x^2) + C$$
$$\therefore \int x\sin(2x^2)dx = -\frac{1}{4}\cos(2x^2) + C$$

6 Evaluate: $\int (1-\cos x)^5 \sin x dx$

Solution:

$$\begin{aligned}
&\int (1-\cos x)^5 \sin x dx \\
&\text{Let } u = 1-\cos x \\
&\Rightarrow \frac{du}{dx} = \sin x \\
&\Rightarrow dx = \frac{du}{\sin x} \\
&= \int u^5 \cdot \frac{du}{\sin x} \\
&= \int u^5 du \\
&= \frac{u^6}{6} + C \\
&= \frac{(1-\cos x)^6}{6} + C \\
&\therefore \int (1-\cos x)^5 \sin x dx = \frac{(1-\cos x)^6}{6} + C
\end{aligned}$$

7 Evaluate: $\int x\sqrt{2-3x^2}dx$

Solution:

$$\begin{aligned}
&\int x\sqrt{2-3x^2}dx \\
&\text{Let } u = 2-3x^2 \\
&\Rightarrow \frac{du}{dx} = -6x \\
&\Rightarrow dx = -\frac{du}{6x} \\
&= -\frac{1}{6}\int x\sqrt{u}\cdot\frac{du}{6x} \\
&= -\frac{1}{6}\int \sqrt{u}du \\
&= -\frac{1}{6}\cdot\frac{2}{3}u^{3/2} + C \\
&= -\frac{1}{9}(2-3x^2)^{3/2} + C \\
&\therefore \int x\sqrt{2-3x^2}dx = -\frac{1}{9}(2-3x^2)^{3/2} + C
\end{aligned}$$

8 Evaluate: $\int \frac{1}{\sqrt{7x+5}}dx$

Solution:

$$\int \frac{1}{\sqrt{7x+5}}dx$$
$$\text{Let } u = 7x+5$$
$$\Rightarrow \frac{du}{dx} = 7$$
$$\Rightarrow dx = \frac{du}{7}$$
$$= \frac{1}{7}\int \frac{1}{\sqrt{u}} \cdot \frac{du}{7}$$
$$= \frac{1}{7}\int u^{-\frac{1}{2}}du$$
$$= \frac{1}{7} \cdot 2u^{\frac{1}{2}} + C$$
$$= \frac{2}{7}(7x+5)^{\frac{1}{2}} + C$$
$$\therefore \int \frac{1}{\sqrt{7x+5}}dx = \frac{2}{7}(7x+5)^{\frac{1}{2}} + C$$

9 Evaluate: $\int \cos^3 x \sin x dx$

Solution:

$$\begin{aligned}
&\int \cos^3 x \sin x dx \\
&\text{Let } u = \cos^3 x, dv = \sin x dx \\
&= \cos^3 x(-\cos x) - \int (-\cos x)(-3\cos^2 x \sin x)dx \\
&= -\cos^4 x + 3\int \cos^4 x \sin x dx \\
&\text{Let } v = \cos^4 x, dv/dx = 4\cos^3 x \sin x \\
&= -\cos^4 x + 3\int v \cdot \frac{dv}{4} \\
&= -\cos^4 x + \frac{3v^2}{8} + C \\
&= -\cos^4 x + \frac{3}{8}\cos^8 x + C \\
&\therefore \int \cos^3 x \sin x dx = -\cos^4 x + \frac{3}{8}\cos^8 x + C
\end{aligned}$$

10 Evaluate: $\int \tan^3 x \sec^2 x dx$

Solution:

$$\begin{aligned}
&\int \tan^3 x \sec^2 x dx \\
&\text{Let } u = \tan x, du = \sec^2 x dx \\
&= \int u^3 du \\
&= \frac{u^4}{4} + C \\
&= \frac{\tan^4 x}{4} + C \\
&\therefore \int \tan^3 x \sec^2 x dx = \frac{\tan^4 x}{4} + C
\end{aligned}$$

11 Evaluate: $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$

Solution:

Let $u = \sqrt{x}$.
Then $x = u^2$ and $dx = 2udu$.

