rational number whose square is 2. (THE PROOF could be continued and the factor 2 removed endlessly.) How, then, is such a number to be defined?

74. THE $\sqrt{2}$ DEFINED AS A SECTION OR CUT.

While it has been proved that the equation $a^2 = b^2$ has no solution, a and b integers, it has been shown that all rational numbers, a/b, can be separated into two classes so that if $a^2 > 2b^2$, a/b is in the upper class, B, and if $a^2 > 2b^2$, a/b is in the lower class, C. For example, $1.415 = 1415/1000$, $(1415)^2 > (2)(10)^3$. As we go endlessly in each sequence the members of class B will approach in magnitude constantly closer to the number of class C. We can therefore find a number which multiplied by itself will approximate as closely as desired to the number 2. Hence, for actual purposes of measurement the rational number system is adequate. In fact, the number $\sqrt{2}$ is often used as if it were a definite rational number. For example, we consult a table of squares, cubes, and roots in a handbook. For $N=2$ we find under $\sqrt{N}$, 1.404214. Actually all the (B) value means is that a member selected from the lower class of rationals as a satisfactory approximation.

While we were unable to find in either sequence a number which multiplied by itself yielded the number 2, only by Euclid's proof do we basically know that no member of either class, B or C, will ever be the square root of 2. We have an infinite number of rational numbers between 1 and 2, yet none of them, multiplied by itself, yields the number 2. This is astounding but basically unassailable. The set of rational numbers has, however, been separated into two classes B and C. They are infinite sequences, the square of whose members approach closer and closer to 2, one from above, the other from below. A number, $\sqrt{2}$, is thus invented which has the unique property, $(\sqrt{2})(\sqrt{2}) = 2$. It is defined as a cut or section (usually called a Dedekind cut) in the rational numbers. Since $\sqrt{2}$ is not a rational number, the number system is enlarged and called the set of real numbers, of which rational numbers are subset.

→ $\sqrt{2}$ DEFINED

1D $2^{\frac{1}{2}}$ note: ☒

circle + 4($\frac{1}{4}$ π) +

→ $\sqrt{2}$ as section cut

L $(\sqrt{2})(\sqrt{2}) = 2$

→ def of $\mathbb{R}$ as section of $\mathbb{Q}$

75. IRRATIONAL NUMBERS.

■ 21/02/2019

The fact that the equation $a^2 = 2b^2$ is not solvable, a and b integers, is not a special property of 2. Let $x^2 = a/b$, a/b an arbitrary positive rational number in its lowest terms. The equation $a^2 = pb^2$, p is prime number is not solvable, a and b integers, and if we let $a^N = pb^N$, N and b integers, N greater than 1, $b \neq 0$, there is no