Aspects of Decision Making in Cost-effectiveness Modelling

Nathan Green (n.green@ucl.ac.uk) (with thanks to Gianluca Baio, Chris Jackson, Nicky J. Welton, Mark Strong, Anna Heath)

24th November 2022

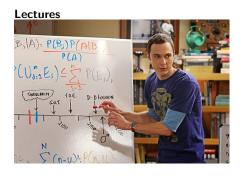
Preliminaries

University College London



- UCL was rated 2nd in the UK for research power in the Research Excellence Framework 2021
- UCL is ranked 8th in the 2022 QS World University Rankings
- The Department of Statistical Science has played a major role in the development of the subject ever since its foundation in 1911 as the Department of Applied Statistics

Objectives



- Introduction to Health economics modelling
 - Decision trees
 - Markov models
- Introduction to sensitivity analyses
 - Deterministic
 - ★ One-way & multi-way
 - ★ Scenario
 - Probabilistic

Objectives

Computer practicals



- Emphasis on practical examples
 - Decision tree and Markov models
 - using R programming language

Timetable

- 2:00-3:00 Health Economics modelling lecture
- 3:00 3:45 Decision tree and Markov model practical
- BREAK
- 3:50 4:20 Sensitivity analysis lecture
- 4:20-5:00 Sensitivity analysis practical

More Bayesian Health Economics...



- This course is only a small part of an annual week-long summer school
 - usually in Florence, Italy
- Several books available
- Edition two of BCEA book in the pipeline and a Health Economic in R book close to being finished!

Lecture 2

Introduction to health economic evaluations

Summary

- Health economic evaluation
 - What is health economics?
 - Why do we need health economics?
- A framework for health economic evaluation
 - Statistical modelling
 - ► Economic modelling
 - Decision analysis
 - Uncertainty analysis
- Standard vs Bayesian HTA
 - ► Two-stage vs integrated approach
- Decision-making
 - Cost-effectiveness plane
 - ▶ ICER
 - EIB

References

Bayesian Methods in Health Economics, chapter 1.
Baio et al (2017). Bayesian Cost-Effectiveness Analysis with the R package BCEA



Objective: Combine costs & benefits of a given intervention into a rational scheme for allocating resources



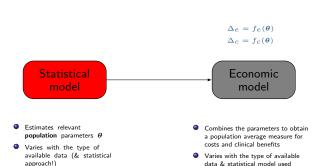
Objective: Combine costs & benefits of a given intervention into a rational scheme for allocating resources

Statistical model

- Estimates relevant population parameters θ
- Varies with the type of available data (& statistical approach!)

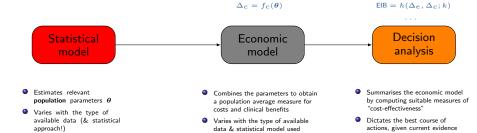


Objective: Combine costs & benefits of a given intervention into a rational scheme for allocating resources





Objective: Combine costs & benefits of a given intervention into a rational scheme for allocating resources

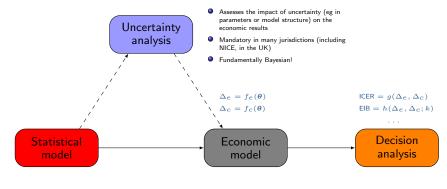


 $\Delta e = f_e(\theta)$

 $ICER = q(\Delta_e, \Delta_c)$

Standardised process

Objective: Combine costs & benefits of a given intervention into a rational scheme for allocating resources



- Estimates relevant population parameters θ
- Varies with the type of available data (& statistical approach!)

- Combines the parameters to obtain a population average measure for costs and clinical benefits
- Varies with the type of available data & statistical model used

- Summarises the economic model by computing suitable measures of "cost-effectiveness"
- Dictates the best course of actions, given current evidence
- Standardised process



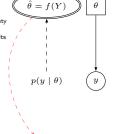
"Standard" approach to HTA — "Two-stage"

Uncertainty analysis

- Assesses the impact of uncertainty (eg in parameters or model structure) on the economic results
- Mandatory in many jurisdictions (including NICE, in the UK)
- Fundamentally Bayesian!

Statistical model

- Estimates relevant population parameters θ
- Varies with the type of available data (& statistical approach!)



1. Estimation (base-case)

 Combines the parameters to obtain a population average measure for costs and clinical benefits

Economic

model

- Varies with the type of available data & statistical model used
- Summarises the economic model by computing suitable measures of "cost-effectiveness"

Decision

analysis

- Dictates the best course of actions, given current evidence
- Standardised process

"Standard" approach to HTA — "Two-stage"

Uncertainty analysis

- Assesses the impact of uncertainty (eg in parameters or model structure) on the economic results
- Mandatory in many jurisdictions (including NICE, in the UK)
- Fundamentally Bayesian!

$\hat{\theta} = f(Y) \qquad \theta \qquad p(\theta) \iff g(\hat{\theta}) \qquad \theta$ $p(y \mid \theta) \qquad y$ Economic model Decision analysis

Statistical model

- Estimates relevant population parameters θ
- Varies with the type of available data (& statistical approach!)

 Combines the parameters to obtain a population average measure for costs and clinical benefits

1. Estimation (base-case)

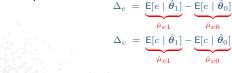
- Varies with the type of available data & statistical model used
- Summarises the economic model by computing suitable measures of "cost-effectiveness"

2. Probabilistic sensitivity analysis

- Dictates the best course of actions, given current evidence
- Standardised process

2./3. Economic modelling+Decision analysis

Cost-effectiveness plane



 Δ_e

Cost differential

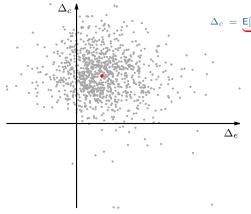
Effectiveness differential

2./3./4. ...+Uncertainty analysis*

Cost-effectiveness plane



Cost differential



^{*}Induced by $g(\hat{\theta}_0), g(\hat{\theta}_1)$

Effectiveness differential

What's wrong with this?...

- Potential correlation between costs & clinical benefits [Individual Level + Aggregated Level Data]
 - Strong positive correlation effective treatments are innovative and result from intensive and lengthy research ⇒ are associated with higher unit costs
 - Negative correlation more effective treatments may reduce total care pathway costs e.g. by reducing hospitalisations, side effects, etc.
 - Because of the way in which standard models are set up, bootstrapping generally only approximates the underlying level of correlation MCMC does a better job!

What's wrong with this?...

- Potential correlation between costs & clinical benefits [Individual Level + Aggregated Level Data]
 - Strong positive correlation effective treatments are innovative and result from intensive and lengthy research ⇒ are associated with higher unit costs
 - Negative correlation more effective treatments may reduce total care pathway costs e.g. by reducing hospitalisations, side effects, etc.
 - Because of the way in which standard models are set up, bootstrapping generally only approximates the underlying level of correlation MCMC does a better job!
- Joint/marginal normality not realistic

[Mainly ILD]

- lacktriangle Costs usually skewed and benefits may be bounded in [0;1]
- Can use transformation (e.g. logs) but care is needed when back transforming to the natural scale
- Should use more suitable models (e.g. Beta, Gamma or log-Normal) generally easier under a Bayesian framework
- ▶ Particularly relevant in presence of partially observed data more on this later!

What's wrong with this?...

- Potential correlation between costs & clinical benefits [Individual Level + Aggregated Level Data]
 - Strong positive correlation effective treatments are innovative and result from intensive and lengthy research ⇒ are associated with higher unit costs
 - Negative correlation more effective treatments may reduce total care pathway costs e.g. by reducing hospitalisations, side effects, etc.
 - Because of the way in which standard models are set up, bootstrapping generally only approximates the underlying level of correlation MCMC does a better job!
- Joint/marginal normality not realistic

[Mainly ILD]

- Costs usually skewed and benefits may be bounded in [0; 1]
- Can use transformation (e.g. logs) but care is needed when back transforming to the natural scale
- Should use more suitable models (e.g. Beta, Gamma or log-Normal) generally easier under a Bayesian framework
- ▶ Particularly relevant in presence of partially observed data more on this later!
- Particularly as the focus is on decision-making (rather than just inference), we need
 to use all available evidence to fully characterise current uncertainty on the model
 parameters and outcomes
 - A Bayesian approach is helpful in combining different sources of information
 - ► Propagating uncertainty is a fundamentally Bayesian operation!

