HW8

Wednesday, February 8, 2023 4:44 PM

Problem 1.
Use mathematical induction to prove
(for all integers n > 0):
P(n): 1 + 3 + 6 + ... + n(n+1)/2 = n(n+1)
(n+2)/6

P(n): 1 + 3 + 6 + … + n(n+1)/2 = n(n+1)(n+2)/6

Base Case: n = 1

1(1+1)(1+2)/6
1(2)(3)/6
6/6 = 1

Assume p(k):

1 + 3 + 6 + … + k(k+1)/2 = k(k+1)(k+2)/6

Prove p(k+1)

1 + 3 + 6 + … + (k+1)((k+1)+1)/2 = (k+1)((k+1)+1)((k+1)+2)/6 = (k+1)(k+2)(k+3)/6

Substitute

k(k+1)(k+2)/6 + (k+1)((k+1)+1)/2
k(k+1)(k+2)/6 + (k+1)(k+2)/2
k(k+1)(k+2)/6 + 3(k+1)(k+2)/6
[k(k+1)(k+2) + 3(k+1)(k+2)] / 6
(k+1)(k+2)(k+3) / 6

Equal, therefore we've proven this is true.

Problem 2.
Use mathematical induction to prove
(for all integers n > 0):
P(n): 1*1! + 2*2! + ... + n*n! = (n+1)! - 1

Base Case: n = 1

(1+1)! - 1
(2)! - 1
2 - 1
1

Assume p(k):

1*1! + 2*2! + … + k*k! = (k+1)! - 1

Prove p(k+1)

1*1! + 2*2! + … + (k+1)*(k+1)! = ((k+1)+1)! - 1 = (k+2)! - 1

Substitute

(k+1)! - 1   +   (k+1)*(k+1)!

(k+1)! + (k+1)*(k+1)! - 1

(1 + k+1)(k+1)! - 1

(k+2)(k+1)! - 1

<u>(k+2)! - 1</u>