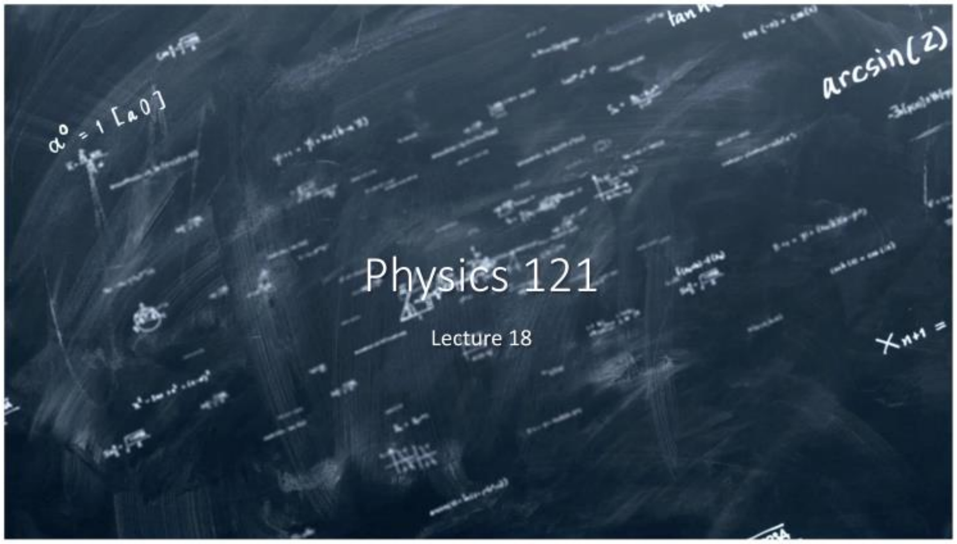
Lecture 18 - 3.14

Wednesday, March 15, 2023 9:51 AM



Not ready to give up, next goal:
General Conference attendance – or bust!

Hi Eric, I thought you would like to know that my ~500 students at BYU have prayed for you and your family since we met. We all wish you happiness and lasting joy. Hope you have a great weekend and will be able to join your ward for sacrament tomorrow! Best wishes, Dennis

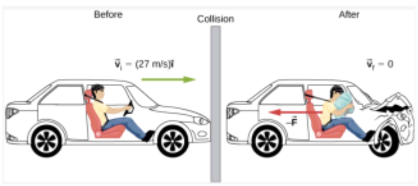
Review – Lecture 17

Momentum $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

LAW OF CONSERVATION OF MOMENTUM

The total momentum of a closed system is conserved:

$$\sum_{j=1}^{N} \vec{\mathbf{p}}_j = \text{constant.}$$



Two kinds of collisions

ELASTIC
- Objects collide and bounce
- Kinetic energy is conserved
- No permanent deformation of the objects

INELASTIC
- Objects collide and stick
- Kinetic energy is NOT conserved
- Objects stick because they lock together, or are permanently bent, or chemical reaction, or…

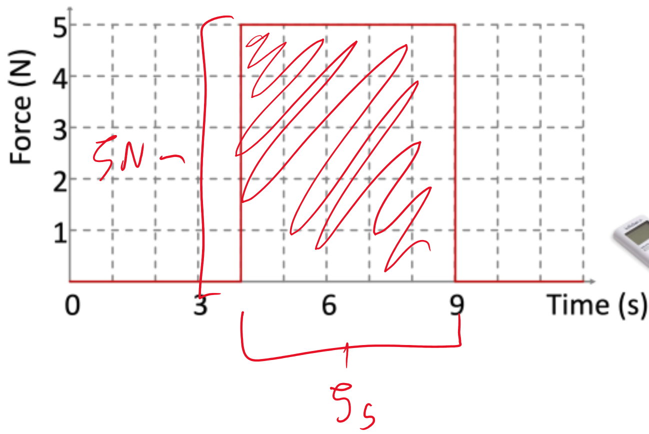
"Impulse"

$$\Delta p = (F) \times (\Delta t) = p_{\text{final}} - p_{\text{initial}}$$

What is the impulse for this interaction?

$$\Delta \mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \mathbf{F} \times \Delta t$$

A) 10 Ns
B) 25 Ns
C) 50 Ns
D) 100 Ns
E) No impulse, only force!