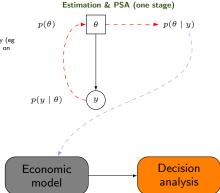
Bayesian approach to HTA — "Integrated"

Uncertainty analysis

- Assesses the impact of uncertainty (eg in parameters or model structure) on the economic results
- Mandatory in many jurisdictions (including NICE, in the UK)
- Fundamentally Bayesian!

Statistical model

- Estimates relevant population parameters θ
- Varies with the type of available data (& statistical approach!)



- Combines the parameters to obtain a population average measure for costs and clinical benefits
- Varies with the type of available data & statistical model used
- Summarises the economic model by computing suitable measures of "cost-effectiveness"
- Dictates the best course of actions, given current evidence
- Standardised process

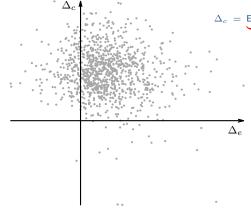


2./4. Economic modelling+Uncertainty analysis*



$$\begin{split} \Delta_e \ = \ \underbrace{\mathbb{E}[e \mid \theta_1]}_{\mu_{e1}} - \underbrace{\mathbb{E}[e \mid \theta_0]}_{\mu_{e0}} \\ \Delta_c \ = \ \mathbb{E}[c \mid \theta_1] - \mathbb{E}[c \mid \theta_0] \end{split}$$

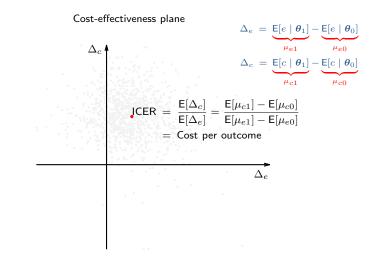
Cost differential



Effectiveness differential

^{*}Induced by $p(\theta \mid data)$

3. Decision analysis

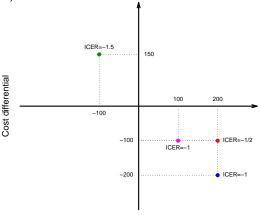


Effectiveness differential

Sost differential

Decision-making based on the ICER

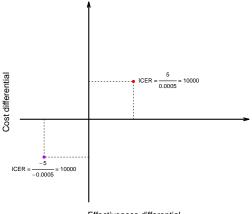
- ICER is not an ordered statistic
 - ► -200/200 better than -100/200 better than -100/100 in terms of decision, but ratios are -1, -1/2, -1
 - ► ICERs in the NW quadrant indicate an intervention that is dominated (+ costs/-effectiveness)



Effectiveness differential

Decision-making based on the ICER

- Equivalent ICERs can mean very different things!
 - $(E[\Delta_e], E[\Delta_c]) = (0.0005, 5)$, indicates that the new treatment produces on average an increase in effectiveness of 0.0005 units at the cost of extra £10 000
 - \blacktriangleright (E[\triangle_e], E[\triangle_c]) = (-0.0005, -5): new intervention less effective, but cheaper
 - ► In both cases, ICER = £10 000



Decision-theoretic approach to HTA

- Analytic framework for decision-making in the face of uncertainty
- Considers a set of prescriptive axioms to ensure rationality in decision-making
- Identifies the best course of action given:
 - ► Model specification
 - ► Current evidence



Decision-theoretic approach to HTA

- Analytic framework for decision-making in the face of uncertainty
- Considers a set of *prescriptive* axioms to ensure **rationality** in decision-making
- Identifies the best course of action given:
 - ► Model specification
 - Current evidence

Process of rational decision-making

- Describe uncertainty on all unknown quantities by means of a (possibly subjective) probability distribution
- **②** For each intervention t, outcomes o = (e, c) are valued by means of a pre-specified *measure of utility*
- Select as the most "cost-effective" the intervention that is associated with the maximum expected utility

$$p(\boldsymbol{\omega}) = p(e, c \mid \boldsymbol{\theta}) p(\boldsymbol{\theta})$$

$$\mathcal{U}^t = \mathsf{E}_{\boldsymbol{\omega}}[u(e,c;t)]$$

Decision-theoretic approach to HTA

- Analytic framework for decision-making in the face of uncertainty
- Considers a set of *prescriptive* axioms to ensure **rationality** in decision-making
- Identifies the best course of action given:
 - Model specification
 - ► Current evidence

Process of rational decision-making

- Describe uncertainty on all unknown quantities by means of a (possibly subjective) probability distribution
- **②** For each intervention t, outcomes o = (e, c) are valued by means of a pre-specified measure of utility
- Select as the most "cost-effective" the intervention that is associated with the maximum expected utility

$$p(\boldsymbol{\omega}) = p(e, c \mid \boldsymbol{\theta}) p(\boldsymbol{\theta})$$

$$\mathcal{U}^t = \mathsf{E}_{\boldsymbol{\omega}}[u(e,c;t)]$$

- Typical utility function in HTA: Monetary Net Benefit $u(e,c;t)=nb_t=ke_t-c_t$
 - lacktriangleright k is the "willingness to pay", i.e. the cost per extra unit of effectiveness gained
 - Fixed, *linear* form, which simplifies computations
 - ▶ Assumes decision-maker is *risk neutral*. Not necessarily true!



ICER vs EIB

Under the MNB, the expected utility is

$$\begin{split} \mathcal{U}^t &= \mathcal{NB}_t = \mathsf{E}_{\pmb{\omega}}[u(e,c;t)] \\ &= k \mathsf{E}_{\pmb{\omega}}[e_t] - \mathsf{E}_{\pmb{\omega}}[c_t] \\ &= k \mathsf{E}_{\pmb{\theta}}[e \mid \pmb{\theta}_t] - \mathsf{E}_{\pmb{\theta}}[c \mid \pmb{\theta}_t] = k \mathsf{E}[\mu_{et}] - \mathsf{E}[\mu_{ct}] \end{split}$$

NB: The expectation is taken with respect to $p(\omega)$ so \mathcal{NB}_t is a pure number!



ICER vs EIB

Under the MNB, the expected utility is

$$\begin{split} \mathcal{U}^t &= \mathcal{NB}_t = \mathsf{E}_{\pmb{\omega}}[u(e,c;t)] \\ &= k \mathsf{E}_{\pmb{\omega}}[e_t] - \mathsf{E}_{\pmb{\omega}}[c_t] \\ &= k \mathsf{E}_{\pmb{\theta}}[e \mid \pmb{\theta}_t] - \mathsf{E}_{\pmb{\theta}}[c \mid \pmb{\theta}_t] = k \mathsf{E}[\mu_{et}] - \mathsf{E}[\mu_{ct}] \end{split}$$

NB: The expectation is taken with respect to $p(\omega)$ so \mathcal{NB}_t is a pure number!

ullet Assuming we are considering only two interventions t=(0,1), decision-making can be effected by looking at the *Expected Incremental Benefit*

$$\begin{split} \mathsf{EIB} &= & \mathcal{N}\mathcal{B}_1 - \mathcal{N}\mathcal{B}_0 \\ &= & (k\mathsf{E}[\mu_{e1}] - \mathsf{E}[\mu_{c1}]) - (k\mathsf{E}[\mu_{e0}] - \mathsf{E}[\mu_{c0}]) \\ &= & k\mathsf{E}[\Delta_e] - \mathsf{E}[\Delta_c] \end{split}$$

ICER vs EIB

Under the MNB, the expected utility is

$$\begin{split} \mathcal{U}^t &= \mathcal{NB}_t = \mathsf{E}_{\pmb{\omega}}[u(e,c;t)] \\ &= k \mathsf{E}_{\pmb{\omega}}[e_t] - \mathsf{E}_{\pmb{\omega}}[c_t] \\ &= k \mathsf{E}_{\pmb{\theta}}[e \mid \pmb{\theta}_t] - \mathsf{E}_{\pmb{\theta}}[c \mid \pmb{\theta}_t] = k \mathsf{E}[\mu_{et}] - \mathsf{E}[\mu_{ct}] \end{split}$$

NB: The expectation is taken with respect to $p(\omega)$ so \mathcal{NB}_t is a pure number!

