Tyler Nevell

101039497

tyler.nevell@colorado.edu

Problem Set 6

1ai.

We have 87 cents. The quarter at 25 cents can go into 87 3 times, leaving us leftover at 12 cents. 3 coins so far.

We have 12 cents. The dime at 10 cents can go into 12 once, leaving us with 2 cents. 4 coins so far.

We have 2 cents. The nickel at 5 cents can not go into 2 cents, leaving us with 2 cents. 4 coins so far.

We have 2 cents. The penny at 1 cent can go into 2 cents twice, leaving us with 0 cents. We used 6 coins total.

1bi.

We have 80 cents. The quarter at 25 cents can go into 80 3 times, leaving us leftover at 5 cents. 3 coins so far.

We have 5 cents. The nickel at 5 cents can go into 5 1 time, leaving us with 0 cents. We used 4 coins total.

1aii.

We have 87 cents. The quarter at 25 cents can go into 87 3 times, leaving us leftover at 12 cents. 3 coins so far.

We have 12 cents. The dime at 10 cents can go into 12 once, leaving us with 2 cents. 4 coins so far.

We cannot use nickels. 4 coins so far.

We have 2 cents. The penny at 1 cent can go into 2 cents twice, leaving us with 0 cents. We used 6 coins total.

The greedy algorithm returned the optimal solution for this situation.

1bii.

We have 80 cents. The quarter at 25 cents can go into 80 3 times, leaving us leftover at 5 cents. 3 coins so far.

We have 5 cents. We cannot use nickels.

We have 5 cents. The penny at 1 cent can go into 5 cents 5 times, leaving us with 0 cents. We used 8 coins total.

The greedy algorithm did not return the optimal solution for this situation. The optimal solution would have used 2 Quarters and 3 Dimes for a total of 5 coins.

Problem 2

2a.

Rewrite as $f(n) = (50n)^{1/3} + 40n + log(25) + log(n)$

So, we can determine:

$$(50n)^{1/3} \leq n$$
$$k = 8$$

and:

$$40n \leq 40n$$
k = 0

and:

$$log(n) \leq n$$
$$k = 0$$

and:

$$log(25) \leq n$$
$$k = 2$$

So:

$$g(n) = n + 40n + 2n$$

$$O(g(n) = 43n$$

Where $C = 43$ and $k = 8$

2b.

$$log(n) \leq n$$
$$g(n) = (log(n))^a$$
$$(log(n)^a \leq n^a$$

Because we know that $log(n) \leq n^p$ when $p > 0$, we know there is a C and a k such that $log(n)$ is $O(n^p)$. From this, we can infer:

$$(log(n))^a \leq (Cn^p)^a \text{ when } p > 0$$

We have to pick a p to get rid of a. So we pick $1/a$ for our p and get the following:

$$(log(n))^a \leq C^a n^{1/a*a}$$
$$C^a = d$$
$$(log(n))^a \leq d * n$$

Therefore: $g(n)$ is $O(n)$ where our $C = d$ and $k = k$

2c.

$h(n) = 5n^2 + (log(n))^3 - nlog(n^3)$ is $\Theta(n2)$ by showing that it is $O(n2)$ and $\Omega(n2)$.

So rewrite it as:

$$h(n) = 5n^2 + (logn)^3 - 3 * nlog(n)$$

Using last problems solution, we know we can use the hint given previously to let $p = 2/3$ and use the rule to show:

$$(logn)^3 \leq (Cn^{\frac{2}{3}})^3$$
$$(logn)^3 \leq C^3n^2 \text{ where } C = 1 \text{ and } k = 0$$

For $5n^2$:

$$5n^2 \leq 5n^2 \text{ where } C = 5 \text{ and } k = 0$$

For $3 * nlog(n^3)$, it is less than zero because it is negative and therefore just disappears at $k = 1$ So...

$$5n^2 + n^2 - 0 \leq 6n^2$$

So $h(n) = 5n^2 + (logn)^3 - 3nlogn$ is $O(n^2)$ where $C = 6andk = 1$.

For $\Omega(n2)$:

We know that $5n^2 \geq 5n^2$ at $k = 0$

For $(logn)^3$:

$$(logn)^3 \geq 0 \text{ at } k = 1$$

For $3nlogn$

$$nlogn \leq n^2$$

Multiply both sides by -3:

$$-3nlogn \geq -3n^2 \text{ at } k = 1$$

So:

$$5n^2 + 0 - 3n^2 \geq 2n^2$$

So $h(n) = 5n^2 + (logn)^3 - 3nlogn$ is $\Omega(n2)$ where $C = 2$ and $k = 1$

Problem 3

3a.

From the example shown on the worksheet, we're given that to multiply an mxn matrix by an nxk matrix, we use the equation $m * n * k$ to find total number of multiplications. Using this, we find AB uses 32 multiplications. From this, we can derive the rule that the number of multiplications used in a matrix multiplication is the product of the number of entries in the first matrix multiplied by the number of columns found in the second matrix. We apply the same rule when multiplying by the additional matrix C. We then add the number of multiplications from the first equation to the number of multiplications from the second equation to find the total number of multiplications used to find the product of $(AB)C$.

So, for $P = (AB)C$ we find that total multiplications is

$$m * n * k + m * k * p$$

3b.

We again, find the total entries of B which is $n * k$ and multiply that product by the amount of columns in C which is p. Then, we multiply that total entries of A which is $m * n$, since it is the first matrix in this equation, by the amount of columns found in BC which is p. Then we add the products of both matrix multiplications together.

So, for $P = A(BC)$ we find that total multiplications is

$$n * k * p + m * n * p$$

3c. So for $P = (AB)C$ we find that

$$4 * 2 * 4 + 4 * 4 * 2 = 64 \text{ multiplications}$$

And for $P = A(BC)$ we find that

$$2 * 4 * 2 + 4 * 2 * 2 = 32 \text{ multiplications}$$

So we will use $A(BC)$ to find P on a seperate page.

$$BC = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 9 \\ 8 & 9 \end{bmatrix}$$

$2 + 2 + 3 + 2 = 9$
$1 + 2 + 3 + 3 = 9$ 16 multis ✓
$2 + 1 + 1 + 4 = 8$
$1 + 1 + 1 + 6 = 9$

$$A(BC) = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 9 & 9 \\ 8 & 9 \end{bmatrix} = \begin{bmatrix} 25 & 27 \\ 17 & 18 \\ 42 & 45 \\ 17 & 18 \end{bmatrix}$$

$9 + 16 = 25$
$9 + 18 = 27$
$9 + 8 = 17$
$9 + 9 = 18$ 16 multis ✓
$18 + 24 = 42$
$18 + 27 = 45$
$9 + 8 = 17$
$9 + 9 = 18$

Scanned by CamScanner