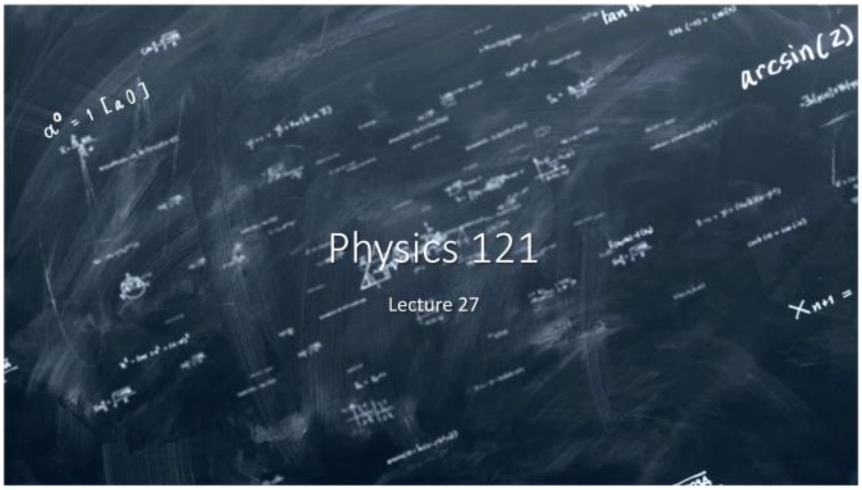
Lecture 27 - 4.13

Thursday, April 13, 2023 9:28 AM



Devotional Thought

Technologies change our lives circumstances, but spiritual goals remain constant.

Cast away every feeling of superiority or guilt.

- I know better
- I am not good enough

→ Just obey and be happy!

$T = \frac{2\pi}{\omega}$ $\quad f = \frac{1}{T}$

Review

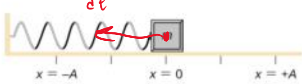
General equation for Oscillation

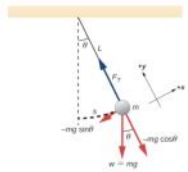
$$x(t) = A\cos(\omega t + \phi)$$

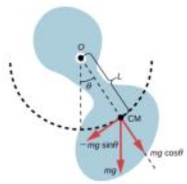
Oscillatory motion has changing acceleration – welcome Differential Equations!

$-k\,x = m \cdot a \;;\; a = \frac{d^2x}{dt^2}$

$\frac{d^2x}{dt^2} = \frac{-k}{m}x$







$\omega = \sqrt{\frac{k}{m}}$ $\quad T = 2\pi\sqrt{\frac{m}{k}}$

$\omega = \sqrt{\frac{g}{L}}$ $\quad T = 2\pi\sqrt{\frac{L}{g}}$

$\omega = \sqrt{\frac{mgL}{I}}$ $\quad T = 2\pi\sqrt{\frac{I}{mgL}}$

HW 14 has 2 important problems
oscillation problem
planetary motion problem

Generalizing Gravity

NEWTON'S LAW OF GRAVITATION

Newton's law of gravitation can be expressed as

$$\vec{\mathbf{F}}_{12} = G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{12} \quad (13.1)$$

gravitational Force

where $\vec{\mathbf{F}}_{12}$ is the force on object 1 exerted by object 2 and $\hat{\mathbf{r}}_{12}$ is a unit vector that points from object 1 toward object 2.

$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$ ← Universal gravitational Constant "Big 6"

$$\vec{\mathbf{F}}_{12} = G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{12}$$

Connection to g:

F = m * a = m * g

$$g = G\frac{M_E}{r^2}$$

Plugging in Earth's m & r, we get 9.801 m/s².

$$\cancel{\vec{F}}_{1,2} = G\,\frac{m_1 \cdot m_2}{r^2} \cdot \cancel{\hat{r}_{1,2}}$$

An 80 kg guy sits 0.5 m away from his 60 kg date in a movie theater. What is the gravitational attractive force between them?

$$G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$$

80kg ● ——— 0.5m ——— ● 60kg

$$= \frac{6.67 \cdot 10^{-11} \cdot 80 \cdot 60}{0.5^2}$$

$$= 6.67 \cdot 10^{-11} \cdot \underbrace{4800 \cdot 4}_{\sim 20{,}000} \approx 13 \cdot 10^{-7}\ N$$

Quiz: How much does g change between sea level and an airplane at 3km height?

A) ~0.1 %
B) ~1 %
C) ~10 %
D) ~100 %
E) This is way too hard of a problem to solve rapidly…

$$g = G\frac{M_E}{r^2}$$

$$g_{Air} = \frac{G\,\frac{M_E}{(r_E+3)^2}}{\cancel{G}\,\frac{\cancel{M_E}}{r_E^2}} = \frac{(r_E+3)^2}{r_E^2}$$

g_E

g_A/g_E

Let's practice!

