rational number.

69. Addition

$a/b \pm c/d = (ad \pm bc)/bd$. Between any two rational numbers, $a/b > c/d$, a third rational number can always be found. One method is the following:

$$\frac{a/b + c/d}{2} = \frac{ad+bc}{2bd}$$

$$a/b > \frac{ab+bc}{2bd} > cd$$

Between any two integers, as, for example, 1 and 2, there exist an infinite number of rational numbers. Two ~~equal~~ rational numbers are ~~either~~ either equal or unequal. → Quantum?
Between any two unequal rational numbers, no matter how small their numerical difference, there always exists an endless number of unequal rational numbers. Technically, the rational numbers are "dense". Hence, we can speak of the rational numbers as a linearly ordered dense set.

Geometrically there is an infinite number of $\mathbb{R}$ points on any line segment. ~~Between two~~ Chose two rational numbers whose numerical difference is as small as you desire. Irrespective of whatever we use the words, we are no nearer to expressing a totality of rational numbers between them than if we had used the word one, for there is no totality. Limit def.

The symbol ∞ is not a number. It's merely a shorthand expression for the concept of endless numbers. $\lim_{n\to\infty} 1/n = 0$ as the integer n assumes constantly and endlessly increasing values, the rational number, $1/n$, approaches, as a limiting value, zero, although it never reaches that value.

70. Multiplication — 14/02/2009

$(a/b)(c/d) = ac/bd$. In particular, if $a \neq 0$, $b \neq 0$, $(ab/ab) = ab/ab = 1$. Two rational numbers whose product is unity are called inverse elements under multiplication, or reciprocals. Every $\mathbb{R}$ except zero has a unique inverse element.

$bx = a$, $b \neq 0$, is always uniquely solvable.

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