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Problem Set 5

Problem 1

1a. Find a recurrence for T(n)

Base Case: n = 0 and $T_{(0)} = 1$

Recurrence:

$$T_n = T_{(n-1)} + (n+1)$$

1b.

Inductive Hypothese: If $T_n = \frac{(n+1)(n+2)}{2}$ then $T_{(n-1)} = \frac{((n-1)+1)((n-1)+2)}{2} = \frac{n(n+1)}{2}$ when $n > 0$

Proof:

$$T_{(n-1)} + (n+1) = T_n$$

IH for $T_{(n-1)}$:

$$\frac{n(n+1)}{2} + (n+1) = T_n$$

Multiplication:

$$\frac{n(n+1)}{2} + \frac{2(n+1)}{2} = T_n$$

Additive

$$\frac{n(n+1) + 2(n+1)}{2} = T_n$$

Distributive by removing n+1

$$(n+1)\frac{n+2}{2} = T_n$$
$$\frac{(n+1)(n+2)}{2} = T_n$$

Therefore $T_n = \frac{(n+1)(n+2)}{2}$

Problem 2

$A = (1, 5, 7, 8, 3, 6, 9)$

$B = (2, 10, 3, 6, 9)$

Problem 3

3a.

$$|A \cup B \cup C| = |A| + |B \cup C| - |A \cap (B \cup C)|$$
$$|A \cup B \cup C| = |A| + |B| + |C| - |B \cap C| - |A \cap (B \cup C)|$$

Change $|A \cap (B \cup C)|$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = |A \cap B| + |A \cap C| - |A \cap B \cap C|$$

Plug this equality back into the original

$$|A \cup B \cup C| = |A| + |B| + |C| - |B \cap C| - |A \cap B| - |A \cap B| + |A \cap B \cap C|$$

3b. Plug in all the values to the corresponding sets.

$$12 + 5 + 8 - 2 - 6 - 3 + 1 = 15$$

Take that total and subtract from the total subjects to find how many people passed.

$$41 - 15 = 26$$

26 students passed.

Problem 4

4a.

$$f(n) = n^3 - 1$$
$$f : Z- > Z$$

Strategy: Show that there is one pair m and n that is $m = n$ and $f(m) = f(n)$.

Assume: $n^3 - 1 = m^3 - 1$

Add $+1$ to both sides

$$n^3 - 1 + 1 = m^3 - 1 + 1$$
$$\sqrt[3]{n^3} = \sqrt[3]{m^3}$$
$$n = m$$

Therefore: it is one-to-one.

It is not onto, because it cannot reach the integer 2 for example.

4b.

$f(m,n) = 2m - n$

Strategy: Given an element of the codomain, give a procedure for finding the addociated element of the domain.

Proof: Let b be an element of the codomain Z

For any element of the codomain b, arbitrarily choose an m and $2m - b$ will always equal to n.

Not one-to-one because we can reach 20 when m = 9 and n = -2 and when m = 8 and n = -4.

Therefore: It is onto because every element of the codomain can be reached, but not one-to-one because different elements can reach the same results.