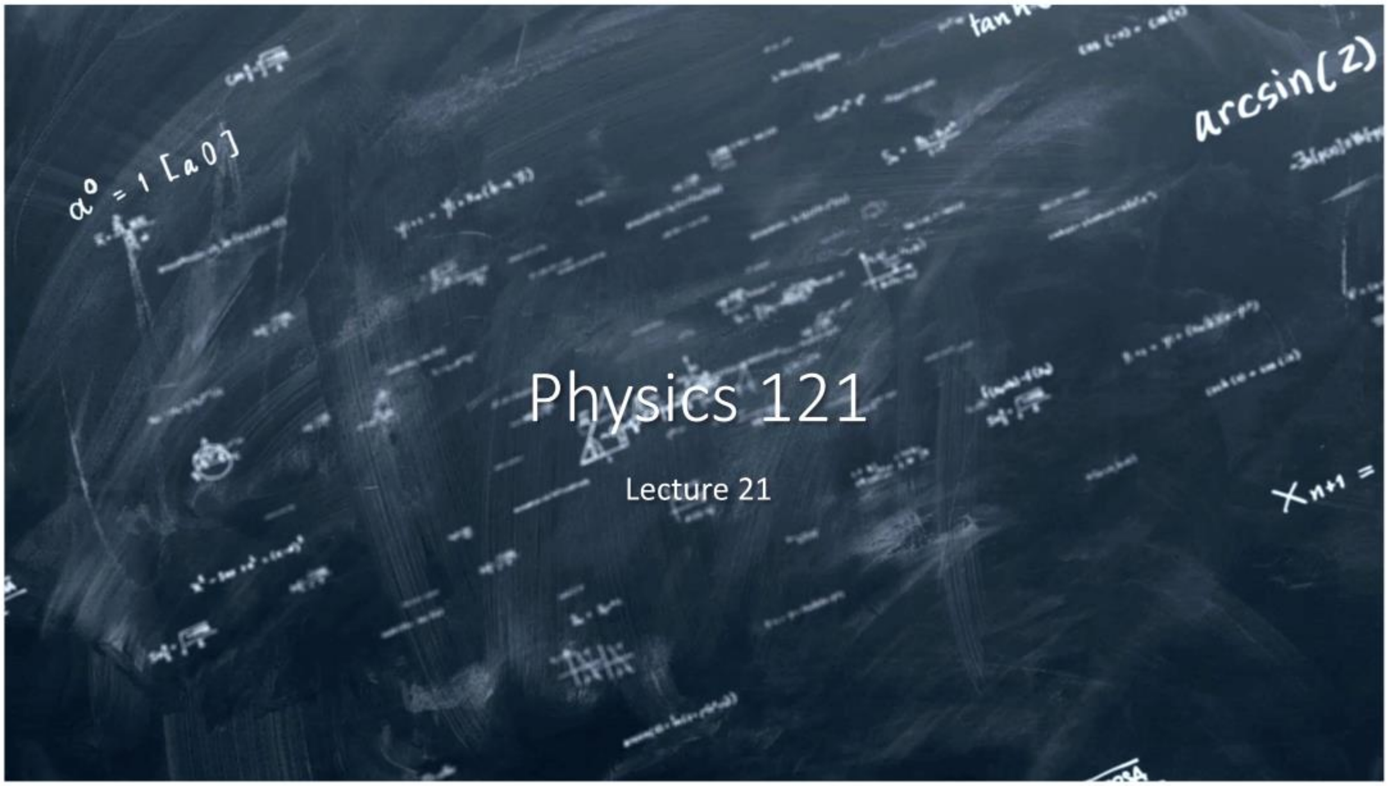
Lecture 21 - 3.23

Tuesday, March 28, 2023 8:06 AM



Study tips

Hard math:

- Redo Homework – from blank paper
- Redo midterm problems – see solution videos
- Redo all in-class quizzes and problems

Hard concepts:

- Create teaching opportunities with study buddy
- Record youtube videos of you teaching concepts

