Wednesday, December 15, 2021 2:52 PM

#1.) Indicate whether $A \in O(B)$, $A \in \Omega(B)$, or $A \in \Theta(B)$

	A	B	O	Ω	Θ
1)	$n^{\frac{1}{\log n}}$	1	Yes	Yes	Yes
2)	$\sqrt{\log n}$	$\log\log n$	NO	Yes	no
3)	$T(n)$	$\frac{1}{5}n$	Yes	Yes	Yes

$g(n) \in O(f(n)) \rightarrow g(n) \leq c \cdot f(n)$

$g(n) \in \Omega(f(n)) \rightarrow g(n) \geq c \cdot f(n)$

$g(n) \in \Theta(f(n)) \rightarrow d \cdot f(n) \leq g(n) \leq c \cdot f(n)$

1) $n^{\frac{1}{\log n}} \in O(1) \rightarrow n^{\frac{1}{\log n}} \leq 1 \cdot c$

$n^{\frac{1}{\log n}} \in \Theta(1)$

2) $\sqrt{\log n} \in O(\log\log n)$

$(\log n)^{1/2} \leq c \cdot \log(\log n)$

$c = 100$

$(\log(2))^{1/2} \leq 100 \cdot \log(\log(2))$

$1^{1/2} \leq 100 \cdot \underbrace{\log(1)}_{0}$ $n > 1$

3) $T(n) \in \underline{O(\frac{1}{5}n)} \rightarrow 5T(\frac{n}{5}) + 2 \leq c \cdot \frac{1}{5}n$

$$T(n) = \begin{cases} 5T(n/5) + 2 & n > 1 \\ 1 & n = 1 \end{cases}$$

$T(n) = a \cdot T(\frac{n}{b}) + c \cdot n^k$

$a = 5 \quad b = 5 \quad c = 2 \quad k = 0$

if $\log_5 5 \leq 5^0$

$1 = 1$ ✓

$= \Theta(n)$

Use definitions to prove:

$n + 3n^2 \in O(n^2)$

Upper Bound $\quad n + 3n^2 \leq c \cdot n^2 \rightarrow \frac{n+3n^2}{c} \leq n^2$

Let $C = 1000$

Let $N = 1$

$\frac{(1)+3(1)^2}{1000} \leq 1$ ✓ $\therefore n+3n^2 \in O(n^2)$

lower bound $\quad n + 3n^2 \geq c \cdot n^2$

$c = 1 \quad N = 1$

$(1) + 3(1)^2 \geq (1) \cdot (1)^2$ ✓

$\boxed{\therefore n + 3n^2 \in \Theta(n^2)}$

Prove: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\hookrightarrow \sum_{i=1}^{n} i = 1 + 2 + 3 + 4 \ldots n = \frac{n(n+1)}{2}$

B.C: $n = 1 \rightarrow 1 \rightarrow \frac{(1)(1+1)}{2} = 1$ √ True

I.H: Assume that $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

I.S: W.T.S: $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

$\hookrightarrow \sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + k+1$

$\rightarrow = \frac{k(k+1)}{2} + \frac{k+1}{1}$

$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$

$\frac{k(k+1) + 2(k+1)}{2}$

$$\frac{k(k+1) + 2(k+1)}{2}$$

$$= (k+1)\left\{\frac{k}{2} + \frac{2}{2}\right\} \boxed{= \frac{(k+1)(k+2)}{2}}$$

What are six steps one must do when designing an algorithm?

1. Idea/intuition
2. Pseudocode
3. Code
4. Experimental run time
5. Prove run time
6. Prove correctness