

But lines  $PQ$ ,  $P'Q'$ ,  $P'Q''$ , and  $PQ''$ , are all equal in length. Some of them, however, differ in direction, e.g. us

$$P'Q' = PQ \quad P'Q'' = -PQ \quad P'Q''' = -PQ$$

Therefore, it is possible to express the sines of angles in the second, third and fourth quadrants in terms of the sine of a related angle in the first quadrant. These relations follow: — P70

Quadrant II  $\sin(180^\circ - A) = P'Q' = \sin A$

Quadrant III  $\sin(180^\circ + A) = P'Q'' = -PQ = -\sin A$

Quadrant IV  $\sin(360^\circ - A) = P'Q''' = -PQ = -\sin A$

We list the following examples (using a four place table) — Plot it with calculator

$$\sin 130^\circ = \sin(180^\circ - 50^\circ) = \sin 50^\circ = 0.7660$$

$$\sin 230^\circ = \sin(180^\circ + 50^\circ) = -\sin 50^\circ = -0.7660$$

$$\sin 310^\circ = \sin(360^\circ - 50^\circ) = -\sin 50^\circ = -0.7660$$

By means of the same device, similar relations can be derived for the cosine and tangent.

We now have at our command sufficient knowledge of trigonometry to appreciate and solve problems in some of the most important fields in applied mathematics. In navigation, one meets such problems as the following:

A pilot wishes to make a good course in the direction  $220^\circ$  (with the northward direction). A 25 mph wind is blowing from  $80^\circ$ . If his air speed is 200 mph, in what direction must he head the plane and what will be the ground speed?

By use of the sine law and a table of the values of the trigonometric functions one calculates the desired direction to be  $213^\circ 13'$  and the desired speed to be 218.5 mph, but we shall omit the details here.

#### 4. Graphs of Trigonometric Functions.

A further use of trigonometry becomes apparent from a graphical representation (graph) of the equation  $y = \sin x$  on a pair of perpendicular lines as indicated in Figure 17. When a value is assigned to  $x$ , the corresponding value of  $y$  can be determined from triangles or from tables. For example, if  $x = 30^\circ$ , then  $y = \sin 30^\circ = 0.5$ . Other pairs of values are shown below:

$x$	$0^\circ$	$30^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$150^\circ$	$210^\circ$	$240^\circ$	$270^\circ$	$300^\circ$	$330^\circ$	$360^\circ$	$180^\circ$
$y$	0	.5	.866	1	.866	.5	-.5	-.866	-1	-.866	-.5	0	0

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