approximation of the square root of 2 accurate to the (n-1)th decimal place. For example, the fifth member of either sequence is an accurate approximation of the square root of 2 to the fourth decimal place.

If the square root of 2 is a rational number it can be represented either by a terminated decimal, or by a pure or recurring decimal. If it is represented by a terminating decimal a member of sequence B or C will eventually be reached whose square is exactly 2. If it is represented by a pure or mixed infinite recurring decimal, a member of each sequence will eventually reach periodic forms of the type 0.3, 0.99, or 0.142857. The fact is that we can carry out this process endlessly without obtaining such result. There is no least member of B since we can always find a member, N, further out in the sequence, whose square exceeds 2 by an ever decreasing positive amount. $N^2 = 2 + \delta$, where δ is a small positive number. There is no greatest member of C since a member, M, can always be found whose square is less than 2 by an ever decreasing amount. $M^2 = 2 - \delta$, i.e., δ is never zero. However, this is merely a statement and not a proof.

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Remember: Statement ≠ Proof

21/02/2019

73. The square root of 2 is not a rational number.

Centuries ago Euclid proved that no rational number squared is equal to 2. In the interpretation of Euclid's proof, Algebraic Symbolism will be used although that symbolism did not appear upon the scene until centuries after his death. First he assumed that there is a rational number, a/b, from which all factors common to both numerator and denominator have been removed, and that the square of this rational number is 2.

$$(a/b)^2 = 2. \quad a^2 = 2b^2$$

If $a^2 = 2b^2$, then a^2 is an even integer, because it has a factor 2, shown in the right member. If a^2 is an even integer, then a is an even integer. If a is an even integer it is divisible by 2. Let $a = 2m$. Then $(2m)^2 = 2b^2$; $4m^2 = 2b^2$; $2m^2 = b^2$. Now b^2 is an even integer since it has a factor 2 shown in the left member. Hence, b is even. But it was assumed that a/b was a rational number with no factor common both to numerator and denominator. The logical conclusion is that there is no

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