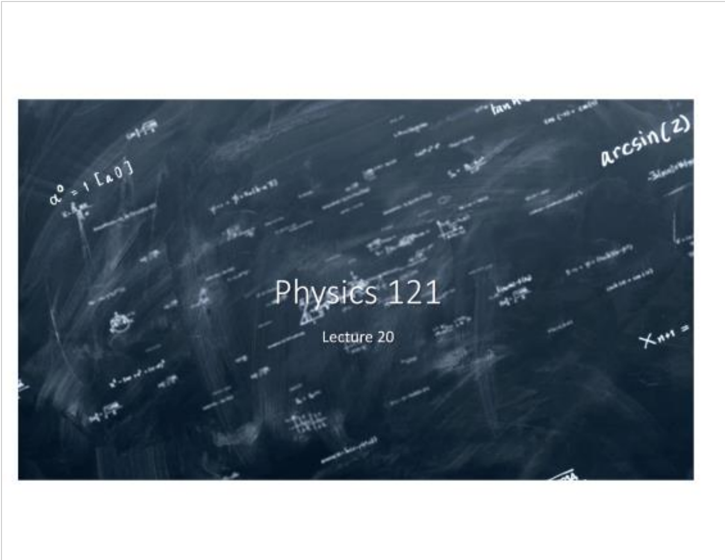
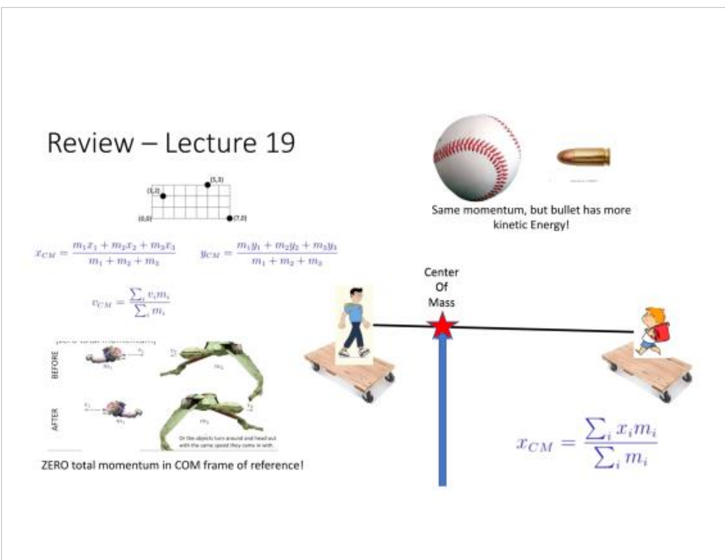
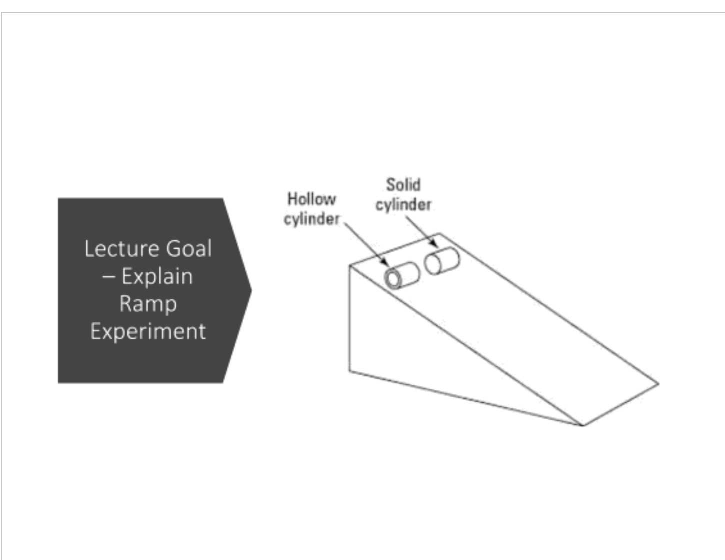
Lecture 20 - 3.21

Tuesday, March 21, 2023 8:17 AM





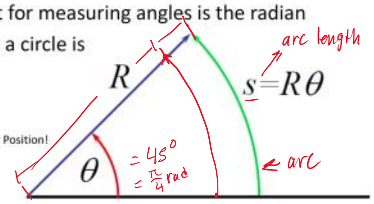


$\cancel{\theta} = 360°$ $s = 2\pi \cdot r$

$\theta = 45° \cdot \frac{2\pi}{360°} = \frac{\pi}{4}$ rad

Arc length

- The "natural" unit for measuring angles is the radian
- A length of arc on a circle is



Theta is called: Angular Position!

