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CSCI 2824 Problem Set 2

# Problem 1

1a.

i. $N(S, D) \vee N(S, V)$

ii. $N(V, D) \wedge \neg(N(V, S) \vee N(V, F))$

iii. $\neg N(D, F) \rightarrow N(V, F)$

iv. $N(F, D) \leftrightarrow \neg N(D, S)$

1b.

Velma states at first that she refuses to sit next to Shaggy or Fred. In the next statement, she declares that she will. These statements contradict each other. Therefore, not all conditions can be satisfied and the problem is not solvable for the group.

# Problem 2

2a. True. This statement is declaring that for *every* number $m$, there is *at least one* number $n$ that is larger than $m$. Because numbers are infinite, there will always be at least one number that is larger than $m$.

2b. False. There exists no $n$ for which 0 is greater than.

2c. True. This statement declares that there exists *at least one* $n$ where every number is larger than it if $n$ is not equal to $m$. We know the natural number 0 exists and that all other natural numbers are larger than it. Therefore, we know this predication to be true.

2d. False. This state declares that every value for $m$ and every value for $n$ is greater when multiplied together than one of the variables if $n$ and $m$ are greater than zero. This is not true. We know that if we assign 1 to $n$ and 12 to $m$ and multiply them together, that value will not be greater than $m$. 12 times 1 is not greater than 12.

# Problem 3

3a. i. $x$ represents students of the university. $E(x)$ represents student of College of Engineering. $A(x)$ represents student of College of Arts & Sciences. $T(x)$ represents student required completion of a Senior Capstone.

$$\begin{array}{c} \forall x(A(x) \lor E(x)) \\ \forall x(E(x) \to T(x)) \\ \hline \forall x(\neg T(x) \to A(x)) \end{array}$$

3a. ii. $x$ represents numbers. $P(x)$ represents numbers that are prime numbers. $N(x)$ represents numbers that are natural numbers. $E(x)$ represents even numbers and $O(x)$ represents odd numbers.

$$\begin{array}{r} \forall x(P(x) \to N(x)) \\ \forall x(N(x) \to (E(x) \lor O(x))) \\ \neg\forall x(P(x) \to O(x)) \\ \hline \exists(P(x) \land E(x)) \end{array}$$

3b. i.

| $Step$ | $Premises$ | $Equivalences$ |
|---|---|---|
| 1 | $\forall x(A(x) \lor E(x))$ | $--$ |
| 2 | $\forall x(E(x) \to T(x))$ | $--$ |
| 3 | $A(c) \lor E(c)$ | $UI(1)$ |
| 4 | $E(c) \to T(c)$ | $UI(2)$ |
| 5 | $\neg A(c) \to E(c)$ | $RBI(3)$ |
| 6 | $\neg T(c) \to \neg E(c)$ | $Contraposition(4)$ |
| 7 | $\neg E(c) \to A(c)$ | $Contraposition(5)$ |
| 8 | $\neg T(c) \to A(c)$ | $HSyllogism(6,7)$ |
| 9 | $\forall x(\neg T(x) \to A(x))$ | $UG(8)$ |

3b. ii.

| $Step$ | $Premises$ | $Equivalences$ |
|---|---|---|
| $1$ | $\forall x(P(x) \rightarrow N(x))$ | $--$ |
| $2$ | $\forall x(N(x) \rightarrow (E(x) \vee O(x)))$ | $--$ |
| $3$ | $\neg\forall x(P(x) \rightarrow O(x))$ | $--$ |
| $4$ | $P(c) \rightarrow N(c)$ | $UI(1)$ |
| $5$ | $N(c) \rightarrow (E(c) \vee O(c))$ | $UI(2)$ |
| $6$ | $\neg(P(c) \rightarrow O(c))$ | $UI(3)$ |
| $7$ | $P(c) \wedge \neg O(c)$ | $Table7Rule5(6)$ |
| $8$ | $P(c)$ | $Simp(7)$ |
| $9$ | $\neg O(c)$ | $Simp(7)$ |
| $10$ | $\neg N(c) \vee E(c) \vee O(c)$ | $RBI(5)$ |
| $11$ | $\neg P(c) \vee N(c)$ | $RBI(4)$ |
| $12$ | $N(c)$ | $DSyllogism(8, 11)$ |
| $13$ | $\neg N(c) \vee E(c)$ | $DSyllogism(9, 10)$ |
| $14$ | $E(c)$ | $DSyllogism(12, 13)$ |
| $15$ | $P(c) \wedge E(c)$ | $Conjunction(8, 14)$ |
| $16$ | $\exists x(P(x) \wedge E(x))$ | $EG(15)$ |