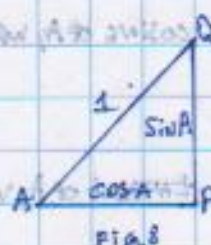


with radius one unit, in such way that the radius appears in the denominator of the trigonometric ratios. Then we need observe only the changes which occur in single lines as the angle increases or decreases.

In Figure 7, $AR = AQ = AQ' = 1$. In triangle APQ ,

$$\sin A = \frac{PQ}{AQ} = \frac{PQ}{1}$$

$$\cos A = \frac{AP}{AQ} = \frac{AP}{1}$$



$$\sin A = \frac{PQ}{AQ} = \frac{PQ}{1}$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{\left(\frac{AP}{1}\right)} = \frac{1}{AP}$$

$$\csc A = \frac{1}{\sin A} = \frac{1}{\left(\frac{PQ}{1}\right)} = \frac{1}{PQ}$$

As angle A increases, PQ assumes a series of positions in the circle, one of which is shown at $P'Q'$. This indicates that the sine of an angle increases as the angle increases. It may also be seen from the diagram that $\sin A$ will reach a maximum value of 1 when A becomes 90° . Similarly, as angle A decreases PQ also decreases, and when A becomes 0° PQ reaches a minimum value of 0. In the case of cosine exactly the reverse is true. As angle A increases AP decreases, and when A reaches 90° the cosine, AP , becomes 0, and when A is 0° it will have a maximum value of 1. To observe corresponding changes in tangent it is necessary to consider another triangle, namely ARS . In this triangle, $\tan A = \frac{RS}{AR} = \frac{RS}{1}$. Like the sine, the tangent increases with the angle, but, as the figure shows, the increase is much more rapid. As the angle A approaches 90° this increase is without limit, and there is no tangent of 90° since it would involve division by 0.

29. Relations between the trigonometric functions

The triangle in Figure 8, which resembles the one in unit circle, shows that the functions are so related to each other that given any one the other two can be determined. As an illustration we will find $\cos A$ and $\tan A$ in terms of $\sin A$. Since we