Scientists want to send a space probe to orbit Mars. They want the probe to orbit at an elevation of 4x the planet's radius above the surface. $M_{Mars} = 0.107 \times M_E = 6.4e23$ kg, $R_{Mars} = 0.53 \times R_E = 3395000$ m.

What is the gravitational acceleration on the surface of Mars?

.38 * 9.801 = 3.7244

$$g_M = G \cdot \frac{M_M}{R_M} = G \cdot \frac{0.107\, M_E}{(0.53 R_E)^2}$$

$$= \frac{0.107}{0.53^2} \cdot \underbrace{\left(G \frac{M_E}{R_E^2} \right)}_{9.801} = 0.38 \cdot g = 0.38 \cdot 9.801$$

$$= \boxed{3.7244}$$

$R_{Total} = R_E + h$

a_c must be gravitational pull by Earth, or g_s

Calculate Period of a Satellite in Circular Orbit

A satellite is orbiting around the Earth at an elevation $h = 1.04e4$ km (above the surface of the Earth). The mass of the satellite is 525 kg, the mass of the Earth is 5.98×10^{24} kg, the radius of the Earth is 6370 km, and the universal gravitational constant is 6.67×10^{-11} N·m^2/kg^2.

$$\underset{\text{period}}{T} = \frac{2\pi R_S}{V_t}$$

$$a = g_s = G \cdot \frac{M_E}{R_S^2} = a_c$$

$$a_c = \frac{V_t^2}{R_S}$$

$$G \cdot \frac{M_E}{R_S^2} = \frac{V_t^2}{R_S}$$

Calculate Escape Velocity

$$\vec{\mathbf{F}}_{12} = G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{12}$$

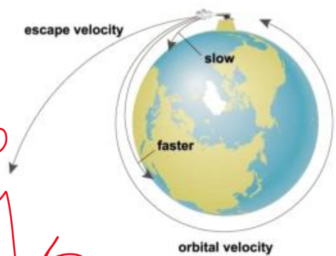
$$U = -\frac{GM_E m}{r}.$$

$$-\frac{dU}{dr} = F \qquad \cancel{U = m \cdot g \cdot h}$$

only works at Earth's surface w/ constant g

Strategy:

1. Conversation of Energy
2. Use correct potential Energy
3. Solve for needed velocity



$$U_1 + K_1 = U_2 + K_2 = 0$$

$$-\frac{2 G \cdot M_E \cdot \cancel{m}}{r} + \frac{1}{2}\cancel{m} v_1^2 = 0$$

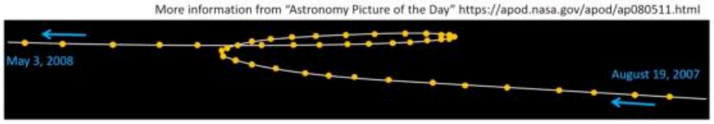
$$\boxed{v_1 = \sqrt{\frac{2 \cdot G \cdot M_E}{r}}}$$

escape velocity

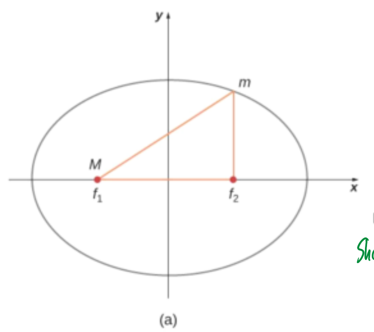
In the late 1500s, Mars had a problem

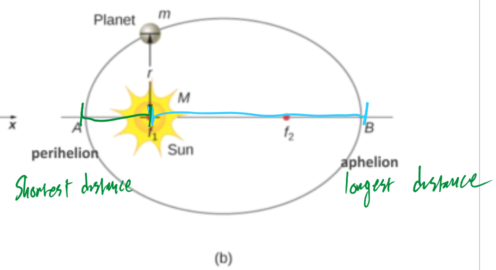
- Johannes Kepler went to work for Tyco Brahe measuring the orbit of Mars.
- Mars sometimes moved backwards in the sky!
- This is because Earth "passes" Mars on an inside track.
- Johannes Kepler wanted all the planets to have circular orbits.
- Mother Nature had a different opinion.

