

THE TREE OF MATHEMATICS — GLENN JAMES (1951)

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1. FUNDAMENTALS OF BEGINNING ALGEBRA

1.1 ALGEBRA MATH = LANGUAGE → LEARNING → UNDERSTANDING

1.2 ALGEBRAIC STATEMENT OF ADDITION: ARITHMETIC WE USE

NUMBERS, FROM 0 to 9, AS IN ALGEBRA WE USE LETTERS TO

GENERALIZE ARITHMETIC:

• $3+5$, OR $4+7$ COULD BE DENOTED AS $a+b$

1.3 ALGEBRAIC STATEMENT OF MULTIPLICATION:

• 3×5 , OR 3×7 AS ALGEBRAIC STATEMENT: $3a$ OR $3b$

THEN IF $a=5$, $3a=15$; OR IF $a=7$, $3a=21$, ETC.

• 4×3 , OR 6×7 AS ALGEBRAIC STATEMENT: $a \times b$ OR ab

THEN IF $a=4$ AND $b=3$, $ab=12$; OR IF $a=6$ AND $b=7$, $ab=42$.

1.4 FURTHER SIMPLIFICATION BY USE OF ALGEBRAIC LANGUAGE:

• $5=3+2$, OR $5 \times 2+7$ AS ALGEBRAIC STATEMENT: $5a+b$

THEN IF $a=3$ AND $b=8$, $5a+b=5 \times 3+8=23$; ETC.

• MULTIPLICATION AND SUBTRACTION:

$3 \times 2-4$, OR $3 \times 5-7$ AS ALGEBRAIC STATEMENT: $3a-b$

THEN IF $a=2$ AND $b=4$, $3a-b=3 \times 2-4=2$; ETC.

• ADDITION OF PRODUCTS:

$2 \times 3+3 \times 4$, OR $2 \times 4+3 \times 6$ AS ALGEBRAIC STATEMENT: $2a+3b$

THEN IF $a=3$ AND $b=4$, $2a+3b=2 \times 3+3 \times 4=18$; ETC.

• SUBTRACTION OF PRODUCTS:

$3 \times 4-2 \times 3$, OR $3 \times 2-2 \times 4$ AS ALGEBRAIC STATEMENT: $3a-2b$

• PRODUCTS OF PRODUCTS:

$(4 \times 2) \times (2 \times 3) = 4 \times 2 \times 2 \times 3$ AS ALGEBRAIC STATEMENT: $4a \times 2b$

SINCE a AND b REPRESENT NUMBERS, WE CAN REARRANGE THE NUMBERS AND LETTERS AS WE PLEASE, THAT IS:

$$4a \times 2b = 4 \times 2 \times a \times b = 8ab$$

• QUOTIENTS OF PRODUCTS:

$\frac{2 \times 12}{3 \times 4}$; $\frac{2 \times 9}{3 \times 2}$ AS ALGEBRAIC STATEMENT: $\frac{2a}{3b}$

THEN IF $a=12$ AND $b=4$, $\frac{2a}{3b} = \frac{2 \times 12}{3 \times 4} = 2$

1.5 MULTIPLICATION OF A SUM:

ENGLISH STATEMENT: THREE TIMES THE QUANTITY FOUR PLUS SEVEN.

ARITHMETIC STATEMENT: $3(4+7) = 3 \times 11$, OR $3 \times 4 + 3 \times 7 = 12 + 21 = 33$

ALGEBRAIC STATEMENT: $a(b+c) = ab+ac$

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