Moment Generating Function: For any random variable X, its moment generating function, denoted by $M_X(t)$ is defined as $M_X(t) = E(e^{Xt})$.

Properties of MGF:

- $M_X(0) = 1$
- $M_X^{(i)}(0) = E(X^i)$, that is, the i^{th} derivative of the MGF evaluated at 0 gives the i^{th} moment.
- Positivity: $M_X(t) \geq 0, \forall t \in \mathbb{R}$
- Translation: $M_{X+b}(t) = e^{bt} M_X(t)$
- Scaling: $M_{aX}(t) = M_X(at)$
- Sum: For any two independent random variables X and Y, $M_{X+Y}(t) = M_X(t)M_Y(t)$
- The MGF determines the distribution of the random variable, so two random variables with the same MGFs must have the same distribution. Mathematically, if $M_X(t) = M_Y(t), \forall t \in \mathbb{R}$, then, $F_X(a) = F_Y(a), \forall a \in \mathbb{R}$.
- Limits of MGF: For a sequence of random variables X_n and another random variable X, if $M_{X_n}(t) \to M_X(t)$, then, $f_{X_n} \to f_X$. Note: this property has deliberately been phrased a little vaguely because we haven't really discussed what notion of 'convergence' of functions are we using.

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