HW7

Sunday, February 5, 2023 9:40 PM

1. Use the truth table method to show that the following arguments are sound.

a)

$P \Rightarrow Q$
$P \Rightarrow R$

$P \Rightarrow Q \wedge R$

b)

$P \vee Q$
$P \Rightarrow R$
$Q \Rightarrow R$

R

a)

P	Q	R	P-> Q	P-> R	Prem	Conlc	Prem->Concl
T	**T**	**T**	T	T	T	T	T
T	**T**	**F**	T	F	F	F	T
T	**F**	**T**	F	T	F	F	T
T	**F**	**F**	F	F	F	F	T
F	**T**	**T**	T	T	T	T	T
F	**T**	**F**	T	T	T	T	T
F	**F**	**T**	T	T	T	T	T
F	**F**	**F**	T	T	T	T	T

b)

P	Q	R	P and Q	P-> R	Q-> R	Prem	Conlc	Prem->Concl
T	**T**	**T**	T	T	T	T	T	T
T	**T**	**F**	T	F	F	F	F	T
T	**F**	**T**	F	T	T	F	T	T
T	**F**	**F**	F	F	T	F	F	T
F	**T**	**T**	F	T	T	F	T	T
F	**T**	**F**	F	T	F	F	F	T
F	**F**	**T**	F	T	T	F	T	T
F	**F**	**F**	F	T	T	F	F	T

2. Do the following proofs deductively. Justify each step in your proof with a law or inference rule.

a) If $P \Rightarrow Q$, $\neg R \Rightarrow \neg Q$, and P then prove R.

b) If $P \Rightarrow (Q \wedge R)$ and $\neg R \wedge Q$ then prove $\neg P$.

a)

1	P->Q	Premise
2	!R -> !Q	Premise
3	P	Premise
4	Q	Modus Ponsens - 1,3
5	!!R / R	Modus Tollens - 2,4

b)

1	P->(Q and R)	Premise
2	!R and Q	Premise
3	!P or (Q and R)	Implication - 1
4	(!P or Q) and (!P or R)	Distributive - 3
5	(!P or R) and (!P or Q)	Commutative - 4
6	!P or R	Simplification - 5
7	!R	Simplication - 2
8	!P	Disjunctive Syllogism - 6,7

b)

1	P->(Q and R)	Premise
2	!R and Q	Premise
3	!R	Simplification - 2
4	!(Q and R)	Conjunction Elimination - 2, 1
5	!P	Modus Tollens 1,4

5. Given the following premises:

"if cows fly then
there is a man in the moon"
"if there is a man in the moon then
the earth is flat"
"cows fly"

Prove the following conclusion:
"the earth is flat."

Justify each step in your proof with a law or an inference rule.

C = cows fly
M = man in the moon
E = Earth is flat

C->M
M->E
C

1	C->M	Premise
2	M->E	Premise
3	C	Premise
4	M	Modus Ponens - 1,3
5	E	Modus Ponens - 2,4

6. For each of the following arguments, indicate which of the rules of inference are used (Modus ponens, Disjunctive syllogism, etc.).

a) If Mr. Smith or Mrs. Smith earns more than $30,000 per year, the Smith family can afford holidays in Hawaii. Since I know that either Mr. Smith or his wife earns more than $30,000, I conclude that the family can afford a holiday in Hawaii.

b) If John was at the party yesterday, he will sleep in. John did not sleep in. Consequently, he was not at the party.

c) If Susan studies hard, she'll get an A and if she gets an A, she'll be happy. Hence, if Susan studies hard, then she'll be happy.

a) Modus Ponens
   a. Money -> Hawaii
   b. Money
   c. Therefore, Hawaii

b) Modus Tollens
   a. Party -> Sleep
   b. !Sleep
   c. Therefore, !Party

c) Hypothetical Syllogism
   a. Study->A
   b. A->Happy
   c. Therefore, Study -> Happy