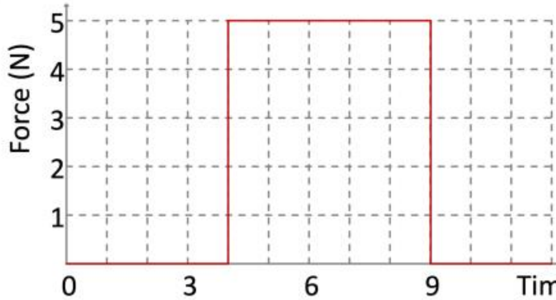
$5s \cdot 5N = 25\ Ns$

iClicker: Is the change in velocity positive or negative?

A: positive B: negative C: cannot tell

$\Delta \mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \mathbf{F} \times \Delta t$ $= m \cdot v_f - m \cdot v_i = 25\ Ns$

$v_f > v_i$
positive



iClicker: If the mass is 7 kg, and the initial velocity is -3 m/s, what is the final velocity? (choose the most appropriate range)

A: between -3 and 0 m/s B: between 0 and 3 m/s C: between 3 and 10 m/s

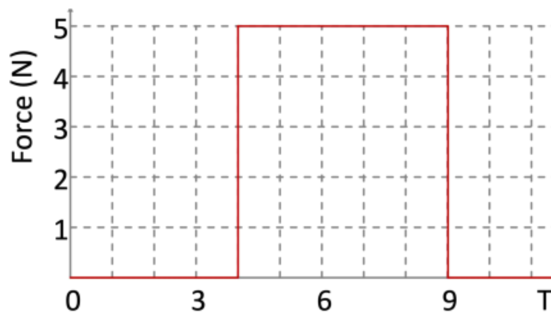
$\Delta \mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \mathbf{F} \times \Delta t$

$25\ Ns = 7\ kg \cdot v_f - 7\ kg \cdot (-3\ m/s)$

$25\ Ns = 7 v_f + 21\ Ns$

$4\ Ns = 7 v_f$

$\frac{4\ Ns}{7\ kg} = v_f = 0.6\ m/s$



Elastic collisions between two point particles

If the initial masses and velocities are known, what are the final velocities?

m_1 $\vec{v}_1$ m_2 $\vec{v}_2$

$v_{1f} = ?$ $v_{2f} = ?$

Conservation of Momentum

$$m_1 \cdot v_1 + m_2 \cdot v_2 = m_1 \cdot v_{1f} + m_2 \cdot v_{2f}$$

Conservation of Kinetic Energy

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Elastic collisions between two point particles

If the initial masses and velocities are known, what are the final velocities?

m_1 $\vec{v}_1$ m_2 $\vec{v}_2$

$\eta = \frac{m_2}{m_1}$

$v_{1f} = ?$ $v_{2f} = ?$

Conservation of Momentum

$$\cancel{m_1} v_1 + \underset{m_1}{m_2} v_2 = \cancel{m_1} v_{1f} + \underset{m_1}{m_2} v_{2f}$$

$$v_1 + \eta v_2 = v_{1f} + \eta v_{2f}$$

$$\boxed{\underbrace{v_1 - v_{1f}} = \eta(v_{2f} - v_2)}$$

Conservation of Kinetic Energy

$$\cancel{\tfrac{1}{2}} \cancel{m_1} v_1^2 + \cancel{\tfrac{1}{2}} \underset{m_1}{m_2} v_2^2 = \cancel{\tfrac{1}{2}} \cancel{m_1} v_{1f}^2 + \cancel{\tfrac{1}{2}} \underset{m_1}{m_2} v_{2f}^2$$

$$v_1^2 + \eta v_2^2 = v_{1f}^2 + \eta v_{2f}^2$$

$$v_1^2 - v_{1f}^2 = \eta(v_{2f}^2 - v_2^2)$$

$a^2 - b^2 = (a+b)(a-b)$

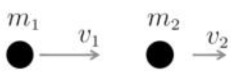
$$(v_1 - v_{1f})(v_1 + v_{1f}) = \eta(v_{2f} - v_2)(v_{2f} + v_2)$$

$$\boxed{v_1 + v_{1f} = v_{2f} + v_2}$$

these are equal, so they cancel out.