$2\pi \cdot \cancel{R} = \cancel{R} \cdot 360$

$2\pi = 360°$

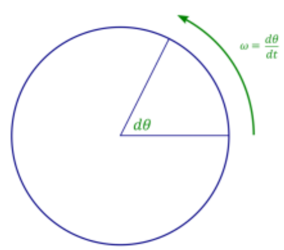
$1 = \frac{2\pi}{360°}$

- In a way you already know this. If you go all the way around the circle, the path length is the circumference, $2\pi R$ (path length = angle times radius)

Angular Velocity

ω = change in angular position over time

aka Angular Velocity



Symmetry:

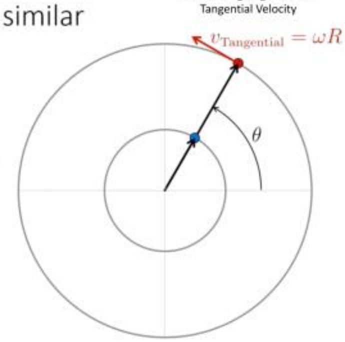
- Change in position is velocity

- Change in angular position is angular velocity

In a way, all circles are similar

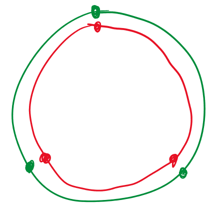
- Think of someone walking in a circle near you
- Think of someone else walking in a circle far from you
- Think about the person farther away from you "hiding" behind the person who is close to you.
- The person who is farther away is walking faster
- But they have the same angular position on the circle (theta)

Connecting Angular with Tangential Velocity



A circular workout

Q2: So, suppose you are jogging with your roommate around a circular track. You run in the inside lane and your roommate runs in the outside lane. You and your roommate run exactly side-by-side. Which of the following is true?



A. You have the same angular speed but you are running faster than your roommate.

B. You have the same angular speed but you are running slower than your roommate.

C. You run the same speed but you have a greater angular speed than your roommate.

D. You run the same speed but you have a slower angular speed than your roommate.

Quiz

A wheel rotates by 597 degrees. What is the path length traveled by a particle located at a radius of 8 cm?

A. About 8 cm

B. About 16 cm

C. About 30 cm

D. About 50 cm

E. About 80 cm

$S = \theta \cdot R$

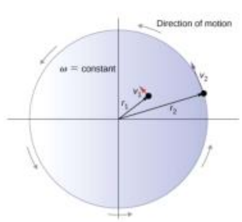
$597^\circ \cdot \frac{2\pi}{360} \cdot 8$

≈ 83.36

Quiz

Two particles rotate on a disk. Which one has greater angular velocity?

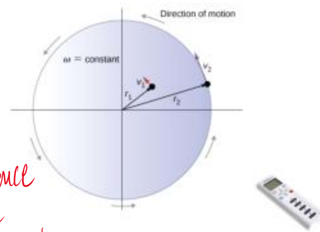
A) Particle 1
B) Particle 2
C) Both the same
D) Impossible to answer



Quiz

Two particles rotate on a disk. Which one has greater kinetic energy?

A) Particle 1
B) Particle 2
C) Both the same
D) Impossible to answer

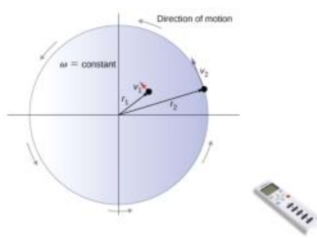


Technically D bc no masses are given, though the difference would have to be huge for 2 not to have more.

Quiz

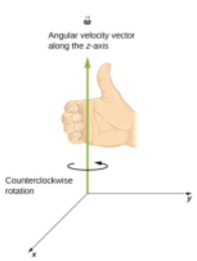
Two particles rotate on a disk. Which one has greater tangential velocity?

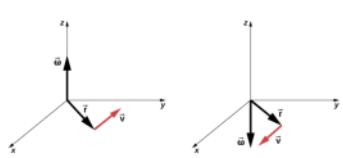
A) Particle 1
B) Particle 2
C) Both the same
D) Impossible to answer



Vector Excursion

$\vec{v} = \vec{\omega} \times \vec{r}$. cross product

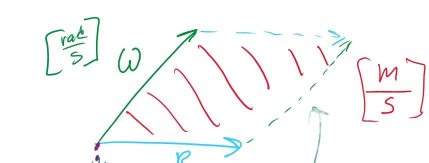




Cross Products:
- magnitude is the area of enclosed parallelogram
- direction is perpendicular to the plane
- sign is positive for counterclockwise rotation
  - depends on order A x B = - (B x A)

right-hand rule
(ccw) counter-clockwise: +
(cw) clock wise : −

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 b_1 - a_2 b_2 \\ a_2 b_1 - a_1 b_2 \end{pmatrix}$$



$\vec{V}$ (pointing at us) equal to area
$\left[\frac{m}{s}\right]$

$= \omega \cdot R$

Quiz

What is the tangential velocity for a wheel of radius 1m spinning at 10 RPM?

