

31. RADICALS AND IMAGINARY NUMBERS.

NOTE THAT THE LAST SOLUTION RESULTED IN AN INDICATED ROOT WHICH CANNOT BE FOUND EXACTLY ($\sqrt{37}$). SINCE $\sqrt{37}$ IS APPROXIMATELY 6 ($\sqrt{36}=6$), APPROXIMATE VALUES OF x ARE $x = \frac{-5+6}{2} = \frac{1}{2}$ AND $x = \frac{-5-6}{2} = -\frac{11}{2}$

ILLUSTRATION:

1) $x^2 + 5x + 9 = 0$, WHAT IS THE VALUE OF x ?

SOLUTION: $b=5$, $c=9$. THEREFORE:

$$x = \frac{-5 \pm \sqrt{25 - 36}}{2} = \frac{-5 \pm \sqrt{-1}}{2}$$

THIS TIME WE HAVE A MINUS SIGN UNDER THE RADICAL SIGN. THUS THE SOLUTION OF QUADRATIC EQUATIONS INTRODUCES TWO TYPES OF NUMBERS WE HAVE NOT DEALT BEFORE. FIRST A CHILD MEETS THE WHOLE NUMBERS, THEN THE FRACTIONS. NOW WE HAVE ARRIVED AT A POINT WHERE THESE TWO TYPES OF NUMBERS NO LONGER SUFFICE. WE HAVE NUMBERS SUCH AS $\sqrt{2}$ AND $\sqrt{34}$, WHICH CANNOT BE EXPRESSED EXACTLY AS THE QUOTIENT OF TWO INTEGERS - THESE WE CALL IRRATIONAL NUMBERS, AND WE ALSO HAVE $\sqrt{-7}$, WHICH ARE CALLED UNFORTUNATELY, IMAGINARY NUMBERS.

RADICALS OF SECOND ORDER SUCH AS $\sqrt{37}$, USUALLY HAVE THE 2 OMITTED BY CONVENIENCE. THE SIGN $\sqrt{\quad}$ IS A CONTRADICTION OF $\sqrt{\quad}$ AND THE VINICULUM.

EX: $x = \sqrt{A}$, IF $A=4$ $x = \sqrt{4}=2$; IF $A=5$, $x = \sqrt{5}$

32. APPROXIMATING ROOTS BY THE MECHANICS RULE.

THERE ARE SEVERAL METHODS OF APPROXIMATING SQUARE ROOTS. ONE OF THE MOST INTERESTING AND PRACTICAL IS "THE MECHANICS RULE". SUPPOSE, FOR EXAMPLE, THAT WE MUST FIND $\sqrt{175}$. FIRST, CONSIDER 175 AS THE AREA OF A SQUARE, THEN FIND THE APPROXIMATE LENGTH OF THE SIDE OF SUCH SQUARE SINCE $13^2=169$ AND $14^2=196$, THE SIDE IS EVIDENTLY BETWEEN 13 AND 14. SUPPOSE WE ESTIMATE IT TO BE 13, THEN THE RESULT OF DIVIDING 175 BY 13 SHOULD BE 13.46. HOWEVER, SINCE WE HAVE UNDERESTIMATED THE LENGTH SIDE,