ullet Assuming we are considering only two interventions t=(0,1), decision-making can be effected by looking at the *Expected Incremental Benefit*

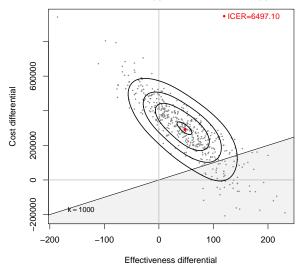
$$\begin{split} \mathsf{EIB} &= \mathcal{N}\mathcal{B}_1 - \mathcal{N}\mathcal{B}_0 \\ &= (k\mathsf{E}[\mu_{e1}] - \mathsf{E}[\mu_{c1}]) - (k\mathsf{E}[\mu_{e0}] - \mathsf{E}[\mu_{c0}]) \\ &= k\mathsf{E}[\Delta_e] - \mathsf{E}[\Delta_c] \end{split}$$

ullet The reference treatment t=1 is more cost-effective than the comparator t=0 if

$$\mathsf{EIB} > 0 \Rightarrow k > (<) \frac{\mathsf{E}[\Delta_c]}{\mathsf{E}[\Delta_e]} = \mathsf{ICER} \quad \mathsf{if} \; \mathsf{E}[\Delta_e] > (<) 0$$

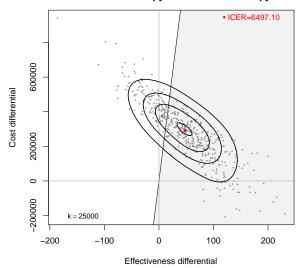
Cost-effectiveness plane vs EIB vs ICER

Cost effectiveness plane New Chemotherapy vs Old Chemotherapy

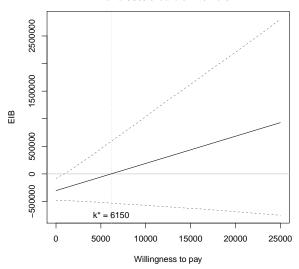


Cost-effectiveness plane vs EIB vs ICER

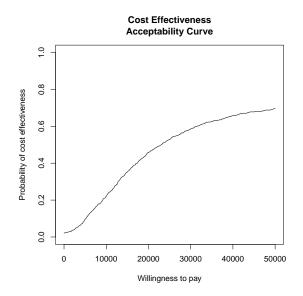
Cost effectiveness plane New Chemotherapy vs Old Chemotherapy



Expected Incremental Benefit and 95% credible intervals



Cost effectiveness acceptability curve



Lecture 3

Decision trees

Model-based economic evaluation

- Trial-based economic evaluation
 - assess cost-effectiveness based on individual-level data on costs and effects
 - shorter-term, selected population
- Model-based economic evaluation
 - construct a model-based representation of disease/clinical history
 - ► State-transition (usually *Markov*) models for clinical histories very common
 - "populate" model with available + relevant data.
- Model-based evaluation typically used to estimate
 - long-term cost-effectiveness
 - in wider population

by combining data from trials with other sources of evidence.

References

Bayesian Methods in Health Economics, chapter 5.4. Welton et al (2012). Evidence Synthesis for Decision Making in Healthcare



Summary

- Decision tree model
 - Strengths and limitations
 - ▶ When to use and when not use
- How to calculate on a decision tree
 - 'Forward' method
 - 'Backward' method
- Examples

References

Decision Making in Health and Medicine, Weinstein et al, Cambridge University Press



A decision tree is a visual representation of a decision analysis:

• Structure the tree

- Structure the tree
- Estimate *probabilities*

- Structure the tree
- Estimate probabilities
- Estimate payoffs (assign values to costs and outcomes)

- Structure the tree
- Estimate probabilities
- Estimate payoffs (assign values to costs and outcomes)
- Analyse the tree

- Structure the tree
- Estimate probabilities
- Estimate payoffs (assign values to costs and outcomes)
- Analyse the tree
 - ► Evaluate the tree
 - ► Explore *uncertainty*

A decision tree is made up of nodes, branches and outcomes

Nodes:

A decision tree is made up of nodes, branches and outcomes

- Nodes:
 - Decision node (square): Describes the problem. Deterministic choice.

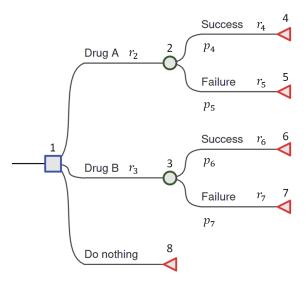
A decision tree is made up of nodes, branches and outcomes

- Nodes:
 - ▶ Decision node (square): Describes the problem. Deterministic choice.
 - ▶ Chance node (circle): Represents the point at which several possible events can occur.

A decision tree is made up of nodes, branches and outcomes

- Nodes:
 - ▶ Decision node (square): Describes the problem. Deterministic choice.
 - ▶ Chance node (circle): Represents the point at which several possible events can occur.
 - ▶ Terminal node (triangle): Represents the end of a tree with a payoff attached.

A hypothetical example is of a comparison between two drugs, 'Drug A' and 'Drug B'. Each drug has different costs associated with them and different performance in terms of the chance that the drug is successful at treating the patient.



- Branches issuing from a chance node represent possible events patients may
 experience at that point in the tree.
- Branch probabilities represent the likelihood of each event.
- The sequence of chance nodes from left to right usually follows the sequence of events.
- The events stemming from a chance node must be *mutually exclusive* and probabilities should sum to 1.

Advantages of Decision Trees

 They enable the economic question to be structured in a meaningful and visual manner.

Advantages of Decision Trees

- They enable the economic question to be structured in a meaningful and visual manner.
- They allow data informing the model parameters to be assimilated and, where appropriate, synthesised.

Advantages of Decision Trees

- They enable the economic question to be structured in a meaningful and visual manner.
- They allow data informing the model parameters to be assimilated and, where appropriate, synthesised.
- They are relatively simple to undertake and suitable for:
 - Diseases that occur only once.
 - Decisions about acute care.
 - Decisions with short time frames.

Limitations of Decision Trees

- They do not explicitly account for passage of time:
 - Passage of time accounted for by outcome measure.
 - Limited ability to account for long term outcomes.

Limitations of Decision Trees

- They do not explicitly account for passage of time:
 - ▶ Passage of time accounted for by outcome measure.
 - ▶ Limited ability to account for long term outcomes.
- Possible to add branches but results in a complex model.
- Other modelling techniques can handle repeated events better.
- Structure of tree only allows for one-way progression of patient through model: Not movement back and forth between states.
- Decision trees can still be useful as a sub-model.



Practice » Guidelines

Tuberculosis—diagnosis, management, prevention, and control: summary of updated NICE guidance

BMJ 2016; 352 doi: https://doi.org/10.1136/bmj.h6747 (Published 13 January 2016) Cite this as: BMJ 2016;352:h6747

Article Related content Metrics Responses

Lucy Elizabeth Hoppe, technical analyst (clinical) ¹, Rachel Kettle, technical advisor (public health) ², Michael Eisenhur, consultant paediatrician and member of the Guideline Development Group ³, Ibrahim Abubakar, professor of infectious disease epidemiology and co-chair of the Guideline Development Group ⁴ on behalf of the Guideline Development Group

Author affiliations >

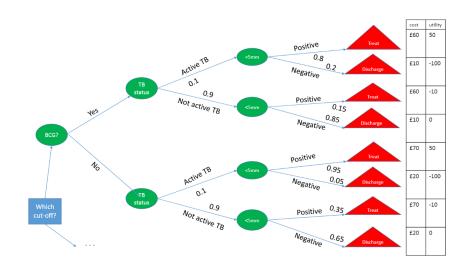
Correspondence to: L E Hoppe lucy.hoppe@nice.org.uk

What you need to know

Undertake tuberculosis (TB) testing in close contacts of people with pulmonary or laryngeal TB, people who
are immunocompromised and at high risk of TB, and new entrants from high incidence countries who
present to healthcare services



Simple decision tree example: New TST guidelines



Calculations on a decision tree

- Conditional probabilities two or more events
- Probability of both A and B occurring, divided by the probability of B occurring
- The joint probability of A and B measures the probability that A and B occur together at the same moment.
- The marginal probability of A is the individual probability of A, ignoring any value of event B.

