Every number of B is greater in magnitude than every number of A. Every number, B_m, is called an upper bound of A, since no matter what number m you select in B, the number m+1 is such that b_{m+1} is less than b_m. The set of rational numbers is dense. ~~To be specific, the rational number 1/3 will be defined.~~ — p 77

— Really dense ooo 18/02/2019

Arbitrarily we now include in class B the rational number 1/3. Note that there is now a least upper bound in B for A. If we select any rational number r, less than 1/3, we can always find in A, a number x such that x is greater than r. The number 1/3 is the least upper bound in the set of rational numbers.

$1/3 > A$

→ Upper bound

More technically, the set of all rational numbers ~~for the~~ ~~set of rational numbers~~ can be separated into ~~another~~ two subsets or classes, A and B, in such a manner that every chosen rational number belongs to A or to B, but not to both. Neither set is empty and a_n in A and b_m in B implies a_n is less than b_m. There exists a cut number c such that a_n in A implies a_n is less than or equal to c and b_m in B implies c is less than or equal b_m. The rational number 1/3 could have equivalently been included as member of class A. Then 1/3 would have been the greatest lower bound in A for B.

— 19/02/2019

$A \neq 0$, $B \neq 0$

$a_n < b_m$

$a_n \leq c$

$b_m \geq c$

→ lower bound

Technically, the separation of all ~~number~~ rational numbers into two classes such that every number in one class is less than every number in other class, is called section or cut. The rational number 1/3 has thus been defined unambiguously as a section or cut in the rational numbers. In a similar manner any other rational number can be exactly defined. For example, zero is the cut between all positive rational numbers and zero, and the negative rational numbers.

→ Section cut def.

This definition depends upon ~~the~~ the two converging sequences, A and B, having the same limit, i.e., they define the same number. Two rational sequences, B and C, also converge to the same limit and define the same number. The two decimal sequences, A and B, ~~having~~ also converge to the same limit and define the same number, $0.\dot{3} = 1/3$. The difference between the corresponding members of B and A, $b_1 - a_1$, $b_2 - a_2$, ... forms a sequence which must also converge to a limit, zero. Two

→ Limit