1.

$$\ddot{y} + 7\dot{y} + 10y = \dot{u} + 3u$$

a)

Zero at $t = 0$

Laplace transform:

$$s^2Y(s) + 7sY(s) + 10Y(s) = sU(s) + 3U(s)$$
$$\Rightarrow Y(s)(s^2 + 7s + 10) = U(s)(s + 3)$$

$$\Rightarrow \frac{Y(s)}{U(s)} = \frac{s+3}{s^2 + 7s + 10}$$

b)

Unit step in laplace space is $U(s) = \frac{1}{s}$

$$Y(s) = \frac{s+3}{s^2 + 7s + 10} \cdot \frac{1}{s}$$

Final value theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$$

$$sY(s) = \frac{s+3}{s^2 + 7s + 10}$$

$$\lim_{s\to 0} \frac{s+3}{s^2 + 7s + 10} = \frac{3}{10}$$

$$y(\infty) = 0.3$$

c)

$$Y(s) = \frac{s+3}{s(s^2 + 7s + 10)} = \frac{s+3}{s(s+2)(s+5)}$$

$$= \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+5}$$

$$\Rightarrow s + 3 = A(s+2)(s+5) + Bs(s+5) + Cs(s+2)$$

$$\begin{cases} s = 0: \;\; 3 = A \cdot 2 \cdot 5 & \Rightarrow A = \frac{3}{10} \\ s = -2: \; 3 - 2 = B \cdot (-2) \cdot 3 & \Rightarrow B = -\frac{1}{6} \\ s = -5: \; 3 - 5 = C \cdot (-5) \cdot (-3) & \Rightarrow C = -\frac{2}{15} \end{cases}$$

$$\Rightarrow Y(s) = \frac{3/10}{s} - \frac{1/6}{s+2} - \frac{2/15}{s+5}$$

$$\Rightarrow \; y(t) = \frac{3}{10} - \frac{1}{6}e^{-2t} - \frac{2}{15}e^{-5t}, \;\; t \geq 0$$

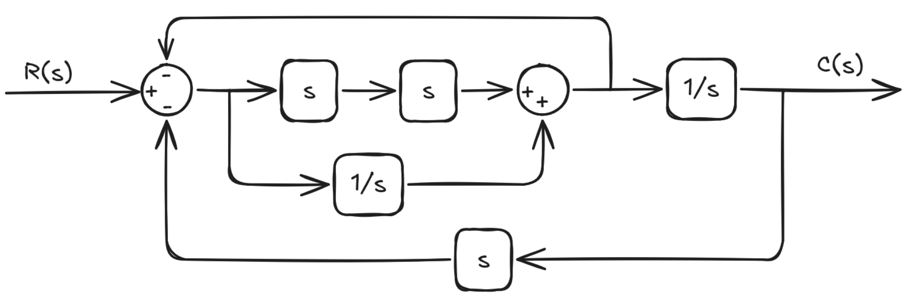
d)

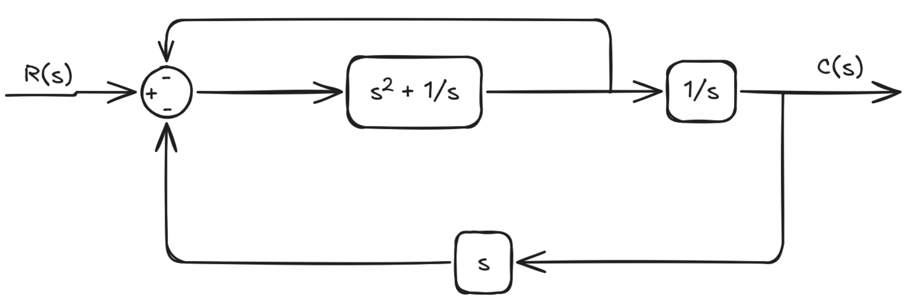
$$\lim_{t \to \infty} y(t) = \frac{3}{10} - \frac{1}{6} \cdot 0 - \frac{2}{15} \cdot 0 = \frac{3}{10}$$

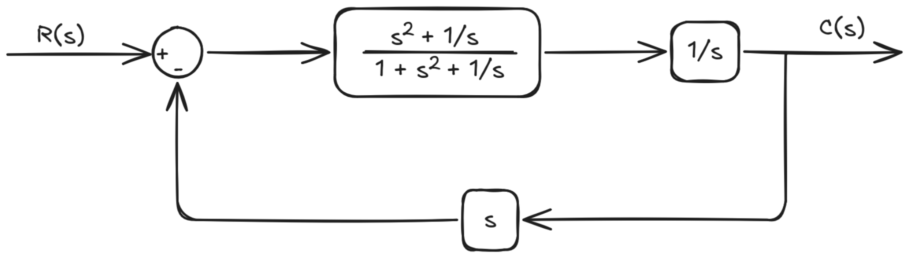
Matches with final value theorem

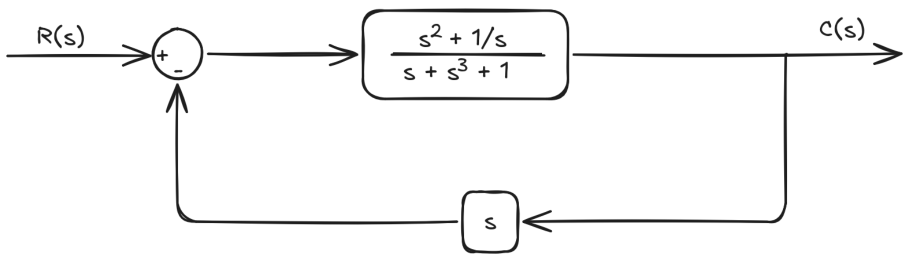
2

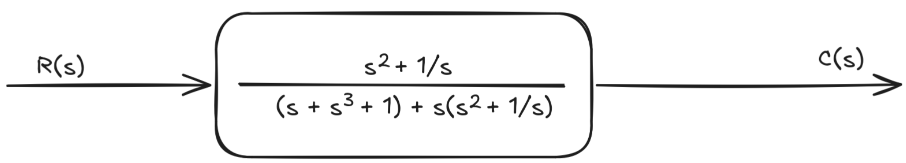
a)











Simplified:

$$\frac{C(s)}{R(s)} = \frac{s^2 + \frac{1}{s}}{2s^3 + s + 2} = \frac{s^3 + 1}{2s^4 + s^2 + 2s}$$

b)

Characteristic equation:

$$2s^4 + s^2 + 2s = 0$$

$$\Rightarrow s(2s^3 + s + 2) = 0$$

Pole at $s = 0$

Poles from $2s^3 + s + 2 = 0$:

Numeric solver on calculator gives root at $s \approx -0.84$

$$\Rightarrow (s + 0.84)(2s^2 - 1.67s + 2.39) = 0$$

solving for roots in $2s^2 - 1.67s + 2.39 = 0$ gives:

$$s \approx 0.42 \pm \sqrt{0.42^2 - 1.20} \approx 0.42 \pm 1.17j$$

Since the system roots av positive real parts the system is **Unstable**.