integer a which satisfies the equations. Since any integer greater than unity is either a prime, or expressible as a product of primes, it is quite easy to believe that the square root of many integers is not a rational number. The square root of 2 is merely the simplest example.

Felix Klein, the German mathematician, has left us an interesting derivation. He stated that the word "irrational" is without doubt the translation into Latin of the Greek "$\alpha\lambda o\gamma o\varsigma$". The Greek ~~word could not, like rational~~ however, means "inexpressible", and implies that the new numbers could not, like rational numbers, be expressed by the ratio of two whole numbers. The misunderstanding put upon the Latin "Ratio", that it could only convey the meaning "reason", gave to "irrational" the meaning "unreasonable", which of course, is a misnomer. — Greek "$\alpha\lambda o\gamma o\varsigma$"

However as we seen, it is possible to express every irrational number as a limit of a sequence of rational numbers. Toward the end of the 16th century decimals were introduced. It was then found that rational numbers could be expressed as finite and infinite decimals. It was then found that rational numbers could be expressed as finite and infinite ~~series~~ decimals. The infinite decimals ~~are~~ were always periodic or recurring. It is easy to set up decimals which are infinite and ~~not recurring~~ non periodic. If we assume that all these decimals are numbers, then this subset of decimals constitutes the irrational numbers. — p81 — Irrational number def.

However, the proof that a given number, such as π, is an irrational number is not always so simple as that for $\sqrt{2}$.

As mentioned previously, the entire set of rationals and irrationals numbers is called set of real numbers. — $\mathbb{R}[(\mathbb{Q}),(\mathbb{I})]$

76. Operations upon Irrational Numbers.

We operate upon these irrational numbers just as we do upon rational numbers. This is a very important matter. We consider

1. irrational numbers between ever narrowing rational limits and perform upon these rational limits and operations. The result will also be enclosed between ever narrowing limits.

As example, separate all rational numbers, a/b, into two classes such that $a^2 < 5b^2$ constitutes the lower class, and $a^2 > 5b^2$ defines the upper class. The cut number is the square root of 5. Separate all rational numbers, c/d,

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