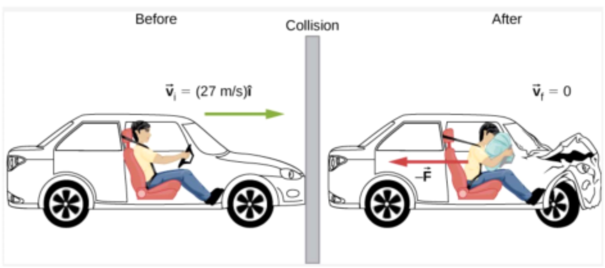
Review – Lecture 17

Momentum $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

LAW OF CONSERVATION OF MOMENTUM

The total momentum of a closed system is conserved:

$$\sum_{j=1}^{N} \vec{\mathbf{p}}_{\mathbf{j}} = \text{constant.}$$



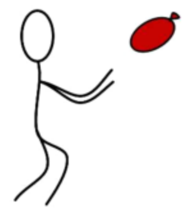
Two kinds of collisions

ELASTIC

- Objects collide and bounce
- Kinetic energy is conserved
- No permanent deformation of the objects

INELASTIC

- Objects collide and stick
- Kinetic energy is NOT conserved
- Objects stick because they lock together, or are permanently bent, or chemical reaction, or…

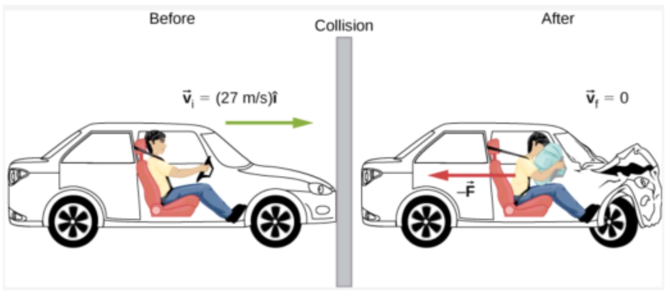


"Impulse"

$$\Delta p = (F) \times (\Delta t) = p_{\text{final}} - p_{\text{initial}}$$

INELASTIC COLLISION:
A 10 g bullet moving with a speed of 500 m/s hits and sticks in a stationary 1 kg wooden block. After the collision, the block+bullet system slides across a frictionless surface.
Where did that kinetic energy go?

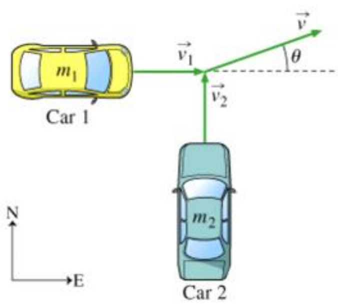
Useful application = Airbags



Mass of driver = 80 kg
Without airbag 0.2 sec until he rests (forever).
With airbag 2.5 sec until rest.

