

$(1/b)(b)x = (1/b)a$ .  $x = a/b$ . WE ARE ASSUMED THAT THE NUMBER  $1/b$  EXISTS AND HENCE WE CAN MULTIPLY BY  $1/b$ . HENCE, EXCEPT FOR DIVISION BY ZERO, THE SET OF RATIONAL NUMBERS IS CLOSED UNDER THE OPERATION OF DIVISION.

## 71. THE RATIONAL NUMBERS AS SECTIONS OR CUTS.

THE NUMBER IN WHICH WE NOW DEFINE RATIONAL NUMBERS WILL ~~SEEM~~ <sup>SEEM</sup> FORGOTTEN AND QUITE ARTIFICIAL. THE FOLLOWING DEFINITION IS VALID IF ~~THE~~ <sup>THE</sup> SET IS LINEARLY ORDERED AND DENSE. FOR EXAMPLES:

THE RATIONAL  $1/3$  WILL BE DEFINED. FOR WHICH THERE'S FOUR SEQUENCES

A  $99/300$ ;  $999/3,000$ ;  $9999/30,000$ ;  $99999/300,000$ ...

B  $102/300$ ;  $10002/3000$ ;  $100002/30000$ ;  $1000002/300000$ ...

C  $0.33$ ;  $0.333$ ;  $0.3333$ ;  $0.33333$ ...

D  $0.34$ ;  $0.334$ ;  $0.3334$ ;  $0.33334$ ...

IN EACH SEQUENCE THE NUMBER OF MEMBERS IS ENDLESS. AND

B ARE WRITTEN IN RATIONAL FRACTIONS. C AND D IN AN EQUIVALENT

DECIMAL REPRESENTATION. THE METHOD OF FORMATION OF EACH SEQUENCE

AND MEMBER IS CLEARLY SHOWN. FOR EXAMPLE IN A THE NUMERATOR OF THE

GENERAL MEMBER,  $a_n$ , CONSISTS OF  $(n+1)$  DIGITS, EACH BEING 9. THE

DENOMINATOR CONSISTS OF 3 FOLLOWED BY  $(n+1)$  ZEROS. IF  $n=4$ , THE

MEMBER IS THE NUMBER  $99999/300000$ .

EACH MEMBER OF A, AFTER THE FIRST MEMBER, IS GREATER IN MAGNITUDE

THAN THE PRECEDING MEMBER. THE ORIGINAL REPRESENTATION, C AND D,

SHOW THIS CLEARLY. ANY MEMBER OF A IS LESS THAN  $1/3$ . ANY MEMBER

OF CLASS B IS GREATER THAN  $1/3$ . THIS IS READILY SEEN IF WE SET

THE FOLLOWING SEQUENCES, A' B' FOR CONVENIENCE, LET THE MEMBERS

OF B BE  $b_1, b_2, b_3, \dots$  WITH GENERAL TERM  $b_m$ .

$$A' \quad 1/3 - a_1; 1/3 - a_2; 1/3 - a_n; \dots$$

$$B' \quad b_1 - 1/3; b_2 - 1/3; b_3 - 1/3; \dots$$

FOR ANY GIVEN POSITIVE RATIONAL NUMBER  $\delta$ , THE POSITIVE

NUMBERS,  $1/3 - a_n$  AND  $b_m - 1/3$ , CAN BE MADE LESS THAN  $\delta$

BY CHOOSING  $n$  OR  $m$  SUFFICIENTLY LARGE. THEN IS POSSIBLE TO

APPROXIMATE TO  $1/3$  AS CLOSE AS DESIRED. FOR EXAMPLE, LET  $\delta = 1/10^6$ .

THE DIFFERENCE BETWEEN  $1/3$  AND  $a_n$  MUST BE LESS THAN  $0.000001$ . IF

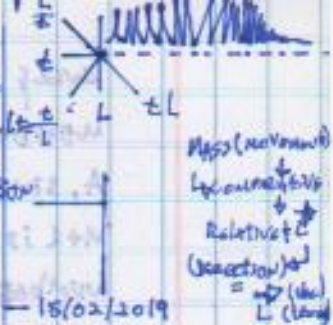
$n=4$ , WE HAVE  $(1/3 - 999999/3000000) > 1/10^6$ . IF  $n=5$ ,  $(1000000/3000000 - 1/3) < 1/10^6$ . SINCE THE DIFFERENCE BETWEEN  $1/3$  AND  $b_m$

CANNOT BE ZERO WITH EXCEPTION OF  $1/3$  THE RATIONAL NUMBERS ARE

THUS SEPARATED IN TWO CLASSES, A, EVERY MEMBER OF WHICH IS

LESS THAN  $1/3$ . BY OUR CLASS SELECTION, WE THEREFORE INSURE THAT

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15/02/2019

FORGOTTEN = 0.76

TODAY I SHALL NOTE A

BIT MORE HERE, ESPECIALLY

BECAUSE OF MY CHAIRMAN FOR

WRITING IT DOWN AS IT

MOST BE TOLD, BEING

READY, AS I WAS

COPYING THE FIRST PART

OF SECTION 71.

I COPY IN ORDER TO

TRAIN COPYING SKILLS AND

THE CORRECTED FIELD, AS

WELL TO BETTER UNDERSTAND

THE BOOK, BEING CAPABLE

TO TAKE FORTY-FOUR NOTES.

I HAVE NOTIFIED VERY LITTLE

TILL NOW. FROM HERE

ON, MY SEMESTER

BEGINS AGAIN, SO,

I SHALL JUST GO ON.

HOPE TO DO AGAIN

(LARGE) N

1/3

1/3

1/3

1/3

1/3

1/3

1/3

1/3

1/3

1/3

1/3