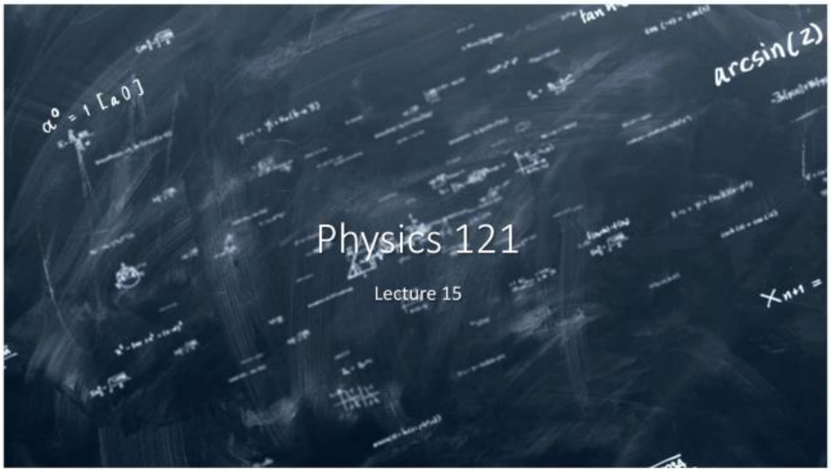
Lecture 15 - 3.2

Thursday, March 2, 2023 9:10 AM



Forumtional Thought

We must reconcile the apparent paradox that we are both controlled by our circumstances and able to control our circumstances.

Angela L. Duckworth

"You could argue that the moral of my mom's story is this: You can choose to change your circumstances."

Dealing with contradictions and paradoxes

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QM and **GR** completely **disagree**. Despite decades of the best minds in science working on this problem, there is no solution in sight. It is one (of many) significant scientific problems (or research areas).

In spite of this conflict, no one in the science community is going "inactive" over this issue. We press forward in faith, eagerly pursuing missing knowledge.

Review

Work is defined as the energy transferred to or from an object.

For changing Forces and crooked paths: $W = \int_a^b \mathbf{F}(\mathbf{s}) \cdot d\mathbf{s}$ For constant Forces and straight paths: $W = Fs.$

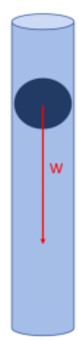
We call the ability of an object to do work energy!

Power = Work / Time

- We talked about kinetic energy and gravitational potential energy.

$$K = \frac{1}{2}mv^2 \qquad U_{\text{grav}} = mgh$$

Total Energy is conserved!



Conservative Forces = Force that conserves total energy. Doesn't matter the path it will take

If the work done by a force is independent of path → conservative!

Examples:

- gravity is conservative
- friction is not conservative (longer path = more heat lost)

Forces can be tested mathematically, by checking if their work done are "exact differentials". This takes multivariate calculus, so don't bother here.

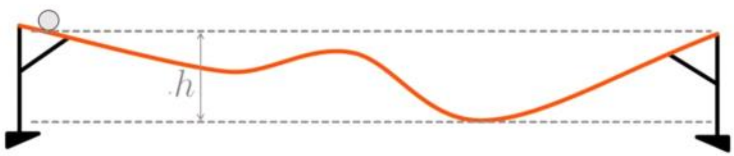
Energy Skate Park – Energy is conserved!

https://phet.colorado.edu/sims/html/energy-skate-park-basics/latest/energy-skate-park-basics_en.html

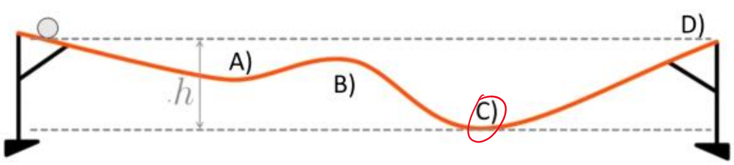
When no friction is present, the sum of K and U_{grav} is constant

Let's imagine a ball rolling back and forth on a frictionless track

- Initially the ball has its maximum U_{grav} but no kinetic energy
- As the ball rolls, it exchanges U_{grav} for K sometimes, and K for U_{grav} at other times.