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx = \int \cos(u) \cdot \frac{1}{u} \cdot 2udu$$
$$= 2 \int \cos(u) du$$
$$= 2\sin(u) + C$$

12 Evaluate: $\int_0^{\frac{\pi}{2}} (1 + 3\sin x)^{\frac{3}{2}} \cos x dx$

Solution:

$$\int_0^{\frac{\pi}{2}} (1 + 3\sin x)^{\frac{3}{2}} \cos x dx$$
(Let $u = 1 + 3\sin x, du = 3\cos x dx$)
$$= \int_0^{\frac{\pi}{2}} u^{\frac{3}{2}} \cdot \frac{1}{3\cos x} du$$
$$= \int_0^{\frac{\pi}{2}} u^{\frac{3}{2}} \cdot \frac{1}{3} \cdot \sec x dx$$
$$= \frac{1}{3} \int_0^{\frac{\pi}{2}} u^{\frac{3}{2}} d(\sin x)$$
$$= \frac{1}{3} \cdot \frac{2}{5} \left[u^{\frac{5}{2}} \right]_0^{\frac{\pi}{2}}$$
$$= \frac{1}{15} \left[(1 + 3\sin x)^{\frac{5}{2}} \right]_0^{\frac{\pi}{2}}$$
$$= \frac{1}{15} \cdot (4 - 0)$$
$$= \frac{4}{15}$$

13 Evaluate: $\int_0^2 e^{\sin x} \cos x dx$

Solution:

$$\int_0^2 e^{\sin x} \cos x dx$$

Let $u = e^{\sin x}, dv = \cos x dx$

$$= e^{\sin x} \sin x - \int \sin x \cdot e^{\sin x} \cos x dx$$

$$= e^{\sin x} \sin x - C \int e^{\sin x} dx$$

(Repeated integral; indefinite solution)

$$= e^{\sin x} \sin x - Ce^{\sin x} + D$$

$$\therefore \int_0^2 e^{\sin x} \cos x dx = e^{\sin x}(\sin x - C) + D$$

14 Evaluate: $\int \frac{\cot \sqrt{y} \csc \sqrt{y}}{\sqrt{y}}$

Solution:

Let $u = \sqrt{y}$.

Then $y = u^2$ and $dy = 2udu$.

$$\int \frac{\cot(\sqrt{y}) \csc(\sqrt{y})}{\sqrt{y}} dy = \int \cot(u) \csc(u) \cdot \frac{1}{u} \cdot 2udu$$

$$= 2 \int \cot(u) \csc(u) du$$

$$= -2 \csc(u) + C$$

$$\therefore \int \frac{\cot \sqrt{y} \csc \sqrt{y}}{\sqrt{y}} = -2 \csc(\sqrt{y}) + C$$

15 Evaluate: $\int e^{-\cot(x)} \csc^2 x dx$

Solution:

Let $u = -\cot(x)$.
Then $du = -\csc^2(x)dx$.

$$\int e^{-\cot(x)} \csc^2(x)dx = \int e^u \cdot \csc^2(x) \cdot \frac{-1}{\csc^2(x)} du$$
$$= -\int e^u du$$
$$= -e^u + C$$

$$\therefore \int e^{-\cot(x)} \csc^2 x dx = -e^{-\cot(x)} + C$$

16 Evaluate: $\int \tan^4(3x) \sec^2(3x)dx$

Solution:

Let $u = \tan(3x)$.
Then $du = 3\sec^2(3x)dx$.

$$\int \tan^4(3x) \sec^2(3x)dx = \int u^4 \cdot \sec^2(3x) \cdot \frac{1}{3\sec^2(3x)} du$$
$$= \frac{1}{3} \int u^4 du$$
$$= \frac{1}{15} u^5 + C$$

$$\therefore \int \tan^4(3x) \sec^2(3x)dx = \frac{1}{15} \tan^5(3x) + C$$

17 Evaluate: $\int \frac{(\ln t)^{10}}{t} dt$

Solution:

$$\text{Let } u = \ln t.$$
$$\text{Then } du = \frac{1}{t} dt.$$

$$\int \frac{(\ln t)^{10}}{t} dt = \int u^{10} \cdot t \cdot \frac{1}{t} du$$
$$= \int u^{10} du$$
$$= \frac{u^{11}}{11} + C$$

$$\therefore \int \frac{(\ln t)^{10}}{t} dt = \frac{(\ln t)^{11}}{11} + C$$

18 Evaluate: $\int \frac{1+(\sqrt{x})^4}{\sqrt{x}} dx$

Solution:

$$\int \frac{1 + (\sqrt{x})^4}{\sqrt{x}} dx = \int (1 + x^2/\sqrt{x}) dx$$
$$= \int (1 + x^{\frac{5}{2}}) dx$$
$$= x + \frac{2}{7} x^{\frac{7}{2}} + C$$