More information from "Astronomy Picture of the Day" https://apod.nasa.gov/apod/ap080511.html



Kepler's 1st Law: Planets orbit on Ellipses





Vocabulary

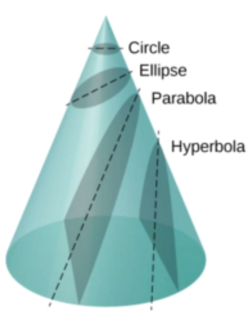
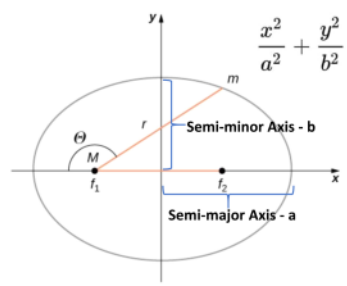


Figure 13.18 All motion caused by an inverse square force is one of the four conic sections and is determined by the energy and direction of the moving body.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

e is called the eccentricity.

how Circular or stretched it is,

If *e = 0* → *circle*

If e ~ 1 → *Very oblong ellipse*

Constants α and *e* are determined by the total energy and angular momentum of the satellite

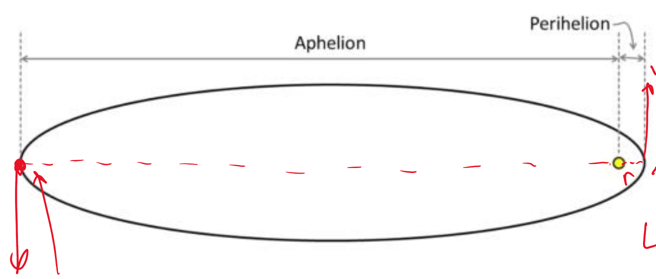
Quiz: What must be true for *e = 1?*

A) Semi-major axis is a lot bigger than semi-minor axis
B) Semi-minor axis is a lot bigger than semi-major axis
C) One of the two axis is close to 0
D) All of the above
E) None of the above

irrelevant for Final

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Homework hint: Applying conservation of angular momentum.



$L_p = r \times p$

$L = r \cdot p$
$= r \cdot m \cdot v$

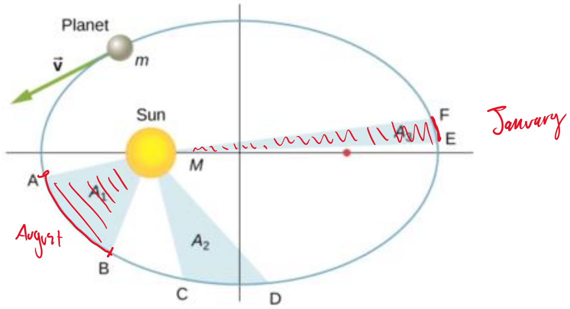
$L_A = r \cdot m \cdot V$

$L_A = L_P$

$r_A \cdot \cancel{m} \cdot v_A = r_p \cdot \cancel{m} \cdot v_p$

$r_A \cdot v_A = r_p \cdot v_p$

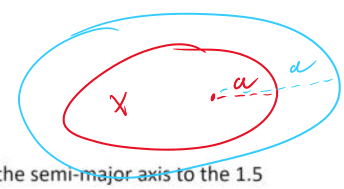
Kepler's 2nd Law



A planet sweeps out equal areas in equal times.

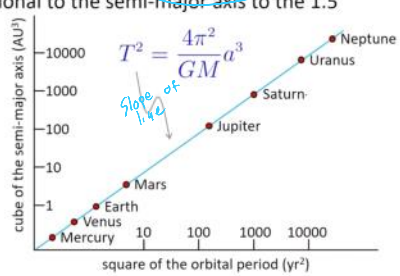
Distance is different, but Area is the same

Kepler's 3rd law



- The orbital period is proportional to the semi-major axis to the 1.5 power

$$T^2 = \frac{4\pi^2}{GM}a^3$$



See an example at http://hyperphysics.phy-astr.gsu.edu/hbase/kepler.html

Not super necessary, easier ways to solve same thing

The moon orbits the earth in roughly a circle with an orbital radius of $r = 385{,}000$ km and an orbital period of 27.3 days. Use this information to determine the mass of the earth.

$$G = 6.67 \times 10^{-11}\ \mathrm{N\ m^2/kg^2}$$

T

$$a_c = G\frac{m_E}{R^2} = \frac{v_T^2}{R}$$

$$T = \frac{2\pi \cdot R}{v} \Rightarrow v = \frac{2\pi R}{T}$$

$$G \cdot \frac{m_E}{R^2} = \frac{1}{\cancel{R}} \cdot \frac{2^2\pi^2 R^{\cancel{2}}}{T^2}$$

$$m_E = \frac{R^3 \cdot 4\pi^2}{G \cdot T^2}$$

Homework hint: Binary Star problem

14-7 A binary star system consists of two equal mass stars that revolve in circular orbits about their center of mass. The period of the motion, $\boxed{T = 15.3 \text{ days}}$ and the orbital speed $v = 220$ km/s of the stars can be measured from telescopic observations. What is the mass of each star?

5 pts

$$a_c = g$$

$$\frac{v_T^2}{r} = G \cdot \frac{m}{(2r)^2}$$

$$T = \frac{2\pi r}{v} \Rightarrow r = \frac{T \cdot v}{2\pi}$$

THAT'S IT!

Exit Poll

- Please provide a letter grade for todays lecture:

A. A
B. B
C. C
D. D
E. Fail