LET US EXAMINE EQUATIONS OF THIRD DEGREE SUCH AS:

$$x^3 - 5x^2 + 2x + 8 = 0$$

39. Solution of cubes by trial.

SUCH EQUATION CAN BE SOLVED BY TRIAL AS FOLLOWS:

Value of x	Left Side	Right Side = 0
1	$1 - 5 + 2 + 8 = 6$	0
-1	$-1 - 5 + 2 + 8 = 0$	0 YES
2	$8 - 20 + 4 + 8 = 0$	0 YES
-2	$-8 - 20 + 4 + 8 = -24$	0
4	$64 - 80 + 8 + 8 = 0$	0 YES
-4	$-64 - 80 - 8 + 8 = -144$	0
8	$512 - 320 + 16 + 8 = 216$	0
-8	$-512 - 320 - 16 + 8 = -840$	0

THEREFORE -1, 2 AND 4 ARE VALUES OF x THAT MAKE BOTH SIDES HAVE THE SAME VALUE. AS A MATTER OF FACT, ONE CAN SHOW THAT SUCH AN EQUATION HAS ONLY AS MANY ROOTS, AS THE DEGREE OF THE EQUATION. THAT IS A THIRD DEGREE EQUATION HAS THREE ROOTS.

40. Solution of cubics by factoring.

STARTING WITH THE TRIAL METHOD, AS SOON AS THE EXACT VALUE OF THE UNKNOWN HAS BEEN FOUND, WE FACTOR THE GIVEN EQUATION. IF NO EXACT VALUE IS FOUND BY TRIAL, IT IS NECESSARY TO USE METHODS AS WILL BE SHOWN LATER. CONSIDER THE EQUATION $x^3 - 5x^2 + 2x + 8 = 0$. IF $x = -1$, THE LEFT SIDE EQUALS THE RIGHT SIDE SO WE SUSPECT THAT $x + 1$ IS A FACTOR. WE DIVIDE $x^3 - 5x^2 + 2x + 8$ BY $x + 1$ TO FIND IF IT IS AND CO FIND OTHER FACTORS, JUST AS WE DIVIDE 1655 by 5, ONCE WE NOTE THAT 5 IS A FACTOR OF 1655. THIS IS DONE AS FOLLOWS:

$$5\,\overline{)\,1655\,}$$

$$331$$

$$\begin{array}{r} 15 \\ \hline 15 \\ 15 \\ \hline 5 \\ 5 \\ \hline \end{array}$$

$$x + 1\,\overline{)\,x^3 - 5x^2 + 2x + 8\,}$$ quotient $x^2 - 6x + 8$

$$\underline{x^3 + x^2}$$
$$-6x^2 + 2x$$
$$\underline{-6x^2 - 6x}$$
$$8x + 8$$
$$\underline{8x + 8}$$

SINCE $x^3 = x \times x^2$, put x^2 in the quotient. Then MULTIPLY $x + 1$ by x^2 putting the product bellow the dividend. Subtract, THEN REPEAT

THE PROCESS UNTIL THERE IS NO EXPRESSION LEFT. THUS

$$x^3 - 5x^2 + 2x + 8 = (x + 1)(x^2 - 6x + 8) = 0$$

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