Elastic collisions between two point particles

If the initial masses and velocities are known, what are the final velocities?



$$\eta = \frac{m_2}{m_1} \qquad v_{1f} = ? \qquad v_{2f} = ?$$

Conservation of Momentum

$$v_1 - v_{1f} = \eta(v_{2f} - v_2)$$

Conservation of Kinetic Energy

$$v_1 + v_{1f} = v_{2f} + v_2$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

Elastic Collisions in 1D

Ball 1 has a velocity of 10 m / s.

Ball 2 is at rest. Both have the same mass.

After a head on collision, what are the final velocities of both balls?

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V_i

V_0

$$\left.\begin{array}{l} V_{1f} = V_2 \\ V_{2f} = V_1 \end{array}\right\}$$

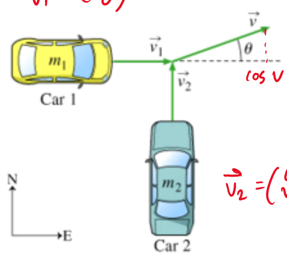
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

Inelastic Crashes in 2 Dimensions

$\vec{v}_1 = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$

Two cars collide at a right angle. What is the angle theta in terms of m1, m2, v1, and v2?



$\vec{v}_2 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$

$$x: \quad m_1 \vec{v}_1 = (m_1 + m_2) \cos\theta \cdot v_f$$

$$y: \quad m_2 \vec{v}_2 = (m_1 + m_2) \sin\theta \cdot v_f$$

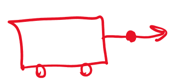
$$\frac{y}{x}: \quad \frac{m_2 v_2}{m_1 v_1} = \tan\theta$$

$$\Rightarrow \tan^{-1}\left(\frac{m_2 v_2}{m_1 v_1}\right) = \theta$$

Rubber bugs and your windshield

Q3: A 3 gram grasshopper bounces elastically off your car's windshield. The grasshopper was initially at rest and your car (1000 kg) was initially moving 25 m/s. What is the final speed of the grasshopper?

A) 25 m/s
B) 50 m/s
C) 100 m/s
D) 150 m/s
E) 200 m/s



$$m_c \cdot v_c + \cancel{m_b \cdot v_b}^{\,0} = m_{cf} \cdot v_{cf} + m_{bf} \cdot v_{bf}$$

$$\cancel{\tfrac{1}{2}} m_c v_c^2 = \cancel{\tfrac{1}{2}} m_c v_{cf}^2 + \cancel{\tfrac{1}{2}} m_b v_{bf}^2$$

$$v_{2f} = \frac{2m_c}{m_c + m_b} \cdot v_c + \cancel{\frac{m_b - m_c}{m_c + m_b} \cdot v_b}^{\,0}$$

$$= \frac{2 \cdot 1000\,\text{kg}}{1000.003\,\text{kg}} \cdot 25\,\text{m/s}$$

$$\approx 2 \cdot 25\,\text{m/s} = 50\,\text{m/s}$$

In two dimensions…

Conserve momentum in x and y separately

Two cars on a frictionless surface collide and lock together. Car 1, mass = 1500 kg, was initially traveling east at 10 m/s. Car 2, mass 1000 kg, was initially traveling north at 6 m/s.

What fraction of the initial kinetic energy is lost in this collision?