A) 10 m / s
B) 5 m / s
C) 1 m / s
D) 0.28 m /s
E) Impossible to answer

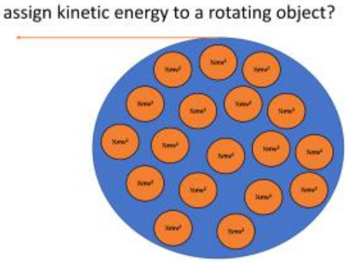
$$\vec{v} = \vec{\omega} \cdot \vec{r}$$
$$= 10\,RPM \cdot 1m$$
$$= 1\cancel{0}\,\frac{2\pi \cdot 1}{6\cancel{0}} = \frac{2\pi}{6} \approx \boxed{1\,m/s}$$

Experiment – Axis matter!

Rotate a ball around itself or around a pole

Rotational Kinetic Energy

How to assign kinetic energy to a rotating object?



$$\omega = \frac{v_t}{r}$$

$$K = \sum_j \frac{1}{2} m_j v_j^2$$
$$= \sum_j \frac{1}{2} m_j (r_j \omega_j)^2$$

$$K = \frac{1}{2}\underbrace{\left(\sum_j m_j r_j^2\right)}_{I = \sum_j m_j r_j^2} \omega^2$$

Moment of Inertia

Overthinking Inertia

Inertia = Opposition to getting into motion

Linear Inertia = Plain old mass. Heavy objects don't want to move.

Rotational Inertia = masses do not like to rotate around far away points. We call this opposition to rotation *Moment of Inertia*

Key takeaway:

Just as ***Masses*** don't want to move, ***Moment of Inertia*** don't want to rotate!

$$K = \frac{1}{2} m v_t^2 \qquad K = \frac{1}{2} I \omega^2$$

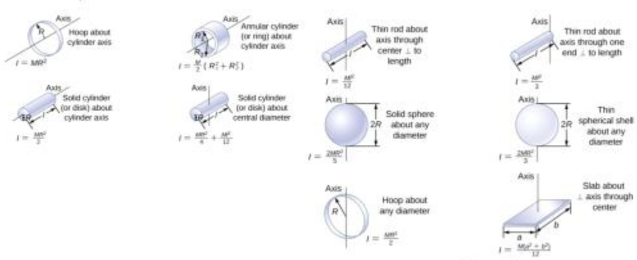
Kind of like a mass, it resists the change of rotation

More Symmetry!

Rotational	**Translational**
$I = \sum_j m_j r_j^2$	m
$K = \frac{1}{2} I \omega^2$	$K = \frac{1}{2} m v^2$

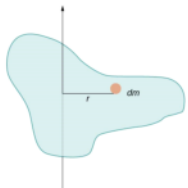
Table 10.4 Rotational and Translational Kinetic Energies and Inertia

Moment of Inertia can be looked up for many shapes!

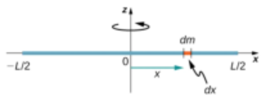


Notice that they all have the same general form: $I = [\,]mR^2$

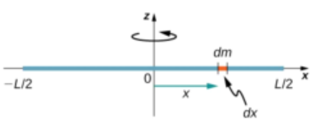
How to find it yourself! – Advanced topic



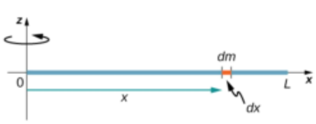
$$I = \sum_i m_i r_i^2 \text{ becomes } I = \int r^2 dm.$$



Let's do it for a thin rod rotating around center!



Now for rotation around axis!



Easier Way? **The Parallel-Axis Theorem**

PARALLEL-AXIS THEOREM

Let m be the mass of an object and let d be the distance from an axis through the object's center of mass to a new axis. Then we have

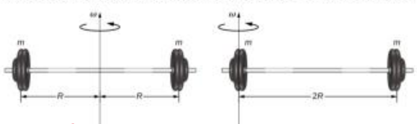
$$I_{\text{parallel axis}} = I_{\text{center of mass}} + md^2. \quad (10.20)$$

$$I_{\text{end}} = I_{\text{center of mass}} + md^2 = \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2 = \left(\frac{1}{12} + \frac{1}{4}\right)mL^2 = \frac{1}{3}mL^2$$

Jakob Steiner

Quiz

Is it easier to rotate a barbell around the center or an end-point?



A) center B) end-point

at center of mass is smallest I

There is energy in the rotation too

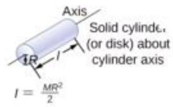
- Total kinetic energy = KE for center of mass + KE for rotation

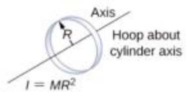
$$KE_{\text{rot}} = \frac{1}{2}I\omega^2$$

- Conservation of energy in one of your homework problems

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Revisiting our Experiment





$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$I = [\ ]mR^2 \quad \omega = \frac{v}{R}$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}[\ ]\cancel{m}\cancel{R^2}\left(\frac{v}{\cancel{R}}\right)^2$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}[\ ]\cancel{m}v^2$$

solve for v

$$v = \sqrt{\frac{2gh}{1+[\ ]}}$$

the I formula is the only thing that matters

Notice: There is no dependence on the mass OR the radius!

Sphere's factor : 2/5

Hoop's factor : 1

Exit Poll

- Please provide a letter grade for todays lecture:

A. A
B. B
C. C
D. D
E. Fail