• Complete set of all possible outcomes is set of terminal nodes on the RHS

- Complete set of all possible outcomes is set of terminal nodes on the RHS
- By chaining the conditional probabilities for a given pathway we obtain the total or
 joint probability of reaching a terminal node. Each pathway through the tree is a
 mutually exclusive sequence of events. Consider one such pathway through the tree
 with the following n nodes.

$$x_0 \to x_{[1]} \to x_{[2]} \to \cdots \to x_{[n]}$$

where

- x₀ is the root node (often the decision node)
- $lackbox{} x_{[n]}$ is the terminal node and the square brackets indicate that this is the ith ordered nodes in the sequence.

- Complete set of all possible outcomes is set of terminal nodes on the RHS
- By chaining the conditional probabilities for a given pathway we obtain the total or *joint* probability of reaching a terminal node. Each pathway through the tree is a mutually exclusive sequence of events. Consider one such pathway through the tree with the following n nodes.

$$x_0 \to x_{[1]} \to x_{[2]} \to \cdots \to x_{[n]}$$

where

- ► x₀ is the root node (often the decision node)
- $ightharpoonup x_{[n]}$ is the terminal node and the square brackets indicate that this is the ith ordered nodes in the sequence.
- By the product rule joint probability of this path is

$$p(x_{[1]}, x_{[2]}, \dots, x_{[n]}) = p(x_{[2]} \mid x_{[1]}) p(x_{[3]} \mid x_{[2]}) \cdots p(x_{[n]} \mid x_{[n-1]}) = \prod_{i=1}^{[n-1]} p(x_{[i+1]} \mid x_{[i]})$$



- Complete set of all possible outcomes is set of terminal nodes on the RHS
- By chaining the conditional probabilities for a given pathway we obtain the total or *joint* probability of reaching a terminal node. Each pathway through the tree is a mutually exclusive sequence of events. Consider one such pathway through the tree with the following n nodes.

$$x_0 \to x_{[1]} \to x_{[2]} \to \cdots \to x_{[n]}$$

where

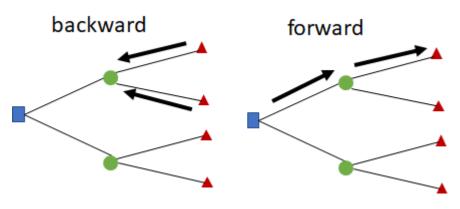
- x₀ is the root node (often the decision node)
- $ightharpoonup x_{[n]}$ is the terminal node and the square brackets indicate that this is the ith ordered nodes in the sequence.
- By the product rule joint probability of this path is

$$p(x_{[1]}, x_{[2]}, \dots, x_{[n]}) = p(x_{[2]} \mid x_{[1]}) p(x_{[3]} \mid x_{[2]}) \cdots p(x_{[n]} \mid x_{[n-1]}) = \prod_{i=1}^{[n-1]} p(x_{[i+1]} \mid x_{[i]})$$

• For the above pathway, the corresponding cost and effects are $c_{[1]}, c_{[2]}, \ldots, c_{[n]}$ and $e_{[1]}, e_{[2]}, \ldots, e_{[n]}$.

How to calculate expected values

There are two alternative approaches used for calculating the expected cost and effectiveness on a decision tree.



- Weighted average of the total values of the child nodes of a parent chance node
 - ▶ Weights are the probability of traversing each branch to the child nodes.

- Weighted average of the total values of the child nodes of a parent chance node
 Weights are the probability of traversing each branch to the child nodes.
- Act of stepping backwards in this way through the tree is called folding back (or rolling back).

- Weighted average of the total values of the child nodes of a parent chance node
 - Weights are the probability of traversing each branch to the child nodes.
- Act of stepping backwards in this way through the tree is called folding back (or rolling back).
- So called because the children are folded up or collapsed, so that the chance node is now represented by its expected value.

- Weighted average of the total values of the child nodes of a parent chance node
 - Weights are the probability of traversing each branch to the child nodes.
- Act of stepping backwards in this way through the tree is called folding back (or rolling back).
- So called because the children are folded up or collapsed, so that the chance node is now represented by its expected value.
- Starting at the right-most terminal nodes the expected values at each chance node are calculate in turn and the tree can thus be folded back all the way to the decision node

- Weighted average of the total values of the child nodes of a parent chance node
 - ▶ Weights are the probability of traversing each branch to the child nodes.
- Act of stepping backwards in this way through the tree is called folding back (or rolling back).
- So called because the children are folded up or collapsed, so that the chance node is now represented by its expected value.
- Starting at the right-most terminal nodes the expected values at each chance node are calculate in turn and the tree can thus be folded back all the way to the decision node
- Example of something called a recursive function which is a function that calls itself during its execution.
 - ▶ A well-known example is when calculating the Fibonacci series.
- This approach is part of a whole field of stochastic optimisation in applied probability called Markov decision process (MDP).

• Recall the conditional probabilities $p_{ij} = p(x_j \mid x_i)$, then the expected value is

$$\mathbb{E}[V_i] = \left\{ \begin{array}{ll} r_i & \text{if} & i \in \mathcal{S}_{term} \\ r_i + \sum p_{ij} \mathbb{E}[V_j] & \text{otherwise} \end{array} \right.$$

where

- lacktriangle $\mathbb E$ is the weighted average of the values, V is the random variable total node value, e.g. cost or QALYs
- r_i is the (unit) value at node i
- $ightharpoonup \mathcal{S}_{term}$ is the set of all terminal nodes. Note that at a terminal node the expected value is simply the value at that node.

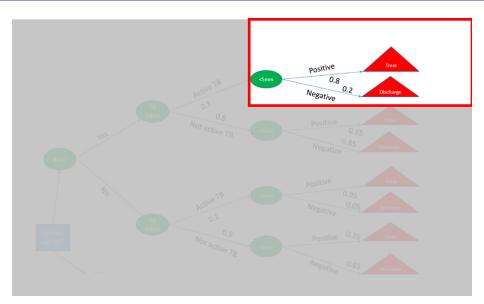
• Recall the conditional probabilities $p_{ij} = p(x_j \mid x_i)$, then the expected value is

$$\mathbb{E}[V_i] = \left\{ \begin{array}{ll} r_i & \text{if} & i \in \mathcal{S}_{term} \\ r_i + \sum p_{ij} \mathbb{E}[V_j] & \text{otherwise} \end{array} \right.$$

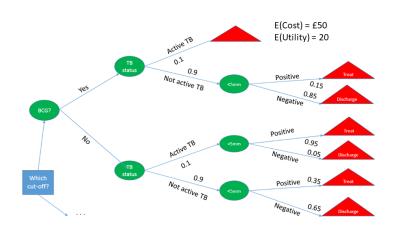
where

- lacktriangle $\mathbb E$ is the weighted average of the values, V is the random variable total node value, e.g. cost or QALYs
- r_i is the (unit) value at node i
- $ightharpoonup \mathcal{S}_{term}$ is the set of all terminal nodes. Note that at a terminal node the expected value is simply the value at that node.
- An advantage of using this method is that total expected values can be obtained at each node and so if there are multiple decision node not at the root of the tree the recursive approach can fold-back sub-trees.

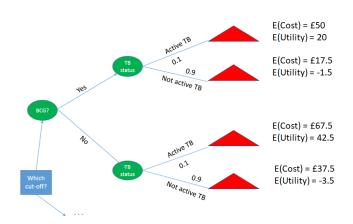
Simple decision tree example: New TST guidelines



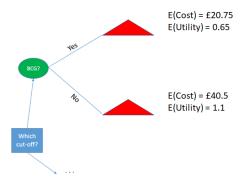
Simple decision tree example: New TST guidelines



Simple decision tree example: New TST guidelines



Simple decision tree example: New TST guidelines



• May be more intuitive



- May be more intuitive
- Calculate the total health and costs, and joint probability along all of the distinct pathways of the tree corresponding to a decision.

- May be more intuitive
- Calculate the total health and costs, and joint probability along all of the distinct pathways of the tree corresponding to a decision.
- The weighted average of the costs or health values give the expected value at the decision node.