19 Evaluate: $\int \frac{e^x}{1+e^{2x}} dx$

Solution:

$$\begin{aligned}
&\text{Let } u = e^x. \\
&\text{Then } du = e^x dx.
\end{aligned}$$

$$\begin{aligned}
&\int \frac{e^x}{1+e^{2x}} dx = \int \frac{1}{1+u^2} \cdot \frac{1}{e^x} du \\
&= \int \frac{1}{u(1+u^2)} du \\
&= \tan^{-1}(e^x) + C
\end{aligned}$$

$$\therefore \int \frac{e^x}{1+e^{2x}} dx = \tan^{-1}(e^x) + C$$

20 Evaluate: $\int \frac{x}{\sqrt{e^{2x^2}-1}} dx$

Solution:

$$\begin{aligned}
&\text{Let } u = e^{x^2}. \\
&\text{Then } du = 2xe^{x^2} dx.
\end{aligned}$$

$$\begin{aligned}
&\int \frac{x}{\sqrt{e^{2x^2}-1}} dx = \int \frac{x}{\sqrt{u^2-1}} \cdot \frac{1}{2xe^{x^2}} du \\
&= \frac{1}{2} \int \frac{1}{\sqrt{u^2-1}} du \\
&= \frac{1}{2} \sin^{-1}(u) + C \\
&\therefore \int \frac{x}{\sqrt{e^{2x^2}-1}} dx = \frac{1}{2} \sin^{-1}(e^{x^2}) + C
\end{aligned}$$

21 Evaluate: $\int \frac{e^{2x}}{1+e^{4x}} dx$

Solution:

Let $u = e^{2x}$.
Then $du = 2e^{2x} dx$.

$$\int \frac{e^{2x}}{1+e^{4x}} dx = \int \frac{1}{1+u^2} \cdot \frac{1}{2e^{2x}} du$$
$$= \frac{1}{2} \int \frac{1}{u(1+u^2)} du$$
$$= \frac{1}{2} \tan^{-1}(u) + C$$

$$\therefore \int \frac{e^{2x}}{1+e^{4x}} dx = \frac{1}{2} \tan^{-1}(e^{2x}) + C$$

22 Evaluate: $\int \frac{3x}{\sqrt{1-x^4}} dx$

Solution:

Let $u = x^2$.
Then $du = 2x dx$.

$$\int \frac{3x}{\sqrt{1-x^4}} dx = \int \frac{3x}{\sqrt{1-u^2}} \cdot \frac{1}{2x} du$$
$$= \frac{3}{2} \int \frac{1}{\sqrt{1-u^2}} du$$
$$= \frac{3}{2} \sin^{-1}(u) + C$$

$$\therefore \int \frac{3x}{\sqrt{1-x^4}} dx = \frac{3}{2} \sin^{-1}(x^2) + C$$

23 Evaluate: $\int \frac{1}{(1+t^2)\tan^{-1} t} dt$

Solution:

$$\text{Let } u = \tan^{-1} t,\ dv = \frac{1}{1+t^2} dt$$
$$du = \frac{1}{1+t^2} dt,\ v = \tan^{-1} t$$

$$\int \frac{1}{(1+t^2)\tan^{-1} t} dt = \tan^{-1} t \cdot \tan^{-1} t - \int \tan^{-1} t \cdot \frac{1}{1+t^2} dt$$
$$\Rightarrow 2I = \tan^{-2} t$$
$$\Rightarrow I = \frac{1}{2} \tan^{-2} t + C$$