1. What is the magnitude of the impulse?
2. What is the difference in average force with and without airbag?

Review – Lecture 18



$$v_{1f} = 2v_{CM} - v_1$$

Two kinds of collisions

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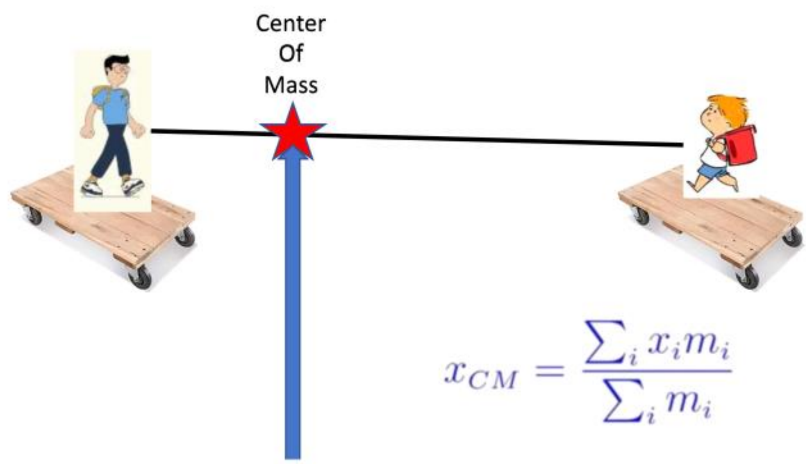
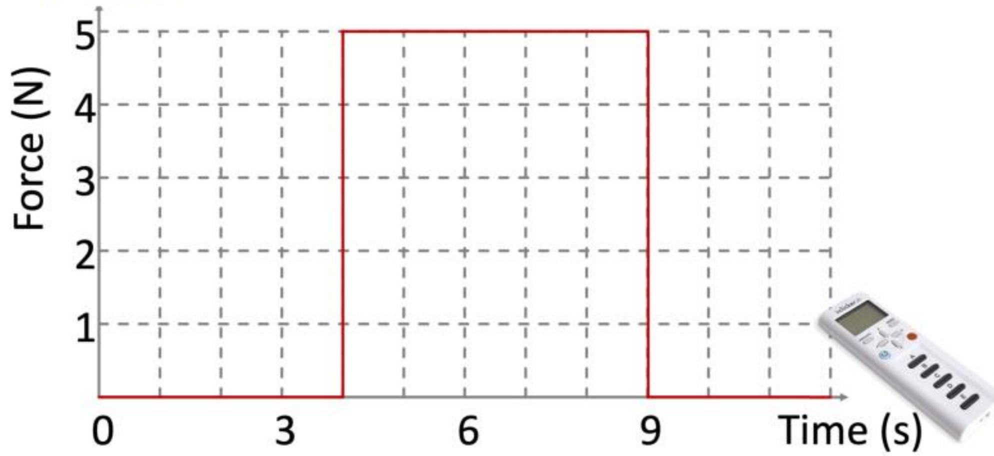


$$x_{CM} = \frac{\sum_i x_i m_i}{\sum_i m_i}$$

iClicker: If the mass is 7 kg, and the initial velocity is -3 m/s, what is the final velocity? (choose the most appropriate range)

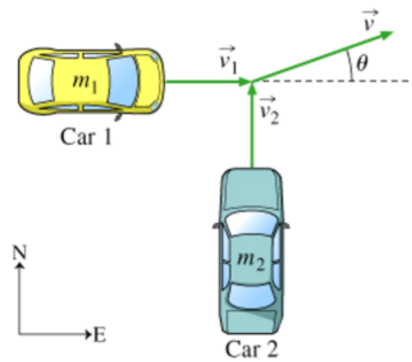
A: between -3 and 0 m/s B: between 0 and 3 m/s C: between 3 and 10 m/s

$$\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \mathbf{F} \times \Delta t$$

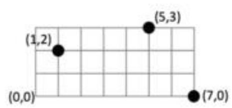


Inelastic Crashes in 2 Dimensions

Two cars collide at a right angle. What is the angle theta in terms of m1, m2, v1, and v2?

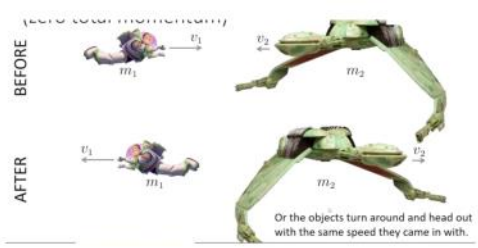


Review – Lecture 19



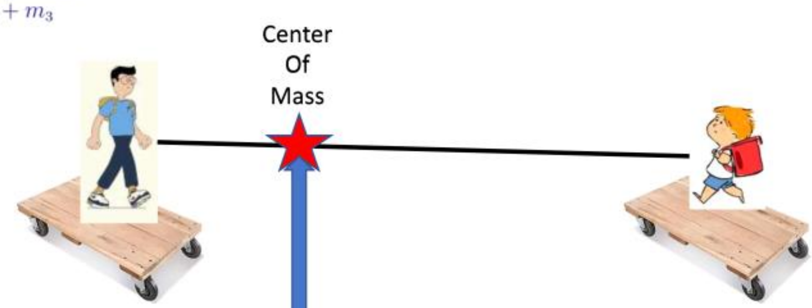
$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} \qquad y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$v_{CM} = \frac{\sum_i v_i m_i}{\sum_i m_i}$$



ZERO tota momentum in COM frame of reference!

