

DEFINED  $\tan A$  AS (SIDE OPPOSITE)/(SIDE ADJACENT), EVIDENTLY  $\tan A = (\sin A)/(\cos A)$ . IT FOLLOWS ALSO, FROM THE PYTHAGOREAN THEOREM, WHICH STATES THAT THE SUM OF THE SQUARES OF THE LEGS OF A RIGHT TRIANGLE IS EQUAL TO THE SQUARE OF THE HYPOTENUSE, THAT:

$$\sin^2 A + \cos^2 A = 1.$$

FROM THIS EQUATION:  $\cos^2 A = 1 - \sin^2 A$

WHENCE:  $\cos A = \sqrt{1 - \sin^2 A}$

~~SUBSTITUTING~~ SUBSTITUTING THE LATTER IN  $\tan A = (\sin A)/(\cos A)$  GIVES:

$$\tan A = \frac{\sin A}{\sqrt{1 - \sin^2 A}}$$

$$\tan A = \frac{\sin A}{\cos A}$$

## 60. TRIGONOMETRIC FUNCTIONS OF $45^\circ$ , $30^\circ$ , AND $60^\circ$ .

THE NUMERICAL VALUES OF THE FUNCTIONS OF ANY ANGLE COULD BE APPROXIMATED BY MEANS OF CAREFULLY CONSTRUCTED LINGERS, FOR INSTANCE, BY MEASURING THE LINES PQ, AP AND SR IN FIGURE 7.

HOWEVER, THE VALUES OF THE TRIGONOMETRIC FUNCTIONS ARE IN GENERAL BETTER APPROXIMATED BY MEANS OF TABLES DERIVED IN OTHER BRANCHES

OF MATHEMATICS. BUT THE VALUES OF FUNCTIONS OF CERTAIN USEFUL ANGLES CAN BE DETERMINED EXACTLY FROM THE PROPERTIES OF CERTAIN

SPECIFIC TRIANGLES. LET US CONSIDER, FOR EXAMPLE, AN ISOSCELES RIGHT TRIANGLE, THAT IS, ONE HAVING TWO EQUAL LEGS. SUPPOSE

WE DENOTE THE EQUAL LEGS (FIGURE 9) ~~TO BE~~ <sup>BY</sup> ANY LETTER, SAY  $b$ .

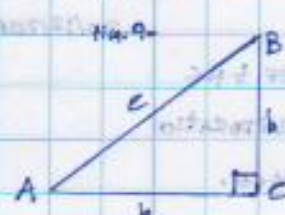
THEN WE FIND THE HYPOTENUSE BY THE PYTHAGOREAN THEOREM:

$$c^2 = b^2 + b^2, \text{ WHENCE } c^2 = 2b^2 \text{ AND } c = b\sqrt{2}.$$

→ WHAT BEACHES?

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$$\rightarrow \text{PYTHAGOREAN THEOREM} \\ c^2 = b^2 + b^2$$



SINCE THE SUM OF THE ANGLES OF ANY TRIANGLE IS  $180^\circ$  AND THE ANGLES OPPOSITE THE EQUAL SIDES OF A TRIANGLE ARE EQUAL, EACH OF THE ACUTE ANGLES MUST BE  $45^\circ$ , SO WE CAN WRITE THE VALUES OF THE FUNCTION OF AN ANGLE OF  $45^\circ$ :

$$\sin 45^\circ = \frac{b}{b\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1.4142}{2} = 0.7071$$

$$\cos 45^\circ = \frac{b}{b\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.7071$$

$$\tan 45^\circ = \frac{b}{b} = 1.0000$$

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