- May be more intuitive
- Calculate the total health and costs, and joint probability along all of the distinct pathways of the tree corresponding to a decision.
- The weighted average of the costs or health values give the expected value at the decision node.
- Formally

$$\mathbb{E}[V] = \sum_{j \in S_{term}} r_j^* p_j^*$$

where

- $\blacktriangleright \ r_j^* = r_{[1]} + r_{[2]} + \dots + r_{[n]}$ is the total of the values along each pathway with terminal node j
- $p_j^* = p(x_{[1]}, x_{[2]}, \dots, x_{[n]})$ is the joint probabilities of traversing the unique path with terminal node j.
- ightharpoonup The set of possible terminal nodes if the decision-maker takes a given decision is denoted S_{term} .

TB Combined probabilities for each branch

If probability of BCG is 0.1 (c.f. NHS Immunisation Statistics, England 2012-13) then

$$\begin{array}{lll} p_1^* & = & 0.1 \times 0.1 \times 0.8 = 0.008 \\ p_2^* & = & 0.1 \times 0.1 \times 0.2 = 0.002 \\ p_3^* & = & 0.1 \times 0.9 \times 0.15 = 0.0135 \\ p_4^* & = & 0.1 \times 0.9 \times 0.85 = 0.0765 \\ p_5^* & = & 0.9 \times 0.1 \times 0.95 = 0.0855 \\ p_6^* & = & 0.9 \times 0.1 \times 0.05 = 0.0045 \\ p_7^* & = & 0.9 \times 0.9 \times 0.35 = 0.2835 \\ p_8^* & = & 0.9 \times 0.9 \times 0.65 = 0.5265 \end{array}$$

TB expected cost and health impact

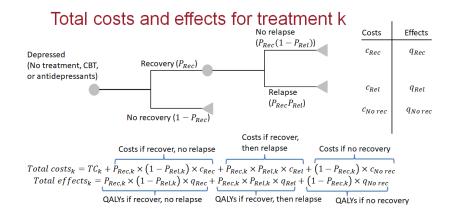
Cost:

$$\begin{aligned} &0.008\times60+0.003\times10+0.0135\times60+0.0765\times10\\ &+0.0855\times70+0.0045\times20+0.2835\times70+0.5265\times20\\ &=38.535 \end{aligned}$$

Health:

$$\begin{array}{l} 0.008\times50+0.003\times(-100)+0.0135\times(-10)+0.0765\times0\\ +0.0855\times50+0.0045\times(-100)+0.2835\times(-10)+0.5265\times0\\ =0.955 \end{array}$$

Decision tree practical



Lecture 4

Markov models

Outline of this lecture

- Theory of Markov modelling
- Example: Estimating Markov model transition probabilities from individual-level counts of transitions
- Bayesian inference ideas
 - Dirichlet and Multinomial distributions
 - Advantages of Bayes: including prior information, natural quantification of uncertainty (probabilistic sensitivity analysis).
- Implementing Markov models in R
- Brief discussion of decision modelling more generally, from Bayesian perspective

Markov models in discrete time

Assume

- ullet a set of S "clinically relevant" states: exhaustive and mutually exclusive
- ullet a discrete time axis, indexed by "cycles" or time units j
- a set of allowed transitions in a diagram of the states

Arrows connecting two states denote that a person can move

- ullet from the state at the start of the arrow at time j
- ullet to the state at the end of the arrow at time j+1
- Absence of an arrow implies transition is not allowed by model

Markov models in discrete time

Assume

- ullet a set of S "clinically relevant" states: exhaustive and mutually exclusive
- ullet a discrete time axis, indexed by "cycles" or time units j
- a set of allowed transitions in a diagram of the states

Arrows connecting two states denote that a person can move

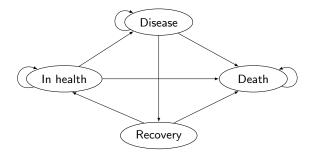
- ullet from the state at the start of the arrow at time j
- ullet to the state at the end of the arrow at time j+1
- Absence of an arrow implies transition is not allowed by model

Movements occur according to suitable transition probabilities

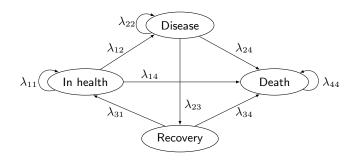
$$\pi_j = \pi_{j-1} \Lambda_j$$

- π_i is the vector of probabilities of occupying each state at time j
- $\Lambda_j = [\Lambda_{j;s,s'}]$ is the transition probability matrix at time j: s,s' entry is the probability of moving from state s to state s' at time j

1. Define a structure



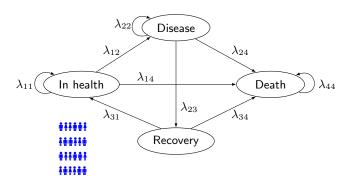
2. Estimate the transition probabilities from available, relevant data Define costs and utilities associated with occupying each state s at each time j



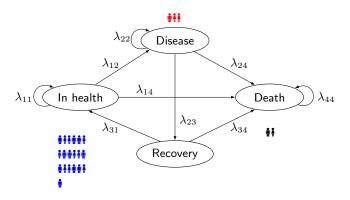
3. Run the simulation, recording:

proportion of people in each state \rightarrow expected costs and effects, at each time:

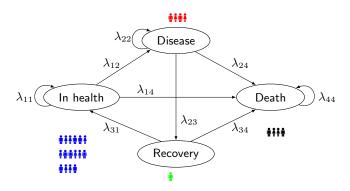
$$j = 0$$



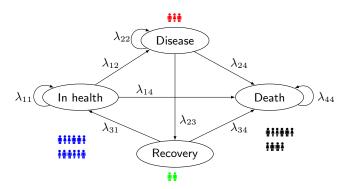
3. Run the simulation: j=1



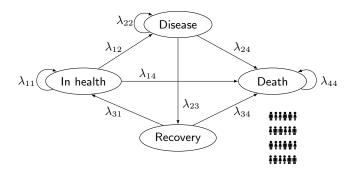
3. Run the simulation: j=2



3. Run the simulation: j=3



3. Run the simulation: j = J



Markov assumption (discrete time)

Markov models are multi-state models in which next transition depends only on the current state

$$\Pr(\mathsf{state}\ s'\ \mathsf{at}\ \mathsf{time}\ j+1\ |\ \mathsf{history}\ \mathsf{of}\ \mathsf{the}\ \mathsf{process}) = \\ \Pr(\mathsf{state}\ s'\ \mathsf{at}\ \mathsf{time}\ j+1\ |\ \mathsf{state}\ s\ \mathsf{at}\ \mathsf{time}\ j)$$

Markov assumption (discrete time)

Markov models are multi-state models in which next transition depends only on the current state

$$\Pr(\mathsf{state}\ s'\ \mathsf{at}\ \mathsf{time}\ j+1\ |\ \mathsf{history}\ \mathsf{of}\ \mathsf{the}\ \mathsf{process}) = \\ \Pr(\mathsf{state}\ s'\ \mathsf{at}\ \mathsf{time}\ j+1\ |\ \mathsf{state}\ s\ \mathsf{at}\ \mathsf{time}\ j)$$

A Markov model is time-homogeneous if this transition probability is the same for all times j.

• Counterexample: risk of death depends on age.

Markov assumption (discrete time)

Markov models are multi-state models in which next transition depends only on the current state

$$\Pr(\mathsf{state}\ s'\ \mathsf{at}\ \mathsf{time}\ j+1\ |\ \mathsf{history}\ \mathsf{of}\ \mathsf{the}\ \mathsf{process}) = \\ \Pr(\mathsf{state}\ s'\ \mathsf{at}\ \mathsf{time}\ j+1\ |\ \mathsf{state}\ s\ \mathsf{at}\ \mathsf{time}\ j)$$

A Markov model is time-homogeneous if this transition probability is the same for all times j.

• Counterexample: risk of death depends on age.