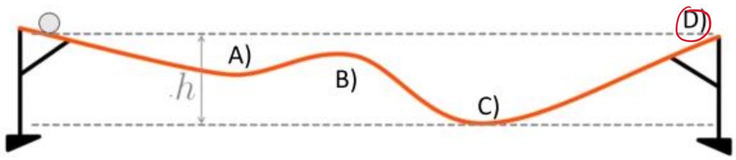


Where is the ball moving the fastest?



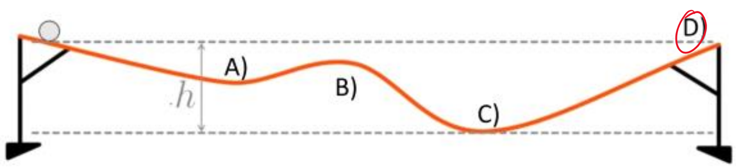
C is where all E_{pot} is converted into Kinetic.

Where is the ball moving the slowest?



D is where all K is converted back into Vg / E pot

Where is the ball turn around?

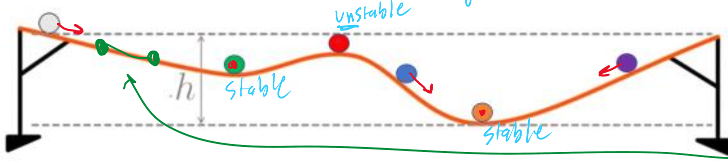


D is where Vg starts converting back into K

Connecting Energy with Forces

Discussion: Consider the 6 balls drawn below at rest.
What do you expect them to do when released?
Why would they do that?

unstable - any tiny force will set it in motion



W and N are unbalanced, so the weight pulls it in the direction of misbalance

Think of this image as a graph of Potential Energy

Potential Decreases and K increased the same amount

$W = F \cdot d \quad = m \cdot g \cdot h \qquad F = \frac{m \cdot g \cdot h}{d} = \frac{E_{pot}}{d}$

$$F = \frac{-\Delta E_{pot}}{\Delta x} = \frac{-d\, E_{pot}}{d\, x}$$

Super cool, important, and awesome:

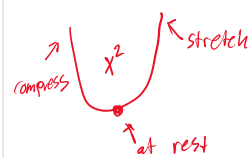
The force in a given direction!

The potential or forcefield!

$$F_x = -\frac{dU}{dx}$$

The change in position (gradient)!

U can be any potential, spring, gravity, etc.

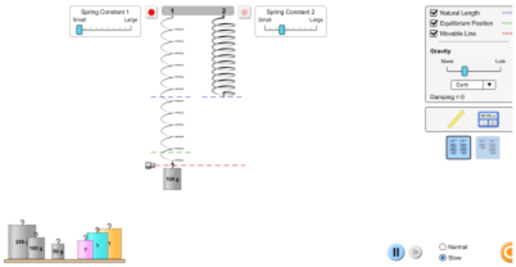


$U = x^2$

$F = \frac{dU}{dx} = 2x$

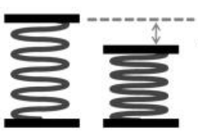
Spring Constant = K

Fun with Springs



https://phet.colorado.edu/sims/html/masses-and-springs/latest/masses-and-springs_en.html

Math with Springs



$$\Delta x = x - x_0$$

$$F_{\text{spring}} = -k(x - x_0)$$ Hooke's Law

$$F_x = -\frac{dU}{dx}$$

$$U_{\text{spring}} = \tfrac{1}{2}k(x - x_0)^2$$

$dU = -F_x dx$

$\int dU = \int -F_x dx$

$U = \frac{1}{2} k(x - x_0)^2$

Bungee jumping

https://www.youtube.com/watch?v=kJ-slNvmFYA#t=3m5s

iClicker

A bungee cord hanging down from a cliff is 50 meters long. If a person grabs hold of the end, the cord stretches to 80 meters. What is the spring constant of the bungee cord? The person has a mass of 81 kg.

