



By Definition (Pg. 62):

$$AR = AQ = AP = 1$$

$$\text{Then } \sin A = \frac{PQ}{AQ} = \frac{PQ}{1} = PQ;$$

$$\cos A = \frac{AR}{AQ} = \frac{AR}{1} = AR;$$

$$\tan A = \frac{PQ}{AR} = \frac{PQ}{1} = PQ;$$

for triangle ASR:

$$\sin A = \frac{SR}{AS}; \cos A = \frac{AR}{AS}; \tan A = \frac{SR}{AR}; \text{ Then,}$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{\left(\frac{AR}{AS}\right)} = \left(\frac{1}{\frac{AR}{AS}}\right) = \frac{AS}{AR}$$

$$\tan A = \frac{SR}{AR} = \frac{SR}{1} = SR \quad \sec^2 A = 1 + \tan^2 A$$

By Definition (Pg. 64):

$$\sec^2 A = AS^2 \quad \tan^2 A = SR^2 \quad \text{So } AS^2 = SR^2 + AR^2$$

$$AS^2 = SR^2 + 1 \quad \text{Then } \sec^2 A = 1 + \tan^2 A = 1 + AS^2 - SR^2$$

2. In Figure 7 draw a line perpendicular to AB at B, intersecting AS in T. Show that  $BT = \cot A$  ( $\angle BTA = \angle TAP$ ) and that  $AT = \csc A$ . In triangle BTA, prove the relation  $\csc^2 A = 1 + \cot^2 A$ .

$$BT = \cot A \quad \text{Then } BT = \frac{1}{\tan A}$$

$$\frac{1}{\tan A} = \frac{1}{\left(\frac{BT}{AB}\right)} = \left(\frac{1}{\frac{BT}{AB}}\right) = \frac{AB}{BT}$$

$$\frac{AP}{\sin A} = \frac{TP}{\sin A} = \frac{AT}{\sin B} \quad \frac{AT}{\sin B} = \frac{BT}{\sin A}$$

$$\cot A = \frac{1}{\tan A}$$

$$\tan A = \frac{BT}{AB} = \frac{QP}{AP} \quad \cot A = \frac{1}{\tan A} = \frac{AB}{BT}$$

$$\frac{QP}{AP} = \frac{SR}{AR} = \frac{SR}{1} \quad \text{So } \cot A = \frac{1}{\left(\frac{SR}{1}\right)} = \frac{1}{SR} = \frac{1}{BT}$$

$$\csc A = \frac{1}{\sin A} = \frac{1}{\left(\frac{BT}{AT}\right)}$$

$$\sin A = \frac{BT}{AT} = \frac{SR}{AS} = \frac{PQ}{AQ}$$

$$\csc A = \frac{1}{\sin A} = \frac{1}{\left(\frac{BT}{AT}\right)} = \frac{AT}{BT} \quad \tan A = \frac{BT}{AB} = \frac{SR}{AR} = \frac{BT}{1}$$

$$\angle BTA = \angle APQ = \angle ARS \quad \cot A = \frac{1}{\tan A}$$

$$\cot^2 A = \frac{1}{\tan^2 A} = \frac{1}{\left(\frac{BT}{AB}\right)^2} = \frac{AB^2}{BT^2}$$

$$\csc^2 A = \frac{1}{\sin^2 A} = \frac{1}{\left(\frac{BT}{AT}\right)^2} = \frac{AT^2}{BT^2}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\left(\frac{BT}{AB}\right)} = \frac{AB}{BT} \quad \tan A = \frac{BT}{AB} = \frac{BT}{1}$$

