Function Exercises

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December 2, 2023

1 Functions

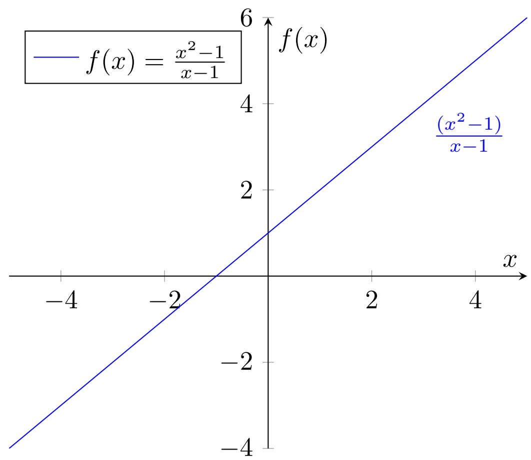
1.1 Find the domain and range of $f(x) = \frac{x^2-1}{x-1}$

Domain is all possible inputes for which the function is defined. In this case, we need to avoid values of x that makes the denominator equal to zero, as division by zero is undefined. Thus the domain is all real numbers except $x = 1$, because that would make the denominator $x - 1 = 0$

$$\text{Domain: } x \in \mathbb{R} \setminus \{1\}$$

Let function

$$f(x) = \frac{(x^2-1)}{x-1}$$
$$\Rightarrow f(x) = \frac{(x-1)(x+1)}{x-1}$$
$$\Rightarrow f(x) = x + 1$$
$$\therefore \text{Range: } f(x) \in \mathbb{R}$$



1.2 Find the domain and range of

$$f(x) = \begin{cases} 2x+6 & \textbf{if } -3 \le x \le 0 \\ 6 & \textbf{if } 0 \le x \le 2 \\ 2x-6 & \textbf{if } 2 \le x \le 5 \end{cases}$$

Domain:

The domain of $f(x)$ is set to all possible inputes values for which the function is defined. In this case, the function is defined in the intervals $[-3,0]$, $[0,2]$, $[2,5]$, and for any other values, it is not explicitly defind. So the domain is:

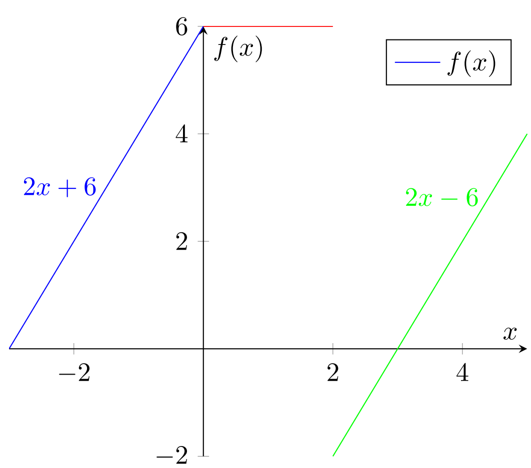
$$\text{Domain: } x \in [-3,0] \cup [0,2] \cup [2,5]$$

$$x \in [-3,5]$$

Range:

For $-3 \le x \le 0$, $f(x) = 2x+6$ where the output varies from 0 to 6
For $0 \le x \le 2$, $f(x) = 6$
For $2 \le x \le 5$, $f(x) = 2x-6$ where the ouptu varies from -2 to 4
So the Range is:

$$\text{Ragne: } f(x) \in [0,6] \cup [-2,4] \rightarrow f(x) \in [-2,6]$$



1.3 Find the domain and range of

$$f(x) = \begin{cases} x-1 & \textbf{if } x>0 \\ -\frac{1}{2} & \textbf{if } x=0 \\ x+1 & \textbf{if } x<0 \end{cases}$$

Domain:

Domain of this function $f(x)$ can take any input.

$$\therefore \text{Domain: } x \in \mathbb{R}$$
$$\text{Domain: } x \in (-\infty, \infty)$$