A. 1.6 N/m
B. 2.7 N/m
C. 10 N/m
D. 16 N/m
E. 27 N/m

$F = ma \quad = -k(x - x_0)$

$F = 81_{kg} \cdot 10_{m/s^2} = k(80_m - 50_m)$

$F = 810N \quad = -k(30_m)$

$F = \frac{810N}{30m} = k = 27\ N/m$

$T = -kx = W = m \cdot g$

$x = 30$

$W = -m \cdot g$

$m \cdot g = -kx$

$\Rightarrow \quad k = \frac{+m \cdot g}{30} = \frac{81 \cdot 10}{30} = 27_{N/m}$

$E_T = m \cdot g \cdot \Delta H$

$E_T = \frac{-1}{2} k \Delta H^2$

$m \cdot g \cdot \cancel{\Delta H} = \frac{-1}{2} k \Delta H^{\cancel{2}}$

$k = \frac{2mg}{\Delta H}$

this won't happen if forces are conserved

One more comment about potential energy

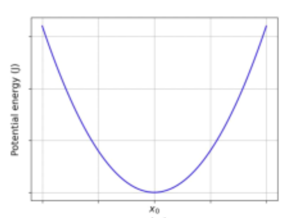
- The potential energy for the spring is parabolic, concave up.

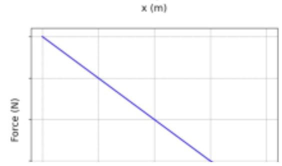
$$U_{\text{spring}} = \frac{1}{2}k(x - x_0)^2$$

- The force that the spring exerts is linear in the displacement

$$F_{\text{spring}} = -k(x - x_0)$$

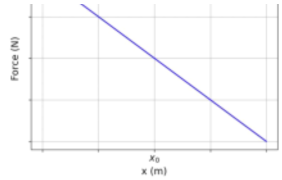
- Wherever the particle goes, the force on it always pushes the particle back towards the





$$F_{\text{spring}} = -k(x - x_0)$$

- Wherever the particle goes, the force on it always pushes the particle back towards the origin.

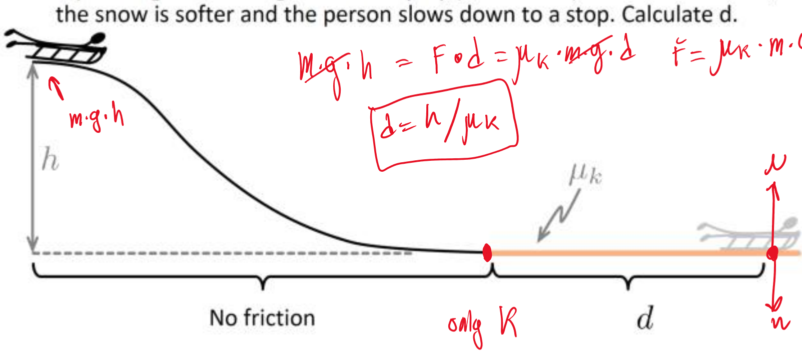


Practice problem

A person goes sledding down a very icy (frictionless) hill. At the bottom, the snow is softer and the person slows down to a stop. Calculate d.

$\cancel{m}\cdot\cancel{g}\cdot h = F\cdot d = \mu_k\cdot\cancel{m}\cdot\cancel{g}\cdot d \qquad F = \mu_k\cdot m\cdot g$

$\boxed{d = h/\mu_k}$



Exit Poll

- Please provide a letter grade for todays lecture:

A. A
B. B
C. C
D. D
E. Fail