$a^2 \times a^3 = a \cdot a \times a \cdot a \cdot a$ or a^5; wich means we merely ~~add~~ added the exponents.

In divison tree types of results are met:

Illustrations:

1) $a^5 \div a^2 = \dfrac{a^5}{a^2} = \dfrac{aaaaa}{aa} = a^3$; or $a^{5-2} = a^3$

2) $a^2 \div a^2 = \dfrac{a^2}{a^2} = \dfrac{aa}{aa} = 1$

3) $a^2 \div a^5 = \dfrac{a^2}{a^5} = \dfrac{aa}{aaaaa} = \dfrac{1}{aaa} = \dfrac{1}{a^3}$

35. Negative exponents.

In illustration 2) above, zero exponents are introduced into our mathematical vocabulary. Since $a^2 \div a^2 = 1$ and $a^2 \div a^2 = a^0$, it is convenient to define the 0 power of any quantity to be equal to 1.

Similiarly, in illustration 3) above, negative exponents are introduced. Since, by 3),

$$a^2 \div a^5 = \frac{1}{a^3} \quad \text{and} \quad a^2 \div a^5 = a^{2-5} = a^{-3},$$

we define the negative exponent to be reciprocal of the same power with a positive exponent, i.e. $a^{-n} = \dfrac{1}{a^n}$, where n can be any number. — p35

36. Fractional exponents.

Let us multiply $a^{1/2}$ by $a^{1/2}$. Adding the exponents we hae $a^{1/2} \times a^{1/2} = a$. This would indicate that $a^{1/2}$ is one of two equal factors of a, wich is the definition of the square root of a. So we define $a^{1/2}$ to be the same as $\sqrt{a}$. In like manner, since $a^{1/3} \times a^{1/3} \times a^{1/3} = a$, we shall say that $a^{1/3}$ is the cube root of a (i.e. $\sqrt[3]{a} = a^{1/3}$).

This definition of fractional exponents ~~give us a powerful tool for simplifying computation~~ and the application of the same rules for multiplication and division as were used for integral ~~number~~ exponents and 0, to devise a method for multiplying and dividing radicals of different order. ~~Note~~ that, in order to add or subtract fractional exponents, they must be changed to a common denominator.

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