— 04/01/2019

62. THE SINE LAW.

AS ANGLE OF A TRIANGLE CHANGES, THE SIDE OPPOSITE THAT ANGLE CHANGES WITH IT. MATHEMATICALLY, WE ARE CONCERNED WITH THE EXACT NATURE OF THIS VARIATION AND IT WILL BE SHOWN THAT THE SIDE INCREASES OR DECREASES IN DIRECT PROPORTION WITH THE SINE OF THE OPOSITE ANGLE. TO SOLVE THE PROBLEM WE NEED ANOTHER RELATION BETWEEN THE SIDES AND ANGLES OF A TRIANGLE. FROM FIGURE 15 WE HAVE, BY THE DEFINITION OF THE SINE OF AN ANGLE,

$$\frac{h}{b} = \sin A \text{ AND } \frac{h}{a} = \sin B$$

$\frac{h}{b} = \sin A$

WHENCE $h = b\sin A$ AND $h = a\sin B$

$\frac{h}{a} = \sin B$

EQUATING THESE TWO VALUES OF h, WE HAVE

$$b\sin A = a\sin B$$

FIG. 15

$\frac{b\sin B}{b\sin B} = \frac{a\sin A}{b\sin B}$

DIVIDING BOTH SIDES BY $b\sin B$ GIVES

$\frac{\sin A}{\sin B} = \frac{a}{b}$

$$\frac{b\sin A}{b\sin B} = \frac{a\sin B}{b\sin B}$$ CANCELLING OUT LIKE TERMS IN NUMERATORS AND DENOMINATORS,

WE OBTAIN: $$\frac{\sin A}{\sin B} = \frac{a}{b}$$

STATED IN WORDS: THE SIDES OF A TRIANGLE ARE PROPORTIONAL TO THE SINES OF THE OPPOSITE ANGLES. IN ITS COMPLETE FORM THE LAW OF SINES MAY BE WRITTEN: → SINE LAW

— 868

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

NOW LET US PROCEED WITH THE SOLUTION OF THE PROBLEM OF FIGURE 15. SUPPOSE, UPON MEASUREMENT, THE ANGLES OF ELEVATION AT A AND B PROVE TO BE 34° AND 50°, RESPECTIVELY, AND SIDE c MEASURE 3000 FEET. THEN ANGLE ABC IS 180° − 50° OR 130° AND ANGLE

$$C = 180° - (130° + 34°) = 16°.$$

APPLYING THE LAW OF SINES,

$$\frac{a}{\sin A} = \frac{c}{\sin C},$$

WE HAVE IN THIS PROBLEM

$$\frac{a}{\sin 34°} = \frac{3000}{\sin 16°}.$$

MULTIPLYING BOTH SIDES BY $\sin 34°$, AND USING A FOUR PLACE TABLE WE OBTAIN