Please note: The duration of the exam is 100 minutes. You may use a calculator. Please give a numeric answer (rounded to 4 decimal places). You are expected to **write all steps** taken in getting the final answer along with a **mention of properties/theorems used** in these steps. You are not allowed to leave the examination hall during the first 30 minutes of the exam.

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1. Shrek wants to buy a birthday gift for Fiona. Assume that the amount of time spent deciding what to get for her takes X days, where the distribution of the continuous random variable X has the memoryless property and its range is $[0, \infty)$. The probability that he takes more than 3 days to decide what to buy is $e^{-6} \approx 0.0025$. When he has decided to get a certain gift, then the amount of time taken to procure it takes an additional e^X days. What is the expected amount of days it takes Shrek to get Fiona's gift (including both the decision making time and the time required for procuring it)?

2. The 'This week's 49' news show comprises of the discussion of 49 most important news items of the week. Assume that the amount of time spent on discussing each news item has independent, identical distribution, each with an expected value of 2 minutes. The probability that the total show time is between 98 and 100 minutes is 0.17. Determine the (approximate) standard deviation of the time taken in discussing a single news item. (Assume that there is no time lag between two consecutive news items).

3. The joint probability mass function of random variables X and Y is given below:

Y \ X	0	2	5
2	α	0.2	0.1
3	0.2	0.25	β

   for some real numbers $\alpha, \beta \in [0, 1]$. We know that the events $\{X = 0\}$ and $\{Y = 2\}$ are independent. Determine α and β. Compute the covariance of X and Y.

4. Let the moment generating function of random variables X and Y be $e^{\frac{t^2}{2}}$ and e^{2t^2} respectively. Let $Z = X + Y$. We further know that X and Y are independent. Find an estimate for $P(Z > a)$ using Chernoff's inequality (note that the answer is not numerical). For $a = \sqrt{10}$, find the exact probability of $P(Z > a)$ and compare it with the estimate.

5. In a horse riding competition, competitors are each given a random horse and they have to clear 12 hurdles with it. Any one horse clears each of the 12 hurdles with identical probability, and clearing different hurdles are independent events. Depending on the horse, let this probability of clearing a hurdle be denoted by the continuous random variable V.

   (a) Assume that $V = p$ for a given horse. What is the probability, in terms of p, that a competitor with this horse cannot clear at most 1 of the 12 hurdles?

   (b) V has the following distribution function:

$$F_V(x) = \begin{cases} 3x^2 - 2x^3 & \text{if } 0 < x \leq 1 \\ 0 & \text{if } x \leq 0 \\ 1 & \text{otherwise} \end{cases}$$

   Determine the probability that a competitor will not clear at most 1 of the 12 hurdles.

6.* Bob conducts independent identical experiments one after another, each with a $\frac{1}{2}$ probability of success. Let X denote the number of experiments conducted until (and including) the second success. Determine the distribution ($P(X = k)$) and the expected value of X.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Name	Range	$P(X=i)$ or $F_X(x)$	f_X	$E(X)$	σ_X	MGF
Indicator $1(p)$	{0,1}	$P(X=1)=p$		p	$\sqrt{pq}$	$1-p+pe^t$
Binomial $Bin(n,p)$	{0,1,...,n}	$\binom{n}{i}p^i(1-p)^{n-i}$		np	$\sqrt{np(1-p)}$	$(1-p+pe^t)^n$
Poisson $Pois(\lambda)$	{0,1,...}	$\frac{\lambda^i}{i!}e^{-\lambda}$		λ	$\sqrt{\lambda}$	$e^{\lambda(e^t-1)}$
Geometric $Geo(p)$	{1,2,...}	$=(1-p)^{i-1}p$		$\frac{1}{p}$	$\frac{\sqrt{1-p}}{p}$	$\frac{pe^t}{1-(1-p)e^t}, e^t < \frac{1}{1-p}$
Uniform $U(a,b)$	(a,b)	$\frac{x-a}{b-a}$	$\frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{b-a}{2\sqrt{3}}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
Exponential $Exp(\lambda)$	$\mathbf{R}^+$	$1-e^{-\lambda x}$	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda}$	$(1-t\lambda^{-1})^{-1}, t<\lambda$
Normal $N(\mu,\sigma^2)$	$\mathbf{R}$	$\Phi(\frac{x-\mu}{\sigma})$	$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ	$e^{t\mu+\frac{1}{2}\sigma^2t^2}$
Multivariate Normal $N_n(\boldsymbol{\mu},\boldsymbol{\Sigma})$	$\mathbf{R}^n$	$f_{\boldsymbol{X}}(\boldsymbol{x}) = \frac{1}{(2\pi)^{\frac{n}{2}}det(\boldsymbol{\Sigma})^{\frac{1}{2}}}e^{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})}$		$\boldsymbol{\mu}$	Covariance matrix $\boldsymbol{\Sigma}$	