Reviewing My Work of Proving of Goldbach's Conjecture

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Concepts

The Even Number 2n

n is any positive integer greater than 2, so 2*n is the even number greater than 4.

The Integral Array 0 to 2n

```
n = 10;
Range[0, 2 n]
```

Out[ ]= {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

The Number Pair

There are total of n pairs of two integers which sum is 2n and which value is in between 0 and 2n.

```
In[ ]:= Map[{#, 2 n - #} &, Range[0, n]]
```

Out[ ]= {{0, 20}, {1, 19}, {2, 18}, {3, 17}, {4, 16},
{5, 15}, {6, 14}, {7, 13}, {8, 12}, {9, 11}, {10, 10}}

```
In[ ]:= n = 10;
Grid[{Range[0, n], Table[20 - x, {x, 0, n}]}]
```

Out[ ]=

0	1	2	3	4	5	6	7	8	9	10
20	19	18	17	16	15	14	13	12	11	10

The Prime Pair

the pair which both number are prime are prime pair.

```
In[ ]:= Map[If[PrimeQ[#], Framed[#], #] &, {Range[0, n], Table[20 - x, {x, 0, n}]}, {2}];
Grid[%]
```

Out[ ]=

0	1	$\boxed{2}$	$\boxed{3}$	4	$\boxed{5}$	6	$\boxed{7}$	8	9	10
20	$\boxed{19}$	18	$\boxed{17}$	16	15	14	$\boxed{13}$	12	$\boxed{11}$	10

In[ ]:=
```
Map[{#, 2 n - #} &, Range[0, n]];
Select[%, PrimeQ[#[[1]]] && PrimeQ[#[[2]]] &]
```
Out[ ]=
```
{{3, 17}, {7, 13}}
```

The Prime Factor

Any positive integer (>=2) can be considered as a composition of one or more prime numbers. e.g.

In[ ]:=
```
FactorInteger[20]
```
Out[ ]=
```
{{2, 2}, {5, 1}}
```

Merging same value item, will get

In[ ]:=
```
Map[#[[1]] &, FactorInteger[20]]
```
Out[ ]=
```
{2, 5}
```

Applying this function to all integers in range 2n

In[ ]:=
```
Range[2 n];
FactorInteger[%];
Map[#[[1]] &, %, {2}]
```
Out[ ]=
```
{{1}, {2}, {3}, {2}, {5}, {2, 3}, {7}, {2}, {3}, {2, 5},
 {11}, {2, 3}, {13}, {2, 7}, {3, 5}, {2}, {17}, {2, 3}, {19}, {2, 5}}
```

Similar to the problems “color balls in a bag” in statistics, different combination will get different result, same combination always get same result.

Multiple same value prime factors are trivial here for this problem.

from above, conclude that in an integer array in rang 2n there are 2n/2 items with prime factor 2, roughly about 2n/3 items with prime factor 3 (integral divide), etc.

The Cyclic Distribution of Prime Factors

all kind of prime factors, starting from 0, equally cyclically distributed in array.

any two prime factor is independent to each other which means if removed all number with factor 2, then factor 3, 5, 7 ... etc will still be remained at proportion roughly about 1/3, 1/5, 1/7... etc (integral divide). “roughly about” I mean although in some circumstances it has the same quotient as result but their remainder can be any number between 0 and divisor-1.

an integer array which is removed prime factors $\{p_1,p_2,...p_n\}$, which is a cyclic function, which produced a cyclic image with repeat length of $p_1 p_2 ... p_n$. e.g. removing factor {2, 3} will produce following image:

In[ ]:=
```
Map[If[Mod[#, 2] ≠ 0 && Mod[#, 3] ≠ 0, Framed[#], #] &, Range[0, 25]]
```
Out[ ]=

$\{0, \boxed{1}, 2, 3, 4, \boxed{5}, 6, \boxed{7}, 8, 9, 10, \boxed{11}, 12, \boxed{13}, 14, 15, 16, \boxed{17}, 18, \boxed{19}, 20, 21, 22, \boxed{23}, 24, \boxed{25}\}$

in addition, find that any two sections with same cyclic length in the array always contains exactly same amount of remain numbers. e.g. after removing factor {2, 3}, any section for example section

[1,6] remains 2 numbers 1 and 5. section [7,12] remains 2 numbers 7 and 11. And section [16, 21] also remains remains 2 numbers 17 and 19.

