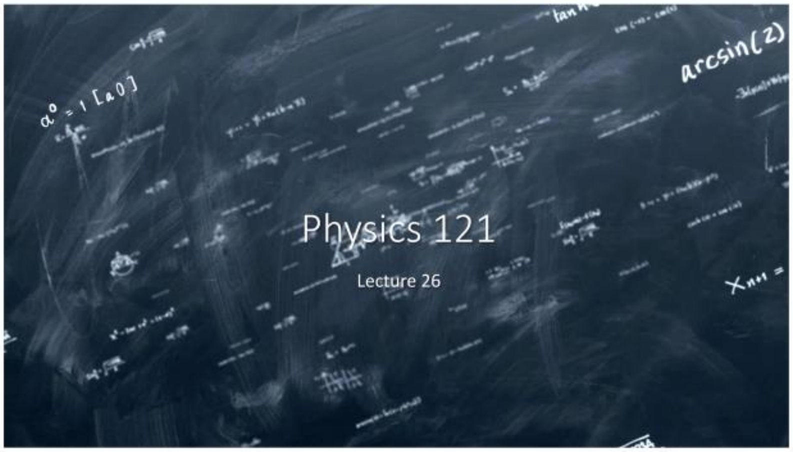
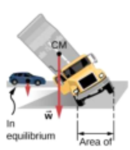
Lecture 26 - 4.11

Tuesday, April 11, 2023 9:31 AM



Review



FIRST EQUILIBRIUM CONDITION

The first equilibrium condition for the static equilibrium of a rigid body expresses *translational* equilibrium:

$$\sum_k \vec{\mathbf{F}}_k = \vec{\mathbf{0}}. \qquad \boxed{12.2}$$

SECOND EQUILIBRIUM CONDITION

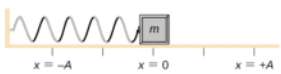
The second equilibrium condition for the static equilibrium of a rigid body expresses *rotational* equilibrium:

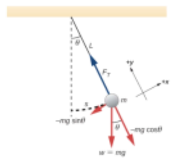
$$\sum_k \vec{\tau}_k = \vec{\mathbf{0}}. \qquad \boxed{12.5}$$

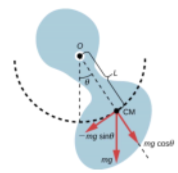
Choose the origins = pivot points most advantageously!

Learning Outcome

Find the Angular Frequency, Period of Motion, and Frequency of the following Oscillators:







Let’s Fill our Toolbox!

What does this Equation describe?

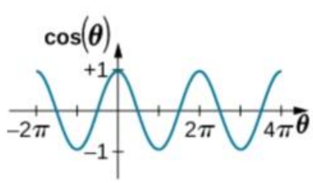
position over time

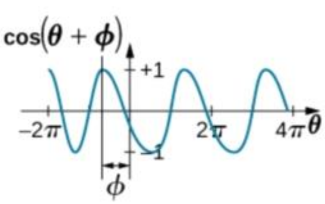
frequency

$$x(t) = A\cos(\omega t + \phi)$$

Amplitude

phase shift





amplitude
starting position
(without this, its just 1, which is why cos(θ) goes up to 1 and down to -1)

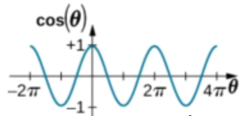
frequency
how long it takes to complete a whole cycle.
the larger this value, the more streched out the wave is

phase-shift
how far shifted left/right

Let's Fill our Toolbox!

What else can we learn from this Equation?

$$x(t) = A\cos(\omega t + \phi)$$



$v(t) = \frac{dx}{dt} = -A\sin(\omega t + \phi)\cdot\omega = -A\omega\sin(\omega t + \phi) \Rightarrow v_{max} = A\cdot\omega$

$a(t) = \frac{dv}{dt} = -A\cos(\omega t + \phi)\cdot\omega^2 = -A\omega^2\cos(\omega t + \phi) \Rightarrow A_{max} = A\omega^2$

Let's Fill our Toolbox!

$x(t) = A\cos(\omega t + \phi)$	$x_{max} = A$
$v(t) = -v_{max}\sin(\omega t + \phi)$	$v_{max} = A\omega$
$a(t) = -a_{max}\cos(\omega t + \phi)$	$a_{max} = A\omega^2.$

$[\omega] = \frac{rad}{s}$

$[\phi] = rad$

$v = \frac{d}{t}$

$\omega = \frac{2\pi}{T}$

Let's Fill Our Toolbox

- We define **periodic motion** to be any motion that repeats itself at regular time intervals. Angular speed (frequency) is cycle / time:

$$\omega = \frac{2\pi}{T} \qquad T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T}.$$

$$1\ \text{Hz} = 1\frac{\text{cycle}}{\text{s}} \quad \text{or} \quad 1\ \text{Hz} = \frac{1}{\text{s}} = 1\ \text{s}^{-1}$$

A cycle is one complete **oscillation**

here T is period, not Tension

frequency is how many times it fulfills an oscillation in one second

Let's Practice!

