Chapter 4.1 Integration

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Definition 4.1 – Definition of Antiderivatice A function F is an antiderivative of f on an interval I when F'(x) = f(x) for all x in I

Theorem 4.1 – Representation of Antiderivatives If F is an antiderivatice of f on an interval I, then G is an antiderivative of f on the interval I if an only if G is of the form $G(x) = F(x) + C$ for all x in I where C is constant.

Problem 4.1 – Solving a Differential Equation Find the general solution of the differential equation $y' = 2$

Problem 4.2 – Describing Antiderivatives Solve. $\int 3x dx$

Problem 4.3 – Rewriting Before Integration Rewrite, Integrate, Simplify

- $\int 1/x^3 dx$
- $\int \sqrt{x} dx$

Problem 4.4 – Integrating Polynomial Functions Solve.

- $\int dx$
- $\int (x + 2) dx$
- $\int (3x^4 - 5x^2 + x) dx$

Problem 4.5 – Rewriting Before Integrating Solve. $\int \frac{(x+1)}{(\sqrt{x})} dx$

Problem 4.6 – Rewriting Before Integrating $\int \frac{sin(x)}{cos^2(x)} dx$

Problem 4.7 – Rewriting Before Integrating Rewrite, Integrate, Simplify.

- $\int \frac{2}{\sqrt{x}} dx$
- $\int (t^2+1)^2 dt$
- $\int \frac{x^3+3}{x^2} dx$
- $\int \sqrt[3]{x}(x-4) dx$

Definition 4.2 – Particular Solution In many applications of integration, you are given enough information to determine a particular solution. To do this, you need only know the value of y = F(x) for one value of x. This information is called an **initial condition**.

Definition 4.3 – Initial Condition The value of y = F(x) for one value of x.

Problem 4.8 – Finding a Particular Solution Find the **general solution**: $F'(x) = e^x$ and find the particular solution that satisfies the **initial condition**: $F(0) = 3$

NOTE: So far in this section, you have been using x as the variable of integration. In applications, it is often convenient to use a different variable. For instance, in the next example, involving *time*, the **variable of integration** is t.

Problem 4.9 – Solving a Vertical Motion Problem A ball is thrown upward with an initial velocity of 64 feet per second from an initial height of 80 feet. (a) Find the **position function giving the height s as a function of the time t**. (b) **When** does the ball hit **the ground**?

Basic Integration Rules

Differentiation Formula	*Integration Formula*	
$\frac{d}{dx}[C] = 0$	$\int 0\,dx = C$	
$\frac{d}{dx}[kx] = k$	$\int k\,dx = kx + C$	
$\frac{d}{dx}[kf(x)] = kf'(x)$	$\int kf(x)\,dx = k\int f(x)\,dx$	
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$	
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	Power Rule
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\,dx = \sin x + C$	
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x\,dx = -\cos x + C$	
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$	
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$	
$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$	
$\frac{d}{dx}[\csc x] = -\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$	
$\frac{d}{dx}[e^x] = e^x$	$\int e^x\,dx = e^x + C$	
$\frac{d}{dx}[a^x] = (\ln a)a^x$	$\int a^x\,dx = \left(\frac{1}{\ln a}\right)a^x + C$	
$\frac{d}{dx}[\ln x] = \frac{1}{x}, x > 0$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$	