Please note: The duration of the exam is **100** minutes. You may use a calculator. Please give a numeric answer (rounded to 4 decimal places). You are expected to **write all steps** taken in getting the final answer along with a **mention of properties/theorems used** in these steps. You are not allowed to leave the examination hall during the first 30 minutes of the exam.

1. Troll-tree is under threat from Chef again, who has planted a troll catching device under the tree. Of the trolls that the device catches, Chef finds only those trolls delectable whose height is at most 2cm. You know that the heights of the trolls has probability density function $\frac{x}{4}$, if $1 < x < 3$ and 0 otherwise (where x is measured in cm). Assume that the device can catch one troll at a time and heights of trolls caught on different occasions are totally independent. Let A be the event that the first troll caught by the device is delectable, B the second and C the third. Determine $\mathbb{P}(A \cup B \cup C)$.
Solution: (0 points) Let X denote the height of a troll.
(2 points) $\mathbb{P}(A) = \mathbb{P}(B) = \mathbb{P}(C) = \mathbb{P}(1 < X < 2)$ (or $\mathbb{P}(X < 2)$, using $\leq$ also gets all points)
(3 points) $\mathbb{P}(1 < X < 2) = \int_1^2 f_X(x)\mathrm{d}x$ (No points are awarded if there are mistakes in this step. Exceptions: if limits are a and b instead of 1 and 2.)
(2 points) $= \int_1^2 \frac{x}{4}\mathrm{d}x = \left[\frac{x^2}{8}\right]_1^2 = \frac{2^2}{8} - \frac{1^2}{8} = \frac{3}{8}$
Solution I:
(2 points) $\mathbb{P}(A \cup B \cup C) = 1 - \mathbb{P}(\overline{A \cup B \cup C})$
(1 points) using de Morgan laws:
(3 points) $= 1 - \mathbb{P}(\overline{A} \cap \overline{B} \cap \overline{C})$
(1 points) A, B, C are totally independent, so $\overline{A}, \overline{B}, \overline{C}$ are also totally independent, so,
(3 points) $= 1 - \mathbb{P}(\overline{A})\,\mathbb{P}(\overline{B})\,\mathbb{P}(\overline{C})$
(1 points) $\mathbb{P}(\overline{A}) = \mathbb{P}(\overline{B}) = \mathbb{P}(\overline{C}) = 1 - \frac{3}{8} = \frac{5}{8}$
(2 points) $\mathbb{P}(A \cup B \cup C) = 1 - \left(\frac{5}{8}\right)^3 = 1 - \frac{125}{512} = \frac{387}{512} \approx \underline{\underline{0.7559}}$
Solution II:
(2 points) Using Inclusion-Exclusion:
(4 points) $\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(A \cap B) - \mathbb{P}(A \cap C) - \mathbb{P}(B \cap C) + \mathbb{P}(A \cap B \cap C)$
(2 points) Since A, B, C are totally independent, so,
(3 points) $= \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(A)\,\mathbb{P}(B) - \mathbb{P}(A)\,\mathbb{P}(C) - \mathbb{P}(B)\,\mathbb{P}(C) + \mathbb{P}(A)\,\mathbb{P}(B)\,\mathbb{P}(C)$
(2 points) $= 3 \cdot \frac{3}{8} - 3 \cdot \left(\frac{3}{8}\right)^2 + \left(\frac{3}{8}\right)^3 = \frac{387}{512} \approx \underline{\underline{0.7559}}$

