

HW 9

Wednesday, February 15, 2023 9:57 AM

1. Translate the following into predicate calculus. For each answer, also state your assumed universe of discourse.

a) "Anyone who was an ancient Roman and tried to kill Caesar was not loyal to Caesar."

b) "All cats which are calico, are female."

c) "Some Texans have never left the state of Texas."

a) Universe of Discourse: All individuals/people

$$(\forall x) ((\text{AncientRoman}(x) \wedge \text{TriedKillCaesar}(x)) \rightarrow \sim \text{LoyalToCaesar}(x))$$

b) Universe of Discourse: All cats

$$(\forall x) ((\text{Cat}(x) \wedge \text{Calico}(x)) \rightarrow \text{Female}(x))$$

c) Universe of Discourse: All individuals/people

$$(\exists x) (\text{Texan}(x) \wedge \text{LeftTexas}(x))$$

2. A universe contains the three individuals a, b, and c. For these individuals, a predicate $Q(x, y)$ is defined, and its truth values are given by the following table:

$x \backslash y$	a	b	c
a	T	F	T
b	F	T	F
c	F	T	T

Write each of the following expressions without quantifiers (i.e. convert them to expressions with ANDs and ORs or both) and then evaluate the expression.

a) $\forall x \exists y Q(x, y)$

b) $\forall y Q(y, b)$

c) $\forall y Q(y, y)$

a) $(\forall x) (\exists y) Q(x, y)$

$$(Q(a, a) \vee Q(a, b) \vee Q(a, c)) \wedge (Q(b, a) \vee Q(b, b) \vee Q(b, c)) \wedge (Q(c, a) \vee Q(c, b) \vee Q(c, c))$$

$$(TRUE \wedge TRUE \wedge TRUE)$$

TRUE

b) $(\forall y) Q(y, b)$

$$Q(a, b) \wedge Q(b, b) \wedge Q(c, b)$$

$$FALSE \wedge TRUE \wedge FALSE$$

FALSE

c) $(\forall y) Q(y, y)$

$$Q(a, a) \wedge Q(b, b) \wedge Q(c, c)$$

$$TRUE \wedge TRUE \wedge TRUE$$

TRUE

1. Algebraically transform:

$\neg \forall x(P(x) \wedge Q(y) \Rightarrow \exists zR(z))$ to
 $\exists x \forall z(P(x) \wedge Q(y) \wedge \neg R(z))$

Justify each step with one or more laws.

1	$\neg \forall x(P(x) \wedge Q(y) \Rightarrow \exists zR(z))$	Given
2	$\neg \forall x(\neg(P(x) \wedge Q(y)) \vee \exists zR(z))$	Implication
3	$\neg \forall x(\neg P(x) \vee \neg Q(y) \vee \exists zR(z))$	DeMorgan's
4	$\exists x \neg(\neg P(x) \vee \neg Q(y) \vee \exists zR(z))$	DeMorgan's
5	$\exists x(P(x) \wedge Q(y) \wedge \neg \exists zR(z))$	DeMorgan's
6	<u>$\exists x \forall z(P(x) \wedge Q(y) \wedge \neg R(z))$</u>	<u>DeMorgan's</u>