Non-Markov models can often be made Markov by adding extra states

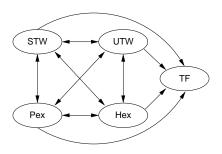
- \bullet e.g. states health \rightarrow disease \rightarrow death
- suppose risk of death changes with time spent with disease
- ullet split disease into "early disease" \to " late disease", with differing risks of death

Remember these models are only approximations to a true process that is usually continuous-time, continuous-state

Example: asthma¹

Five-state model for management of asthma:

- STW: Successfully treated week
- UTW: Unsuccessfully treated week
- Hex: Hospital-managed exacerbation
- Pex: Primary care-managed exacerbation
- TF: Treatment failure enters a "usual care" pattern (absorbing)



¹Briggs A, Ades AE and Price MJ. Medical Decision Making (2003); 23:341-350



Day 1

Data

Estimate the Markov model transition probabilities using individual-level transition data

- From a RCT with 2 treatments
 - ▶ SFC: Salmeterol (50 μ g) / fluticasone propionate (100 μ g) in combination
 - \star new treatment. t=2
 - ▶ FP: Fluticasone propionate alone $(100\mu g)$
 - \star existing treatment. t=1
- 12 week trial
- \bullet For each arm we count the number of transitions between states from week j to j+1
- From the Markov assumption, these can be considered independent



SFC									
	Num	ber in sta	Total in state						
$r_{ss'}$	STW	UTW	Hex	Pex	TF	at week j (n_s)			
STW	210	60	0	1	1	272			
UTW	88	641	0	4	13	746			
Hex	0	0	0	0	0	0			
Pex	1	0	0	0	1	2			
TF	0	0	0	0	81	81			
·	FP								
STW	66	32	0	0	2	100			
UTW	42	752	0	5	20	819			
Hex	0	0	0	0	0	0			
Pex	0	4	0	1	0	5			
TF	0	0	0	0	156	156			

These are summed over all weeks



Estimating transition probabilities

Could estimate (s,s^\prime) transition probability by dividing total

- ullet number of transitions observed from s to s^\prime , by
- ullet number of weeks observed in state s

But can't estimate transition rates out of Hex since nobody went into Hex! Also very few went into Pex

• estimates / SEs for rates out of Pex will be unstable.

Economic model needs to

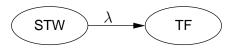
- include possibility of Hex / Pex expensive states!
- account for *uncertainty* in the transition rates.
- ightarrow use Bayesian inference
 - combine priors on Hex/Pex/other states with data
 - ullet ightarrow posterior distribution of transition probabilities.



Binomial distribution for an event

Suppose there are two states in the model (successfully treated week / treatment failure).

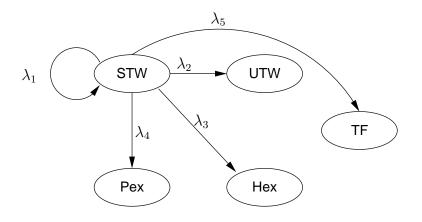
Out of n people currently under treatment, the following week r people have had treatment failure.



Total in STW at j	Number in state at $j+1$				
at week j	STW TF				
n	n-r r				

Beta distribution is a convenient prior for λ (conjugacy)

Multinomial model for several events



Nu	ımber in	Total in STW at j			
STW	UTW	Hex	Pex	TF	at week j
r_1	r_2	r_3	r_4	r_5	n

Bayesian estimation of multinomial model

Likelihood

$$\begin{array}{lcl} r_1,\ldots,r_5 & \sim & \mathsf{Multinomial}(\lambda_1,\ldots,\lambda_5,n) \\ p(\pmb{r}\mid \pmb{\lambda}) & = & \frac{n!}{r_1!\cdots r_5!}\lambda_1^{r_1}\ldots\lambda_5^{r_5} & \propto & \lambda_1^{r_1}\ldots\lambda_5^{r_5} \\ \sum \lambda_s & = & 1 \end{array}$$

Prior distribution for five transition probabilities

$$\begin{array}{lcl} (\lambda_1,\ldots,\lambda_5) & \sim & \mathsf{Dirichlet}(a_1,\ldots,a_5) \\ p(\lambda_1,\ldots,\lambda_5) & = & \frac{\Gamma(a_1+\cdots+a_5)}{\Gamma(a_1)\cdots\Gamma(a_5)}\lambda_1^{(a_1-1)}\cdots\lambda_5^{(a_5-1)} & \propto & \lambda_1^{(a_1-1)}\cdots\lambda_5^{(a_5-1)} \end{array}$$

Posterior distribution

$$\begin{array}{ccc} p(\pmb{\lambda} \mid \pmb{r}) & \propto & \lambda_1^{(a_1+r_1-1)} \cdots \lambda_5^{(a_5+r_5-1)} \\ \pmb{\lambda} & \sim & \mathsf{Dirichlet}(a_1+r_1, \dots, a_5+r_5) \end{array}$$

Dirichlet prior distribution

• General prior for a set of numbers that add up to 1.

$$(p_1,\ldots,p_S) \sim \mathsf{Dirichlet}(a_1,\ldots,a_S)$$

- $ightharpoonup a_s$ proportional to expected probability p_s of outcome s
- lacktriangle scale of the a_s indicates prior *precision* of (p_1,\ldots,p_S)
- **NB**: Dirichlet(1, ..., 1) is a flat prior.
- \bullet Given a sample size $\sum a_s,\,a_s$ is your prior expectation for the number of patients you would expect in each state s

Simulating from a Dirichlet in R

However the Dirichlet posterior is known here.

We can simulate directly from a known Dirichlet distribution in R by exploiting that a Dirichlet random vector is a function of Gamma random variables:

- Simulate $Z_s \sim \mathsf{Gamma}(a_s,1)$ for $s=1,\ldots,S$. Then
- $(Z_1,\ldots,Z_S)/\sum_{i=1}^S Z_i \sim \mathsf{Dirichlet}(a_1,\ldots,a_S)$

Simulating from a Dirichlet in R

However the Dirichlet posterior is known here.

We can simulate directly from a known Dirichlet distribution in R by exploiting that a Dirichlet random vector is a function of Gamma random variables:

- Simulate $Z_s \sim \mathsf{Gamma}(a_s,1)$ for $s=1,\ldots,S$. Then
- $(Z_1,\ldots,Z_S)/\sum_{i=1}^S Z_i \sim \mathsf{Dirichlet}(a_1,\ldots,a_S)$

For example, to simulate 1000 draws from a Dirichlet(1,1,1,1):

Simulating from a Dirichlet in R

However the Dirichlet posterior is known here.

We can simulate directly from a known Dirichlet distribution in R by exploiting that a Dirichlet random vector is a function of Gamma random variables:

- Simulate $Z_s \sim \mathsf{Gamma}(a_s,1)$ for $s=1,\ldots,S.$ Then
- $(Z_1, \ldots, Z_S) / \sum_{i=1}^S Z_i \sim \mathsf{Dirichlet}(a_1, \ldots, a_S)$

For example, to simulate 1000 draws from a Dirichlet(1,1,1,1):

Or functions to simulate directly from the Dirichlet are provided in several R add-on packages available from CRAN, e.g.

```
install.packages("VGAM") # if not already installed
library(VGAM)
dir_rand <- rdiric(1000, a)</pre>
```



Estimated transition probabilities versus observed data

After obtaining the posterior Dirichet distributions

	Raw transition counts					Bayesian posterior mean				
To:	STW	UTW	Hex	Pex	TF	STW	UTW	Hex	Pex	TF
From:	SFC									
STW	210	60	0	1	1	0.76	0.22	0.004	0.01	0.01
UTW	88	641	0	4	13	0.12	0.85	0.001	0.01	0.02
Hex	0	0	0	0	0	0.20	0.20	0.20	0.20	0.20
Pex	1	0	0	0	1	0.29	0.14	0.14	0.14	0.28
TF	0	0	0	0	81	0	0	0	0	1
	FP									
STW	66	32	0	0	2	0.64	0.31	0.01	0.01	0.03
UTW	42	752	0	5	20	0.05	0.91	0.001	0.00	0.03
Hex	0	0	0	0	0	0.20	0.20	0.20	0.20	0.20
Pex	0	4	0	1	0	0.10	0.50	0.10	0.20	0.10
TF	0	0	0	0	156	0	0	0	0	1

Information from the prior has been combined with the data

Cost-effectiveness modelling

Calculate expected costs and expected benefits of each treatment over the long term:

- Cohort simulation: Estimate proportion of population in each state at each successive time (from Markov transition probabilities)
 - ▶ at the start, everyone in "base" state (e.g. just received treatment)
 - ▶ usually in the long run, everyone will be in the "absorbing" state (e.g. death)
- Assign cost and benefit to each state in model
- Accumulate expected cost and benefit over times

NB: Check BMHE 5.4 + practical!