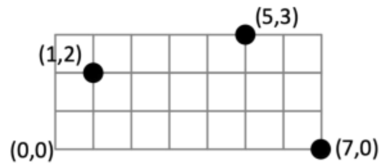
Same momentum, but bullet has more kinetic Energy!



$$x_{CM} = \frac{\sum_i x_i m_i}{\sum_i m_i}$$

If we have a bunch of masses, we can define a "center of mass"

- Think about three particles, equal mass.



$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \qquad y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

Three rods

Three thin rods each of length L are arranged in an inverted "U", as shown. The two side rods have a mass M. The top rod has a mass 3M. Where is the center of mass of the system?

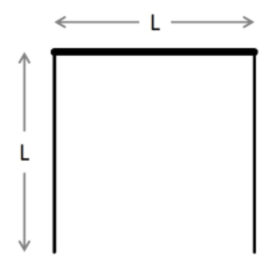
The "vertical" location of the center of mass is:

A. -L/5

B. -L/4

C. -L/3

D. -L/2



Review Lecture 20

$$\theta = \frac{s}{r}$$

Angular Position

counterclockwise rotations as being positive and clockwise rotations as negative.

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt},$$

Angular Velocity

$$\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}.$$

Angular Velocity Vector

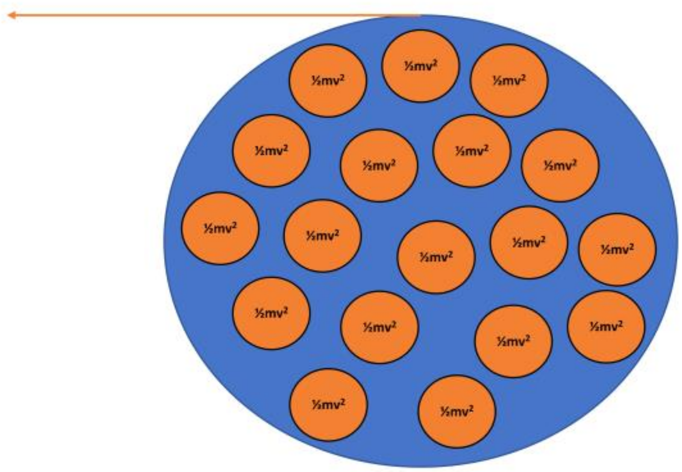
$$v_t = r\omega$$

Tangential Velocity

Rotational
$\theta_f = \theta_0 + \bar{\omega} t$
$\omega_f = \omega_0 + \alpha t$
$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$

Rotational Kinetic Energy

How to assign kinetic energy to a rotating object?



$$\omega = \frac{v_{\mathrm{t}}}{\ }$$

$$K = \sum_j \frac{1}{2} m_j v_j^2$$

$$= \sum_j \frac{1}{2} m_j (r_j \omega_j)^2$$

$$K = \frac{1}{2} \underbrace{\left(\sum_j m_j r_j^2 \right)}_{I = \sum_j m_j r_j^2} \omega^2$$

Moment of Inertia

Overthinking Inertia

Inertia = Opposition to getting into motion

Linear Inertia = Plain old mass. Heavy objects don't want to move.

Rotational Inertia = masses do not like to rotate around far away points. We call this opposition to rotation *Moment of Inertia*

Key takeaway:

Just as ***Masses*** don't want to move, ***Moment of Inertia*** don't want to rotate!