2. Let $c > 0$ be a real number, and let X be a continuous random variable with the following distribution function:
$$F_X : x \mapsto \begin{cases} 0 & \text{if } x \leq c, \\ \ln(x) - \ln(c) & \text{if } c < x \leq ce, \\ 1 & \text{if } ce < x, \end{cases}$$
where e is Euler's constant, ≈ 2.718.
(a) Given that $\mathbb{E}(X) = e - 1$, determine c.
(b) Determine the variance of X.
Solution: (2 points) $f_X(x) = F'_X(x)$, so,
(2 points) $f_X(x) = \frac{1}{x}$, if $c < x < ce$,
(1 points) and 0 otherwise
(2 points) $\mathbb{E}(X) = \int_{-\infty}^{\infty} x f_X(x)\,\mathrm{d}x$ (If the limits are c and ce, then also points are awarded.)
(2 points) $= \int_c^{ce} x\frac{1}{x}\mathrm{d}x = \int_c^{ce} 1\mathrm{d}x = [1]_c^{ce} = c(e-1)$
(1 points) $\Rightarrow c = \underline{\underline{1}}$
(2 points) $Var(X) = \mathbb{E}(X^2) - \mathbb{E}^2(X)$
(3 points) $\mathbb{E}(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,\mathrm{d}x$
(2 points) $= \int_1^e x^2\frac{1}{x}\,\mathrm{d}x = \int_1^e x\,\mathrm{d}x = \left[\frac{x^2}{2}\right]_1^e = \frac{e^2-1}{2}$
(2 points) $\Rightarrow Var(X) = \frac{e^2-1}{2} - (e-1)^2$
(1 points) $= \frac{1}{2}(e-1)(3-e) \approx \underline{\underline{0.2420}}$

3. Barny works at the IT hardware support division of a firm where he is responsible for resolving exactly 18 problems every day. A given problem is said to be 'relevant' if the restart of the machine does not resolve the

problem. Let all problems, independent of each other, have a probability of $\frac{1}{3}$ of being relevant.
(a) What is the distribution of the number of relevant problems resolved during a workday of Barny?
(b) Approximately what is the probability that there are more than 580 and less than 620 relavant problems in a given period of 100 days? (Assume that problems arising on different days are also independent.)
Solution: (1 points) Let X denote the number of relevant problems encountered in a day.
(2 points) X has Binomial distribution,
(1 points) with parameters: $X \sim B(18, \frac{1}{3})$
(1 points) Let $X_1, \dots, X_{100}$ denote the number of relevant problems encountered on different days.
(1 points) $\mathbb{E}(X_i) = 18 \cdot \frac{1}{3} = 6$
(1 points) $Var(X_i) = 18 \cdot \frac{1}{3} \cdot \frac{2}{3} = 4$ and $\sigma_{X_i} = 2$
(1 points) $\mathbb{P}(580 < \sum_{i=1}^{100} X_i < 620) = ?$
(2 points) $= \mathbb{P}(\sum_{i=1}^{100} X_i < 620) - \mathbb{P}(\sum_{i=1}^{100} X_i \leq 580)$
(1 points) $= \mathbb{P}(\sum_{i=1}^{100} X_i - 100 \cdot 6 < 20) - \mathbb{P}(\sum_{i=1}^{100} X_i - 100 \cdot 6 \leq -20)$
(1 points) $= \mathbb{P}\left(\frac{\sum_{i=1}^{100} X_i - 600}{\sqrt{100 \cdot 2}} < 1\right) - \mathbb{P}\left(\frac{\sum_{i=1}^{100} X_i - 600}{\sqrt{100 \cdot 2}} \leq -1\right)$
(2 points) Using the Central Limit Theorem,
(2 points) $\frac{\sum_{i=1}^{100} X_i - 600}{20} \sim N(0, 1)$, (If the previous three steps are done despite no solution for part (a), then also points are awarded.)
(1 points) So the required probability is: $\approx \Phi(1) - \Phi(-1)$
(2 points) $= 2\Phi(1) - 1 \approx 2 \cdot 0.8413 - 1$
(1 points) $= \underline{\underline{0.6826}}$