Discounting

- Costs and outcomes can occur at different times with respect to when the intervention is implemented
 - ▶ Society tends to value benefits that arrive closer to the present time more than those that are achievable later in the future
- Discounting accounts for differential timing by reducing the value of costs & effects in the future
 - Particularly relevant for economic evaluation spanning over a time horizon > 1 year (e.g. Markov models)

Discounting

- Costs and outcomes can occur at different times with respect to when the intervention is implemented
 - Society tends to value benefits that arrive closer to the present time more than those that are achievable later in the future
- Discounting accounts for differential timing by reducing the value of costs & effects in the future
 - Particularly relevant for economic evaluation spanning over a time horizon > 1 year (e.g. Markov models)
- If cost at some future time j is c_{tj} , then discounted cost is $c_{tj}/(1+d)^j$.
- Discount rate d: NICE suggest 3.5% for costs and outcomes
- ullet Present Value of intervention (treatment) t over a time horizon J is

$$\mathsf{PV}_t^c = \sum_{j=0}^J \frac{c_{tj}}{(1+d)^j} \qquad \mathsf{PV}_t^e = \sum_{j=0}^J \frac{e_{tj}}{(1+d)^j}$$

(for costs c and clinical benefits e, respectively)



Simulating prevalences of states over time

- ullet π_{js} is the **expected proportion** of patients in state s at time j
- \bullet $\lambda_{ss'}$ is the probability of going from state s to state s' in one time step
- Thus

$$\pi_{j+1,s} = \pi_{j1}\lambda_{1s} + \dots + \pi_{jS}\lambda_{Ss}$$

which we can write in matrix form as

$$(\pi_{j+1,1},\ldots,\pi_{j+1,S}) = (\pi_{j,1},\ldots,\pi_{j,S}) \begin{pmatrix} \lambda_{11} & \ldots & \lambda_{1S} \\ \vdots & \ddots & \vdots \\ \lambda_{S1} & \ldots & \lambda_{SS} \end{pmatrix}$$
 $\pi_{j+1} = \pi_{j}\Lambda$

• NB: Λ may depend on time j

Simulating prevalences of states over time

- ullet π_{js} is the **expected proportion** of patients in state s at time j
- \bullet $\lambda_{ss'}$ is the probability of going from state s to state s' in one time step
- Thus

$$\pi_{j+1,s} = \pi_{j1}\lambda_{1s} + \dots + \pi_{jS}\lambda_{Ss}$$

which we can write in matrix form as

$$(\pi_{j+1,1},\ldots,\pi_{j+1,S}) = (\pi_{j,1},\ldots,\pi_{j,S}) \begin{pmatrix} \lambda_{11} & \ldots & \lambda_{1S} \\ \vdots & \ddots & \vdots \\ \lambda_{S1} & \ldots & \lambda_{SS} \end{pmatrix}$$
 $\boldsymbol{\pi}_{j+1} = \boldsymbol{\pi}_{j} \boldsymbol{\Lambda}$

- **NB**: Λ may depend on time j
- Instead of calculating **expected** proportions π_{js} , could simulate the **absolute number** m_{js} of patients in a cohort of size M, in state s, at time j.
 - may help to illustrate what model is doing, though decision is generally based on expected cost (average cost for an infinite population)

Coding this in R (see BMHE 5.4)

Markov cost-effectiveness models can be implemented straightforwardly in R.

Many different ways to write code like this: see the questions sheet / solutions for another way.

See also heemod, an R package designed for health economic modelling $\verb|https://CRAN.R-project.org/package=heemod|$



Costs and utilities

• Assign a cost to each state for each treatment

	Cost per week (\pounds)				
Treatment	STW	UTW	Hex	Pex	TF
SFC	7.96	7.96	1821.17	100.79	TF cost (j)
FP	2.38	2.38	1815.58	95.21	TF cost (j)

 \bullet In this example, assume the expected cost for week j in SFC group is

$$\pi_{j1} \times \operatorname{cost}_{(SFC,1)} + \dots + \pi_{j4} \times \operatorname{cost}_{(SFC,4)} + \pi_{j5} \times \mathsf{TF} \operatorname{cost}(j)$$

- ▶ TF $cost(j) = \pi_{j1}cost_{(FP,1)} + \cdots + \pi_{j4}cost_{(FP,4)} =$ average cost over states, weighted by probability of occupying state, under old treatment
- Utility measure is the number of weeks in the successfully controlled state

Expected costs and effects from a Markov model in R

Combining probabilities of state occupancy (pi_sfc) with state-specific costs and utilities, for SFC treatment group (with similar code for FP group)

```
state_costs_sfc <- c(STW=7.96, UTW=7.96, Hex=1821.17, Pex=100.79, TF=NA)
state util sfc <- c(1, 0, 0, 0, 0)
cost sfc <- eff sfc <- numeric(nsim)
## Loop over PSA samples
for (i in 1:nsim){
    cts <- ets <- numeric(J)
    ## Loop over times j
    for (j in 1:J){
        state costs sfc["TF"] <- state costs sfc[1:4] %*% pi sfc[i.1:3.i]
        # costs for time j, averaged over states
        cts[j] <- sum(pi_sfc[i,,j] * state_costs_sfc)
        # effects for time j, averaged over states
        ets[j] <- sum(pi_sfc[i,,j] * state_util_sfc)
    # expected costs and effects summed over all times, and averaged over states
    # for PSA sample i
    cost sfc[i] <- sum(cts)
    eff_sfc[i] <- sum(ets)
```

Summary: Bayesian health economic model

At each iteration of "probabilistic sensitivity analysis", for each arm

- Step 1:
 - ightharpoonup draw a realization of the transition probabilities Λ from their posterior distribution
 - lacktriangle define starting state and set time j=1

Summary: Bayesian health economic model

At each iteration of "probabilistic sensitivity analysis", for each arm

- Step 1:
 - \triangleright draw a realization of the transition probabilities Λ from their posterior distribution
 - define starting state and set time j=1
- Step 2: Markov model
 - ▶ Set time j = j + 1
 - find prevalence of states π_i at this time
 - ightharpoonup combine π_j with state-specific cost and utility to calculate expected cost and utility at this time
 - repeat step 2 up to time horizon j = J (= 13 in this case)

Summary: Bayesian health economic model

At each iteration of "probabilistic sensitivity analysis", for each arm

- Step 1:
 - \triangleright draw a realization of the transition probabilities Λ from their posterior distribution
 - \blacktriangleright define starting state and set time j=1
- Step 2: Markov model
 - ▶ Set time j = j + 1
 - find prevalence of states π_i at this time
 - ightharpoonup combine π_j with state-specific cost and utility to calculate expected cost and utility at this time
 - repeat step 2 up to time horizon j = J(= 13 in this case)
- Step 3: Cost-effectiveness
 - calculate expected costs
 - calculate expected effects (here, proportion of time spent in STW)

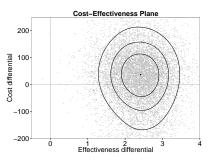
summed over times

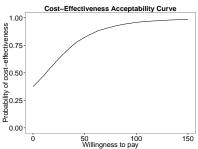
Gives a sample from the **posterior distribution** of expected costs and effects: vectors cost_sfc, eff_sfc, cost_fp, eff_fp in R code

Results

Sample from joint posterior distribution of costs and effects for each treatment gives, e.g.

- incremental cost-effectiveness ratio = mean cost difference / mean benefit difference
- "cost-effectiveness plane"
- incremental net benefit (with CI)
- cost-effectiveness acceptability curve





Bayesian decision modelling more generally

Example given here was simple

 Markov model with all transition probabilities estimated from individual-level transition count data.

Many more complicated examples, e.g.

- Transition probabilities for baseline disease progression estimated from published data
- ullet Treatment effects estimated from RCTs / (network-)meta-analysis o transition probabilities for treated groups
- Combinations of individual-level and published aggregate data...