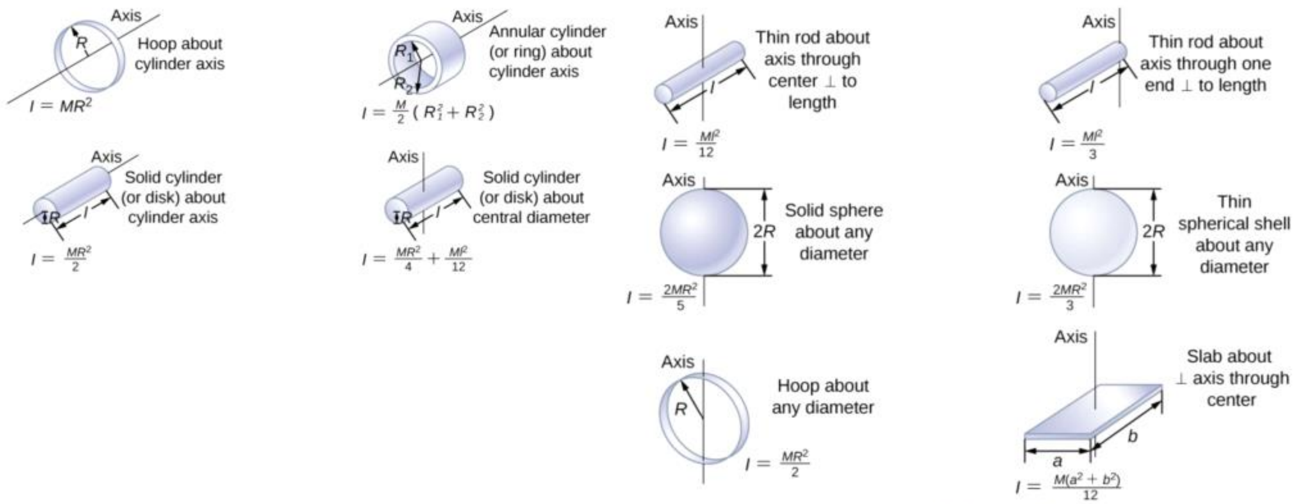
$$K = \frac{1}{2}mv_t^2 \qquad\qquad K = \frac{1}{2}I\omega^2$$

More Symmetry!

Rotational	**Translational**
$I = \sum_j m_j r_j^2$	m
$K = \frac{1}{2} I \omega^2$	$K = \frac{1}{2} m v^2$

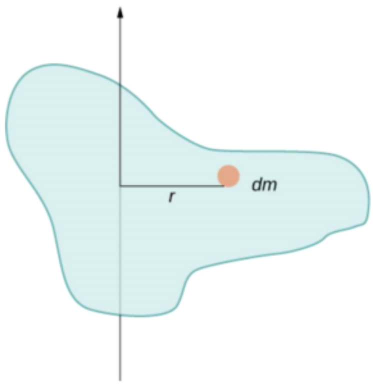
Table 10.4 Rotational and Translational Kinetic Energies and Inertia

Moment of Inertia can be looked up for many shapes!

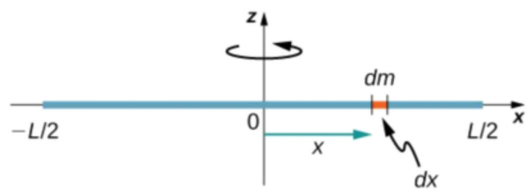


Notice that they all have the same general form: $I = [\ ]mR^2$

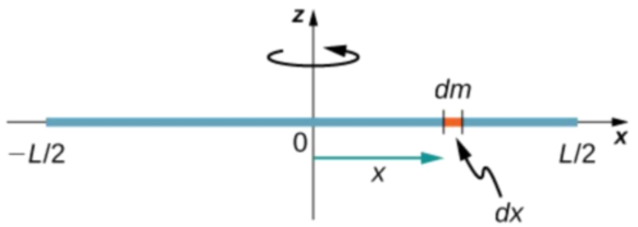
How to find it yourself! – Advanced topic



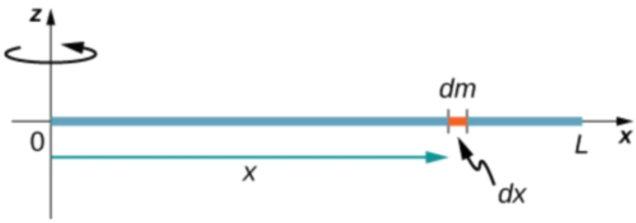
$$I = \sum_i m_i r_i^2 \text{ becomes } I = \int r^2 dm.$$



Let’s do it for a thin rod rotating around center!