Given x(t) = 5m * cos(10 rad/s * t + 3), what is the:

Amplitude = 5 m

Let's Practice!
Given x(t) = 5m * cos(10 rad/s * t + 3), what is the:

Angular Frequency $= 10\ \text{rad/s}$

Let's Practice!
Given x(t) = 5m * cos(10 rad/s * t + 3), what is the:

Period

$$T = \frac{2\pi}{\omega} = \frac{2\pi \cdot s}{10\,\text{rad}} \approx 0.6s$$

Let's Practice!
Given x(t) = 5m * cos(10 rad/s * t + 3), what is the:

Frequency

$f = \frac{1}{T} = \frac{1}{0.6} = 1.6667$

Let's Practice!
Given x(t) = 5m * cos(10 rad/s * t + 3), what is the:

Phase constant $= 3$ rad

Let's Practice!
Given x(t) = 5m * cos(10 rad/s * t + 3), what is the:

Position after 1 ms? $1ms = 0.0001 s = \frac{1}{1000} s$

$$x(1ms) = 5m \cdot \cos\left(10 \frac{rad}{s} \cdot \frac{1}{1000} s + 3 rad\right)$$

$$= 5m \cdot \cos\left(\frac{1}{100} + 3\right)$$

Let's Practice!
Given x(t) = 5m * cos(10 rad/s * t + 3), what is the:

Velocity after 1 ms?

$$v(t) = 5m \cdot 10 \frac{rad}{s} \cdot -\sin\left(10 \frac{rad}{s} \cdot \frac{1}{1000} s + 3\right)$$

$$= -50 \cdot \sin(3.54)$$

Let's Practice!

Given x(t) = 5m * cos(10 rad/s * t + 3), what is the:

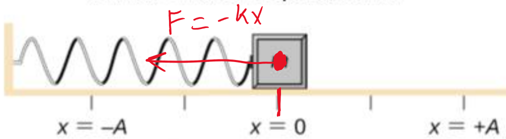
Acceleration after 1 ms?

$$a(1ms) = 5m\left(10\,\frac{rad}{s}\right)^2 \cdot -\cos(3\text{ ish})$$

$$= -500\,\frac{m}{s^2} \cdot \cos(3)$$

Simple Harmonic Oscillator – Meet Differential Equations!

$F = -kx$



$$F_x = -kx$$

$$ma = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$x(t) = A\cos(\omega t + \phi)$$

$$\cancel{-}\cancel{A}\omega^2\cos\cancel{(\omega t + \phi)} = \cancel{-}\frac{k}{m}\cancel{A}\cos\cancel{(\omega t + \phi)}$$

$$\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}} \qquad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\omega^2 = \frac{k}{m}$$

Facts about simple harmonic oscillator

Plugging a Guitar String harder will change the frequency?

- A) True
- B) False
- C) Can't be answered

changes Amplitude, not frequency

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

tension on string, from tuning pegs

mass or thickness of string

Facts about simple harmonic oscillator

Frequency of a SHO depends on:

- ✓ A) Mass
- ✓ B) Force Constant k
- C) Day of the week
- D) Amplitude
- E) More than one of the above

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

Energy in a Simple Harmonic Oscilator

Total Energy = Potential Energy + Kinetic Energy

$$E_{\text{Total}} = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$\begin{aligned} E_{\text{Total}} &= \tfrac{1}{2}kA^2\cos^2(\omega t + \phi) + \tfrac{1}{2}mA^2\omega^2\sin^2(\omega t + \phi) \\ &= \tfrac{1}{2}kA^2\cos^2(\omega t + \phi) + \tfrac{1}{2}mA^2\left(\tfrac{k}{m}\right)\sin^2(\omega t + \phi) \\ &= \tfrac{1}{2}kA^2\cos^2(\omega t + \phi) + \tfrac{1}{2}kA^2\sin^2(\omega t + \phi) \\ &= \tfrac{1}{2}kA^2\left(\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right) \\ &= \underline{\tfrac{1}{2}kA^2}. \end{aligned}$$

Just plug in the x(t) and v(t) equations and do some chugging.

$\omega^2 = \frac{k}{m}$

Use a trig identity!