Examples in:

- Bayesian Methods in Health Economics,
- Evidence Synthesis for Decision Making in Healthcare,
- http://nicedsu.org.uk/technical-support-documents/



Lecture 5

Uncertainty analysis

Handling uncertainty in economic evaluations

- Population uncertainty: Sub-group analysis
- Parameter uncertainty: Sensitivity analysis
- Structural uncertainty: Sensitivity analysis
- Collect more data
- Sensitivity analysis (SA)
- Deterministic sensitivity analysis
 - One-way SA
 - ► Two-way SA
 - Scenario analysis (best or worst case, what if..)
- Probabilistic sensitivity analysis (PSA)
 - Monte Carlo simulation
- Cost-effectiveness acceptability curves (CEAC)

How robust are health economic evaluations

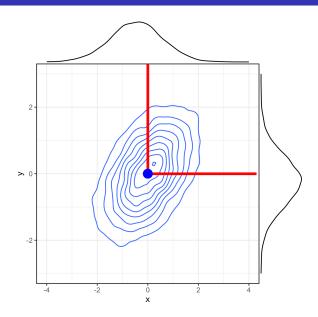
- Do limitations in either the quality or availability of evidence affect the recommended decision?
- If the decision is not altered despite 'reasonable' variations in key assumptions/parameters, then the analysis can be considered to be 'robust'
- Two types of uncertainty:
 - Structural (is the model design correct?)
 - ▶ Parameter (are the values correct?)
- In economic evaluation, some form of sensitivity analysis is frequently carried out in order to allow for uncertainty
- This uncertainty may be present in the evaluation for several reasons:
 - Data are unavailable and assumptions are necessary
 - Available but inaccurate
- In this type of analysis the values recorded for important parameters are varied, usually one at a time, in order to determine whether the results are sensitive to the assumptions made

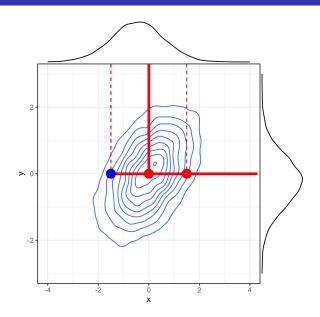
Uncertainty in economic evaluations

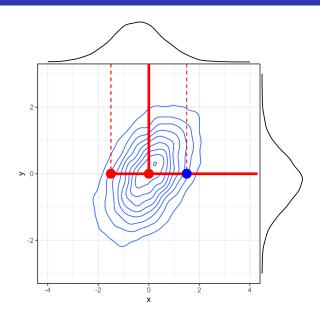
- Structural: scenario analysis
- Re-run the analysis with alternate assumptions and model structures
- Parameter: sensitivity analysis (SA)
- Re-run the analysis with different parameter values
- Type of sensitivity analysis:
 - One-way SA
 - ► Multi-way SA
 - Extreme values SA
 - Probabilistic SA

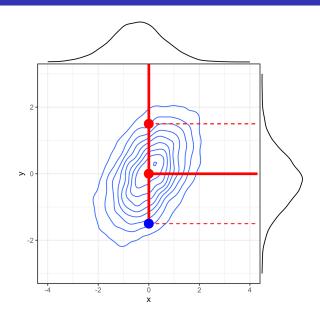
Types of sensitivity analysis

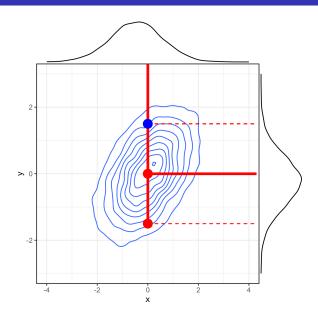
- Simple sensitivity analysis entails varying one or more of the components of an evaluation to see how it affects the results
- Probabilistic sensitivity analysis assigns ranges and distribution to variables and computer programs are used to select values at random from each range and to record the results
- By using these different methods of sensitivity analysis it is possible to show whether
 the results of a particular study over a range of assumptions or hinge on the
 accuracy of particular assumptions

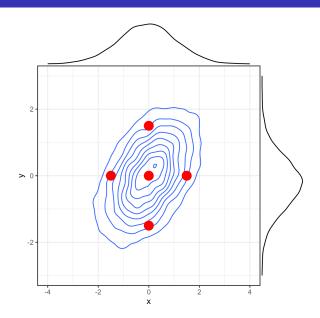


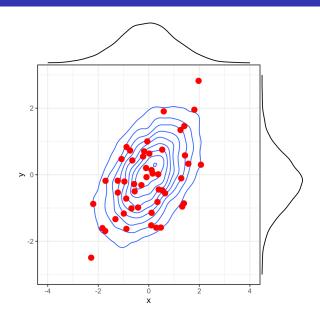




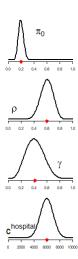








Parameters



Model structure Old chemotherapy



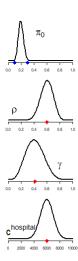
New chemotherapy



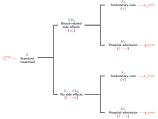
Old chemotherapy		
Benefits	Costs	

New chemotherapy		
Benefits	Costs	

Parameters



Model structure Old chemotherapy



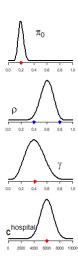
New chemotherapy



Old chemotherapy		
Benefits	Costs	
-		

New chemotherapy		
Benefits	Costs	

Parameters



Model structure Old chemotherapy



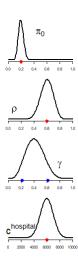
New chemotherapy



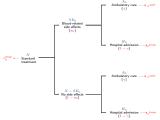
Old chemotherapy			
Benefits	Costs		

New chemotherapy		
Benefits	Costs	

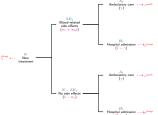
Parameters



Model structure Old chemotherapy



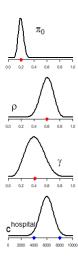
New chemotherapy



Old chemotherapy		
Benefits	Costs	

New chem	notherapy
Benefits	Costs

Parameters



Model structure Old chemotherapy



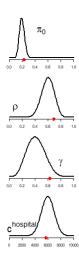
New chemotherapy



Old chemotherapy		
Benefits	Costs	

New chem	notherapy
Benefits	Costs

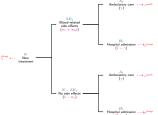
Parameters



Model structure Old chemotherapy



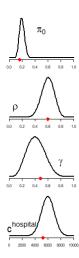
New chemotherapy



Old chemotherapy		
Benefits	Costs	
741	670 382.1	

New chemotherapy	
Benefits	Costs
732	1 131 978

Parameters



Model structure Old chemotherapy



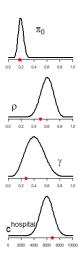
New chemotherapy



Old chemotherapy	
Benefits	Costs
741	670 382.1
699	871 273.3

New chemotherapy		
Benefits	Costs	
732	1 131 978	
664	1 325 654	

Parameters



Model structure Old chemotherapy



New chemotherapy



Old chemotherapy	
Benefits	Costs
741	670 382.1
699	871 273.3
726	425 822.2
716.2	790 381.2

New chemotherapy	
Benefits	Costs
732	1 131 978
664	1 325 654
811	766 411.4
774.5	1 066 849.8

$$ICER = \frac{276\,468.6}{58.3}$$
$$= 6\,497.1$$



Is this all we need? (see Vol)

- The CEAC only deals with the probability of making the "right decision"
- But it does not account for the payoff/penalty associated with making the "wrong" one!



Is this all we need? (see Vol)

- The CEAC only deals with the probability of making the "right decision"
- But it does not account for the payoff/penalty associated with making the "wrong" one!
- **Example 1**: Intervention t = 1 is the most cost-effective, given current evidence
 - ▶ Pr(t = 1 is cost-effective) = 0.51
 - ► If we get it wrong: Increase in costs = £3

 Decrease in effectiveness = 0.000001 QALYs
 - ► Large uncertainty/negligible consequences ⇒ can afford uncertainty

- The CEAC only deals with the probability of making the "right decision"
- But it does not account for the payoff/penalty associated with making the "wrong" one!
- **Example 1**: Intervention t = 1 is the most cost-effective, given current evidence
 - ▶ Pr(t = 1 is cost-effective) = 0.51
 - ▶ If we get it wrong: Increase in costs = £3

 Decrease in effectiveness = 0.000001 QALYs
 - ► Large uncertainty/negligible consequences ⇒ can afford uncertainty
- Example 2: Intervention t=1 is the most cost-effective, given current evidence
 - ▶ Pr(t = 1 is cost-effective) = 0.999
 - ▶ If we get it wrong: Increase in costs = £1 000 000 000 Decrease in effectiveness = 999999 QALYs
 - ► Tiny uncertainty/dire consequences ⇒ probably should think about it...