

$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$ $\sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$ $= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$ $= \frac{\sqrt{6} - \sqrt{2}}{4}$	$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$ $\cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$ $= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$ $= \frac{\sqrt{6} + \sqrt{2}}{4}$	$\sin(2\alpha) = 2 \sin \alpha \cos \alpha$ $\sin(2 \cdot 45^\circ) = 2 \sin 45^\circ \cos 45^\circ$ $= 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = 1$	$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$ $\cos(2 \cdot 45^\circ) = \cos^2 45^\circ - \sin^2 45^\circ$ $= \left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2 = 0$	$\sin(2\alpha) = 2 \sin \alpha \cos \alpha$ $\sin(2 \cdot 30^\circ) = 2 \sin 30^\circ \cos 30^\circ$ $= 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$	$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$ $\cos(2 \cdot 30^\circ) = \cos^2 30^\circ - \sin^2 30^\circ$ $= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$	$\sin(3\alpha) = 3 \sin \alpha - 4 \sin^3 \alpha$ $\sin(3 \cdot 45^\circ) = 3 \sin 45^\circ - 4 \sin^3 45^\circ$ $= 3 \cdot \frac{\sqrt{2}}{2} - 4 \left(\frac{\sqrt{2}}{2}\right)^3 = \frac{3\sqrt{2}}{2} - \sqrt{2} = \frac{\sqrt{2}}{2}$	$\cos(3\alpha) = 4 \cos^3 \alpha - 3 \cos \alpha$ $\cos(3 \cdot 45^\circ) = 4 \cos^3 45^\circ - 3 \cos 45^\circ$ $= 4 \left(\frac{\sqrt{2}}{2}\right)^3 - 3 \cdot \frac{\sqrt{2}}{2} = \sqrt{2} - \frac{3\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$	$\sin(45^\circ) = \frac{\sqrt{2}}{2}$ $\cos(45^\circ) = \frac{\sqrt{2}}{2}$ $\sin(30^\circ) = \frac{1}{2}$ $\cos(30^\circ) = \frac{\sqrt{3}}{2}$ $\sin(60^\circ) = \frac{\sqrt{3}}{2}$ $\cos(60^\circ) = \frac{1}{2}$ $\sin(90^\circ) = 1$ $\cos(90^\circ) = 0$ $\sin(120^\circ) = \frac{\sqrt{3}}{2}$ $\cos(120^\circ) = -\frac{1}{2}$ $\sin(150^\circ) = \frac{1}{2}$ $\cos(150^\circ) = -\frac{\sqrt{3}}{2}$ $\sin(180^\circ) = 0$ $\cos(180^\circ) = -1$ $\sin(210^\circ) = -\frac{1}{2}$ $\cos(210^\circ) = -\frac{\sqrt{3}}{2}$ $\sin(240^\circ) = -\frac{\sqrt{3}}{2}$ $\cos(240^\circ) = -\frac{1}{2}$ $\sin(270^\circ) = -1$ $\cos(270^\circ) = 0$ $\sin(300^\circ) = -\frac{\sqrt{3}}{2}$ $\cos(300^\circ) = \frac{1}{2}$ $\sin(330^\circ) = -\frac{1}{2}$ $\cos(330^\circ) = \frac{\sqrt{3}}{2}$ $\sin(360^\circ) = 0$ $\cos(360^\circ) = 1$	$\sin(15^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos(15^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\sin(75^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(75^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(105^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(105^\circ) = -\frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(135^\circ) = \frac{\sqrt{2}}{2}$ $\cos(135^\circ) = -\frac{\sqrt{2}}{2}$ $\sin(165^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos(165^\circ) = -\frac{\sqrt{6} + \sqrt{2}}{4}$ $\sin(195^\circ) = -\frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(195^\circ) = -\frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(225^\circ) = -\frac{\sqrt{2}}{2}$ $\cos(225^\circ) = -\frac{\sqrt{2}}{2}$ $\sin(255^\circ) = -\frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos(255^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\sin(285^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(285^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(315^\circ) = \frac{\sqrt{2}}{2}$ $\cos(315^\circ) = \frac{\sqrt{2}}{2}$	$\sin(15^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos(15^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\sin(75^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(75^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(105^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(105^\circ) = -\frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(135^\circ) = \frac{\sqrt{2}}{2}$ $\cos(135^\circ) = -\frac{\sqrt{2}}{2}$ $\sin(165^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos(165^\circ) = -\frac{\sqrt{6} + \sqrt{2}}{4}$ $\sin(195^\circ) = -\frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(195^\circ) = -\frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(225^\circ) = -\frac{\sqrt{2}}{2}$ $\cos(225^\circ) = -\frac{\sqrt{2}}{2}$ $\sin(255^\circ) = -\frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos(255^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\sin(285^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(285^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(315^\circ) = \frac{\sqrt{2}}{2}$ $\cos(315^\circ) = \frac{\sqrt{2}}{2}$	$\sin(15^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos(15^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\sin(75^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(75^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(105^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(105^\circ) = -\frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(135^\circ) = \frac{\sqrt{2}}{2}$ $\cos(135^\circ) = -\frac{\sqrt{2}}{2}$ $\sin(165^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos(165^\circ) = -\frac{\sqrt{6} + \sqrt{2}}{4}$ $\sin(195^\circ) = -\frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(195^\circ) = -\frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(225^\circ) = -\frac{\sqrt{2}}{2}$ $\cos(225^\circ) = -\frac{\sqrt{2}}{2}$ $\sin(255^\circ) = -\frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos(255^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\sin(285^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(285^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(315^\circ) = \frac{\sqrt{2}}{2}$ $\cos(315^\circ) = \frac{\sqrt{2}}{2}$	$\sin(15^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos(15^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\sin(75^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(75^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(105^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(105^\circ) = -\frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(135^\circ) = \frac{\sqrt{2}}{2}$ $\cos(135^\circ) = -\frac{\sqrt{2}}{2}$ $\sin(165^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos(165^\circ) = -\frac{\sqrt{6} + \sqrt{2}}{4}$ $\sin(195^\circ) = -\frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(195^\circ) = -\frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(225^\circ) = -\frac{\sqrt{2}}{2}$ $\cos(225^\circ) = -\frac{\sqrt{2}}{2}$ $\sin(255^\circ) = -\frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos(255^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\sin(285^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(285^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\sin(315^\circ) = \frac{\sqrt{2}}{2}$ $\cos(315^\circ) = \frac{\sqrt{2}}{2}$	$\sin(15^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$ $\cos(15^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\sin(75^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$ $\cos(75^\circ) = \frac{\sqrt{6} - \sqrt{$
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