Now for rotation around end-point!



Easier Way? **The Parallel-Axis Theorem**

PARALLEL-AXIS THEOREM

Let m be the mass of an object and let d be the distance from an axis through the object's center of mass to a new axis. Then we have

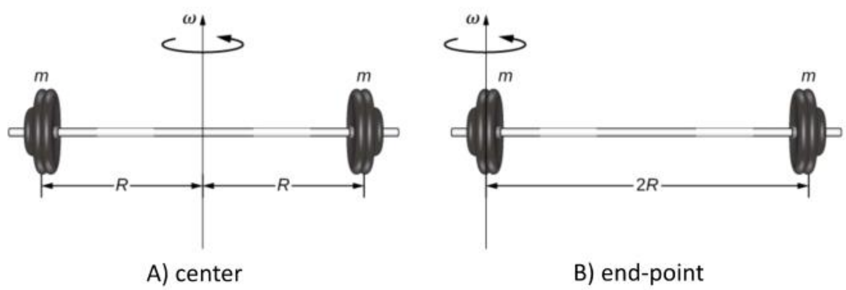
$$I_{\text{parallel-axis}} = I_{\text{center of mass}} + md^2. \tag{10.20}$$

Jakob Steiner

$$I_{\text{end}} = I_{\text{center of mass}} + md^2 = \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2 = \left(\frac{1}{12} + \frac{1}{4}\right)mL^2 = \frac{1}{3}mL^2$$

Quiz

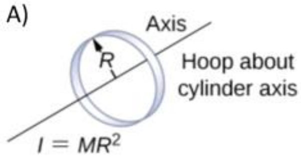
Is it easier to rotate a barbell around the center or an end-point?



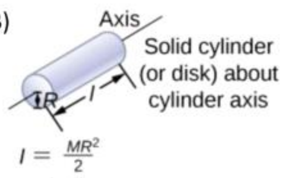
Experiment – Rolling down a track

Opinion poll, what is rolling faster:

A)

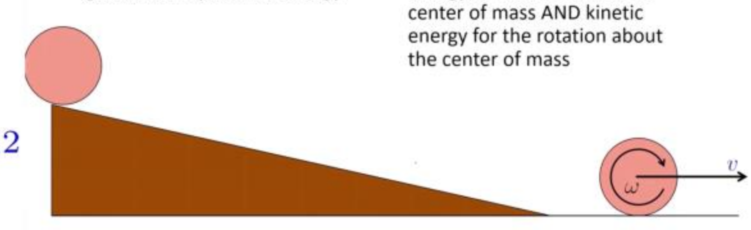


B)

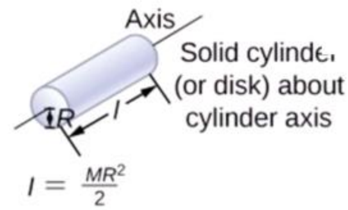


- At the beginning, there is just gravitational potential energy
- At the end there is kinetic energy for the motion of the center of mass AND kinetic energy for the rotation about the center of mass

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$



Revisiting our Experiment



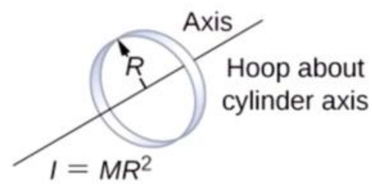
$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

$$I = [\ ]mR^2 \quad \omega = \frac{v}{R}$$

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}[\ ]mR^2\left(\tfrac{v}{R}\right)^2$$

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}[\ ]mv^2$$

$$v = \sqrt{\frac{2gh}{1+[\ ]}}$$



Notice: There is no dependence on the mass OR the radius!

Exit Poll

- Please provide a letter grade for todays lecture:

A. A
B. B
C. C
D. D
E. Fail