The Largest Prime Less Than or Equal to $\sqrt{2n}$

A set of all primes less than or equals to p can sieve all composite number in range p^2. So for integral array in range 2n it only needs a set of primes which only less than or equals to $\sqrt{2n}$. e.g. for integral array 100, it only need 4 primes which {2,3,5,7}. in credible small amount.

Calculation for Each Eratosthenes Sieve Step

To sieve all primes in range (p, 2n], on each step:

$$R - R/p$$

p is the prime divisor which is less than or equals to $\sqrt{2n}$
R the amount of number which still remained on the array
if it is the last step then the division is integral division which keep no remainder.

Lower Bound of Prime Pairs Count Estimation

Combining all knowledge above, following is formula calculating how many prime pairs the even number 2n must have at least at the worst circumstance:

$$n[(2-1)/2][(3-2)/3]\ldots[(p-2)/p]$$

p is the prime divisor which is less than or equals to $\sqrt{2n}$
if it is the last step then the division is integral division which keep no remainder.

for example, to calculate how many prime pairs the even number 100 at least has

```
In[1]:= Floor[50 (1 / 2) ((3 - 2) / 3) ((5 - 2) / 5) ((7 - 2) / 7)]

Out[1]= 3

In[2]:= NumberPairs = Function[n2, Module[{n = n2 / 2},
            Map[{n - #, n + #} &, Range[0, n]]]
          ];
        PrimePairs = Function[n2,
           Select[NumberPairs[n2], PrimeQ[#[[1]]] && PrimeQ[#[[2]]] &]
          ];

In[4]:= PrimePairs[100]
        Length[%]

Out[4]= {{47, 53}, {41, 59}, {29, 71}, {17, 83}, {11, 89}, {3, 97}}

Out[5]= 6
```

the formula results 3. field test results 6.
6>3 means it is ok.

