is possible to solve any quadratic equation.

Ex.1: $x^2+8x+7=0$

x^2+8x can be made a perfect square by adding the square of half the coefficient of x, i.e. 4^2 or 16. Rewriting the original equation transposing the 7 (i.e. subtracting 7 from both sides), we obtain: $x^2+8x=-7$. Adding 16 to both sides, we obtain: $x^2+8x+16=16-7$, or $(x+4)^2=9$. Taking the square root of both sides: $x+4=\pm3$ whence $x=\pm3-4$ or $x=-1$, or -7.

30. The quadratic formula.

(1) $x^2+8x+15=0$

(2) $x^2+bx+c=0$

(3) $x^2+bx=-c$

(4) $x^2+bx+\frac{b^2}{4}=-c+\frac{b^2}{4}$

(5) $\left(x+\frac{b}{2}\right)^2=\frac{b^2-4c}{4}$

(6) $x+\frac{b}{2}=\frac{\pm\sqrt{b^2-4c}}{2}$

(7) $x=-\frac{b}{2}\pm\frac{\sqrt{b^2-4c}}{2}$

(8) $x=\frac{-b\pm\sqrt{b^2-4c}}{2}$

In an equation such as (1), if the arithmetic numbers are replaced by letters, we can have (2), then putting c on the right side (3), adding to both sides the square of one half the coefficient of x (4), whence (5), taking the square root of both sides (6), transposing $\frac{b}{2}$ to the right side (7), combining the terms on the right side, (8).

The right side of the equal sign is an algebraic expression involving the coefficient of x, and the constant c. It can be used to solve any quadratic equation, and is called quadratic formula.

Illustrations:

1) $x^2+8x+15=0$. What is the value of x? ← P32

Solution: $b=8$ and $c=15$. Therefore:

$$x=\frac{-8+\sqrt{64-60}}{2}=\frac{-8+2}{2}=-3,$$

and $$x=\frac{-8-\sqrt{64-60}}{2}=\frac{-8-2}{2}=-5.$$

2) $x^2+5x-3=0$. What is the value of x?

Solution: $b=5$ and $c=-3$. Therefore:

$$x=\frac{-5\pm\sqrt{25+12}}{2}=\frac{-5\pm\sqrt{37}}{2}=\frac{-5\pm\frac{\sqrt{37}}{2}}{2}$$

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