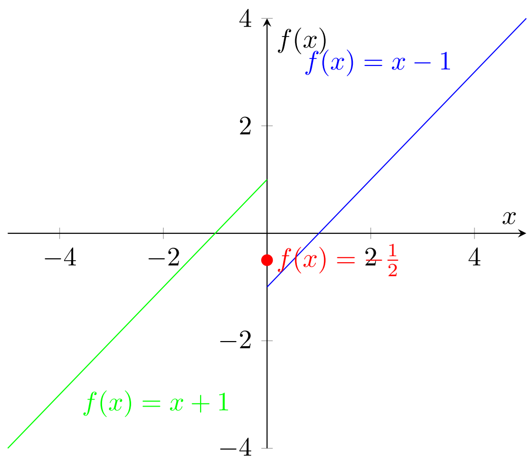
Range:

For $x > 0$, $f(x) = x - 1$ where the result of the function is $[0, \infty]$

For $x = 0$, $f(x) = -\frac{1}{2}$ which is a constant value

For $x < 0$, $f(x) = x + 1$ where the result of the function is $[0, -\infty]$

$$\therefore \text{Range: } x \in [0, -\infty] \cup [\frac{1}{2}] \cup [0, \infty]$$
$$\text{Range: } x \in (-\infty, \infty)$$



1.4 Find the domain and range of

$$f(x) = \begin{cases} -2x+1 & \textbf{if } x < 0 \\ 1 & \textbf{if } 0 \le x < 1 \\ 2x-1 & \textbf{if } x \ge 1 \end{cases}$$

Domain:

Domain of this function $f(x)$ can take any input.

$$\begin{aligned} \therefore \text{Domain: } & x \in \mathbb{R} \\ \text{Domain: } & x \in (-\infty, \infty) \end{aligned}$$

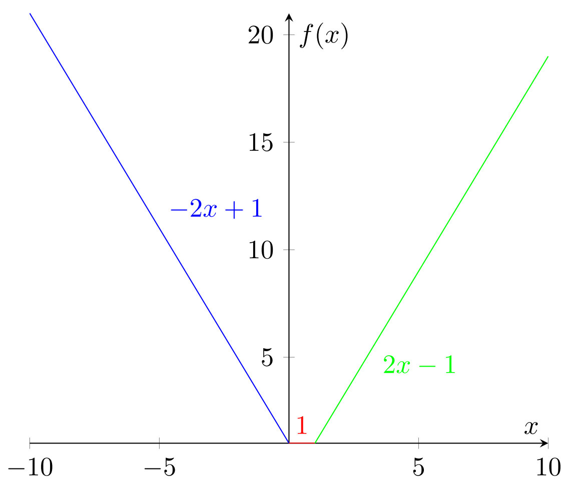
Range:

For $x < 0$, $f(x) = -2x + 1$ where the result of the function is $(-\infty, 1]$

For $0 \le x < 1$, $f(x) = 1$ which is a constant value

For $x \ge 1$, $f(x) = 2x - 1$ where the result of the function is $[1, \infty)$

$$\begin{aligned} \therefore \text{Range: } & x \in (-\infty, 1] \cup [1] \cup [1, \infty) \\ \text{Range: } & x \in (-\infty, \infty) \end{aligned}$$



1.5 Find the domain and range of

$$f(x) = \begin{cases} x & \textbf{if } 0 \leq x < \frac{1}{2} \\ 1 & \textbf{if } x = \frac{1}{2} \\ 1 - x & \textbf{if } \frac{1}{2} < x \leq 1 \end{cases}$$

Domain:

The domain of the function $f(x)$ is all possible input values for which the function is defined. In this case, the intervals are $[0, \frac{1}{2})$, $[\frac{1}{2}]$, $(\frac{1}{2}, 1]$. So the domain is:

$$\text{Domain: } x \in [0, \frac{1}{2}) \cup [\frac{1}{2}] \cup (\frac{1}{2}, 1]$$
$$\text{Domain: } x \in [0, 1]$$

Range:

For $0 \leq x < \frac{1}{2}$, $f(x) = x$ where the result of the function is $[0, \frac{1}{2})$

For $x = \frac{1}{2}$, $f(x) = 1$ which is a constant value. So the range is [1]

For $\frac{1}{2} < x \leq 1$, $f(x) = 1 - x$ where the result of the function is $[\frac{1}{2}, 1]$

$$\therefore \text{Range: } x \in [0, \frac{1}{2}) \cup \{1\} \cup (\frac{1}{2}, 1]$$
$$\text{Range: } x \in [0, \frac{1}{2}) \cup \{1\}$$