4. Let $X \sim \text{Exp}(1)$ be a random variable. After the value of X is determined, we pick a number Y uniformly from the interval $[0, e^X]$.
(a) Determine $\mathbb{P}(Y < X \mid X = x)$.
(b) What is $\mathbb{P}(Y < X)$?
Solution: (2 points) Let Z denote the value of Y conditioning on $\{X = x\}$. Then $Z \sim \text{U}(0, e^x)$ (Points are awarded even if a new variable is not introduced)
(2 points) Z has the following distribution: $F_Z(z) = \frac{z}{e^x}$, if $0 < z \leq e^x$, and is 0, if $z < 0$, and is 1 if $z > e^x$.
(3 points) $\mathbb{P}(Y < X \mid X = x) = \mathbb{P}(Z < x) = F_Z(x) = \frac{x}{e^x}$ (Points are awarded even if its not checked whether $x \leq e^x$.)
(2 points) Using Law of Total Probability,
(3 points) $\mathbb{P}(Y < X) = \int_{-\infty}^{\infty} \mathbb{P}(Y < X | X = x) f_X(x) \,\mathrm{d}x$
(2 points) $= \int_0^{\infty} \frac{x}{e^x} \cdot e^{-x} \,\mathrm{d}x = \frac{1}{2} \int_0^{\infty} x 2e^{-2x} \,\mathrm{d}x$
(4 points) $= \frac{1}{2}\,\mathbb{E}(W)$, where $W \sim \text{Exp}(2)$, so, $\mathbb{E}(W) = \frac{1}{2}$.
or with partial integration, $= \left[\frac{1}{2}x(-e^{-2x})\right]_0^{\infty} - \int_0^{\infty} \frac{1}{2}(-e^{-2x})\mathrm{d} = 0 - \left[\frac{1}{4}e^{-2x}\right]_0^{\infty} = \frac{1}{4}$
(2 points) So, $P(Y < X) = \frac{1}{4} = \underline{\underline{0.25}}$

5. Let $(X, Y) \sim N(\mathbf{0}, \mathbf{\Sigma})$ be a random variable vector with a bivariate (2D) normal distribution, such that,

$$\mathbf{\Sigma} = \begin{bmatrix} 9 & 1 \\ 1 & 4 \end{bmatrix}$$

(a) For what value of $c > 0$ are the random variables $X + cY$ and $X - 2cY$ uncorrelated?
Use the value of c obtained in part (a) for the following two parts.
(b) Are $X + cY$ and $X - 2cY$ independent?
(c) Determine $\mathbb{P}(X + cY > 0, X - 2cY < 0)$.
Solution:
Solution I: The vector $(X + cY, X - 2cY)^T = \begin{pmatrix} 1 & c \\ 1 & -2c \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$,
so is a linear transformation of $(X, Y)^T$ with the transformation matrix being $\boldsymbol{A} = \begin{pmatrix} 1 & c \\ 1 & -2c \end{pmatrix}$.
Then Covariance matrix of vector $(X + cY, X - 2cY)^T$ is $\boldsymbol{A\Sigma A^T}$
which is $= \begin{pmatrix} 1 & c \\ 1 & -2c \end{pmatrix} \begin{pmatrix} 9 & 1 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ c & -2c \end{pmatrix} = \begin{pmatrix} 9 + c & 1 + 4c \\ 9 - 2c & 1 - 8c \end{pmatrix} \begin{pmatrix} 1 & 1 \\ c & -2c \end{pmatrix}$
$= \begin{pmatrix} 9 + c + c(1 + 4c) & 9 + c - 2c(1 + 4c) \\ 9 - 2c + c(1 - 8c) & 9 - 2c - 2c(1 - 8c) \end{pmatrix} = \begin{pmatrix} 9 + 2c + 4c^2 & 9 - c - 8c^2 \\ 9 - c - 8c^2 & 9 - 4c + 16c^2 \end{pmatrix}$

if the coordinates of this transform are uncorrelated, $9-c-8c^2=0$, solving which,
$c=1$ or $-\frac{9}{8}$.
But since $c>0$, so $c=\underline{\underline{1}}$.
Since coordinates of a bivariate normal distribution are independent if and only if their covariance is 0, so $X+Y$ and $X-2Y$ are independent random variables with normal distribution.
Plugging value of $c=1$ in the covariance matrix, we get, $\begin{pmatrix} 15 & 0 \\ 0 & 21 \end{pmatrix}$
Also $\boldsymbol{A0}=\mathbf{0}$, so
$X+Y \sim N(0,15)$ and $X-2Y \sim N(0,21)$ and are independent.
(2 points) Because of independence, $\mathbb{P}(X+Y<0, X-2Y>0)=\mathbb{P}(X+Y<0)\,\mathbb{P}(X-2Y>0)$
(1 points) So, $\mathbb{P}(X+Y<0, X-2Y>0)=(0.5)^2=\underline{\underline{\underline{0.25}}}$

