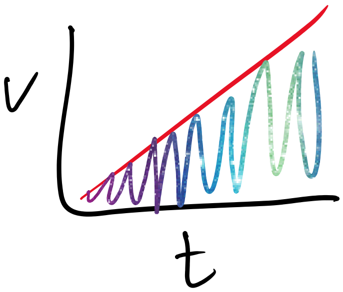
Lecture 2 - 1.12

Thursday, January 12, 2023 10:12 AM

Fundamental Theorem of Calculus



The slope of this line (derivative) is the Acceleration.

The area under this line (integral) is the change of position, or distance travelled.

$v(t) = 10t$

therefore

$a(t) = 10$

The distance covered in 6 seconds is the area under the graph between $0 < t < 6$

6s & 10m/s

$v = \frac{s}{t}$ $\quad s = v \cdot t$

$t = 6s$

$s = \frac{10m}{\cancel{s}} \cdot 6\cancel{s} = 60m$

$$x(t) = \int v(t) \cdot dt$$

$\downarrow\uparrow$

$$\frac{d\,x(t)}{dt} = v(t)$$

Fundamental Theory of Calculus says that you can manipulate these derivatives just like any other algebra

$\cdot dt \qquad \cdot dt$

$$x(t) = \int v(t) \cdot dt$$

The root cause of any movement is acceleration.

Newton's First law says that it will stay moving at the same speed/direction , until more acceleration is applied (change speed or direction)

1. If we know current position and velocity, the acceleration can tell us exactly how it will behave in the future
2. The area under the acceleration curve is velocity
3. The area under velocity curve is position

Kinematic Equations

constant

$$a(t) = a$$

indefinite integral requires constant

initial velocity

$$v(t) = \int a \cdot dt = \boxed{a \cdot t + v_0} \quad \#1$$

$$x(t) = \int v(t) \cdot dt = \int [a \cdot t + v_0] \, dt = \boxed{\frac{1}{2} a \cdot t^2 + v_0 \cdot t + x_0} \quad \#2$$

$v_0 = 0 \qquad a = 5\ m/s^2 \qquad x_0 = 0$

$$x(2) = ?$$

$$= \frac{1}{2}(5)(2)^2 + \cancel{0(2) + 0}$$

$= \frac{1}{2}(5)(2)^2 \cancel{+ 0(2)} \cancel{+0}$

$= \frac{1}{2} \cdot 5 \cdot 4$

$= \frac{1}{2} \cdot 20 = \boxed{10m}$

Velocity (w/o time)

#3

$v^2 = v_0^2 + 2a(x - x_0)$

constant a

$t = 10 \qquad a = 10\ m/s^2 \qquad x_0 = 0 \qquad v_0 = 0$

$X(t) = \frac{1}{2} a \cdot t^2 = \frac{1}{2}(10) \cdot (10)^2$

$= \frac{1}{2} 10 \cdot 100$

$= \frac{1}{2} 1000$

$\boxed{= 500m}$