24 Evaluate: $\int \frac{\tan(\sqrt{x})\sec^2(\sqrt{x})}{\sqrt{x}} dx$

Solution:

$$\text{Let } u = \sqrt{x},\ du = \frac{1}{2\sqrt{x}} dx$$
$$\Rightarrow dx = 2\sqrt{x} du$$

$$\int \frac{\tan(\sqrt{x})\sec^2(\sqrt{x})}{\sqrt{x}} dx = 2 \int \tan(u)\sec^2(u) \cdot u du$$
$$= 2u \sec(u) + C$$
$$= 2\sqrt{x} \sec(\sqrt{x}) + C$$

25 Evaluate: $\int \frac{(2x^2+1)(x^2+\ln x)}{x}dx$

Solution:

$$\int \frac{(2x^2+1)(x^2+\ln x)}{x}dx = \int \frac{2x^4+2x^2\ln x+x^2+\ln x}{x}dx$$
$$= \int 2x^3+2x\ln x+x+\frac{\ln x}{x}dx$$
$$= \frac{1}{2}x^4+x^2\ln x-x^2+\frac{1}{2}x^2+\frac{1}{2}\ln^2 x+C$$

26 Evaluate: $\int \frac{\sin x}{(1+\cos x)^2}dx$

Solution:

$$\text{Let } u = 1+\cos x,\ du = -\sin x dx$$
$$\Rightarrow dx = -\frac{du}{\sin x}$$

$$\int \frac{\sin x}{(1+\cos x)^2}dx = -\int \frac{1}{u^2}du$$
$$= \frac{1}{u}+C$$
$$= \frac{1}{1+\cos x}+C$$

27 Evaluate: $\int \frac{\cos(\ln x)}{x} dx$

Solution:

$$\begin{aligned}
&\text{Let } u = \ln x,\ du = \frac{1}{x}dx \\
&\Rightarrow dx = xdu
\end{aligned}$$

$$\begin{aligned}
&\int \frac{\cos(\ln x)}{x} dx = \int \cos(u) \cdot xdu \\
&= \int \cos(u) du \\
&= \sin(u) + C \\
&= \sin(\ln x) + C
\end{aligned}$$

28 Evaluate: $\int \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}}$

Solution:

$$\begin{aligned}
&\text{Let } u = (\sin^{-1} x)^2,\ dv = \frac{1}{\sqrt{1-x^2}}dx \\
&du = \frac{2\sin^{-1} x}{\sqrt{1-x^2}}dx,\ v = \sin^{-1} x
\end{aligned}$$

$$\begin{aligned}
&\int \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}}dx = (\sin^{-1} x)^3 - 2\int (\sin^{-1} x)^2 \cdot \frac{1}{\sqrt{1-x^2}}dx \\
&\Rightarrow 3I = (\sin^{-1} x)^3 \\
&\Rightarrow I = \frac{1}{3}(\sin^{-1} x)^3 + C
\end{aligned}$$

29 Evaluate: $\int \frac{\ln(\ln x)}{x \ln x} dx$

Solution:

$$\begin{aligned}
&\text{Let } u = \ln(\ln x),\ du = \frac{1}{\ln x} \cdot \frac{1}{x} dx \\
&\Rightarrow dx = \ln x \cdot x du
\end{aligned}$$

$$\begin{aligned}
&\int \frac{\ln(\ln x)}{x \ln x} dx = \int u \cdot \ln x \cdot x du \cdot \frac{1}{x \ln x} \\
&= \int u du \\
&= \frac{1}{2} u^2 + C \\
&= \frac{1}{2} \ln^2(\ln x) + C
\end{aligned}$$

30 Evaluate: $\int \frac{1}{x\sqrt{4-9(\ln x)^2}} dx$

Solution:

$$\begin{aligned}
&\text{Let } u = \ln x,\ du = \frac{1}{x} dx \\
&\Rightarrow dx = x du
\end{aligned}$$

$$\begin{aligned}
&\int \frac{1}{x\sqrt{4-9(\ln x)^2}} dx = \int \frac{1}{\sqrt{4-9u^2}} du \\
&= \frac{1}{3} \sin^{-1}\left(\frac{3u}{2}\right) + C \\
&= \frac{1}{3} \sin^{-1}\left(\frac{3 \ln x}{2}\right) + C
\end{aligned}$$