Lecture 22 - 3.28

Tuesday, March 28, 2023 9:33 AM

The moment a force is applied to anywhere that isn't the center of mass, it will be rotated.

The amount of rotation depends on how far away it is from the center of mass. The further away, the more rotation.

Force multiplied by distance from center of mass

aka <u>Torque</u>

Causes change in angular velocity -> Like Force in linear motion

- Distance from hinge
- Angle of push
- Strength of push

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}.$$

$$|\vec{\tau}| = \left|\vec{\mathbf{r}} \times \vec{\mathbf{F}}\right| = rF\sin\theta$$

Cross Product:

magnitude:

$$|r| \cdot |F| \cdot \sin\theta$$

direction:

right hand rule

iclicker

torque caused by F_3?

0 since it goes through center of mass

torque caused by F_2?

0.5 * 30 = 15 Nm

negative since it is applied right on the edge at a perfectly 90 degree angle, forcing it to go clockwise.

<u>-15 Nm</u>

according to the right hand rule
counter-clockwise is positive
clockwise is negative

torque caused by F_1?

magnitude is 0.5 * 20 * sin(30) = 5.0 Nm

and it pushed counter-clockwise, so it's positive

<u>5.0 Nm</u>

What is the net torque?

-15 + 5 = <u>-10 Nm</u>

Acceleration in Rotational Systems

Uniform Circular Motion has centripetal acceleration

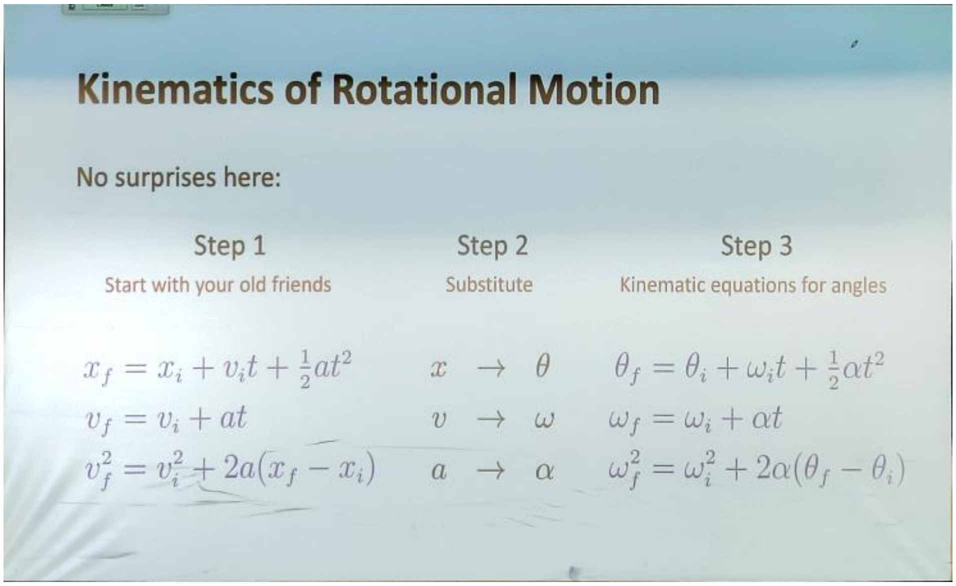
Accelerated Circular Motion

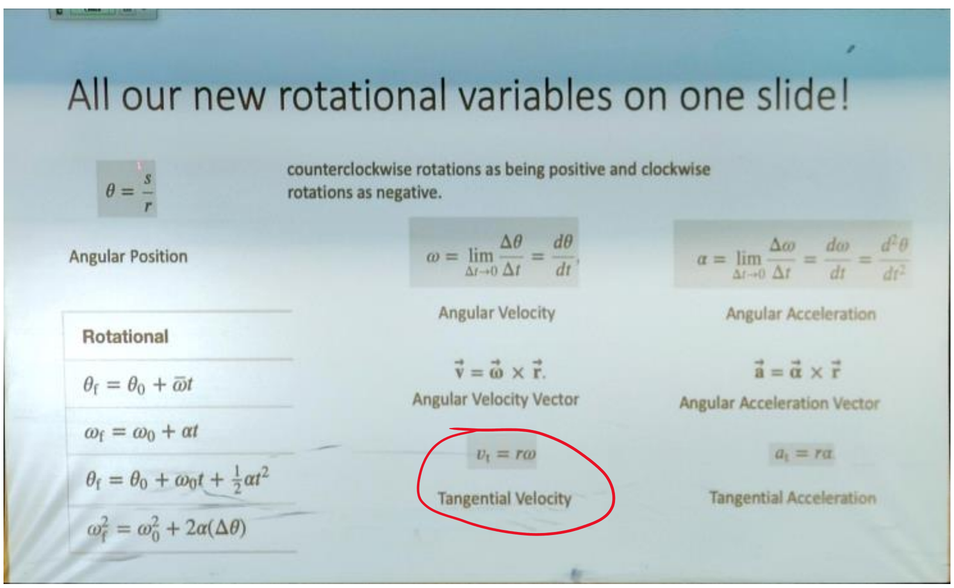
there is a tangentional acceleration that is speeding up the rotation

Combined Acceleration

Add the two acceleration vectors (tangent and centripetal) to get the total/combined acceleration

Kinematics of Rotational Motion





Newton's 2nd Law for Rotations

$$\Sigma \vec{F} = m \cdot \vec{a}$$

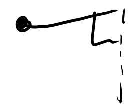
$$a = r \cdot \alpha$$ ← alpha

$$F = m \cdot r \cdot \alpha$$

$$r \cdot F = m \cdot r^2 \cdot \alpha$$ ← $\boxed{I = m \cdot r^2 \text{ for a point-mass}}$

$$\tau = I \cdot \alpha$$

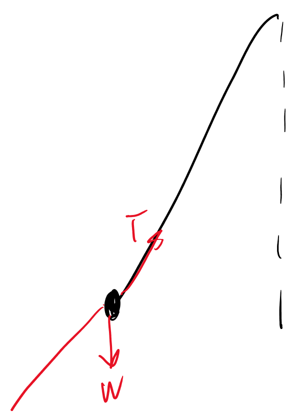
Example at 90^0



$$T = m\cdot g\cdot L = 0.75\,kg\cdot \frac{10\,m}{s^2}\cdot 1.25\,m$$

$$\approx 10\ Nm$$

@ 30^0

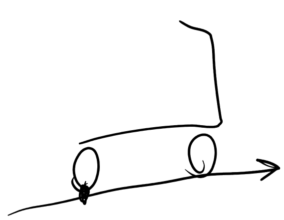


tension doesn't add any torque, since it's at 0 or 180 degrees, and sin(0) and sin(180) are both 0.

$$\sum T = T_W + T_T = W\cdot \sin\theta\cdot L + 0$$

$$= 5\,Nm$$

Homework Hints



$$f = m\cdot a = m\cdot \alpha\cdot R$$

$$m = \frac{f}{\alpha\cdot R}$$

Total Moment of Intertia is just all the individual Intertia's added together.

Key Takeaways

Torque equals moment arm times Force $\tau = r\vec{F}\sin\theta$

The direction of Torque is given by the right hand rule (using the cross product) $\vec{\tau} = \vec{r} \times \vec{F}$

Unbalanced torques cause angular accelerations $\tau = I\alpha$