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Problem Set 3

1. For all n ∈N, n2 mod 4 = 0 or n2 mod 4 = 1

Proof: Let n be any perfect square.

Case 1: n is odd

$$\begin{aligned} n &= (2k+1)^2 \\ n &= 4k^2+4k+1 \\ n &= 4(k^2+k)+1 \\ n(mod4) &= 4(k^2+k)(mod4)+1(mod4) \\ 4(k^2+k)(mod4) &= 0 \\ 1(mod4) &= 1 \\ n(mod4) &= 0+1 \\ n(mod4) &= 1 \end{aligned}$$

Case 2: n is even

$$\begin{aligned} n &= (2l)^2 \\ n &= 4l^2 \\ n(mod4) &= 4l^2(mod4) = 0 \\ n(mod4) &= 0 \end{aligned}$$

$Answer$: Therefore, n is divisible by 4 or $n(mod4) = 1$

2. For all n ∈N, If n is divisible by 2 and n ≥ 4 then n is composite.

Proof: Let $n \geq 4$ and n divisible by 2 (even)

$$\begin{aligned} n &\geq 4 \\ n &= 4m \end{aligned}$$

$$\begin{aligned} n(mod2) &= 2k(mod2) \\ n(mod2) &= 0 \end{aligned}$$

Because n has 2 factors of 4 and m, we know that n is both even (divisible by 2) and is greater than 4, we know that n is a composite by definition.

3. For all n ∈N, n2 +5n+6 is a composite number.

Proof: Let n be a natural number. $n^2 + 5n + 6 is a composite

$$\begin{aligned} n^2+5n+6 &= (n+2)(n+3) \\ (n+2) &= a \\ (n+3) &= b \\ ab &= (n+2)(n+3) \end{aligned}$$

(ab) where both $a > 1$ and $b > 1$ is the definition of composite. Since both $(n+2)$ and $(n+3)$ seperately will be greater than 1 as long as n is a natural number.

4. There exists n ∈N such that 2n −2, 2n −1 and 2n + 1 are prime.

Proof: Let $n = 2$

$$\begin{gathered} 2^2 - 2 = 2 \\ 2^2 - 1 = 3 \\ 2^2 + 1 = 5 \end{gathered}$$

With one number value 2, we were able to prove through each equation that the value will be prime numbers since 2, 3, and 5 are all prime numbers.

Therefore, there exists a natural number n such that the listed above equations are prime numbers.

5. There exists p ∈N such that p,p + 2 are both prime numbers.

Proof: Let p = 3

$$\begin{gathered} p = 3 \\ 3 + 2 = 5 \end{gathered}$$

Because 3 is a prime number and 3 + 2 results in a prime number 5, we have the number to prove that this is true.

6. For all natural numbers n ∈ N, n3 mod 9 equals 0,1 or 8.

Case 1: Proof: Let $nmod9 = 1$

$$\begin{gathered} nmod9 = 1 \\ n^3mod9 = ((nmod9)*(nmod9)*(nmod9))mod9 \\ (1*1*1)mod9 = 1 \end{gathered}$$

Therefore: $n^3mod9 = 1$ in Case 1

Case 2: Proof: Let $nmod9 = 2$

$$nmod9 = 2$$
$$n^3mod9 = ((nmod9) * (nmod9) * (nmod9))mod9$$
$$(2 * 2 * 2)mod9 = 8mod9 = 8$$

Therefore: $n^3mod9 = 8$ in case 2

Case 3: Proof: Let $nmod9 = 3$

$$nmod9 = 3$$
$$n^3mod9 = ((nmod9) * (nmod9) * (nmod9))mod9$$
$$(3 * 3 * 3)mod9 = 27mod9 = 0$$

Therefore: $n^3mod9 = 0$ in case 3

Case 4: Proof: Let $nmod9 = 4$

$$nmod9 = 4$$
$$n^3mod9 = ((nmod9) * (nmod9) * (nmod9))mod9$$
$$(4 * 4 * 4)mod9 = 64mod9 = 1$$

Therefore: $n^3mod9 = 1$ in case 4

Case 5: Proof: Let $nmod9 = 5$

$$nmod9 = 5$$
$$n^3mod9 = ((nmod9) * (nmod9) * (nmod9))mod9$$
$$(5 * 5 * 5)mod9 = 125mod9 = 8$$

Therefore: $n^3mod9 = 8$ in case 5

Case 6: Proof: Let $nmod9 = 6$

$$nmod9 = 6$$
$$n^3mod9 = ((nmod9) * (nmod9) * (nmod9))mod9$$
$$(6 * 6 * 6)mod9 = 216mod9 = 0$$

Therefore: $n^3mod9 = 0$ in case 6

Case 7: Proof: Let $nmod9 = 7$

$$nmod9 = 7$$
$$n^3mod9 = ((nmod9) * (nmod9) * (nmod9))mod9$$
$$(7 * 7 * 7)mod9 = 343mod9 = 1$$

Therefore: $n^3mod9 = 1$ in case 7

Case 8: Proof: Let $nmod9 = 8$

$$nmod9 = 8$$
$$n^3mod9 = ((nmod9) * (nmod9) * (nmod9))mod9$$
$$(8 * 8 * 8)mod9 = 512mod9 = 8$$

Therefore: $n^3mod9 = 8$ in case 8

Case 9: Proof: Let $nmod9 = 9$

$$nmod9 = 9$$
$$n^3mod9 = ((nmod9) * (nmod9) * (nmod9))mod9$$
$$(9 * 9 * 9)mod9 = 729mod9 = 0$$

Therefore: $n^3mod9 = 0$ in case 9

Per the lecture, when dealing with a set exponent and modulo, we see that a pattern will arise regardless of the natural number inputted into n. These cases show that as long as a number n and its product result from being cubed is divisible by one of the natural numbers 1-9 (which are all natural numbers), the modulo 9 of n^3 will be 1, 8, or 9.