numbers are called equal if they differ by less than any preassigned constant, however small. This is strongly evidenced by the fact that 0.9 is equal to unity. Every rational number greater than 1 will $0.\overline{9} = 1$ def always exceed that decimal approximation. Hence, 0.9 is one.

72. THE SQUARE ROOT OF TWO.

Since the set of rational numbers is dense and between two arbitrary _p 78_ rational numbers, e.g. less than R_2, a third R_3, can always be found such that $R_1 < R_3 < R_2$, it should now be possible to find a number which multiplied by itself, yields the number 2. Since $1^2 = 1$, is less than 2, and $2^2 = 4$, is greater than 2, we need only consider the rational number between 1 and 2. The square of any number greater than 2 will be greater than 4.

In investigating the numbers between 1 and 2 to find a number which multiplied by itself, yields the number 2, a methodical process will be used

A	2.25	2.002225	2.00024449	2.00022084
B	1.5	1.42	1.4143	1.41422
C	1.4	1.41	1.4141	1.41421
D	1.96	1.9881	1.99976164	1.9999.899421

_20/02/2019_

A is a sequence, every member¹ which is the square of the corresponding member of B. D is a sequence, every member of which is the square of the corresponding member of C. The first member, 1.4, of the sequence C is chosen so that its square will be less than 2, and so that it differs from the first member of B in the first digit after the decimal point. The square of B is a number greater than 2. The second members of the sequences B and C are selected so that they differ from each other only in the second digit after the decimal point while their squares are respectively greater than and less than 2. The number whose square is 2 most lie between these two members of sequences B and C. The succeeding members of B and C are chosen in a similar manner. Hence, each succeeding member of B decreases in magnitude while each succeeding member of C increases in magnitude. The number whose square is 2 always lies between two corresponding members of B and C. The further we go in each sequence, the more closely do their member approach the square root of 2, one from above, one from below. The nth members of both sequences will differ from each other in the nth digit after the decimal point. The nth number of either sequence gives an

[14]