

[illegible]

[illegible]

Handwritten mathematical work for problem 10, showing trigonometric functions, their derivatives, and graphs.

Problem 10:

a) $y = 2 \sin\left(\frac{1}{4}x\right)$ on $[0, 2\pi]$

Graphs:

- $y = 2 \sin\left(\frac{1}{4}x\right)$ (Graph 1)
- $y = 2 \cos\left(\frac{1}{4}x\right)$ (Graph 2)

Calculus:

1. Derivatives:

- $y' = \frac{1}{2} \cos\left(\frac{1}{4}x\right)$
- $y' = -\frac{1}{2} \sin\left(\frac{1}{4}x\right)$

2. Critical Points:

- $y' = 0 \Rightarrow \cos\left(\frac{1}{4}x\right) = 0 \Rightarrow \frac{1}{4}x = \frac{\pi}{2} \Rightarrow x = 2\pi$
- $y' = 0 \Rightarrow -\sin\left(\frac{1}{4}x\right) = 0 \Rightarrow \frac{1}{4}x = 0 \Rightarrow x = 0$

3. Second Derivatives:

- $y'' = -\frac{1}{4} \sin\left(\frac{1}{4}x\right)$
- $y'' = -\frac{1}{4} \cos\left(\frac{1}{4}x\right)$

4. Concavity:

- $y'' > 0 \Rightarrow \sin\left(\frac{1}{4}x\right) < 0 \Rightarrow \frac{1}{4}x \in (\pi, 2\pi) \Rightarrow x \in (4\pi, 8\pi)$
- $y'' < 0 \Rightarrow \sin\left(\frac{1}{4}x\right) > 0 \Rightarrow \frac{1}{4}x \in (0, \pi) \Rightarrow x \in (0, 4\pi)$

5. Inflection Points:

- $y'' = 0 \Rightarrow \sin\left(\frac{1}{4}x\right) = 0 \Rightarrow \frac{1}{4}x = 0, \pi, 2\pi \Rightarrow x = 0, 4\pi, 8\pi$

6. Local Extrema:

- $y' = 0 \Rightarrow \cos\left(\frac{1}{4}x\right) = 0 \Rightarrow \frac{1}{4}x = \frac{\pi}{2} \Rightarrow x = 2\pi$
- $y' = 0 \Rightarrow -\sin\left(\frac{1}{4}x\right) = 0 \Rightarrow \frac{1}{4}x = 0 \Rightarrow x = 0$

7. Global Extrema:

- $y = 2 \sin\left(\frac{1}{4}x\right)$ on $[0, 2\pi]$
- $y = 2 \cos\left(\frac{1}{4}x\right)$ on $[0, 2\pi]$

8. Graphs:

- $y = 2 \sin\left(\frac{1}{4}x\right)$ (Graph 1)
- $y = 2 \cos\left(\frac{1}{4}x\right)$ (Graph 2)