HW15a - 3.22

Monday, March 20, 2023 5:47 PM

1. Give the following closures, as specified.

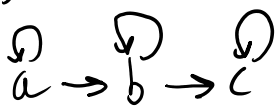
a) Let R be the relation {(a, b), (b, c), (c, c)} on {a, b, c}. Give the reflexive closure as a graph.

b) Let R be the relation {(a, b), (a, c), (b, a), (c, b), (c, c)} on{a, b, c}. Give the symmetric closure as a matrix.

c) Let R be the relation {(a, a), (a, b), (c, d), (d, a)} on{a, b, c, d}. Give the transitive closure as a set of ordered pairs.

d) Let R be the relation {(a, d), (a, b), (c, d), (d, a)} on{a, b, c, d, e}.
Give the reflexive-transitive closure as a matrix.

a)



original pairs
(a,d), (a,b), (c,d), (d,a)

add pairs
(a,a), (c,a), (d,d)

repeat add pairs using new pairs
(a,d), (c,b), (a,b)

repeat add pairs using new pairs
(c,b), (d,b)

repeat add pairs using new pairs
(none)

add reflexive pairs
(b,b), (c,c), (e,e)

result of reflexive transitive closure
{ (a,d), (a,b), (c,d), (d,a), (a,a), (c,a), (d,d), (c,b), (d,b), (b,b), (c,c), (e,e) }

b)

	a	b	c
a	0	1	1
b	1	0	1
c	1	1	1

(a,b) (a,c) (b,a) (c,b) (c,c)
(c,a) (b,c)

c)

original pairs
(a,a), (a,b), (c,d), (d,a)

add pairs
(c,a), (d,b)

repeat add pairs using new pairs
(c,b)

repeat add pairs using new pairs
(none)

result of transitive closure
{ (a,a), (a,b), (c,d), (d,a), (c,a), (d,b), (c,b) }

d)

	a	b	c	d	e
a	1	1	0	1	0
b	0	1	0	0	0
c	1	1	1	1	0
d	1	1	0	1	0
e	0	0	0	0	1

2. For each of the following relations, i) state whether the relation is an equivalence relation and ii) if it is an equivalence relation then list the equivalence classes of its corresponding partition (as a set of sets), and if it is not then list all of the properties (reflexive, symmetric, transitive) it violates. In all cases, the relations are on the set A:
A = {1, 2, 3, 4}.

a) {(1, 1), (1, 4), (2, 2), (3, 3), (4, 1), (4, 4)}

b) A × A

c) {(1, 1), (2, 2), (3, 1), (1, 3), (4, 4)}

d) ({2, 3} × A) ∪ {(1, 1), (4, 4)}

a) { (1, 1), (1, 4), (2, 2), (3, 3), (4, 1), (4, 4) }
   i) Yes
   ii) { {1,4}, {2}, {3} }

b) A x A = { (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4) }
   i) Yes
   ii) { {1,2,3,4} }

c) { (1, 1), (2, 2), (3, 1), (1, 3), (4, 4) }
   i) No
   ii) not reflexive, missing (3,3)
   no transitive, missing (3,3)

d) ({ 2, 3 } x A) U { (1, 1), (4, 4) } =
{ (1,1), (1,2), (1,3), (2,2), (2,3), (3,2), (3,3), (4,2), (4,3), (4,4) }

i) No
ii) Not Symmetric, missing (1,2), (1,3), (4,2), (4,3)

B = ({ 2, 3 } x A) = { (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4) }

=> B U { (1,1), (4,4) } = { (1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,4) }