Solution II: (2 points) $\rho(X+cY, X-2cY)=0 \Leftrightarrow \mathrm{Cov}(X+cY, X-2cY)=0$
(2 points) $\mathrm{Cov}(X+cY, X-2cY)=\mathrm{Cov}(X,X)+\mathrm{Cov}(X,-2cY)+\mathrm{Cov}(cY,X)+\mathrm{Cov}(cY,-2cY)$
(2 points) $=\mathrm{Cov}(X,X)-c\cdot\mathrm{Cov}(X,Y)-2c^2\mathrm{Cov}(Y,Y)$ (If $\mathrm{Cov}(X,X)$ is written as $Var(X)$, then also points are awarded.)
(2 points) $\mathrm{Cov}(X,X)=9$, $\mathrm{Cov}(Y,Y)=4, \mathrm{Cov}(X,Y)=1$
(1 points) $\Rightarrow 9-c-8c^2=0 \Rightarrow c_1=1, c_2=-\frac{9}{8}$,
(1 points) $c>0$, so, $c=\underline{\underline{1}}$
(3 points) Linear transformation of a bivariate normal distribution is also bivariate normal.
(3 points) The vector $(X+cY, X-2cY)$ is a linear transformation, because

$$\begin{bmatrix} 1 & c \\ 1 & -2c \end{bmatrix} \cdot \begin{bmatrix} X \\ X \end{bmatrix} = \begin{bmatrix} X+cY \\ X-2cY \end{bmatrix}$$

(3 points) The coordinates of bivariate normal distribution are independent if and only if their covariance is zero. (points are awarded if correlation is mentioned instead of covariance.)
(1 points) So, $X+cY$ and $X-2cY$ are independent.
(2 points) Because of independence, $\mathbb{P}(X+cY<0, X-2cY>0)=\mathbb{P}(X+cY<0)\,\mathbb{P}(X-2cY>0)$
(3 points) $X+cY$ has normal distribution with mean 0, so, $\mathbb{P}(X+cY<0)=\frac{1}{2}$, and similarly, $\mathbb{P}(X-2cY>0)=0.5$
(1 points) So, $\mathbb{P}(X+cY<0, X-2cY>0)=(0.5)^2=\underline{\underline{\underline{0.25}}}$

6.* Let X, Y be independent random variables, both with distribution $Exp(1)$. Further let $Z=X+Y$. Determine $\mathbb{E}(X+2Y \mid Z)$.
Solution: Solution I:
(2 points) $\mathbb{E}(X+2Y \mid Z)=\mathbb{E}(X \mid Z)+\mathbb{E}(2Y \mid Z)$
(1 points) $=\mathbb{E}(X \mid Z)+2\,\mathbb{E}(Y \mid Z)$
(6 points) $\mathbb{E}(X \mid Z)=\mathbb{E}(Y \mid Z)$ holds by symmetry, because X and Y have the same distribution.
(2 points) So, $\mathbb{E}(X+2Y \mid Z)=3\,\mathbb{E}(Y \mid Z)$
(4 points) $Z=\mathbb{E}(Z \mid Z)$, and since,
(2 points) $\mathbb{E}(Z \mid Z)=\mathbb{E}(X \mid Z)+\mathbb{E}(Y \mid Z)$
(1 points) $=2\,\mathbb{E}(Y \mid Z)$
(1 points) $\Rightarrow \mathbb{E}(Y \mid Z)=0.5Z$
(1 points) $\Rightarrow \mathbb{E}(X+2Y \mid Z)=3\cdot 0.5Z=\underline{\underline{\underline{1.5Z}}}$

Solution II:
(2 points) $\mathbb{E}(X+2Y \mid Z)=\mathbb{E}(X+Y \mid Z)+\mathbb{E}(Y \mid Z)$
(4 points) $\mathbb{E}(X+Y \mid Z)=\mathbb{E}(Z \mid Z)=Z$
Convolution $Z=X+Y$ has the density function $f_Z(z)=\int_{-\infty}^{\infty} f_X(z-y)f_Y(y)dy=\int_0^z e^{-(z-y)}e^{-y}dy=\int_0^z e^{-z}dy=ze^{-z}$ for $z>0$.
(1 points) The joint density function of (X,Y) (using their independence) is :

$$f_{X,Y}(x,y)=f_X(u)f_{Y_2}(v)=e^{-u}e^{-v}=e^{-u-v},$$

if $u,v>0$, and 0 otherwise.
(3 points) The joint distribution function of $(Y, X+Y)$ is:

