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Problem Set 13

Problem 1

(a)

$(a, b) \in R$ if and only if a shares at least one class with b.

$Symmetric$: If a shares a class with b, it can be inferred that b also shares that class with a. Therefore, by definition of symmetry, it is a symmetric relation.

$Non-Reflexive$: a is a person and a person cannot exist in the same space twice in one instance.

$Non-Transitive$: If a shares at least one class with b and b shares at least one class with c, that doesn't necessarily mean that a will share at least one class with c.

(b)

$(a, b) \in R$ if and only if a has a higher GPA than b.

$Non-Reflexive$: Since the rule does not inlude a "or equal to", a cannot have and edge towards itself.

$Non-Symmetric$: If a has a higher GPA than b, then b cannot have a higher GPA than a.

$Transitive$: If a has a higher GPA than b and b has a higher GPA than c, a will have a higher GPA than c.

(c)

$(a, b) \in R$ if and only if a lives in the same home as b.

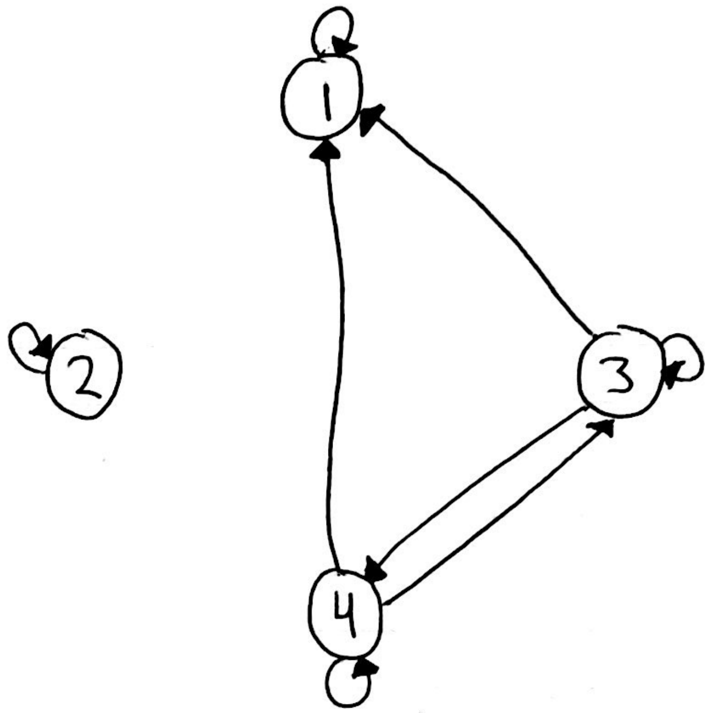
$Symmetric$: It's symmetric since if a exists in the same home as b, b exists in the same home as a.

$Non-Reflexive$: Since a is a singular person, that singular person cannot exist twice in the same space amongst themselves.

$Non-Transitive$: If a lives in the same home as b and b lives in the same home as c, it can be inferred that a also lives in the same home as c.

Problem 2

(a)



Problem 2

(b)

It is only $Reflexive$ since every node has an edge back to itself. It isn't symmetric because there are bidirectional arrows to every node and it is not transitive because you can't reach every node from each node and because 2 is unreachable.

(c)

$i.\ \bar{R} = \{(1,2),(1,3),(1,4),(2,1),(2,3),(2,4),(3,2),(4,2)\}$

$ii.$ If all existing edges are bidirectional in R, R is symmetric. If the complement of R, $\bar{R}$, completes the non existing edges in R, these edges will also have to be bidirectional to satisfy every possible combo in set A.

Problem 3

(a)

$i.$

Reflexive.

Not Symmetric: Imagine S_1 containing {1,2} and S_2 containing {2,3}. The min of S_2 is not less then or equal to the min of S_1.

Not Anti-Symmetric: There will exist sets where both have the same min number and therefore it cannot be anti-symmetric.

Transitive.

$ii.$

Reflexive.

Not Symmetric: There exists sets that will be subsets of another set and smaller than said set, so that means it cannot go in the other direction.

Not Anti-Symmetric: Since both sets can contain the exact same elements and length and therefore be symmetric.

Transitive.

$iii.$

Reflexive.

Symmetric.

Not Anti-Symmetric: Because there is symmetry already.

Transitive.

(b)

It is an equivalence.

Let k,a,b ∈ N and a ≠ b; [k] = {(S_a,S_b) | |S_a| = |S_b| = k}