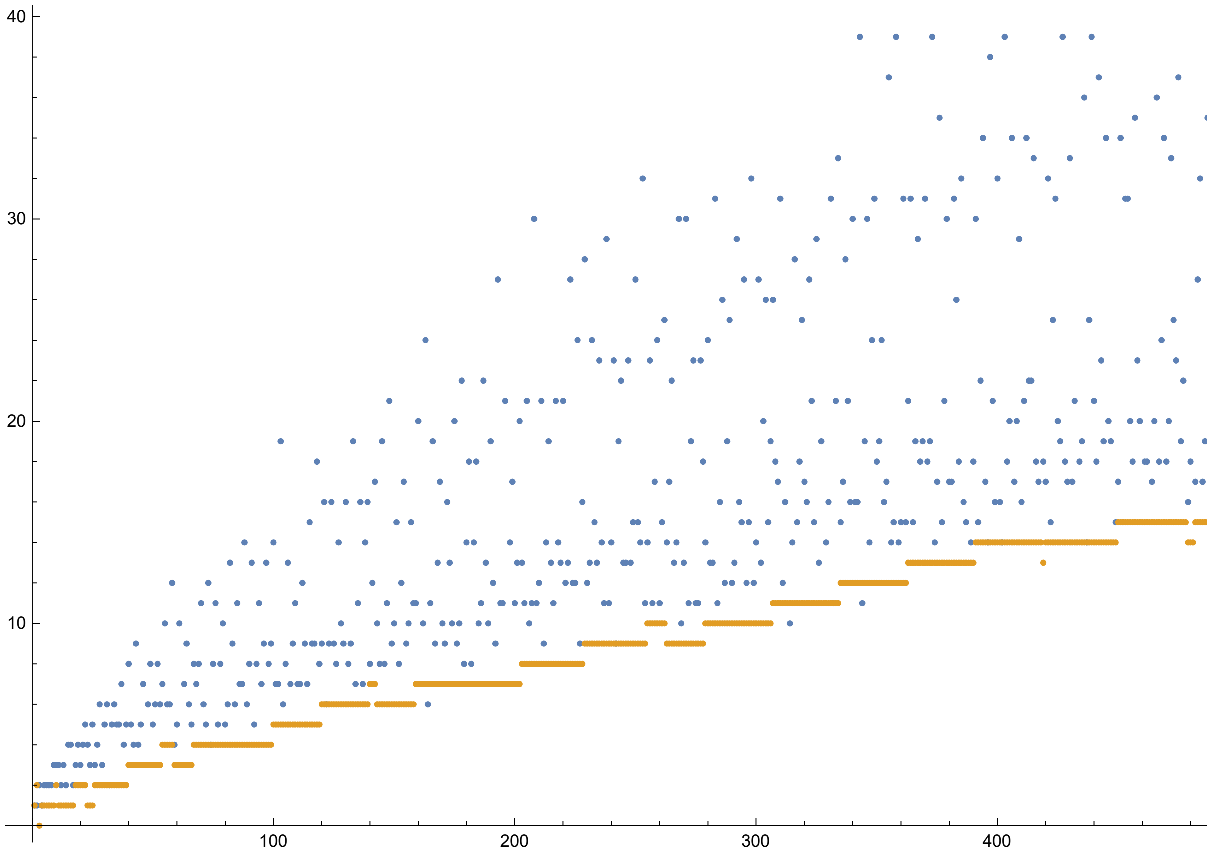
Test at Large

demonstration plot at range [6, π(100)]

```
LowerBoundofPrimePairCount = Function[n2,
   Floor[
    (n2 / 2) 1 / 2 Times @@ ((# - 2) / # & /@ Table[Prime[n], {n, 2, PrimePi[Sqrt[n2]]}])
   ]
  ];
```

In[ ]:=
```
t1 = Table[Length[PrimePairs[2 n]], {n, Prime[2], Prime[100]}];
t2 = Table[LowerBoundofPrimePairCount[2 n], {n, Prime[2], Prime[100]}];
ListPlot[{t1, t2}]
```

Out[ ]=



Exceptions

In[ ]:=
```
If[t1[[#]] < t2[[#]], {2 (# + 2), t1[[#]], t2[[#]]}, Nothing] & /@
 Table[n - 2, {n, Prime[2], Prime[100]}]
```

Out[ ]=
```
{{8, 1, 2}, {332, 6, 7}, {632, 10, 11}, {692, 11, 12}, {992, 13, 15}}
```

at range [6, π(100)], there are 5 exceptions, 2 reasons:

Reason 1: the formula counts 1 as prime which is in fact calculating the average not the worst condition. When the other number paired with 1 coincidently is a prime number, and when at worst circumstances, the estimate value with larger than field test value by 1.

```
In[ ]:= n = 4;
Map[If[PrimeQ[#], Framed[#], #] &, {Range[0, n], Table[2 n - x, {x, 0, n}]}, {2}];
Grid[%]
```

0	1	$\boxed{2}$	$\boxed{3}$	4
8	$\boxed{7}$	6	$\boxed{5}$	4

```
In[ ]:= PrimeQ[8 - 1]
Out[ ]= True

In[ ]:= PrimeQ[332 - 1]
Out[ ]= True

In[ ]:= PrimeQ[632 - 1]
Out[ ]= True

In[ ]:= PrimeQ[692 - 1]
Out[ ]= True

In[ ]:= PrimeQ[992 - 1]
Out[ ]= True
```

Reason 2: the formula keeps remainder on each steps of division which is in fact calculating the average not the worst condition. So when 2n runs at 992, the test result lower than estimation result by 2. 1 is caused by reason 1, and the other 1 is caused by reason 2.

Conclusion

Basically, lower bound of prime pairs count estimation formula is monotone increasing function with rare tiny bumps by remainder and integral division. For Goldbach's conjecture, formula over-fulfills Goldbach's conjecture requirement that for each even number at least equals sum of two prime because the larger 2n the larger lower bound of prime pairs, no exception.

So, by now proven that the statement of Goldbach's conjecture is a true statement.