Five, seven, eight, ten, and in fact, most other integers have no square roots 124

The integers are said to be linearly ordered, that is, they satisfy the following requirements. Let a, b, and c be arbitrary integers.

1) $a \neq b$ implies $a > b$ or $b > a$

2) $a > b$ implies $a \neq b$, i.e., $a > a$ is false

3) $a > b$ and $b > c$ implies $a > c$.

66. Rational Numbers

Let us think of the ratio, a/b, a any integer whatsoever b. any non-zero integer whatsoever.

To insure, in the elementary operations, logical consistency with the integers, the following rules are adopted.

67. Equality

By definition $a/b = c/d$ if, and only if $ad = bc$. The set of rational numbers includes the set of integers as a subset. Merely set $b = 1$. Then $a = b \times 1$. $a/1 = a$

The number $2/4$, $4/8$ and $1/2$ are all merely different ways of presenting the same rational number. The number $1/2$ is considered the simplest because 1 and 2 contains no factors common to both except unity. The number a/b is said to be in its lowest form if a and b are ??? prime, i.e., contain no common factor except unity. $+a/-b = -a/+b$. Hence a rational number can always be written with a common denominator.

68. Inequalities: 12/02/2019

If the rational numbers a/b and c/d are written with positive denominators, $a/b > c/d$ if, and only if $ad > bc$; $a/b < c/d$ if, and only if $ad < bc$. Let a/b be a positive rational number $a > 1$. Then,

$$a/b > a/(b+1) \text{ since } (ab+a) > ab$$

$$a/(b+1) > 1/(b+1) \text{ since } a(b+1) > 1(b+1)$$

$$\therefore a/b > a/(b+1) > 1/(b+1). \text{ Set } n = b+1$$

then $a/b > a/n > 1/n$.

Irrespective of how close to zero we choose the number a/b, a number closer to zero to zero can be found by a suitable selection of n. Similarly, by approaching to zero through negative rational numbers it can be shown that there is no greatest negative

2020/10/16