

OF TWO LINES DETERMINES ONE AND ONLY ONE POINT. TWO POINTS DETERMINE ONE AND ONLY ONE STRAIGHT LINE. A STRAIGHT LINE SEGMENT IS THE SHORTEST SEGMENT JOINING TWO POINTS. AND FINALLY, A GEOMETRIC FIGURE MAY BE MOVED WITHOUT CHANGING ITS SIZE OR SHAPE.

IF A CIRCLE ~~IS DRAWN~~ ^{IS DRAWN} MUST BE DRAWN, WE ASSUME THAT A CIRCLE MAY ALWAYS BE DRAWN WITH A GIVEN POINT AS CENTER, AND A GIVEN LINE SEGMENT AS RADIUS. INCIDENTALLY, THIS BRINGS UP THE PROBLEM OF TOOLS. IN PLANE GEOMETRY, WE ARE PERMITTED, IN ADDITION TO WRITING MATERIALS, ONLY TWO AIDS IN CONSTRUCTING FIGURES: AN UNMARKED STRAIGHTEDGE TO DRAW LINES, AND A PAIR OF COMPASSES TO DRAW CIRCLES. THE THREE PROBLEMS THAT OCCUPIED THE THOUGHTS OF MANY OF EARLY GREEK GEOMETERS, LEADING THEM TO DISCOVERIES IN THE FIELD, WERE: (1) THE TRISECTION OF ANY ANGLE. (2) THE QUADRATURE OF THE CIRCLE (CONSTRUCTING A SQUARE EQUAL IN AREA TO A GIVEN CIRCLE). (3) THE DUPLICATION OF THE CUBE (CONSTRUCTING A CUBE TWICE THE VOLUME OF A GIVEN CUBE). IT WAS FINALLY PROVED IN THE NINETEENTH CENTURY THAT THESE THREE TASKS ARE IMPOSSIBLE TO ACCOMPLISH WITH THE TOOLS OF PLANE GEOMETRY. THEY MAY, HOWEVER, BE SOLVED BY THE USE OF OTHER INSTRUMENTS.

43. Reasoning About Plane Figures.

WITH THE STATEMENT OF OUR AXIOMS AND POSTULATES AND THE ACCEPTANCE OF OUR UNDERLYING ELEMENTS — POINT, LINE, PLANE — WE ARE NOW READY TO DEVELOP OUR SYSTEM OF REASONING ABOUT PLANE FIGURES. THE SIMPLEST FIGURE, OTHER THAN A STRAIGHT LINE, WHICH CAN BE FORMED IN A PLANE SURFACE OCCURS WHEN TWO STRAIGHT LINES INTERSECT. SUCH A FIGURE FORMS FOUR ANGLES (THE SYMBOL \angle IS USED TO DESIGNATE "ANGLE"). THERE ARE RELATIONSHIPS BETWEEN THESE ANGLES AND WE SHALL INVESTIGATE (FIRST, OUR PATTERN OF PROCEDURE, WE MUST DEFINE SOME TERMS. AN ANGLE IS A



GEOMETRIC FIGURE FORMED BY THE INTERSECTION OF TWO LINES; THE POINT OF INTERSECTION IS CALLED THE VERTEX OF THE ANGLE AND LINES FORM ITS SIDES. THERE ARE TWO POSSIBLE RELATIVE POSITIONS WITH PAIRS OF ANGLES AS SHOWN IN THIS PICTURE. THEY