$$\mathbb{P}(Y<y, X+Y<z)=\iint\limits_{\{(u,v)\mid 0<u<y, 0<u+v<z\}} e^{-u-v}\mathrm{d}u\mathrm{d}v=\int_0^y\int_0^{\max(0,z-u)} e^{-u-v}\mathrm{d}v\mathrm{d}u$$

$$= \int_0^y \left[-e^{-u-v}\right]_{v=0}^{\max(0,z-u)} \mathrm{d}u = \int_0^y \left(-e^{-u-(\max(0,z-u))} + e^{-u-0}\right) \mathrm{d}u$$

$$= \int_0^{\min(y,z)} \left(-e^{-u-(z-u)} + e^{-u}\right) \mathrm{d}u + \int_{\min(y,z)}^{z} \left(-e^{-u-0} + e^{-u}\right) \mathrm{d}u$$

$$= \left[-ue^{-y} - e^{-u}\right]_{u=0}^{\min(y,z)} + 0 = -\min(y,z)e^{-y} - e^{-\min(y,z)} - (-0-1)$$

$$\text{if } 0 \le y < z: \qquad = 1 - ye^{-z} - e^{-y}$$

$$\text{if } y \ge z \ge 0: \qquad = 1 - ze^{-z} - e^{-z}$$

(2 points) The joint density function canbe obtained by taking the derivative with respect to both variables. This is non-zero only when $y < z$. So:

$$f_{Y,Z}(y,z) = e^{-z} \qquad \text{if } 0 < y < z$$

(2 points) $f_{Y|Z}(y|z) = \frac{f_{Y,Z}(y,z)}{f_Z(z)}$
(1 points) $f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\mathrm{d}x$ (If the two formulas appear together, then also points are awarded. If limits are 0 and y, then also points are awarded.)
(1 points) $f_Y(y) = \int_0^y e^{-y}\mathrm{d}y = [ye^{-z}]_0^z = ze^{-z}$, if $z > 0$ and 0 otherwise.
(1 points) $f_{Y|Z}(y|z) = \frac{e^{-z}}{ze^{-z}} = \frac{1}{z}$, if $0 < y < z$, and 0 otherwise.
(1 points) $\mathbb{E}(Y \mid Z = z) = \int_0^z x\frac{1}{z}\mathrm{d}y = \left[\frac{y^2}{2z}\right]_0^z = \frac{1}{2}z$
(1 points) $\mathbb{E}(X_2 \mid Z) = \frac{1}{2}Z$
(1 points) So, $\mathbb{E}(X + 2Y \mid Z) = Z + \frac{1}{2}Z = \underline{\underline{1.5Z}}$

Name	Range	$P(X=i)$ or $F_X(x)$	f_X	$E(X)$	σ_X
Indicator $1(p)$	{0,1}	$P(X=1)=p$		p	$\sqrt{pq}$
Binomial $Bin(n,p)$	{0,1,...,n}	$\binom{n}{i}p^i(1-p)^{n-i}$		np	$\sqrt{np(1-p)}$
Poisson $Pois(\lambda)$	{0,1,...}	$\frac{\lambda^i}{i!}e^{-\lambda}$		λ	$\sqrt{\lambda}$
Geometric $Geo(p)$	{1,2,...}	$=(1-p)^{i-1}p$		$\frac{1}{p}$	$\frac{\sqrt{1-p}}{p}$
Uniform $U(a,b)$	(a,b)	$\frac{x-a}{b-a}$	$\frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{b-a}{2\sqrt{3}}$
Exponential $Exp(\lambda)$	$\mathbf{R}^+$	$1-e^{-\lambda x}$	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda}$
Normal $N(\mu,\sigma^2)$	$\mathbf{R}$	$\Phi(\frac{x-\mu}{\sigma})$	$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ
Multivariate Normal $N(\boldsymbol{\mu},\boldsymbol{\Sigma})$	$\mathbf{R}^n$	$f_{\boldsymbol{X}}(\boldsymbol{x}) = \frac{1}{(2\pi)^{\frac{n}{2}} det(\boldsymbol{\Sigma})^{\frac{1}{2}}}e^{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})}$		$\boldsymbol{\mu}$	Covariance matrix $\boldsymbol{\Sigma}$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998