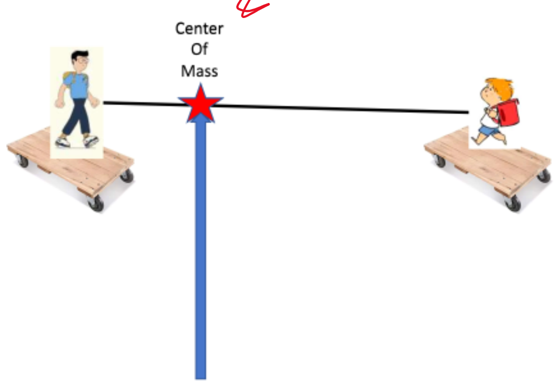
A. About 10%
B. About 22%
C. About 44%
D. About 55%
E. About 90%

Center of Mass Demo

$$x_{cm} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

x_1 m_1

x_2 m_2



The center of mass

If you have a bunch of point masses:

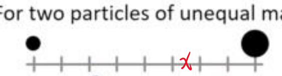
$$x_{CM} = \frac{\sum_i x_i m_i}{\sum_i m_i}$$

For two particles of equal mass:

$x = 0$ $\quad$ $x = 8$

$$x_{CM} = \frac{0 \times m + 8 \times m}{m + m} = 4$$

For two particles of unequal mass



$x_1 = 0$ $\quad$ $x_2 = 8$

$m_1 = 1$ $\quad$ $m_2 = 2$

$$x_{CM} = \frac{0 \times 1 + 8 \times 2}{1 + 2} = \frac{16}{3}$$

If the particles are moving, the center of mass can also move

If you have a bunch of point masses:

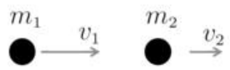
$$v_{CM} = \frac{\sum_i v_i m_i}{\sum_i m_i}$$

$$x_{cm} = \frac{\sum x_i m_i}{\sum m_i}$$

$$\frac{dx_{cm}}{dt} = \frac{\sum \frac{dx_i}{dt} m_i}{\sum m_i} = \frac{\sum v_i m_i}{\sum m_i} = V_{cm}$$

Elastic collisions between two point particles

If the initial masses and velocities are known, what are the final velocities?



$$\eta = \frac{m_2}{m_1} \qquad v_{1f} = ? \quad v_{2f} = ?$$

Conservation of Momentum

$$v_1 - v_{1f} = \eta(v_{2f} - v_2)$$

Conservation of Kinetic Energy

$$v_1 + v_{1f} = v_{2f} + v_2$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

Rewriting our equations

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

Put everything over a common denominator

$$v_{1f} = \frac{m_1 v_1 - m_2 v_1 + 2m_2 v_2}{m_1 + m_2}$$

Add and subtract $2m_1v_1$ (like adding zero)

$$v_{1f} = \frac{-m_1 v_1 - m_2 v_1 + 2m_2 v_2 + 2m_1 v_1}{m_1 + m_2}$$

Group terms creatively

$$v_{1f} = -\cancel{\frac{m_1 + m_2}{m_1 + m_2}} v_1 + 2\frac{m_2 v_2 + m_1 v_1}{m_1 + m_2}$$

Vcm

Simplify using the definition of v_{CM}

$$v_{1f} = 2v_{CM} - v_1$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

$$v_{2f} = \frac{2m_1 v_1 + m_2 v_2 - m_1 v_2}{m_1 + m_2}$$

$$v_{2f} = \frac{2m_1 v_1 + 2m_2 v_2 - m_1 v_2 - m_2 v_2}{m_1 + m_2}$$

$$v_{2f} = 2\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} - \frac{m_1 + m_2}{m_1 + m_2} v_2$$

$$v_{2f} = 2v_{CM} - v_2$$

What are implications of: $v_{1f} = 2v_{CM} - v_1$

A) If center of mass does not move, the final velocity will have same magnitude but opposite direction from initial velocity. $V_{1f} = -V_1$
B) Without initial velocity and a static center of mass, it is impossible to move.
C) It suffices to know the velocity of the center of mass and initial velocity to calculate final velocity of any object in elastic collisions.
D) We should not use this equation for inelastic collisions.
E) All of the above.

Discussion: Given all this, how could a trapped astronaut get unstuck?

$v_{1f} = 2v_{CM} - v_1$

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Exit Poll

- Please provide a letter grade for todays lecture:

A. A
B. B
C. C
D. D
E. Fail