HW6

Thursday, February 2, 2023 9:31 PM

1. Construct the truth table for each of the following expressions. Indicate for each expression whether it is a tautology, a contradiction, or neither (meaning that it is contingent).

a) $(P \wedge (P \Rightarrow Q)) \wedge \neg Q$

b) $(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

c) $(Q \wedge (P \Rightarrow Q)) \Rightarrow P$

a) $(P \wedge (P \Rightarrow Q)) \wedge \neg Q$

P	Q	$(P \Rightarrow Q)$	$P \wedge (P \Rightarrow Q)$	$\neg Q$	$(P \wedge (P \Rightarrow Q)) \wedge \neg Q$
T	T	T	T	F	F
T	F	F	F	T	F
F	T	T	F	F	F
F	F	T	F	T	F

this is a contradition.

b) $(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

P	Q	$(P \Rightarrow Q)$	$\neg P$	$\neg P \vee Q$	$(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

this is a tautology.

c) $(Q \wedge (P \Rightarrow Q)) \Rightarrow P$

P	Q	$(P \Rightarrow Q)$	$Q \wedge (P \Rightarrow Q)$	$(Q \wedge (P \Rightarrow Q)) \Rightarrow P$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

this is neither a tautology nor a contradition.
it is contingent.

5. Reduce the expression

$Q \vee \neg((P \Rightarrow Q) \wedge P)$

to T. ~~Your reduction must be algebraic and~~ you must justify every step with the law (or laws) you use for the step.

Your proof must use logical equivalences (not truth tables). Recall that the textbook gives several tables of logical equivalences in section 1.3.2

$Q \vee \neg((P \Rightarrow Q) \wedge P)$	
$Q \vee \neg((\neg P \vee Q) \wedge P)$	<- Implication
$Q \vee \neg((P \wedge \neg P) \vee (P \wedge Q))$	<-distributive
$Q \vee (\neg(P \wedge \neg P) \wedge \neg(P \wedge Q))$	<- DeMorgan's
$Q \vee ((\neg P \vee \neg\neg P) \wedge (\neg P \vee \neg Q))$	<- DeMorgan's
$Q \vee ((\neg P \vee P) \wedge (\neg P \vee \neg Q))$	<- Double Negation
$(Q \vee(\neg P \vee P)) \wedge (Q \vee(\neg P \vee \neg Q))$	<- Distributive
$(Q \vee \text{TRUE}) \wedge (Q \vee(\neg P \vee \neg Q))$	<- Negation
$\text{TRUE} \wedge (Q \vee(\neg P \vee \neg Q))$	<- Domination
$\text{TRUE} \wedge (Q \vee(\neg Q \vee \neg P))$	<- Commutative
$\text{TRUE} \wedge ((Q \vee\neg Q) \vee \neg P)$	<- Associative
$\text{TRUE} \wedge (\text{TRUE} \vee \neg P)$	<- Negation
$\text{TRUE} \wedge \text{TRUE}$	<- Domination
TRUE	

Use logical equivalences to find ...

6. ~~Algebraically~~ find the conjunctive normal form of the following expression.

$P \Rightarrow ((Q \wedge R) \Leftrightarrow S)$

Justify every step with the law (or laws) you use for the step.

P ⇒ ((Q ∧ R) ⇔ S)	Initial
P ⇒ (((Q ∧ R)⇒ S and S ⇒ (Q ∧ R)))	Equivalence
P ⇒ ((!(Q ∧ R) ∨ S) ∧ (!S ∨ (Q ∧ R)))	Implication
!P ∨ ((!(Q ∧ R) ∨ S) ∧ (!S ∨ (Q ∧ R)))	Implication
!P ∨ (((!Q ∨ !R) ∨ S) ∧ (!S ∨ (Q ∧ R)))	DeMorgan's
!P ∨ (((!Q ∨ !R) ∨ S) ∧ ((!S ∨ Q) ∧ (!S ∨ R)))	Distributive
!P ∨ ((!Q ∨ !R ∨ S) ∧ ((!S ∨ Q) ∧ (!S ∨ R)))	Associative
((!P ∨ !Q ∨ !R ∨ S) ∧ ((!P ∨ !S ∨ Q) ∧ (!P ∨ !S ∨ R)))	Distributive
(!P ∨ !Q ∨ !R ∨ S) ∧ (!P ∨ !S ∨ Q) ∧ (!P ∨ !S ∨ R)	Associative