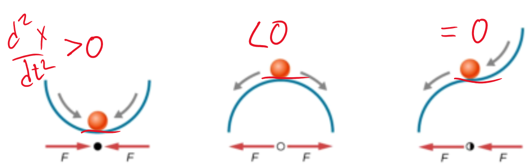
$\cos^2 + \sin^2 = 1$

$$E_{\text{Total}} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

Mix and match

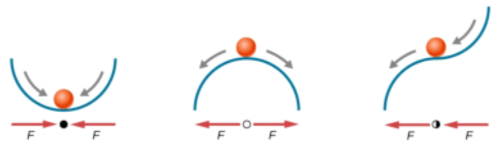
x(t)

$\frac{dx}{dt} = 0$



Stable equilibrium point

Mix and match



Unstable equilibrium point

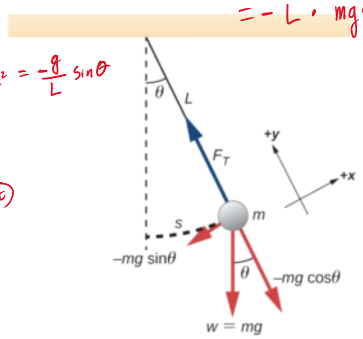
Simple Harmonic Pendulum

$$\tau = L \cdot F_w$$
$$= -L \cdot mg \cdot \sin\theta$$

$$\tau = -L\,(mg \sin\theta)$$

$$I\alpha = \Sigma \tau$$
$$= -L\, mg \sin\theta$$
$$I \cdot \frac{d^2\theta}{dt^2} = \text{''}$$
$$mL^2 \cdot \frac{d^2\theta}{dt} = \text{''}$$

$$\frac{d^2\theta}{dt^2} = \frac{-g}{L} \sin\theta$$



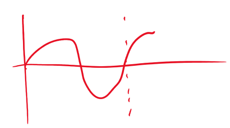
Small Angle Approximation:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta.$$

$$\theta = A\cos(w \cdot t)$$

$$\boxed{\omega = \sqrt{\frac{g}{L}}} \quad \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

$$F_w = -w \cdot \sin\theta$$



Quiz: A simple pendulum has a length of 1m, what is it's frequency? Assume g = 10m/s**2

A) 2s
B)sq 0.5 Hz
C) 3.16 rad / s
D) What is going on here?

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1}{10}}$$

= 2 * pi * sqrt(1/10) = 1.98691765315922
1 / 1.9869 = 0.5033 hz

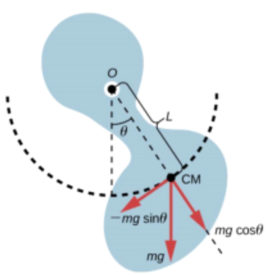
$$\omega = \sqrt{\frac{g}{L}}$$

Physical Harmonic Pendulum

$$I\alpha = \tau_{\text{net}} = L(-mg)\sin\theta.$$

Small Angle Approximation:

$$\begin{aligned} I\alpha &= -L(mg)\theta; \\ I\frac{d^2\theta}{dt^2} &= -L(mg)\theta; \\ \frac{d^2\theta}{dt^2} &= -\left(\frac{mgL}{I}\right)\theta. \end{aligned}$$

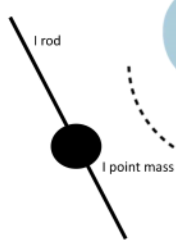


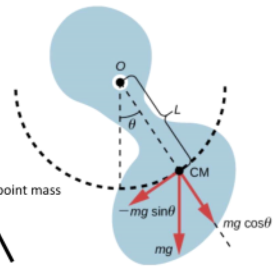
$$\omega = \sqrt{\frac{mgL}{I}}.$$

$$T = 2\pi\sqrt{\frac{I}{mgL}}.$$

Physical Harmonic Pendulum – Homework hint

$$T = 2\pi\sqrt{\frac{I}{mgL}}$$





I is the sum of all the I that make up the pendulum.

$I_{rod} + I_{PM} = I$

L is the distance to Center of mass.

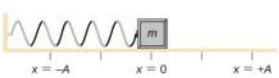
between CM_{rod} and CM_{PM}

m is the total mass of the pendulum.

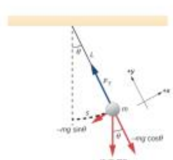
$m_{rod} + m_{pm} = m$

Learning Outcome

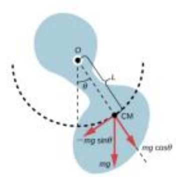
Find the Angular Frequency, Period of Motion, and Frequency of the following Oscillators:



$\omega = \sqrt{\frac{k}{m}}$ $T = 2\pi\sqrt{\frac{m}{k}}$



$\omega = \sqrt{\frac{g}{L}}$ $T = 2\pi\sqrt{\frac{L}{g}}$



$\omega = \sqrt{\frac{mgL}{I}}$ $T = 2\pi\sqrt{\frac{I}{mgL}}$

Exit Poll

- Please provide a letter grade for todays lecture:

A. A
B. B
C. C
D. D
E. Fail